Back-to-back dijets at the EIC: small x and Sudakov resummation

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Low-x 2023

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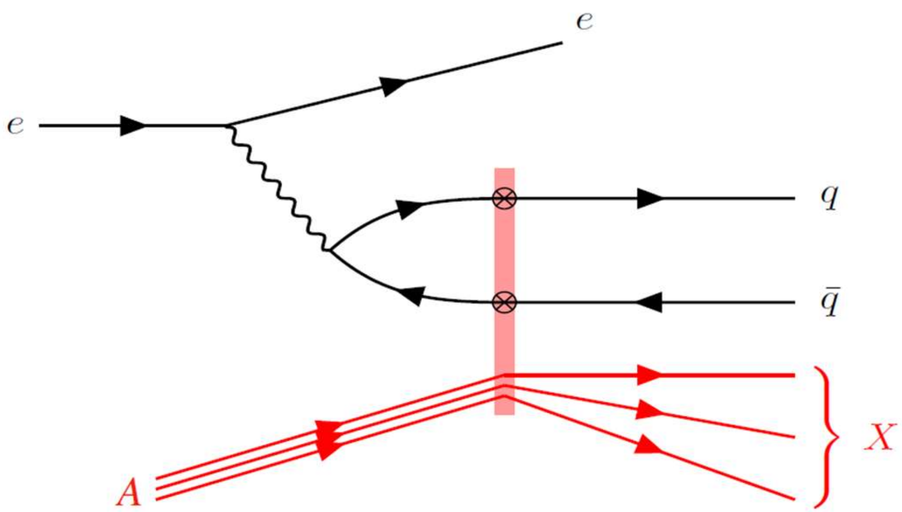
Outline

- Dijet production in DIS
- CGC calculation at LO and NLO
- Back-to-back limit
- Numerical results

Dijet production in DIS

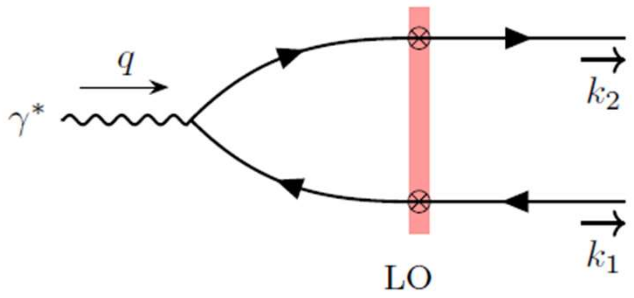


$$\frac{\mathrm{d}\sigma^{e+A\to e'+q\bar{q}+X}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} = \sum_{\lambda=\mathrm{L,T}} f_\lambda(x_{\mathrm{Bj}}, Q^2)\ \frac{\mathrm{d}\sigma^{\gamma^*_\lambda+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2}\,.$$

$$f_{\lambda=\mathrm{L}}(x_{\mathrm{Bj}}, Q^2) = \frac{\alpha_{\mathrm{em}}}{\pi Q^2 x_{\mathrm{Bj}}}(1-y)\,,$$

$$f_{\lambda=\mathrm{T}}(x_{\mathrm{Bj}}, Q^2) = \frac{\alpha_{\mathrm{em}}}{2\pi Q^2 x_{\mathrm{Bj}}}[1+(1-y)^2]\,,$$

CGC calculation at LO



$$\left.\frac{d\sigma^{\gamma_\lambda^\star + A \to q\bar{q} + X}}{d^2\boldsymbol{k_{1\perp}} d^2\boldsymbol{k_{2\perp}} d\eta_1 d\eta_2}\right|_{\mathrm{LO}} = \cdot$$

$$\frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot \boldsymbol{r}_{xx'}} e^{-i\boldsymbol{k_{2\perp}}\cdot \boldsymbol{r}_{yy'}}$$
$$\times\, \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{x}'_\perp, \boldsymbol{y}'_\perp) \mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{x'y'}) \,.$$

Impact factor:

$$\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{x'y'}) = 8 z_1^3 z_2^3 Q^2 K_0(\bar{Q} r_{xy}) K_0(\bar{Q} r_{x'y'}) \,,$$
$$\mathcal{R}^{\mathrm{T}}_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{x'y'}) = 2 z_1 z_2 \left[z_1^2 + z_2^2\right] \frac{\boldsymbol{r}_{xy} \cdot \boldsymbol{r}_{x'y'}}{r_{xy} r_{x'y'}} \bar{Q}^2 K_1(\bar{Q} r_{xy}) K_1(\bar{Q} r_{x'y'}) \,,$$

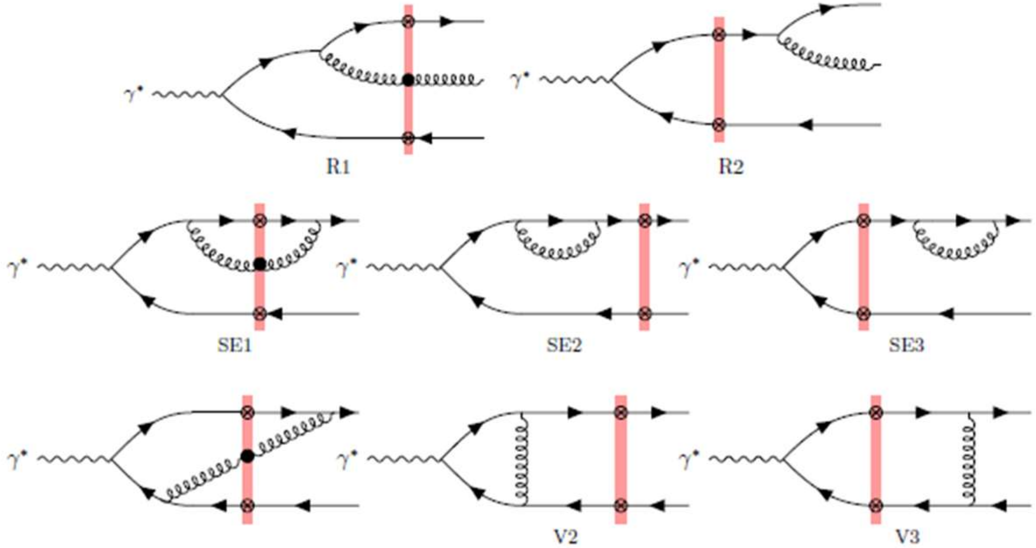
Color correlator:

$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{x}'_\perp, \boldsymbol{y}'_\perp) = \frac{1}{N_c} \left\langle \mathrm{Tr}\left[\left(V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right)\left(V(\boldsymbol{y}'_\perp) V^\dagger(\boldsymbol{x}'_\perp) - \mathbb{1}\right)\right]\right\rangle_Y$$
$$= \left\langle Q_{xy,y'x'} - D_{xy} - D_{y'x'} + 1\right\rangle_Y \,,$$

where Wilson line: $V(\boldsymbol{x}_\perp) = P \exp\left(ig \int dx^- A^+_{\mathrm{cl}}(\boldsymbol{x}_\perp, x^-)\right)$

NLO corrections

P. Caucal, F. Salazar and R. Venugopalan *(2021)*



NLO impact factor:

$$\mathrm{d}\sigma_{\mathrm{R}_2\times\mathrm{R}_2,\mathrm{sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}} \mathcal{R}_{\mathrm{LO}}^{\lambda}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})$$
$$\times C_F \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times \frac{\alpha_s}{\pi}\int_0^1 \frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}}\right]\ln\left(\frac{\boldsymbol{k}_{1\perp}{}^2 \boldsymbol{r}_{xx'}^2 R^2\xi^2}{c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{R}_2\times\mathrm{R}'_2,\mathrm{sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}} \mathcal{R}_{\mathrm{LO}}^{\lambda}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})$$
$$\times \Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times \frac{(-\alpha_s)}{\pi}\int_0^1 \frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy'}}\right]\ln\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{xy'}^2 \xi^2}{z_2^2 c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{R,no-sud,LO}}^{\gamma_{\mathrm{L}}^*+A\to q\bar{q}g+X} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}(4\alpha_s C_F)\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times \frac{e^{-i\boldsymbol{k}_{g\perp}\cdot\boldsymbol{r}_{zz'}}}{(\boldsymbol{k}_{g\perp}-\frac{z_g}{z_1}\boldsymbol{k}_{1\perp})^2}\left\{8z_1 z_2^3(1-z_2)^2 Q^2\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)K_0(\bar{Q}_{\mathrm{R2}} r_{xy})K_0(\bar{Q}_{\mathrm{R2}} r_{x'y'})\delta_z^{(3)}\right.$$
$$\left.-\mathcal{R}_{\mathrm{LO}}^{\mathrm{L}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta(z_1-z_g)\delta_z^{(2)}\right\}+(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma_{\mathrm{R,no-sud,NLO_3}}^{\gamma_{\mathrm{L}}^*+A\to q\bar{q}g+X} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8} \int \mathrm{d}^2\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}(-4\alpha_s)\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times \frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy'}}}{\boldsymbol{l}_\perp^2}\left\{8z_1^2 z_2^2(1-z_2)(1-z_1)Q^2 K_0(\bar{Q}_{\mathrm{R2}} r_{xy})K_0(\bar{Q}_{\mathrm{R2'}} r_{x'y'})\left[1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right]\right.$$
$$\left.\times e^{-i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_{xy'}}\frac{\boldsymbol{l}_\perp\cdot(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)}{(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)^2}\delta_z^{(3)}-\mathcal{R}_{\mathrm{LO}}^{\mathrm{L}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta\left(\frac{c_0^2}{\boldsymbol{r}_{xy'}^2}\geq \boldsymbol{l}_\perp^2\geq \boldsymbol{K}_\perp^2\right)\Theta(z_1-z_g)\delta_z^{(2)}\right\}$$
$$+(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma_{\mathrm{R,no-sud,other}}^{\gamma_{\mathrm{L}}^*+A\to q\bar{q}g+X} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(3)}}{(2\pi)^8} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}} 8z_1^3 z_2^3 Q^2 \int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\frac{\mathrm{d}^2\boldsymbol{z}'_\perp}{\pi}e^{-i\boldsymbol{k}_{g\perp}\cdot\boldsymbol{r}_{zz'}}$$
$$\alpha_s\left\{-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2}} r_{w'y'})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp)\right.$$
$$+\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2'}} r_{w'y'})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp)$$
$$+\frac{1}{2}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$$
$$-\frac{1}{2}\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$$
$$\left.+(1\leftrightarrow 2)+c.c.\right\}-\frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^8}\alpha_s\Theta(z_f-z_g)\times\text{“slow”}$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}} \mathcal{R}_{\mathrm{LO}}^{\lambda}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times\frac{\alpha_s C_F}{\pi}\left\{-\frac{3}{4}\ln\left(\frac{\boldsymbol{k}_{1\perp}{}^2\boldsymbol{k}_{2\perp}{}^2\boldsymbol{r}_{xy}^2\boldsymbol{r}_{x'y'}^2}{c_0^4}\right)-3\ln(R)+\frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right)+\frac{11}{2}+3\ln(2)-\frac{\pi^2}{2}\right\}$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,NLO_3}}^{\lambda=\mathrm{L}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^3\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}} 8z_1^3 z_2^3 Q^2 K_0(\bar{Q}r_{x'y'})$$
$$\times\frac{\alpha_s}{\pi}\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}\left\{K_0(\bar{Q}_{\mathrm{V3}} r_{xy})\left[\left(1-\frac{z_g}{z_1}\right)^2\left(1+\frac{z_g}{z_2}\right)(1+z_g)e^{i(\boldsymbol{P}_\perp+z_g\boldsymbol{q}_\perp)\cdot\boldsymbol{r}_{xy}}K_0(-i\Delta_{\mathrm{V3}} r_{xy})\right.\right.$$
$$\left.-\left(1-\frac{z_g}{2z_1}+\frac{z_g}{2z_2}-\frac{z_g^2}{2z_1z_2}\right)e^{i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy}}\mathcal{J}_{\odot}\left(\boldsymbol{r}_{xy},\left(1-\frac{z_g}{z_1}\right)\boldsymbol{P}_\perp,\Delta_{\mathrm{V3}}\right)\right]$$
$$\left.+K_0(\bar{Q}r_{xy})\ln\left(\frac{z_g P_\perp r_{xy}}{c_0 z_1 z_2}\right)+(1\leftrightarrow 2)\right\}\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)+c.c.$$

