

# Imaging the Odderon from exclusive $\eta_c$ production at the EIC

Sanjin Benić (University of Zagreb)

SB, Horvatić, Kaushik, Vivoda, 2306.10626

Low-x, Leros, Greece, September 4, 2023



**HRZZ**

Croatian Science  
Foundation

# Odderon

- 50 years ago Lukaszuk, Nicolescu: **odderon is a**

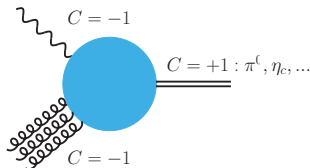
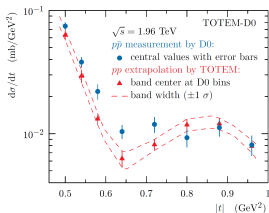
$$C = -1 \text{ exchange}$$

Lukaszuk, Nicolescu, LNC 8 (1973) 405

Joynson, Leader, Nicolescu, Lopez, NCA 30 (1975) 345

Ewerz, 0306137

- QCD: **three** gluons  $\text{tr}_c [\{A^\mu, A^\nu\}A^\rho] \sim d_{abc}$



- elastic  $pp$  vs  $p\bar{p}$

- **Odderon found**  $\rightarrow$  QCD?

TOTEM, D0, PRL 127 (2021) 6, 062003

Royon, Monday, Sep. 4, 14:00

- $ep$  collisions
- exclusive productions of C-even mesons

Czyzewski, Kwiecinski, Motyka,

Sadzikowski, PLB 398 400 (1997)

Bartels, Braun, Colferai, Vacca, EPJC

20 323 (2001)

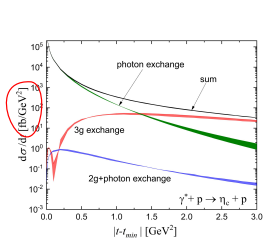
# Odderon in $ep \rightarrow ep\eta_c$

- no experimental confirmation so far  
→ EIC, LHC UPCs?

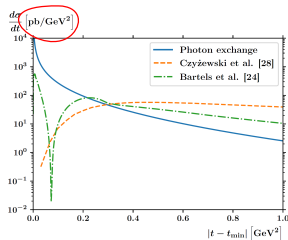
(null result from H1 at HERA:  $\sigma(\gamma^* p \rightarrow \pi^0 N^*) < 49\text{nb}$ )

H1, PLB 544 (2002) 35-43

- more recent computations at moderate  $x \sim 0.1$  lead to somewhat lower cross sections than the original estimates



Dumitru, Stebel, PRD 99 (2019) 9, 094038  
016015



Jia, Mo, Pan, Zhang, PRD 108 (2023) 1,

- this work: take into account evolution effects and consider nuclear targets

# Odderon in the CGC

- high energy collisions  $\rightarrow$  Wilson lines

$$V(\mathbf{z}_\perp) = \mathcal{P} \exp \left[ ig \int_{-\infty}^{\infty} dy^- A^+(y^-, \mathbf{z}_\perp) \right]$$

- Color Glass Condensate (CGC): emergence of a saturation scale  $Q_S^2 \sim A^{1/3}$

$\rightarrow$  better theoretical control for a large nuclei

$$Q_S^2 \gg \Lambda_{\text{QCD}}^2$$

- **odderon** as imaginary part of the dipole

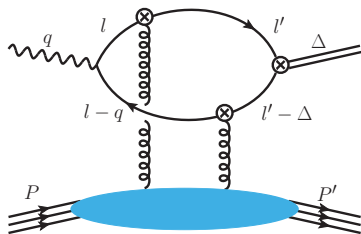
$$\mathcal{O}(\mathbf{x}_\perp, \mathbf{y}_\perp) \equiv -\frac{1}{2iN_c} \text{tr} \left\langle V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) - V(\mathbf{y}_\perp) V^\dagger(\mathbf{x}_\perp) \right\rangle$$

- C-odd:  $\mathbf{x}_\perp \leftrightarrow \mathbf{y}_\perp$   $\mathcal{O}(\mathbf{x}_\perp, \mathbf{y}_\perp) \rightarrow -\mathcal{O}(\mathbf{x}_\perp, \mathbf{y}_\perp)$

