First computation of Mueller-Tang processes using full NLL approach
and the violation of BFKL factorization

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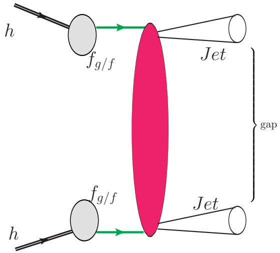
- Mueller-Tang jet process [arXiv:2304.09073]
- Balitsky, Fadin, Kuraev, Lipatov (BFKL) resummation
- Mueller-Tang at the LHC: experiment setup
- NLO impact factor(IF) and Factorization-Breaking
- NLO MT phenomenology

Mueller-Tang process: LL approx.

Probing the High-Energy limit of pQCD at hadron colliders [Mueller, Tang '87]



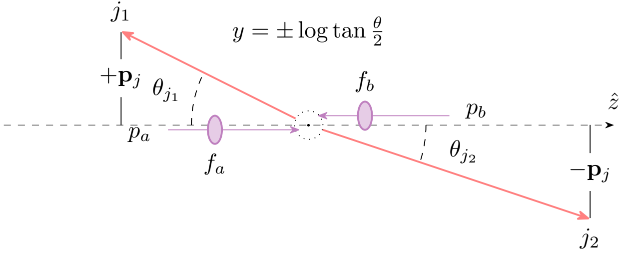
Goal: probing the BFKL Pomeron at finite mom. transferred t

Exclusive dijets: $p_1 + p_2 \to j_1 + j_2 + \text{gap}$

1) large rapidity separation: $Y_j \gg 1$
2) no activity in between: $Y_{\text{gap}} \sim Y_j$

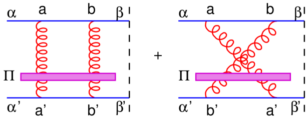
Valid only in first approximation and differs from experiment set up

- $2 \to 2$ elastic scattering $\Leftrightarrow$ back-to-back jets (in transverse plane)
- Large (pseudo)-rapidity $\Leftrightarrow$ very small scattering angles
- $Y \gg 1 \Leftrightarrow$ Need for BFKL resumm
  $Y \gg 1 \to \alpha_s \log(\hat{s}/t) \sim 1$
- $Y_{\text{gap}} \gg 1 \Leftrightarrow$ favors color-singlet exchange

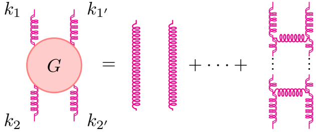


Mueller-Tang jets: LO to LLA

- Lowest-Order Approx.: Box+Crossed diagrams projected onto color-singlet rep.
  $\prod^{ab,a'b'} = \delta_{ab}\delta_{a'b'} / (N_c^2 - 1)$



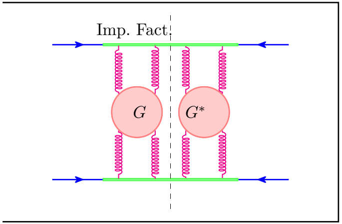
- Leading-Log Approx: Nth-order ladder diagrams not suppressed since $(\alpha_s \log sk^2)^n \sim (\alpha_s Y)^n \sim 1$
- BFKL Resummation → Gluon-Green's function (GGF)



- LL **factorized** cross-section: $\sigma \sim \mathrm{GGF}^2 \otimes \mathrm{IF}$ (IF=Imp. Fact.)

$$\frac{d\hat{\sigma}}{\mathrm{d}Y \mathrm{d}k^2} = \int \mathrm{d}^2 \boldsymbol{\ell}_1 \boldsymbol{\ell}_1' \Phi^2(\boldsymbol{\ell}_1, \boldsymbol{\ell}_1') G^2(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2, k^2, Y) \times \begin{Bmatrix} \ell \to \ell' \\ 1 \to 2 \end{Bmatrix} = \Phi^2 G^2(k, Y)$$

- GGF is universal (process independent)
- GGF is color-singlet
- IF is process dependent
- IF trivial (c-numbers) at LL

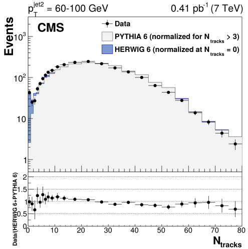


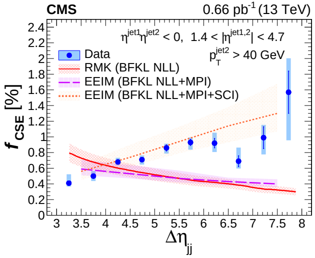
Radiative corrections affect GGF and IF

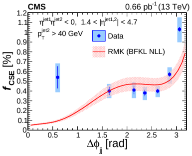
CMS analyses

13 TeV[*C.Baldenegro*:arxiv:2102.06945]
$f_{CSE} \equiv$ color-singlet/jet-gap-jet ratio of -events

7 TeV [EPJC 78,242 (2018)]
Charged-particle multiplicity in the gap region





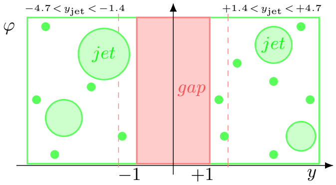


- PYTHIA MC *v.s* HERWIG MC predictions *v.s* data.
- HERWIG 6: include contributions from color singlet exchange (CSE), based on BFKL at LL.
- PYTHIA 6: inclusive dijets, no-BFKL.
- HERWIG *seems* to reproduce the excess in *zero-th* bin $\leftrightarrow$ no particle in gap region

Sensitive to BFKL?

- RMK model (HERWIG) [*C.Royon et al.* PRD83.034036] based on BFKL NLL + LO IFs, gap and survival probability $|S|^2 = 0.1$
  - unsatisfactory agreement with unexpected rise in $Y \equiv \Delta\eta_{jj}$.
  - Better agreement for f_{CSE} vs p_{T_J}.
- EEIM [] based on RMK + soft gap contamination
  - Better agreement then RMK
- Puzzling event excess in decorrelation limit $\Delta\phi_{jj} \to 0$: tentative explanation from factorization violating terms?