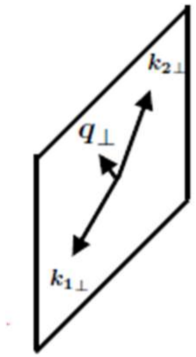
$$\mathrm{d}\sigma_{\mathrm{V,no-sud,other}}^{\lambda=\mathrm{L}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}} 8z_1^3 z_2^3 Q^2 K_0(\bar{Q}r_{x'y'})\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}$$
$$\times\frac{\alpha_s}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\left\{\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO},1}\right.$$
$$-\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2 e^{\gamma_E}}}K_0(\bar{Q}r_{xy})-\Theta(z_f-z_g)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2 e^{\gamma_E}}}K_0(\bar{Q}r_{xy})\right]C_F\Xi_{\mathrm{LO}}$$
$$-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy}^2}\left[\left(1-\frac{z_g}{z_1}\right)\left(1+\frac{z_g}{z_2}\right)\left(1-\frac{z_g}{2z_1}-\frac{z_g}{2(z_2+z_g)}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)\right.$$
$$\left.\left.-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO},1}+(1\leftrightarrow 2)\right\}+c.c..$$

$$\mathrm{d}\sigma_{\mathrm{sud1}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}} \mathcal{R}_{\mathrm{LO}}^{\lambda}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\times\frac{\alpha_s}{\pi}$$
$$\times\left\{C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_f}{z_1}\right)\ln\left(\frac{\boldsymbol{r}_{xx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_f}{z_2}\right)\ln\left(\frac{\boldsymbol{r}_{yy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right.$$
$$\left.+\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_1}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{xy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_2}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{yx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right\}$$

Very hard to do numerics…

F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan (2011)

Back-to-back limit (LO)



$$\boldsymbol{q}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$
$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}$$

Back-to-back limit:

$$q_\perp, Q_s \ll P_\perp$$

High energy limit: $P_\perp \ll W$

Factorization at the LO:

$$\mathrm{d}\sigma_{\mathrm{LO}}^{\gamma_\lambda^\star + A \to q\bar{q} + X} = \alpha_{\mathrm{em}} e_f^2 \alpha_s \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{\lambda, ij}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{G}_Y^{ij}(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp) + \mathcal{O}\left(\frac{q_\perp}{P_\perp}, \frac{Q_s}{P_\perp}\right)$$

where the hard factors:

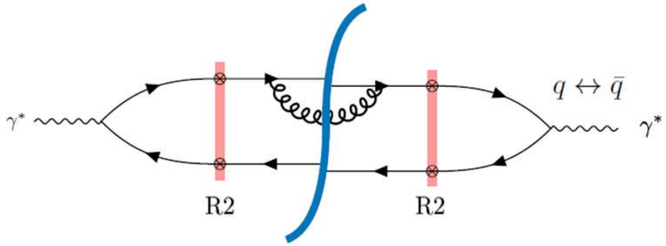
$$\mathcal{H}_{\mathrm{LO}}^{\lambda=\mathrm{L}, ij}(\boldsymbol{P}_\perp) = 16 z_1^3 z_2^3 Q^2 \frac{\boldsymbol{P}_\perp^i \boldsymbol{P}_\perp^j}{(\boldsymbol{P}_\perp^2 + \bar{Q}^2)^4},$$

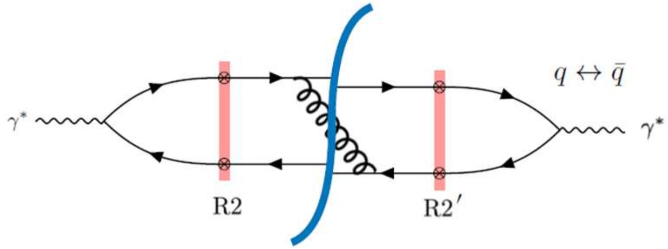
$$\mathcal{H}_{\mathrm{LO}}^{\lambda=\mathrm{T}, ij}(\boldsymbol{P}_\perp) = z_1 z_2 (z_1^2 + z_2^2) \left\{ \frac{\delta^{ij}}{(\boldsymbol{P}_\perp^2 + \bar{Q}^2)^2} - \frac{4\bar{Q}^2 \boldsymbol{P}_\perp^i \boldsymbol{P}_\perp^j}{(\boldsymbol{P}_\perp^2 + \bar{Q}^2)^4} \right\}$$

and Weizsäcker-Williams (WW) distribution:

$$\hat{G}_Y^{ij}(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp) \equiv \frac{-2}{\alpha_s} \left\langle \mathrm{Tr}\left[V(\boldsymbol{b}_\perp) \left(\partial^i V^\dagger(\boldsymbol{b}_\perp)\right) V(\boldsymbol{b}'_\perp) \left(\partial^j V^\dagger(\boldsymbol{b}'_\perp)\right) \right] \right\rangle_Y$$