Kovchegov, Szymanowski, Wallon, PLB 586, 267 (2004)

Hatta, Iancu, Itakura, McLerran, Nucl.Phys.A 760 (2005) 172-207

# Amplitude



$$\mathbf{r}_\perp \equiv \mathbf{x}_\perp - \mathbf{y}_\perp$$

$$\mathbf{b}_\perp \equiv \frac{\mathbf{x}_\perp + \mathbf{y}_\perp}{2}$$

$$x \equiv \frac{(P - P') \cdot q}{P \cdot q}$$

- CGC vertex

$$\tau(p, p') = (2\pi)\delta(p^- - p'^-)\gamma^- \text{sgn}(p^-) \int_{\mathbf{z}_\perp} e^{-i(\mathbf{p}_\perp - \mathbf{p}'_\perp) \cdot \mathbf{z}_\perp} V^{\text{sgn}(p^-)}(\mathbf{z}_\perp)$$

$$\langle \mathcal{M}_\lambda \rangle = eq_c \int_{\mathbf{r}_\perp} \int_{\mathbf{l}''} (2\pi)\delta(l^- - l'^-)\theta(l^-)\theta(q^- - l^-) e^{-i(l'_\perp - l_\perp - \frac{1}{2}\Delta_\perp) \cdot \mathbf{r}_\perp} \\ \times (-iN_c) \mathcal{O}(\mathbf{r}_\perp, \Delta_\perp) \text{tr} [S(l)\not{\epsilon}(\lambda, q)S(l - q)\gamma^- S(l' - \Delta)(i\gamma_5)S(l')\gamma^-]$$

# Amplitude

- only transverse photon polarizations  $\lambda = \pm 1$  survive in the eikonal approximation

$$\langle \mathcal{M}_\lambda \rangle = q^- \lambda e^{i\lambda\phi_\Delta} \langle \mathcal{M} \rangle \quad \frac{d\sigma}{d|t|} = \frac{1}{16\pi} |\langle \mathcal{M} \rangle|^2$$

- polarization independent part of the amplitude

$$\langle \mathcal{M} \rangle = 8\pi i e q_c N_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^\infty r_\perp dr_\perp \mathcal{O}_{2k+1}(r_\perp, \Delta_\perp) \\ \times \mathcal{A}(r_\perp) \left[ J_{2k}(r_\perp \delta_\perp) - \frac{2k+1}{r_\perp \delta_\perp} J_{2k+1}(r_\perp \delta_\perp) \right].$$

- result proportional to  $m_c$

$$\mathcal{A}(r_\perp) = (-1) \frac{\sqrt{2} m_c}{2\pi} \frac{1}{z\bar{z}} [K_0(\varepsilon r_\perp) \partial_{r_\perp} \phi_{\mathcal{P}}(z, r_\perp) - \varepsilon K_1(\varepsilon r_\perp) \phi_{\mathcal{P}}(z, r_\perp)]$$

# BK equation

- **odderon  $\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)$  is explicitly  $b_\perp$ -dependent**  
→ in principle need to solve the **fully impact parameter dependent** Balitsky-Kovchegov (BK) equation

$$\frac{\partial \mathcal{D}(\mathbf{r}_\perp, \mathbf{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int_{\mathbf{r}_{1\perp}} \frac{r_\perp^2}{r_{1\perp}^2 r_{2\perp}^2} [\mathcal{D}(\mathbf{r}_{1\perp}, \mathbf{b}_{1\perp}) \mathcal{D}(\mathbf{r}_{2\perp}, \mathbf{b}_{2\perp}) - \mathcal{D}(\mathbf{r}_\perp, \mathbf{b}_\perp)]$$

$$\mathbf{r}_{2\perp} = \mathbf{r}_\perp - \mathbf{r}_{1\perp} \quad \mathbf{b}_{1\perp} = \mathbf{b}_\perp + \frac{1}{2}(\mathbf{r}_\perp - \mathbf{r}_{1\perp}) \quad \mathbf{b}_{2\perp} = \mathbf{b}_\perp - \frac{1}{2}\mathbf{r}_{1\perp} \quad Y = \ln(1/x)$$

$$\begin{aligned} \mathcal{D}(\mathbf{r}_\perp, \mathbf{b}_\perp) &= \frac{1}{N_c} \text{tr} \left\langle V \left( \mathbf{b}_\perp + \frac{\mathbf{r}_\perp}{2} \right) V^\dagger \left( \mathbf{b}_\perp - \frac{\mathbf{r}_\perp}{2} \right) \right\rangle \\ &= 1 - \mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp) + i\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp) \end{aligned}$$