MT at NLO



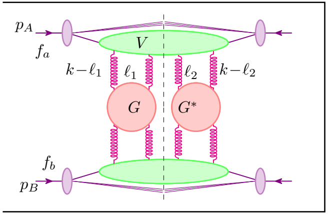
Semi-inclusive dijets: $p_a + p_b \to j_1 + X + \text{gap} + X + j_2$

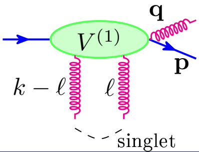
- Central rapidity $-1 < Y_{\rm gap} < 1$ gap: no charged particles *and* no photons or neutral hadrons above energy threshold $p_t > Eth = 0.2$ GeV.
- Tag the 2 hardest jets with $p_j^{jet} > 40$ GeV and $|\eta^{jet}| > 1.5$
- Jet radius $R_{jet} = 0.4$ and anti-k_t jet algorithm.

Cross-section structure: $IF \otimes GGF \otimes GGF^* \otimes IF$

$$\frac{d\hat{\sigma}}{dJ_1 dJ_2 d^2k} = \int \mathrm{d}^2\ell_{1,2}\mathrm{d}^2\ell'_{1,2}\Phi(\boldsymbol{\ell}_{1,2}, \mathbf{k}; J_1)G(\boldsymbol{\ell}_1, \boldsymbol{\ell}'_1, \mathbf{k}, Y)G(\boldsymbol{\ell}_2, \boldsymbol{\ell}'_2, \mathbf{k}, Y)\Phi(\boldsymbol{\ell}'_{1,2}, \mathbf{k}; J_2)$$

- Highly dimensional numerical integrations
- GGF at NLL in full form: *not* averaged over reggeon momenta
  $\propto$ Gauss-Hypergeometric func. and require ad-hoc implementation
  Numerical precision degrades *exponentially* for large conformal spins
  Brute-force solution: use *hundreds* of precision digits





NLO Impact Factors:

- Virtual corrections[Fadin et al.:hep-ph/9908265]: BFKL scale s_0 dependent
- Real corrections[Hentchinski et al.:arxiv1406.5625]: $2 \to 3$ topology
- Interaction is not elastic

BFKL factorization violated?

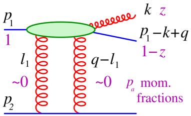
Can MT at NL fit into BFKL framework?

Is BFKL factorization valid at NLO?
Structure of real emission and 1-loop corrections to singlet box+crossed:

- IR singularity must cancel (except the collinear corrections to the PDFs)
- All $\log s$(or Y) factors must reproduce GGF kernel No $\log s$ in IF!

NLO IF from [*Hentchinski et al.*]
D. Colferai and I checked these results using standard Feynman diagram techniques

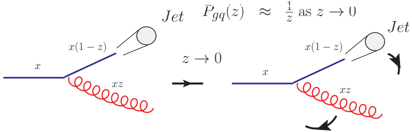
$$\Phi(\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{q}) = \frac{\alpha_{\mathrm{S}}^3}{2\pi(N_c^2-1)} \int_0^1 \mathrm{d}z \int \mathrm{d}^2\boldsymbol{k}$$



$\times$ h.c. $(\boldsymbol{l}_1 \leftrightarrow \boldsymbol{l}_2)$

$$\times\, \boldsymbol{S_J}(\boldsymbol{k}, \boldsymbol{q}, z)\, C_F \frac{1+(1-z)^2}{z}$$

$$\times \left\{ C_F^2 \frac{z^2 \boldsymbol{q}^2}{\boldsymbol{k}^2(\boldsymbol{k}-z\boldsymbol{q})^2} + C_F C_A f_1(\boldsymbol{l}_{1,2}, \boldsymbol{k}, \boldsymbol{q}, z) + C_A^2 f_2(\boldsymbol{l}_{1,2}, \boldsymbol{k}, \boldsymbol{q}) \right\}$$



$$\Phi_{\log} \propto \int \mathrm{d}^2\vec{k} \int_0^1 \frac{\mathrm{d}z}{z}$$

- Central emission incurs in no dynamical suppression

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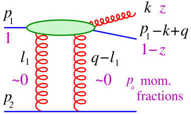

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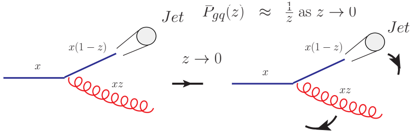

$$\Phi_{\log} \propto \int \mathrm{d}^2\vec{k} \int_0^{Y/2} \mathrm{d}y = Y/2 \frac{E_{th}^2}{E_j^2}$$

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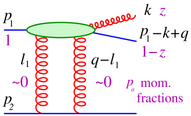

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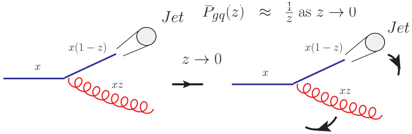
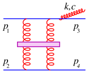


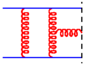
$$\Phi_{\log} \propto \int \mathrm{d}^2\vec{k} \int_0^{Y/2} \mathrm{d}y = Y/2 \frac{E_{th}^2}{E_j^2}$$

- Central emission incurs in no dynamical suppression
- However, gap requirement reduces the size of the violating term
- Formally factorization is violated but small violation in practice

Factorization Breaking



- The theoretical argument:
  "colour-singlet momentum transfer $\Longrightarrow$ no $\log s$ is wrong
- Here colour-singlet either below or above



$\Longrightarrow$ $\log s$ unavoidable!

But we cannot select diagrams!
We can only select final states.