Back-to-back limit (NLO)





One obtains Sudakov logs:

$$\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}(Q,\boldsymbol{P}_\perp)\int \mathrm{d}^2\mathbf{r}_{bb'}\,e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}}$$
$$\left[1 \boxed{+\frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}_\perp^2\mathbf{r}_{bb'}^2}{c_0^2}\right)} + \cdots + \alpha_s\ln\left(\Lambda_f^-/\Lambda^-\right)\mathcal{K}_{LL}\otimes\right]\widetilde{G}_Y(\mathbf{r}_{bb'})$$

But with the wrong (+) sign.

Should be compared to result by Mueller, Xiao, Yuan (2013) for joint small-x and soft gluon resummation:

$$\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}(Q,\boldsymbol{P}_\perp)\int\frac{\mathrm{d}^2\mathbf{r}_{bb'}}{(2\pi)^2}e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}}\widetilde{G}_Y^0(\mathbf{r}_{bb'})e^{-S_{\mathrm{Sud}}(\mathbf{r}_{bb'},\boldsymbol{P}_\perp)}$$

Sudakov factor: $$S_{\mathrm{Sud}}(\mathbf{r}_{bb'},P_\perp) = \frac{\alpha_s N_c}{\pi}\int_{c_0^2/\mathbf{r}_{bb'}^2}^{P_\perp^2}\frac{1}{2}\ln\left(\frac{P_\perp^2}{\mu^2}\right)$$

Taels, Altinoluk, Marquet, Beuf, JHEP 10, 184 (2022),
Caucal, Salazar, Schenke, and Venugopalan, JHEP 11 (2022) 169

Kinematical constraint

One needs to impose kinematical constraint for small-x evolution and get correct Sudakov double log:

$$\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}(Q,\boldsymbol{P}_\perp)\int \mathrm{d}^2\mathbf{r}_{bb'}e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}}$$
$$\left[1 \boxed{-\frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}_\perp^2\mathbf{r}_{bb'}^2}{c_0^2}\right)} - \frac{\alpha_s}{\pi}s_L\ln\left(\frac{\boldsymbol{P}_\perp^2\mathbf{r}_{bb'}^2}{c_0^2}\right) + \boxed{\alpha_s\mathcal{K}_{LL,\mathrm{coll}}}\otimes\right]\widetilde{G}_Y(\mathbf{r}_{bb'}) + \mathcal{O}(\alpha_s)$$

Correct Sudakov double log

Kinematically improved small-x evolution

where kinematically constrained equation for TMD:

$$\frac{\partial\hat{G}^{(0)}_{Y_f}(\boldsymbol{r}_{bb'})}{\partial Y_f} = -\frac{\alpha_s N_c}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{2\pi}\frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2\boldsymbol{r}_{zb'}^2}\Theta\left(-Y_f-\ln(\min(\boldsymbol{r}_{zb}^2,\boldsymbol{r}_{zb'}^2)\mu_\perp^2)\right)$$
$$\times\left\{\hat{G}^{(0)}_{Y_f}(\boldsymbol{r}_{bb'}) + \frac{2}{\boldsymbol{r}_{bb'}^2}\left[1-\frac{2(\boldsymbol{r}_{zb}\cdot\boldsymbol{r}_{zb'})^2}{\boldsymbol{r}_{zb}^2\boldsymbol{r}_{zb'}^2}\right]\hat{G}^{(2)}_{Y_f}(\boldsymbol{r}_{zb'},\boldsymbol{r}_{zb}) + \left[\frac{\boldsymbol{r}^i_{bb'}}{\boldsymbol{r}^2_{bb'}}+\frac{\boldsymbol{r}^i_{zb}}{\boldsymbol{r}^2_{zb}}\right]\hat{G}^{(1),i}_{Y_f}(\boldsymbol{r}_{zb},\boldsymbol{r}_{zb'}) + \left[\frac{\boldsymbol{r}^i_{b'b}}{\boldsymbol{r}^2_{b'b}}+\frac{\boldsymbol{r}^i_{zb'}}{\boldsymbol{r}^2_{zb'}}\right]\hat{G}^{(1),i}_{Y_f}(\boldsymbol{r}_{zb'},\boldsymbol{r}_{zb})\right\}$$

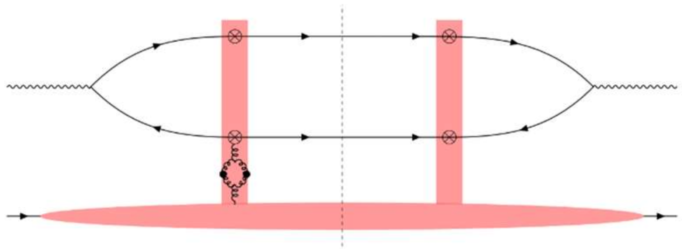
Back-to-back limit (NLO)

$$\left\langle \mathrm{d}\sigma_{\mathrm{LO}}^{(0),\lambda} + \alpha_s \mathrm{d}\sigma_{\mathrm{NLO}}^{(0),\lambda} \right\rangle_{\eta_c} = \frac{1}{2}\mathcal{H}_{\mathrm{LO}}^{\lambda,ii} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0) \Bigg\{ 1 + \frac{\alpha_s(\mu_R)}{\pi} \Bigg[\underbrace{-\frac{N_c}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right)}_{\text{Sudakov double log}}$$