- in general non-local in  $\mathbf{b}_\perp$
- **this work**: local approximation →  $b_\perp$  becomes an external parameter

Kowalski, Lappi, Marquet, Venugopalan, PRC 78 (2008) 045201  
Lappi, Mäntysaari, PRD 88 (2013) 114020

# BK equation

- **odderon**  $\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)$  is explicitly  $b_\perp$ -dependent  
→ in principle need to solve the fully impact parameter dependent Balitsky-Kovchegov (BK) equation

$$\frac{\partial \mathcal{D}(\mathbf{r}_\perp, \mathbf{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int_{\mathbf{r}_{1\perp}} \frac{r_\perp^2}{r_{1\perp}^2 r_{2\perp}^2} [\mathcal{D}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) \mathcal{D}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{D}(\mathbf{r}_\perp, \mathbf{b}_\perp)]$$

$$\mathbf{r}_{2\perp} = \mathbf{r}_\perp - \mathbf{r}_{1\perp} \quad \mathbf{b}_{1\perp} = \mathbf{b}_\perp \quad \mathbf{b}_{2\perp} = \mathbf{b}_\perp \quad Y = \ln(1/x)$$

$$\begin{aligned} \mathcal{D}(\mathbf{r}_\perp, \mathbf{b}_\perp) &= \frac{1}{N_c} \text{tr} \left\langle V \left( \mathbf{b}_\perp + \frac{\mathbf{r}_\perp}{2} \right) V^\dagger \left( \mathbf{b}_\perp - \frac{\mathbf{r}_\perp}{2} \right) \right\rangle \\ &= 1 - \mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp) + i\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp) \end{aligned}$$

- in general non-local in  $\mathbf{b}_\perp$
- **this work**: local approximation →  $b_\perp$  becomes an external parameter

Kowalski, Lappi, Marquet, Venugopalan, PRC 78 (2008) 045201  
Lappi, Mäntysaari, PRD 88 (2013) 114020



# BK equation

- **non-linear terms** couple the Pomeron-Odderon system

$$\begin{aligned}\frac{\partial \mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp)}{\partial Y} &= \frac{\alpha_S N_c}{2\pi^2} \int_{r_{1\perp}} \frac{r_\perp^2}{r_{1\perp}^2 r_{2\perp}^2} \left[ \mathcal{N}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) + \mathcal{N}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp) \right. \\ &\quad \left. + \mathcal{N}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) \mathcal{N}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{O}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) \mathcal{O}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) \right] \\ \frac{\partial \mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)}{\partial Y} &= \frac{\alpha_S N_c}{2\pi^2} \int_{r_{1\perp}} \frac{r_\perp^2}{r_{1\perp}^2 r_{2\perp}^2} \left[ \mathcal{O}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) + \mathcal{O}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp) \right. \\ &\quad \left. - \mathcal{N}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) \mathcal{O}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{O}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) \mathcal{N}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) \right]\end{aligned}$$

Kovchegov, Szymanowski, Wallon, PLB 586, 267 (2004)

Hatta, Iancu, Itakura, McLerran, NPA 760 (2005) 172-207

Motyka, PLB 637, 185 (2006)

Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)

Yao, Hagiwara, Hatta, PLB 790 (2019) 361-366

- **small  $r_\perp$  limit:**  $\mathcal{N} \rightarrow 0$  (linear)  $\rightarrow \mathcal{O} \sim e^{-\#Y}$
- **large  $r_\perp$  limit:**  $\mathcal{N} \rightarrow 1$  (saturation)  $\rightarrow \mathcal{O} \sim e^{-\#Y}$
- in numerical computations we are replacing  $\frac{\alpha_S N_c}{2\pi^2} \frac{r_\perp^2}{r_{1\perp}^2 r_{2\perp}^2}$  by a running coupling kernel with Balitsky's prescription