- Since particles cannot be measured below some energy threshold $E_{\rm th}$
  We can at most require no activity above threshold within the gap
- This prescription is IR safe because inclusive for $E_g < E_{\rm th}$
  But the gluon can have any rapidity $\Longrightarrow \sigma \ni C_A^2 \frac{E_{\rm th}^2}{E_J^2} \log \frac{s}{E_J^2}$

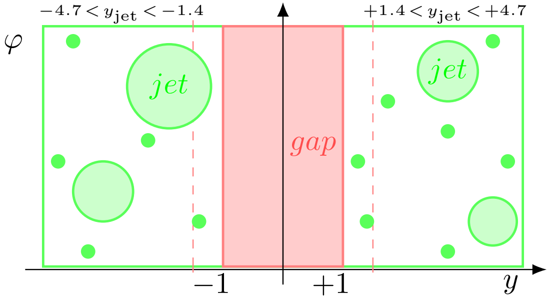
With such "minimal" experimental prescription, BFKL factorization is violated (impact factors depend on s). However violation is expected to be small.

Results

CMS setup

We follow CMS setup:

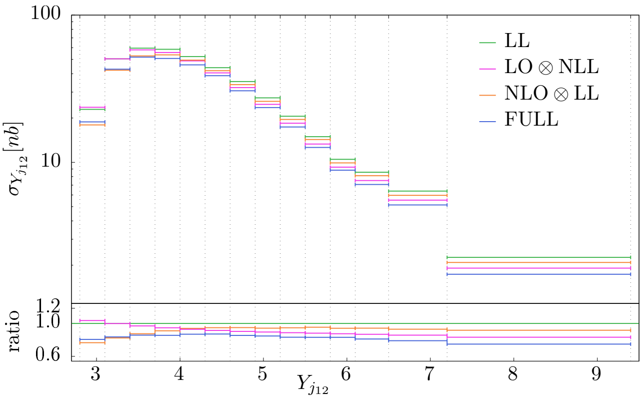
- $\sqrt{s} = 14$ TeV
- $p_j > 40$ GeV
- $2.8 < Y < 9.2$
- $-1 < y_{\rm gap} < 1 \to Y_{\rm gap} = 2$
- $E_{\rm th} = 1$ GeV



Results

Comparing: tot. cross section $\sigma_{Y_{j_1 2}}$ integr. over rapidity bins

- LL(LL GGF x LO IF)
- NLL(NLL GGF x LO IFs)
- FULL (NLL GGF x LO IF + LL GGF x NLO IF)
- NLO IF corrections are large and negative

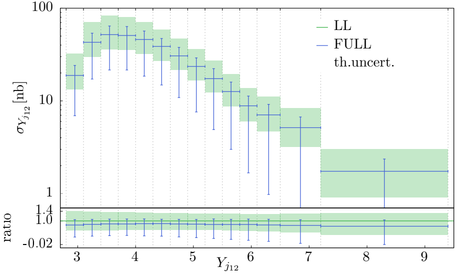


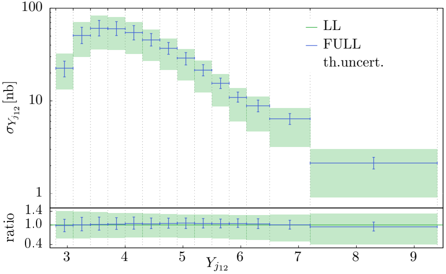
Natural vs Minimal Sensitivity renorm. scale

Th. Uncert.: squared sum of $\Delta\sigma$ variations when $\{s_0, \mu_R, \mu_F\} * 2, /2$

"Natural" scale $\mu_R = p_{j_1} + p_{j_2}$ $\Leftrightarrow$ Minimal sensitivity scale $\mu_R = 4\times(p_{j_1} + p_{j_2})$

σY_{j12}

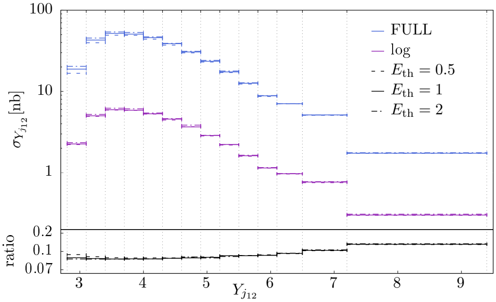




- FULL @ Nat μ_R significantly smaller then LL
- FULL @ MS: μ_R compatible with LL
- FULL @ MS: NLO corrections reduce sensitivity to scale variation

$\sigma_{Y_{j_{12}}}$: sensitivity to gap threshold

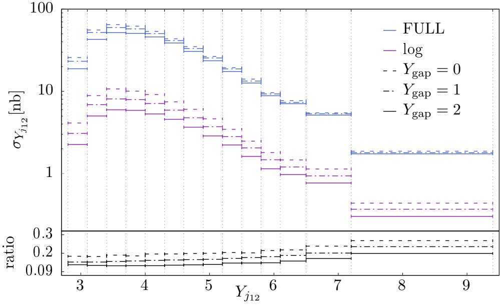
FULL *v.s.* $\propto \log(s)$ violating term: total cross-section as function of jet rapidity separation



- log/FULL $\sim 10\% - 20\%$: small violation
- log term more sensible to gap size:
  gluon emission in log term tend to be more central then the rest
- Minimal sensitivity to $E_{\rm th}$ for both log *and* FULL:
  threshold applied only on central gap region

$\sigma_{Y_{j_{12}}}$: sensitivity to gap size

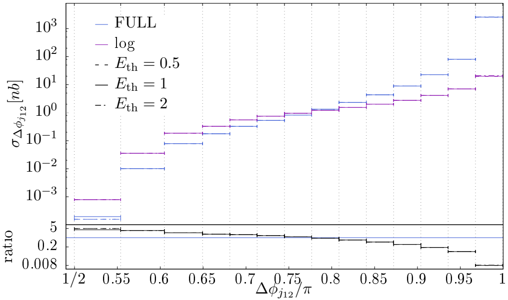
FULL *v.s.* $\propto \log(s)$ **violating term**: total cross-section as function of jet rapidity separation