$$\underbrace{-s_L \ln\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \pi\beta_0 \ln\left(\frac{\mu_R^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right)}_{\text{Sudakov single logs}} + \frac{N_c}{2} f_1^\lambda(\chi, z_1, R) + \frac{1}{2N_c} f_2^\lambda(\chi, z_1, R) \Bigg] \Bigg\}$$

$$+ \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}_{\mathrm{LO}}^{\lambda,ii} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{h}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0) \left\{ \frac{N_c}{2}\left[1 + \ln(R^2)\right] - \frac{1}{2N_c}\ln(z_1 z_2 R^2) \right\} .$$

- Factorized expression even at NLO!
- Single log calculated: coefficient,



$$C_F \log\left(\frac{1}{z_1 z_2 R^2}\right) + N_c \log\left(1 + \frac{Q^2}{M_{q\bar{q}}^2}\right) - \beta_0$$

agrees with result obtained in the collinear Collins, Soper, Sterman (CSS) resummation.

Hatta, Xiao, Yuan, Zhou (2021)

Back-to-back limit (NLO)

$$
\left\langle \mathrm{d}\sigma_{\mathrm{LO}}^{(0),\lambda} + \alpha_s \mathrm{d}\sigma_{\mathrm{NLO}}^{(0),\lambda} \right\rangle_{\eta_f} = \frac{1}{2}\mathcal{H}_{\mathrm{LO}}^{\lambda,ii} \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{G}^0_{\eta_f}(\boldsymbol{r}_{bb'},\mu_0) \left\{ 1 + \frac{\alpha_s(\mu_R)}{\pi} \left[-\frac{N_c}{4} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) \right. \right.
$$

$$
\left. \left. - s_L \ln\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \pi\beta_0 \ln\left(\frac{\mu_R^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \frac{N_c}{2} f_1^\lambda(\chi, z_1, R) + \frac{1}{2N_c} f_2^\lambda(\chi, z_1, R) \right] \right\}
$$

$$
+ \frac{\alpha_s(\mu_R)}{2\pi} \mathcal{H}_{\mathrm{LO}}^{\lambda,ii} \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{h}^0_{\eta_f}(\boldsymbol{r}_{bb'},\mu_0) \left\{ \frac{N_c}{2}\left[1 + \ln(R^2)\right] - \frac{1}{2N_c} \ln(z_1 z_2 R^2) \right\}
$$

Assumption: we can resum the large logs by exponentiation:

$$
\mathcal{S} = \exp\left(- \int_{\frac{c_0^2}{\boldsymbol{r}_{bb'}^2}}^{\mu_h^2} \frac{\mathrm{d}\mu^2}{\mu^2} \frac{\alpha_s N_c}{\pi} \left[\frac{1}{2} \ln\left(\frac{\mu_h^2}{\mu^2}\right) + \frac{s_L - \beta_0}{N_c} \right] \right) ,
$$

NLO hard coefficient functions

Longitudinal polarization of photon:

$$f_1^{\lambda=\mathrm{L}}(\chi,z_1,R)=9-\frac{3\pi^2}{2}-\frac{2\pi^2}{27}-\frac{3}{2}\ln\left(\frac{z_1z_2R^2}{\chi^2}\right)-\ln(z_1)\ln(z_2)-\ln(1+\chi^2)\ln\left(\frac{1+\chi^2}{z_1z_2}\right)$$
$$+\left\{\mathrm{Li}_2\left(\frac{z_2-z_1\chi^2}{z_2(1+\chi^2)}\right)-\frac{1}{4(z_2-z_1\chi^2)}\right.$$
$$\left.+\frac{(1+\chi^2)(z_2(2z_2-z_1)+z_1(2z_1-z_2)\chi^2)}{4(z_2-z_1\chi^2)^2}\ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right)+(1\leftrightarrow 2)\right\}$$

$$f_2^{\lambda=\mathrm{L}}(\chi,z_1,R)=-8+\frac{19\pi^2}{12}+\frac{3}{2}\ln(z_1z_2R^2)-\frac{3}{4}\ln^2\left(\frac{z_1}{z_2}\right)-\ln(\chi)$$
$$+\left\{\frac{1}{4(z_2-z_1\chi^2)}+\frac{(1+\chi^2)z_1(z_2-(1+z_1)\chi^2)}{4(z_2-z_1\chi^2)^2}\ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right)\right.$$
$$\left.+\frac{1}{2}\mathrm{Li}_2(z_2-z_1\chi^2)-\frac{1}{2}\mathrm{Li}_2\left(\frac{z_2-z_1\chi^2}{z_2}\right)+(1\leftrightarrow 2)\right\}$$

where $\chi=\dfrac{Q}{M_{q\bar{q}}}$

Tranverse polarization: similar expressions.

WW TMD's evolution

TMD satisfies kinematically constrained evolution equation which is not closed (involves other than WW-type correlators).