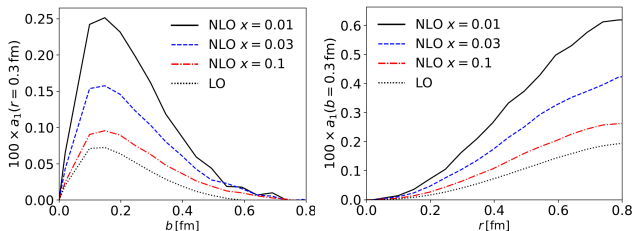
# Initial condition for proton

- $\mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp)$ : HERA fit

$$\mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp) = 1 - \exp \left[ -\frac{1}{4} \mathbf{r}_\perp^2 T_p(\mathbf{b}_\perp) \frac{\sigma_0}{2} Q_{S,0}^2 \log \left( \frac{1}{r_\perp \Lambda_{\text{QCD}}} + e_c e \right) \right]$$

Lappi, Mäntysaari, PRD 88 (2013) 114020

- $\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)$ : recent computation **starting from quark light-cone wavefunctions** at NLO by Dumitru, Mäntysaari, Paatelainen (DMP)



Dumitru, Mäntysaari, Paatelainen, PRD 105 (2022) 3, 036007

Dumitru, Mäntysaari, Paatelainen, PRD 107 (2023) 1, L011501

# Initial conditions for nuclei

- $\mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp)$  same HERA fit + optical Glauber

$$\mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp) = 1 - \exp \left[ -\frac{1}{4} \mathbf{r}_\perp^2 AT_A(\mathbf{b}_\perp) \frac{\sigma_0}{2} Q_{S,0}^2 \log \left( \frac{1}{r_\perp \Lambda_{\text{QCD}}} + e_c e \right) \right]$$

Lappi, Mäntysaari, PRD 88 (2013) 114020

- for  $\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)$  we use the Jeon-Venugopalan (JV) model

$$W[\rho] = \exp \left[ - \int_{\mathbf{x}_\perp} \left( \frac{\delta^{ab} \rho^a \rho^b}{2\mu^2} - \frac{d_{abc} \rho^a \rho^b \rho^c}{\kappa} \right) \right]$$

Jeon, Venugopalan, PRD 71 (2005) 125003

$$\begin{aligned} \mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp) &= \frac{\lambda}{8} \left[ A^{2/3} \frac{dT_A(\mathbf{b}_\perp)}{db_\perp} R_A \frac{\sigma_0}{2} \right] A^{1/2} (Q_{S,0} r_\perp)^3 (\hat{\mathbf{r}}_\perp \cdot \hat{\mathbf{b}}_\perp) \\ &\times \log \left( \frac{1}{r_\perp \Lambda_{\text{QCD}}} + e_c e \right) \exp \left[ -\frac{1}{4} \mathbf{r}_\perp^2 AT_A(\mathbf{b}_\perp) \frac{\sigma_0}{2} Q_{S,0}^2 \log \left( \frac{1}{r_\perp \Lambda_{\text{QCD}}} + e_c e \right) \right] \end{aligned}$$

- in the JV model  $\lambda_{\text{JV}} = -\frac{3}{16} \frac{N_c^2 - 4}{(N_c^2 - 1)^2} \frac{(Q_{S,0} R_A)^3}{\alpha_s^3 A^{3/2}}$

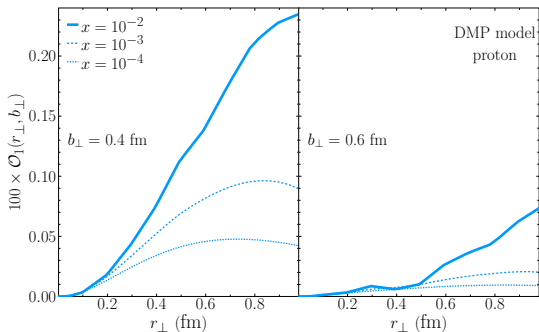
Kovchegov, Sievert, PRD 86 (2012) 034028

Boer, Van Daal, Mulders, Petreska, JHEP 07, 140 (2018)