- log/FULL $\sim 10\% - 20\%$: small violation
- Violation increases at large Y:
  unsuppressed region $Y - Y_{\text{gap}}$ grows with Y
- log term more sensible to gap size:
  gluon emission in log term tend to be more central then the rest

$\sigma_{\Delta\phi_{j_{12}}}$: sensitivity to gap threshold

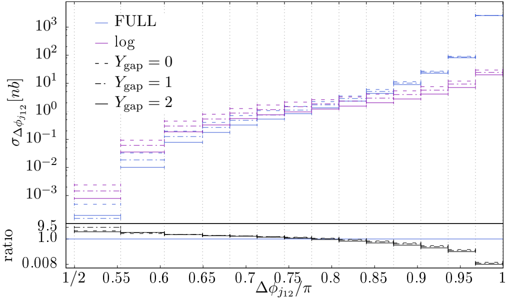
FULL *v.s.* $\propto \log(s)$ violating term: total cross-section as function of jet azimuthal angle



- Azymuthal spread only due to NLO IFs: Recall that $\Delta\phi = \pi$ for LL and NLL contrib.
- Strongly peaked on back-to-back configuration
- Minimal sensitivity to energy threshold
- Violation is small in most events
- Violating term becomes dominant at intermediate to small angles:
  log term is eager to emit gluons at large angles

$\sigma_{\Delta\phi_{j_{12}}}$: sensitivity to gap threshold

FULL *v.s.* $\propto \log(s)$ violating term: total cross-section as function of jet azimuthal angle

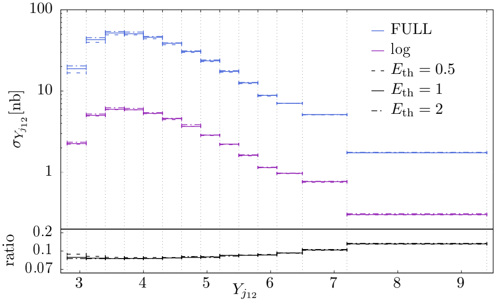


- Azymuthal spread only due to NLO IFs: Recall that $\Delta\phi = \pi$ for LL and NLL contrib.
- Strongly peaked on back-to-back configuration
- Sensitivity to gap size increases towards small angles
- Stronger sensitivity in log term then FULL

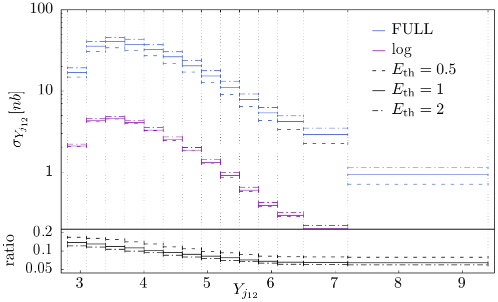
Dynamic vs Central gap

Dynamic gap extends between the whole rapidity range between jets Except for a buffer $y_0 \sim 0.4$ to not interfere with jet $Y_{\rm gap} \propto Y_j$

Central gap

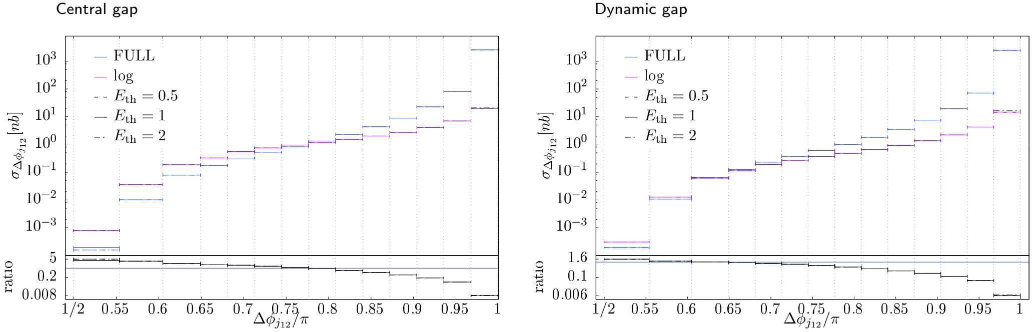


Dynamic gap



- log/FULL drops at large Y
- Violation stays small
- More sensible to threshold
  but could be reduced enlarging the "buffer" zone

Dynamic vs Central gap

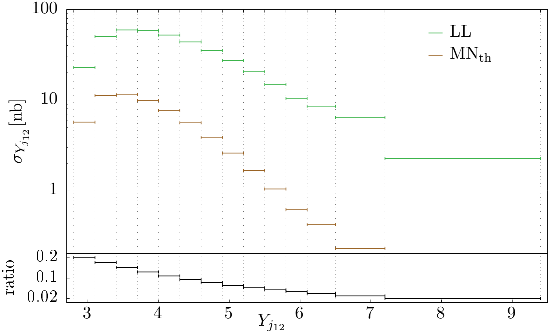


- Violation is small also at small angles
- Effectively no violation with dynamic gap

MN with soft minijets

"Background": Mueller-Navelet emission with transverse energy below threshold:
$\mathrm{MN_{th}}$ *v.s.* MT both at LL

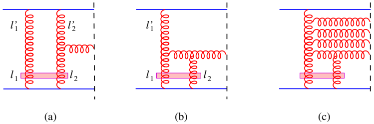
$\sigma_{Y_{j_{12}}}$

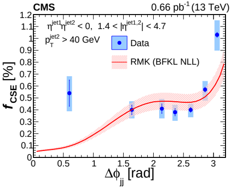


- ratio $\mathrm{MN_{th}}$/MT $> 20\%$
- strongly suppressed at large Ys

Tentative explanation of flattening towards small angles?