For numerical evaluation we assume Gaussian approximation:

$$\hat{G}^{ij}(\boldsymbol{r}_{bb'}) = \frac{2C_F S_\perp}{\alpha_s} \frac{\partial^i \partial^j \Gamma(\boldsymbol{r}_{bb'})}{\Gamma(\boldsymbol{r}_{bb'})} \left[1 - \exp\left(-\frac{C_A}{C_F}\Gamma(\boldsymbol{r}_{bb'})\right)\right]$$

WW TMD

$$\Gamma(\boldsymbol{r}_{bb'}) = -\ln\left(S(\boldsymbol{r}_{bb'})\right) \qquad S = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]\right\rangle_Y$$

S satisfies kinematically constrained BK equation (written in terms of target rapidity η):

$$\frac{\partial \overline{S}_{\eta_f}(\boldsymbol{r}_{bb'})}{\partial \eta_f} = \frac{\alpha_s N_c}{\pi} \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta\left(\ln\left(\frac{1}{x_c}\right) + \ln\left(\frac{\boldsymbol{r}_{bb'}^2}{r_<^2}\right) - \eta_f\right) \Theta\left(\eta_f - \ln\left(\frac{\boldsymbol{r}_{bb'}^2}{r_<^2}\right) - \ln\left(\frac{1}{x_0}\right)\right) \frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2 \boldsymbol{r}_{zb'}^2}$$
$$\times \left[\overline{S}_{\eta_f - \ln(\boldsymbol{r}_{bb'}^2/\boldsymbol{r}_{zb}^2)}(\boldsymbol{r}_{zb}) \overline{S}_{\eta_f - \ln(\boldsymbol{r}_{bb'}^2/\boldsymbol{r}_{zb'}^2)}(\boldsymbol{r}_{zb'}) - \overline{S}_{\eta_f}(\boldsymbol{r}_{bb'})\right].$$

Iancu, Mueller, Soyez, Triantafyllopolous (2019)

BK evolution

$$\frac{\partial \overline{S}_{\eta_f}(\boldsymbol{r}_{bb'})}{\partial \eta_f} = \frac{\alpha_s N_c}{\pi} \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta\left(\ln\left(\frac{1}{x_c}\right) + \ln\left(\frac{\boldsymbol{r}_{bb'}^2}{r_<^2}\right) - \eta_f\right) \Theta\left(\eta_f - \ln\left(\frac{\boldsymbol{r}_{bb'}^2}{r_<^2}\right) - \ln\left(\frac{1}{x_0}\right)\right) \frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2 \boldsymbol{r}_{zb'}^2}$$
$$\times \left[\overline{S}_{\eta_f - \ln(\boldsymbol{r}_{bb'}^2/\boldsymbol{r}_{zb}^2)}(\boldsymbol{r}_{zb}) \overline{S}_{\eta_f - \ln(\boldsymbol{r}_{bb'}^2/\boldsymbol{r}_{zb'}^2)}(\boldsymbol{r}_{zb'}) - \overline{S}_{\eta_f}(\boldsymbol{r}_{bb'})\right] .$$

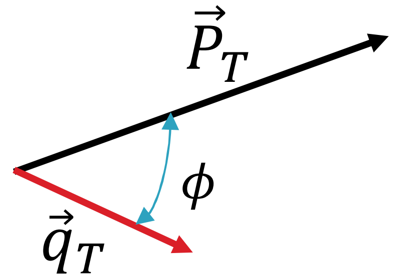
Initial condition at $x_0 = 0.01$ ($\eta_0 = 0$) is MV model:

$$\bar{S}_{\eta_0}(r_\perp) = \exp\left[-\frac{r_\perp^2 Q_{s0}^2}{4} \ln\left(\frac{1}{r_\perp \Lambda} + e\right)\right]$$

where initial saturation scales:

$$Q_{s0}^2 = \begin{cases} 0.1\ \mathrm{GeV}^2 & \text{for proton} \\ \mathrm{A}^{1/3} \times 0.1\ \mathrm{GeV}^2 & \text{for nucleus} \end{cases}$$

Observables for $\gamma^* + p$ or $\gamma^* + A$



$$\frac{\mathrm{d}\sigma^{\gamma_\lambda^*+A\to\mathrm{dijet}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = \mathrm{d}\sigma^{(0),\lambda}(P_\perp, q_\perp, \eta_1, \eta_2) + 2\sum_{n=1}^{\infty} \mathrm{d}\sigma^{(n),\lambda}(P_\perp, q_\perp, \eta_1, \eta_2)\cos(n\phi)\,,$$

Azimuthally averaged cross-section

$v_2 = \frac{d\sigma^{(2)}}{d\sigma^{(0)}}$ anisotropy

Yield:

$$dN_n = \frac{d\sigma^{(n)}}{S_\perp}$$

transverse size of target

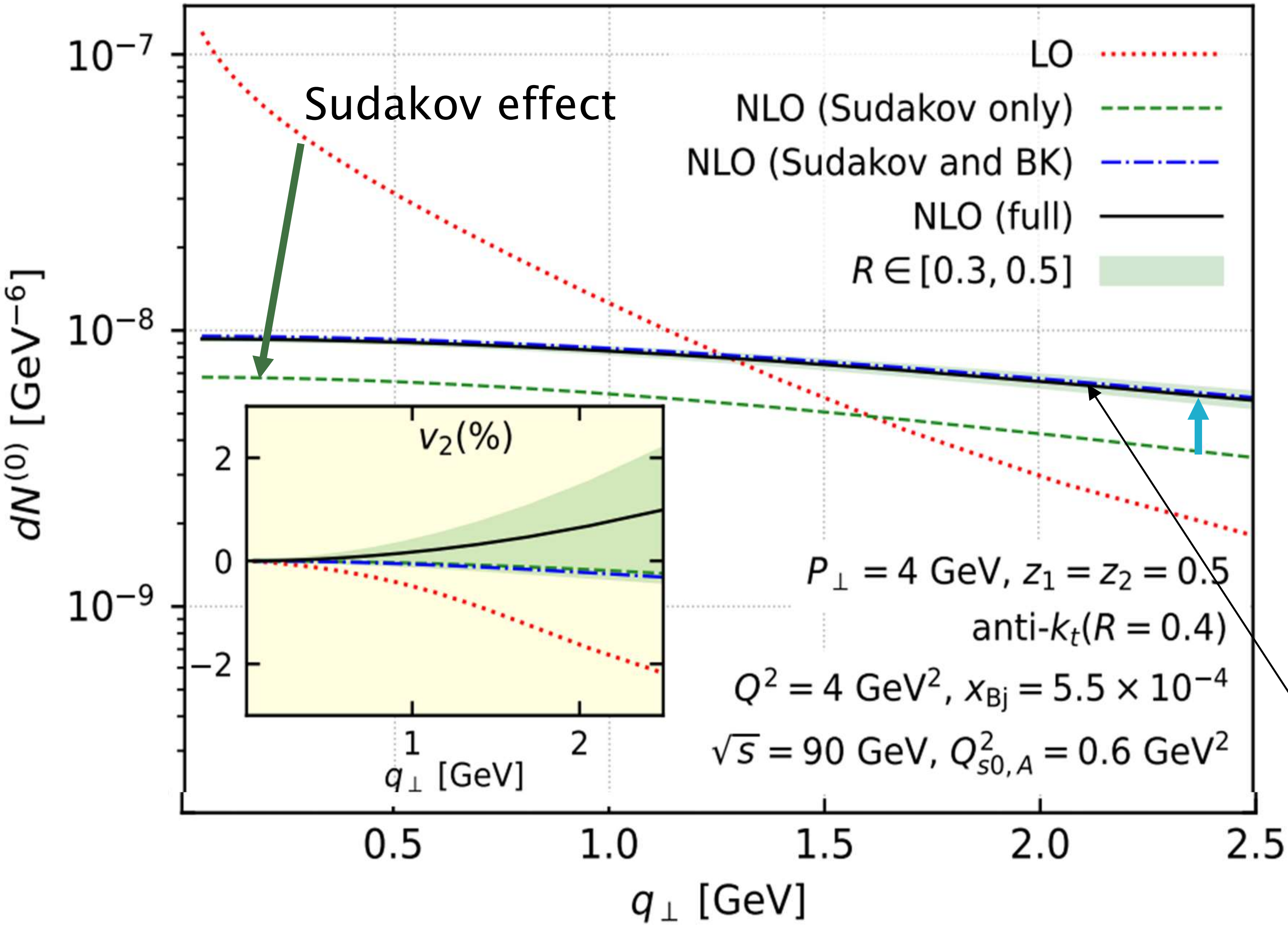
Nuclear modification factor:

$$R_{eA} = \frac{1}{\mathrm{A}^{1/3}} \frac{\mathrm{d}N_0^{e+A}}{\mathrm{d}N_0^{e+p}}$$

where $\mathrm{A}^{1/3} = 6$ and both polarizations of photon are included.

Caucal, Salazar, Schenke, TS and Venugopalan, 2308.00022

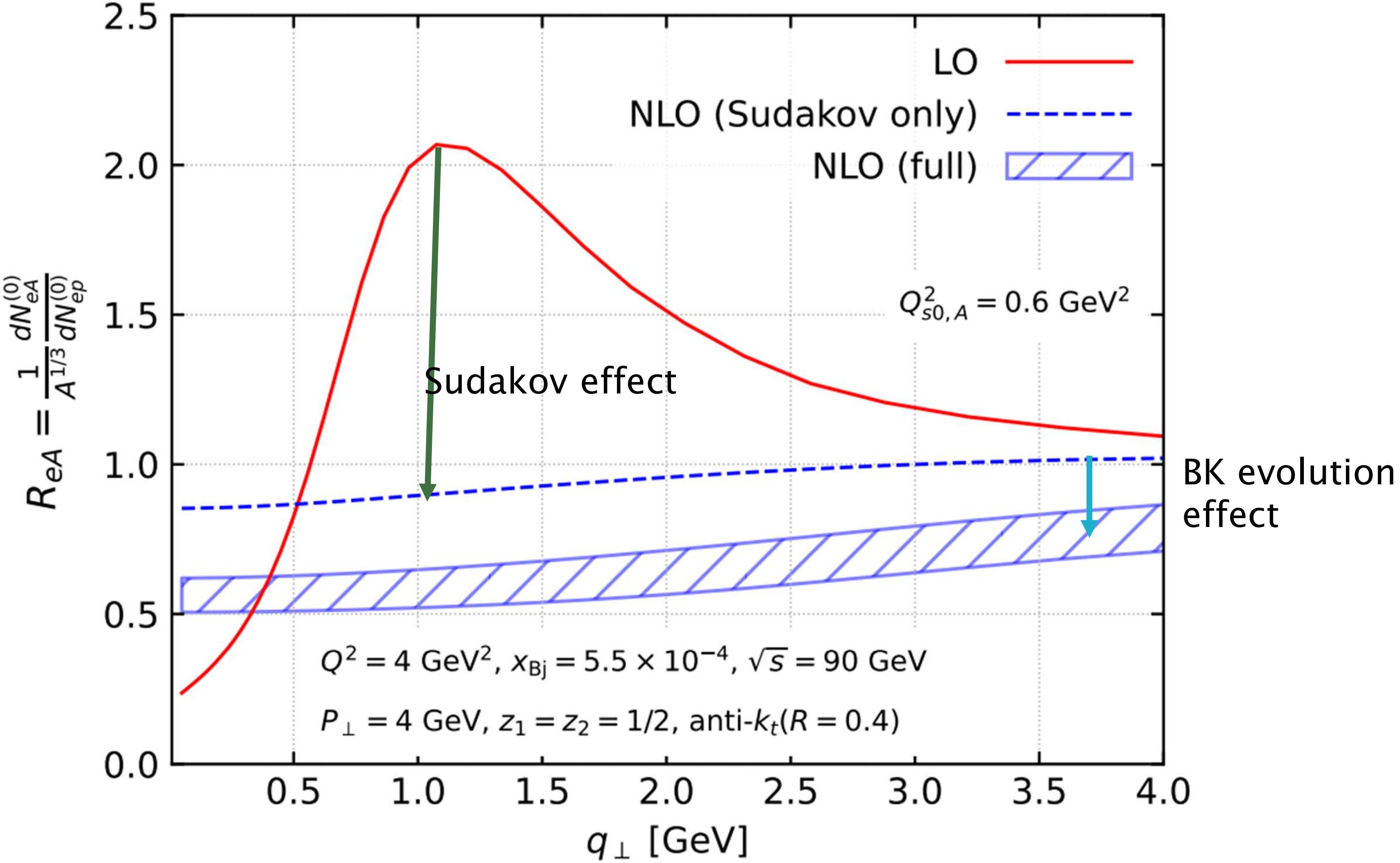
Numerical results: $e + A$ cross-section at EIC