SB, Horvatić, Kaushik, Vivoda, 2306.10626

# Numerical solutions: proton

- rapid drop of the odderon with evolution
- does not obey geometric scaling



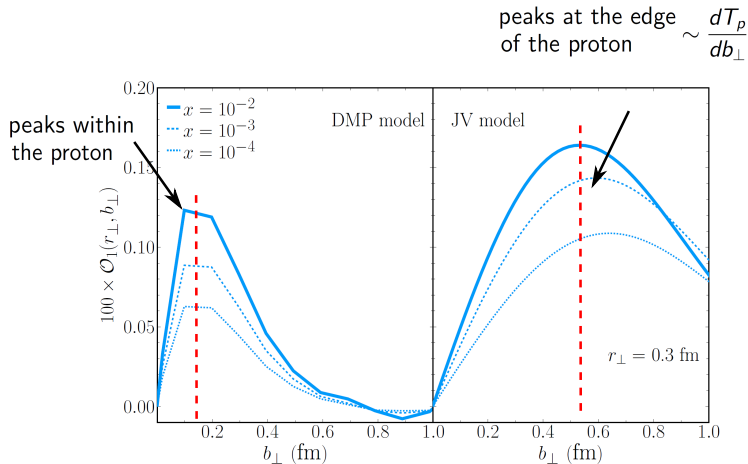
Motyka, PLB 637, 185 (2006)

Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)

Yao, Hagiwara, Hatta, PLB 790 (2019) 361-366

SB, Horvatić, Kaushik, Vivoda, 2306.10626

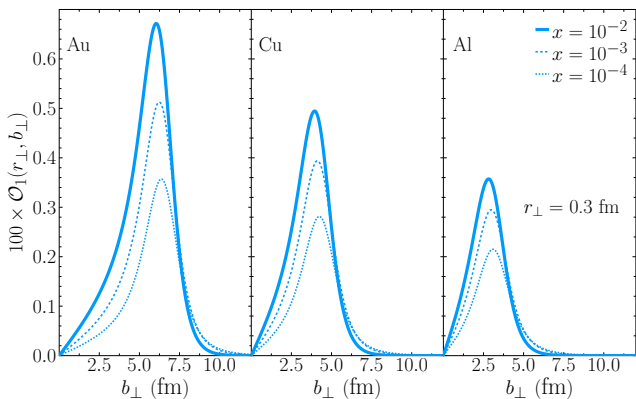
# DMP vs JV for proton



SB, Horvatić, Kaushik, Vivoda, 2306.10626

# Odderon in case of nuclei

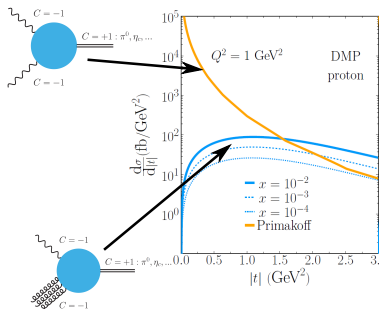
- for nuclei  $T_A(\mathbf{b}_\perp)$  is given as a Woods-Saxon profile



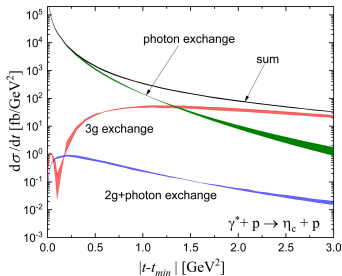
- slight shift of the peak position with evolution to small- $x$

SB, Horvatić, Kaushik, Vivoda, 2306.10626

# Results: proton target



$$x \sim 10^{-2} - 10^{-4}$$



$$x \sim 0.1$$

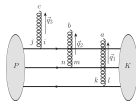
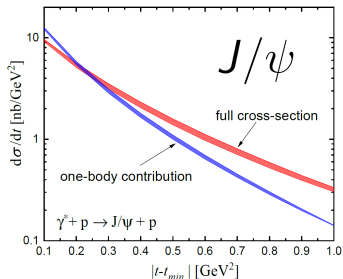
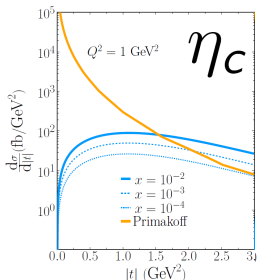
- weak  $|t|$ -dependence: not changed by evolution to small- $x$

→ probes the odderon in the high  $|t| > 1.5$  GeV<sup>2</sup> region

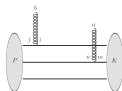
- confirming the general conclusions by Dumitru and Stebel