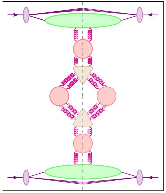
We envision that a more accurate description of uncorrelated $\Delta\phi_{j_{12}} \lesssim \pi/2$ dijet events would require an additional resummation of $\log(s)$ factors in IFs





That would likely increase the number of events with highly uncorrelated jets which may explain the observed puzzling flattening of the cross section towards small azimuthal angles.

Actually, there are other leading diagrams with $\log(s)$ enhancement that have not been considered but can fit into the gap definition.
Since the gap is central and no constraint is imposed on its forward and backward sides we could envision a hybrid process:
Mueller-Navelet $\rightarrow$ Mueller-Tang $\rightarrow$ Mueller-Navelet type of emission.
That could be estimated by a Pomeron loop diagram in the optical limit $t \rightarrow 0$.



Conclusions and outlook

- Complete numerical implementation of MT jets at LHC in NLLA with collinear resummation of BFKL kernel; cross section slightly lower and steeper than in LLA
- BFKL factorization *formally* violated at NLLA
- In practice, violation is small and factorization formula holds (approximately) at LHC energies
- Good stability w.r.t. gap/threshold parameters
- Better description expected with proper renorm scale fixing ($\simeq$ 4 times larger than natural scale)
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- Good stability w.r.t. gap/threshold parameters
- Better description expected with proper renorm scale fixing ($\simeq 4$ times larger than natural scale)
- Need MC implementation to compare to data
- Improvements could include hadronization, resummation of $\log s$ term in IFs, inclusion of gap survival probability and BLM (or others) renorm-scale fixing
- Perhaps with central gap the main contribution come from "cut" Pomeron-loop

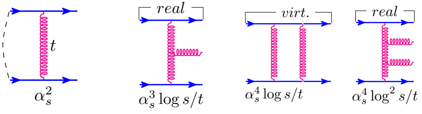
Backup

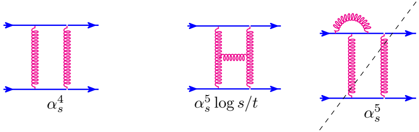
High energy limit of QCD

QCD in the high energy limit shows qualitative new behaviors:

Loop or phase space integrations in scattering amplitudes with precise color structure give rise to $\log(\hat{s}/t) \sim Y$ coefficients

$$\sigma \sim A \log s/t + B + C(s/t)^{-1} \ldots$$

- $(\underline{8}) \in$ 

- $(\underline{1}) \in$ 

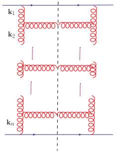
Octet dominates ($\alpha_s^2 \gg \alpha_s^4$) but radiates everywhere. Clearly, $\alpha_s^4 \log s > \alpha_s^4$ and $\alpha_s^n \log s > \alpha_s^n$; what about $\alpha_s^2 \log s > \alpha_s^1$?

- New hierarchy *Balitsky-Fadin-Kuraev-Lipatov*

Effective expansion parameter becomes $\alpha_s \log s/t$

Radiative corrections of order n to the partonic cross sections

$$d\hat{\sigma} \simeq \underbrace{\alpha_s^n \log^n \left(\frac{s}{-t}\right) \sigma^{(0)}}_{\text{Leading Log approx.(LL)}} + \underbrace{\alpha_s^n \log^{n-1} \left(\frac{s}{-t}\right) \sigma^{(1)}}_{\text{Next-to-Leading Log (NLL)}} + \ldots$$



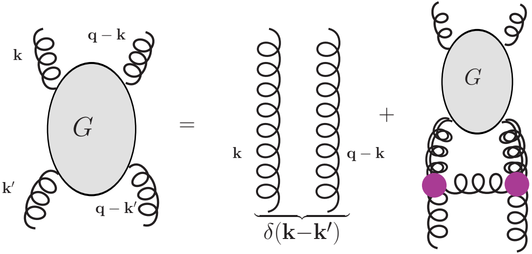
LL Gluon-ladder diagrams

BFKL equation

Recursive integral equation in the form of a Green function equation called BFKL equation.
The ladder diagrams are resummed to all order by iterating the Gluon Green function G.

$$G(\mathbf{k},\mathbf{k}') = \delta^2(\mathbf{k}-\mathbf{k}') + \int d^2\ell \mathcal{K}(\mathbf{k},\ell) G(\ell,\mathbf{k}')$$

G is universal (process independent)

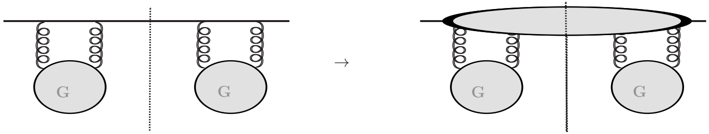


$$G(\mathbf{k},\mathbf{k}',\mathbf{q},Y) = \int_{-i\infty}^{+i\infty} \frac{d\omega}{2\pi i} e^{Y\omega} \sum_{n\in\mathcal{Z}} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi\, i} \frac{E_{\gamma,n}(k) E^*_{\gamma,n}(k')}{\omega - \bar{\alpha}_s \chi(\gamma,n)} \qquad e^{Y\omega} = \left(\frac{s x_1 x_2}{-t}\right)^{\omega}$$

$$E_{n,\nu} \propto \begin{cases} {}_2F_1\Big(a(n,\nu), b(n,\nu), c, z(\mathbf{k},\mathbf{k}',q)\Big), & \text{non-forward, Gauss hypergeometric func.} \\ |\mathbf{k}|^{-\frac{1}{2}+i\nu} e^{in\theta}, & \text{forward limit } q \to 0. \end{cases}$$

NLO impact factors

Several non trivial modifications to the theoretical description needed to accommodate the NLO corrections to the impact factors (IF).