BK evolution effect ($\eta_c \approx 1.3$)

NLO coefficient functions effect (very small)

Numerical results: nuclear modification factor



Summary

- We calculated back-to-back inclusive dijets cross-section up to NLO accuracy:

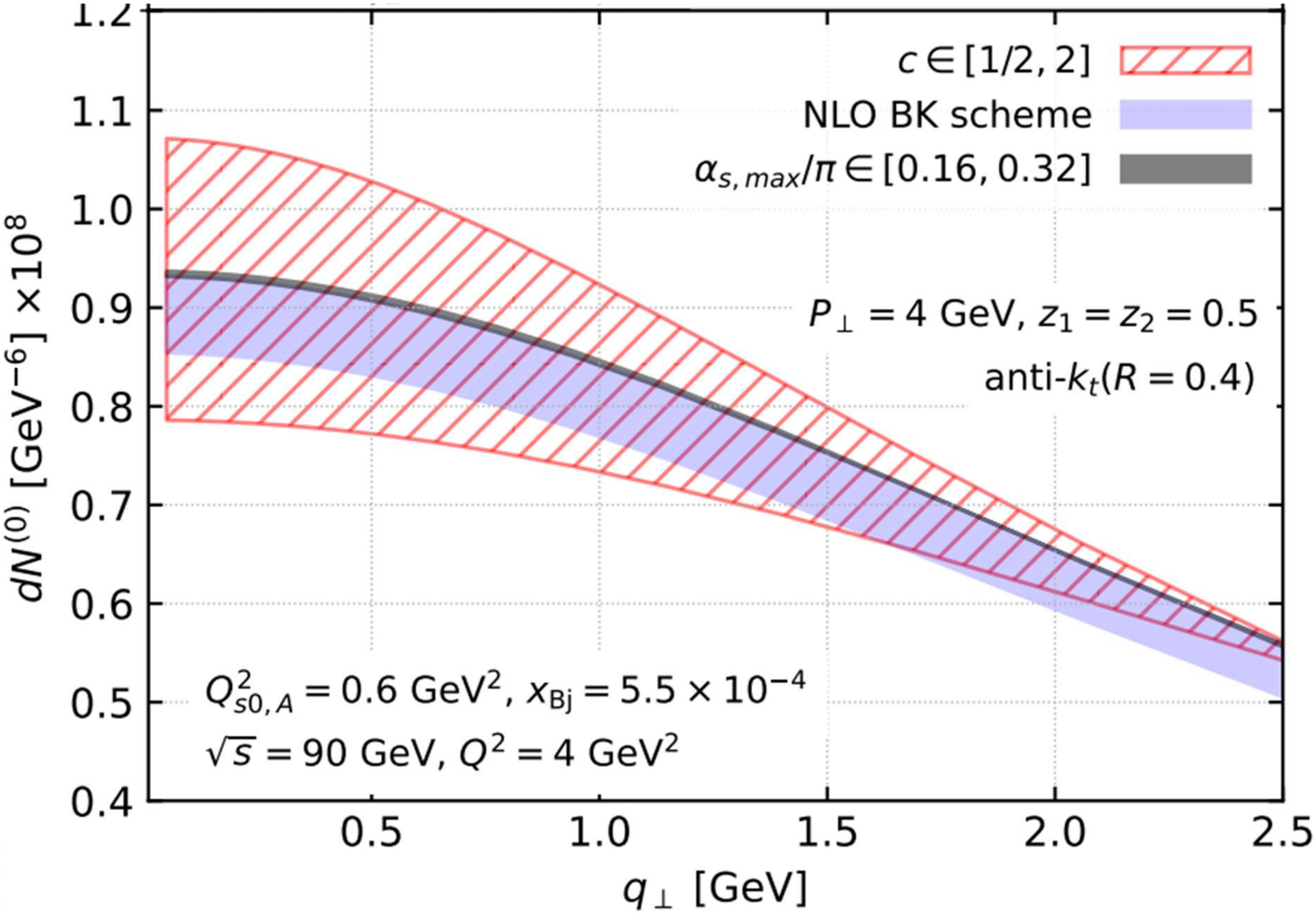
- identify large Sudakov log (both double and single),
- hard coefficient functions are given by analytic expressions.
- kinematical constraint on small-x evolution (BK/JIMWLK).

- Numerical results for EIC:

- Large effect of Sudakov resummation but effect of small-x evolution also visible.
- R_{eA} is below 1 due to the small-x evolution (saturation).

Thank you

Backup slides



Backup slides