Dumitru, Stebel, PRD 99 (2019) 9, 094038  
SB, Horvatić, Kaushik, Vivoda, 2306.10626

# Contrast with $J/\psi$



weak  $|t|$   
dependence



strong  $|t|$   
dependence

- Landshoff mechanism

Donnachie, Landshoff, Z. Phys. C 2 (1979) 55

Czyzewski, Kwiecinski, Motyka, Sadzikowski, PLB 398 400 (1997)

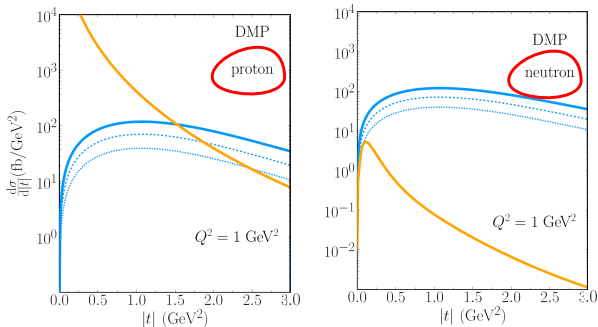
Dumitru, Stebel, PRD 99 (2019) 9, 094038

SB, Horvatić, Kaushik, Vivoda, 2306.10626



# Results: neutron target

→ a way to suppress the Primakoff background!



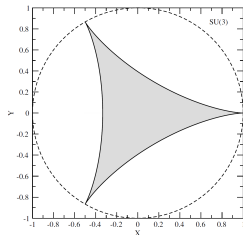
→ **odderon dominance for all  $|t|$ !**

- measurement prospect:  $d$  or  $^3\text{He}$  with spectator proton forward tagging

SB, Horvatić, Kaushik, Vivoda, 2306.10626  
CLAS, PRL 108, 142001 (2012)  
Frišćić et. al. PLB 823, 136726 (2021)

# Nuclear targets: preliminary comments

- assumption: JV model captures the  $A$ -dependence, but **odderon coupling** has some uncertainty



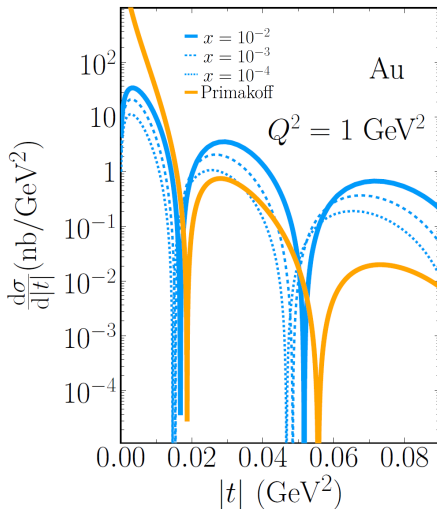
- upper bound by group theory constraint

Kaiser, JPA, 39, 15287 (2006)

Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)

- ultimately to be determined experimentally
- consider the odderon coupling as a free parameter

# Results for nuclear targets



SB, Horvatić, Kaushik, Vivoda, 2306.10626

- result for maximal coupling allowed by group theory
- cross section in  $\text{nb/GeV}^2$ !
- odderon contribution has a **shifted diffractive pattern** with respect to the Primakoff cross section
- mismatch becomes more pronounced at small- $x$ /large  $|t|$

# Origin of the mismatch?

- the odderon cross section at leading twist

$$\frac{d\sigma}{d|t|} \simeq \frac{9\pi q_c^2 \alpha_S^6 A^2 C_{3F}^2 \mathcal{R}_P^2(0)}{N_c m_c^5} \frac{|t| T_A^{\text{strong}}(\sqrt{|t|})^2}{m_c^4}$$

SB, Horvatić, Kaushik, Vivoda, 2306.10626

- contrast with the Primakoff cross section

$$\frac{d\sigma}{d|t|} \simeq \frac{\pi q_c^4 \alpha^3 Z^2 N_c \mathcal{R}_P^2(0)}{m_c^5} \frac{T_A^{\text{charge}}(\sqrt{|t|})^2}{|t|}$$