Non-factorizable. NLO impact factors connect the Gluon Green functions over the "cut"

NLO impact factors have yet to be implemented for phenomenology studies to complete the NLO calculation (BFKL@NLL + impact factors@NLO). Efforts by D. Colferai, F. Deganutti, C. Royon, T. Raben on this direction (private communication), and by U. of Munster coll. (M. Klasen, J. Salomon, P. Gonzlez, M. Kampshoff).

$$\frac{d\hat{\sigma}}{dJ_1 dJ_2 d^2\mathbf{q}} = |A(Y,q)|^2 \quad \Leftrightarrow \quad V_a(\mathbf{k}_1,\mathbf{k}_2,J_1,\mathbf{q}) \otimes G(\mathbf{k}_1,\mathbf{k'}_1,\mathbf{q},Y) \otimes G(\mathbf{k}_2,\mathbf{k'}_2,\mathbf{q},Y) \otimes V_b(\mathbf{k'}_1,\mathbf{k'}_2,J_2,\mathbf{q}),$$

$$A(Y,q) \sim V_a(q)V_b(q) \int d^2k d^2k' G(\mathbf{k},\mathbf{k'},\mathbf{q},Y) \quad \Leftrightarrow \quad \bar{G}\left(Y,\mathbf{q},\frac{k}{k'}\right) \propto \sum_n^{n\ even} \int d\nu \left[\frac{k^{*\bar{h}-2}}{k'^{h-2}}\, {}_2F_1\left(\frac{k}{k'}\right)\, {}_2F_1\left(\frac{k'^*}{k^*}\right) + \{1 \leftrightarrow 2\}\right].$$

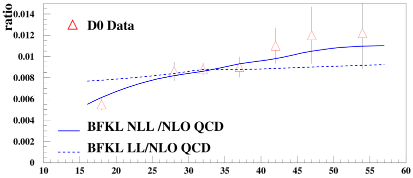
- From squared amplitude to multiple convolution between the the jet vertices and the GGFs.
- LO vertices are c-numbers and can be factorized out of the convolution.
- Average of GGF over the reggeon momenta is *remarkably* simple.

$$A(Y,q) \sim A(Y,q=0)\frac{4}{q^2}$$

$\Big($ ${}_2F_1$ for large conf. spins using ball-arithmetic c-library *https://arblib.org* $\Big)$

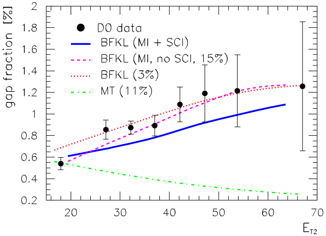
Previous fits and analysis

Fraction of jet-gap-jet events vs inclusive dijets measured by D0 Coll. [Phys.Lett. B440 189 (1998)] well reproduced by BFKL estimates. **NLL order correction are necessary**



[O. Kepka, C. Marquet, C. Royon Phys.Rev. D83.034036 (2011)]

- Ratio $R = \frac{NLL^{*} BFKL}{NLOQCD}$ of jet-gap-jet events to inclusive dijet events as a function of p_t.
- $\mathrm{NLL}^{*} \sim$ NLL (forward) Green Func. + collinear improvement. No NLO Imp. Factors
- Normalization fixed by gap survival probability $|S|^2 = 0.1$.



[R. Enberg, G. Ingelman, L. Motyka Phys.Lett.B524,273 (2002)]

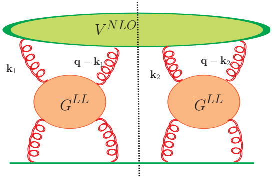
- NLL^{*} BFKL predictions + soft rescattering corrections (EIM models) describe many features of the data (not so good for other observables).
- Different implementations of underlying event:
  Gap survival probability (S),
  Multiple interactions (MI),
  Soft colour interactions (SCI).

non-forward Gluon Green Function

The decision to keep just the pure NL contribution brings some simplification

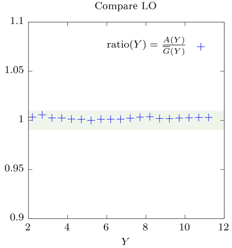
$$\frac{d\hat{\sigma}}{dJ_1 dJ_2 d^2\mathbf{q}} = \int d^2k_1 d^2k_2 V^{NLO}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}; J_1)\times$$

$$\underbrace{\int d^2k'_1 G(\mathbf{k}_1, \mathbf{k}'_1, \mathbf{q}, Y)}_{\overline{G}(\mathbf{k}_1, \mathbf{q}, Y)} \underbrace{\int d^2k'_2 G(\mathbf{k}_2, \mathbf{k}'_2, \mathbf{q}, Y)}_{\overline{G}(\mathbf{k}_2, \mathbf{q}, Y)} V^{LO}(J_2, \mathbf{q})$$



$$\bar{G}(x_1 x_2, q, \Delta\theta, \frac{k}{k'}) \propto \sum_m^{m\ even} \int d\nu \left[k^{*\,\bar{h}-2} k'^{h-2}\, {}_2F_1\left(1-h, 2-h, 2; -\frac{k}{k'}\right)\, {}_2F_1\left(1-\bar{h}, 2-\bar{h}, 2; -\frac{k'^*}{k^*}\right) + \{1 \leftrightarrow 2\}\right].$$

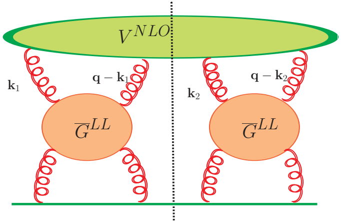
- Integrand is highly oscillatory and slowly falling with ν. $h = \frac{1+n}{2} + i\nu$
- Fast and reliable evaluation of ${}_2F_1(a, b, c; z)$ and for large $Im(a, b)$ notoriously difficult.
- To avoid numerical cancellations for large conformal spin even quadruple precision not enough.



Numerical analysis

The decision to keep just the pure NL contribution brings some simplification

$$\frac{d\hat{\sigma}}{dJ_1 dJ_2 d^2\mathbf{q}} = \int d^2k_1 d^2k_2 V^1(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}; J_1)\times$$
$$\underbrace{\int d^2k'_1 G(\mathbf{k}_1, \mathbf{k}'_1, \mathbf{q}, Y)}_{\overline{G}(\mathbf{k}_1, \mathbf{q}, Y)} \underbrace{\int d^2k'_2 G(\mathbf{k}_2, \mathbf{k}'_2, \mathbf{q}, Y)}_{\overline{G}(\mathbf{k}_2, \mathbf{q}, Y)} V^0(J_2, \mathbf{q})$$



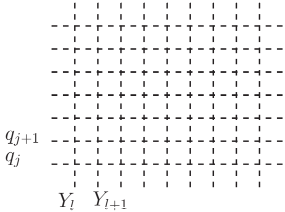
- Large increase in computation time due to the high-dimensional multiple integration.

The full form of the eigenfunction in momentum space is known [Bartels, Braun, Colferai, Vacca].

- The momentum dependence of the eigenfunction is expressed through hypergeometric functions in a region of parameter very sensible to numerical fluctuations. $\;_2F_1(a, b; c, z),\; a - b \in \mathbb{Z}^-$

Numerical analysis

- Calculation of the partonic cross section.
  (1) $\bar{G}$ as a grid of its parameters $\{k_i, q_j, \theta_l, Y_m\}$. It involves a numerical integration over ν and a sum over n for each set of the parameters.
  (2) Partonic cross section as the interpolation of $\bar{G}$ grids and the NLO vertex.

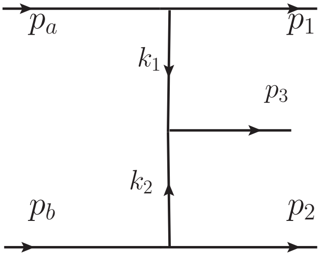


$$\frac{\hat{\sigma}(k_{J_1}, k_{J_2}, \theta_{J_1,J_2}, Y)}{dk_J dY} \propto \sum V(k_{1_i}, k_{2_j}, \theta_{1_n}, \theta_{2_m}, J)\bar{G}(k_{1_i}, q_r, \theta_{1_n}, Y_l)\bar{G}(k_{2_j}, q_r, \theta_{2_m}, Y_l)$$

- Dressing of the initial state and final state hadronization by Herwig
  (1) Proton-proton scattering $\frac{d\sigma^{pp\to JGJ}}{dx_1 dx_2 dq} \propto \sum_{a,b} f_a(x_1, k_{J_1}) f_b(x_2, k_{J_2}) \hat{\sigma}(k_{J_1}, k_{J_2}, \theta_{J_1,J_2}, Y)$
  (2) Fitting of the cross section and its substitution by a sum of analytic functions of the fitting parameters.
  (3) Hadronization from the proto-jet to the detector with a matching procedure to remove the double counted diagrams. The error avoided by this subtraction is predicted to be of NL order.

BFKL

Balitsky, Fadin, Kuraev, Lipatov (BFKL) were the first to consider the Regge limit of QCD.
The large logs come from the integration over the longitudinal momentum fraction bounded by the outermost partons.
Sudakov parametrization $k_i = z_i p^+ + \bar{z}_i p^- + \mathbf{k}$, $p^+ = \frac{p_a}{\sqrt{2}}, p^- = \frac{p_b}{\sqrt{2}}$



On shell conditions $\rightarrow (\mathbf{k}_1, \mathbf{k}_2, z_1)$, $\bar{z}_1 = \mathbf{k}_1/s$, $\bar{z}_2 = \mathbf{q}/z_1 s$.
Positive energies $E > 0 \rightarrow 1 > z_1 > z_2 > 0$.

$$\int d\Pi_3 \propto \int_{z_2}^{1} \frac{dz_1}{z_1} \int dz_2 \delta(z_2 - \mathbf{k}_2/s) = \log(\frac{s}{s_0})$$

Changing s_0 leaves the LL unaltered.

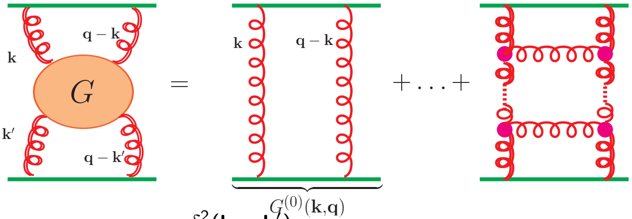
The amplitude is independent from the longitudinal fractions:

- Eikonal approximation $-ig\bar{u}(p_a - k_1)\gamma^\mu u(p_a) \simeq -2igp_a^\mu$.
- $k_1 \rightarrow z_1 p^+ + \mathbf{k}_2, k_1 \rightarrow \bar{z}_2 p^- + \mathbf{k}_2 \rightarrow k_1^2 = (z_1 p^+, 0, \mathbf{k}_1)^2 \rightarrow \frac{1}{k_1^2} \simeq -\frac{1}{\mathbf{k}_1^2}$.

For $s \gg t$ the predominant contribution comes from the strongly ordered region
$1 \gg z_1 \gg z_2 \gg 0 \rightarrow y_1 \gg y_3 \gg y_2$. $y_i = \log(\frac{z_i\sqrt{s}}{|\mathbf{k}_i|})$.