Jia, Mo, Pan, Zhang, PRD 108 (2023) 1, 016015

- in our Woods-Saxon parametrizations

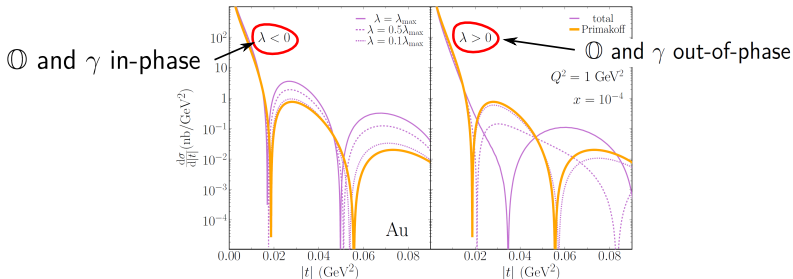
$$R_A^{\text{strong}} = R_A^{\text{charge}} \quad d_A^{\text{strong}} = d_A^{\text{charge}}$$

→ shift is not seen at leading twist

→ origin is the multiple scatterings+small-x evolution in the odderon distribution

# Mismatch as odderon signature

- basic premise: locations of the Primakoff diffractive minima “known” from nuclear physics  
(any plans to measure nuclear charge FFs at the EIC?)



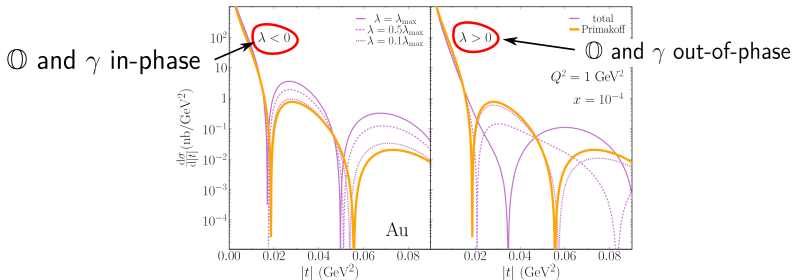
→ **odderon signature**: a mismatch in the diffractive minima of the cross section w.r.t the Primakoff reference

SB, Horvatić, Kaushik, Vivoda, 2306.10626

- large odderon coupling ( $\lambda$ ): Primakoff diffractive dip gets washed away
- sensitivity to the **sign** of  $\lambda$

# Mismatch as odderon signature

- basic premise: locations of the Primakoff diffractive minima “known” from nuclear physics  
(any plans to measure nuclear charge FFs at the EIC?)

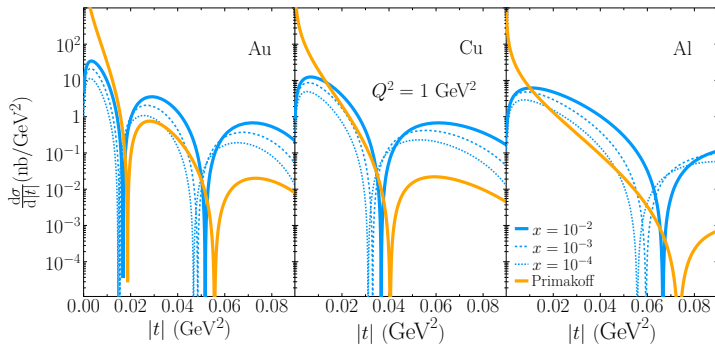


→ **odderon signature**: a mismatch in the diffractive minima of the cross section w.r.t the Primakoff reference

SB, Horvatić, Kaushik, Vivoda, 2306.10626

- can this be measured?
- need fine  $t$ -binning + control over incoherent contribution

# Mismatch for different nuclei species



- diffractive pattern modified across nuclear species as a consequence of the geometry
- somewhat more pronounced in case of lighter nuclei
- **another handle**: nuclei with neutron skins

SB, Horvatić, Kaushik, Vivoda, 2306.10626

# Comments

- can  $\eta_c$  be measured at the EIC (or LHC)?  
→ **proof-of-concept results**, should translate to light mesons ( $\pi^0, \eta$ )/ $C$ -even quarkonia states
- need to fix the odderon coupling  
→ can elastic  $pp$  vs  $p\bar{p}$  help?
- forward limit  $|t| \rightarrow 0$  cross section vanishes  
→ consider the spin-flip contribution from the gluon Sivers function

Boussarie, Hatta, Szymanowski, Wallon, PRL 124 (2020) 17, 172501