LL approximation: LO vertex

At LL accuracy the Gluon green function G resumms to all orders of perturbation theory the ladder diagrams composed by s-channel gluons connected to t-channel reggeizzed gluons through the Lipatov vertex. The normalization of the Gluon Green function fixes the jet vertex leading order.



$$\lim_{Y\to 0} G(\mathbf{k},\mathbf{k}',\mathbf{q},Y) = G(\mathbf{k},\mathbf{k}',\mathbf{q},0) = \frac{\delta^2(\mathbf{k}-\mathbf{k}')}{\mathbf{k}^2(\mathbf{q}-\mathbf{k})^2}.$$

At this order, apart for the jet distribution function S that fixes the jet momentum, the jet vertex is a simple color factors (c-number)

$$V_a(x,\mathbf{q},x_J,\mathbf{k}_J) = S_J^0(x,\mathbf{q};x_J,\mathbf{k}_J)h_a^0,$$

$$h_a^0 = C_{q/g}^2 \frac{\alpha_s^2}{N_c^2-1}, \qquad S_J^{(0)} = x\delta^2(\mathbf{k}_J-\mathbf{q})\delta(x_J-x).$$

The independence of the LO vertices from the reggeon momenta allow for considerable simplification.

details of NLO impact factor

Details of NLO impact factor

$$\frac{d\hat{V}^{(1)}(x,\boldsymbol{k},\boldsymbol{l}_1,\boldsymbol{l}_2;x_J,\boldsymbol{k}_J;M_{X,\max},s_0)}{dJ} =$$

$$= v_q^{(0)}\frac{\alpha_s}{2\pi}\Bigg[S_J^{(2)}(\boldsymbol{k},x)\cdot\Bigg[-\frac{\beta_0}{4}\Bigg[\Bigg\{\ln\left(\frac{\boldsymbol{l}_1^2}{\mu^2}\right)+\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\mu^2}\right)+\{1\leftrightarrow 2\}\Bigg\}-\frac{20}{3}\Bigg]-8C_f$$

$$+\frac{C_a}{2}\Bigg[\Bigg\{\frac{3}{2\boldsymbol{k}^2}\Bigg\{\boldsymbol{l}_1^2\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\boldsymbol{l}_1^2}\right)+(\boldsymbol{l}_1-\boldsymbol{k})^2\ln\left(\frac{\boldsymbol{l}_1^2}{(\boldsymbol{l}_1-\boldsymbol{k})^2}\right)-4|\boldsymbol{l}_1||\boldsymbol{l}_1-\boldsymbol{k}|\phi_1\sin\phi_1\Bigg\}$$

$$-\frac{3}{2}\left[\ln\left(\frac{\boldsymbol{l}_1^2}{\boldsymbol{k}^2}\right)+\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\boldsymbol{k}^2}\right)\right]-\ln\left(\frac{\boldsymbol{l}_1^2}{\boldsymbol{k}^2}\right)\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{s_0}\right)-\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\boldsymbol{k}^2}\right)\ln\left(\frac{\boldsymbol{l}_1^2}{s_0}\right)-2\phi_1^2+\{1\leftrightarrow 2\}\Bigg\}+2\pi^2+\frac{14}{3}\Bigg]\Bigg]$$

$$+\int_{z_0}^1 dz\,\Bigg\{\ln\frac{\lambda^2}{\mu_F^2}S_J^{(2)}(\boldsymbol{k},zx)\left[P_{qq}(z)+\frac{C_a^2}{C_f^2}P_{gq}(z)\right]+\left[(1-z)\left[1-\frac{2}{z}\frac{C_a^2}{C_f^2}\right]+2(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+\right]S_J^{(2)}(\boldsymbol{k},zx)+4S_J^{(2)}(\boldsymbol{k},x)\Bigg\}$$

$$+\int_0^1 dz\int\frac{d^2\boldsymbol{q}}{\pi}\Bigg[P_{qq}(z)\Theta\left(\hat{M}^2_{X,\max}-\frac{(\boldsymbol{p}-z\boldsymbol{k})^2}{z(1-z)}\right)\Theta\left(\frac{|\boldsymbol{q}|}{1-z}-\lambda\right)$$

$$\times\frac{\boldsymbol{k}^2}{\boldsymbol{q}^2(\boldsymbol{p}-z\boldsymbol{k})^2}S_J^{(3)}(\boldsymbol{p},\boldsymbol{q},(1-z)x,x)+\Theta\left(\hat{M}^2_{X,\max}-\frac{\boldsymbol{\Delta}^2}{z(1-z)}\right)S_J^{(3)}(\boldsymbol{p},\boldsymbol{q},zx,x)P_{gq}(z)$$

$$\times\left\{\frac{C_a}{C_f}[J_1(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_1,z)+J_1(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_2,z)]+\frac{C_a^2}{C_f^2}J_2(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_1,\boldsymbol{l}_2)\Theta(\boldsymbol{p}^2-\lambda^2)\right\}\Bigg]\Bigg]$$

NLO impact factors

In general the cross section for these processes is given as a multiple convolution between the the jet vertices and the GGFs.

$$\frac{d\hat{\sigma}}{d\mathcal{J}_1 d\mathcal{J}_2 d^2\mathbf{q}} = \int d^2\mathbf{k}_1 d^2\mathbf{k}'_1 d^2\mathbf{k}_2 d^2\mathbf{k}'_2 V_a(\mathbf{k}_1, \mathbf{k}_2, \mathcal{J}_1, \mathbf{q}) \times$$
$$G(\mathbf{k}_1, \mathbf{k}'_1, \mathbf{q}, Y) G(\mathbf{k}_2, \mathbf{k}'_2, \mathbf{q}, Y) V_b(\mathbf{k}'_1, \mathbf{k}'_2, \mathcal{J}_2, \mathbf{q}), \qquad \mathcal{J} = \{\mathbf{k}_{\mathcal{J}}, x_{\mathcal{J}}\}.$$

Jet Functions for NLO impact factor

$$J_1(q,k,l,z) = \frac{1}{2}\frac{k^2}{(q-k)^2}\left(\frac{(1-z)^2}{(q-zk)^2} - \frac{1}{q^2}\right) - \frac{1}{4}\frac{1}{(q-l)^2}\left(\frac{(l-z\cdot k)^2}{(q-zk)^2} - \frac{l^2}{q^2}\right)$$
$$- \frac{1}{4}\frac{1}{(q-k+l)^2}\left(\frac{(l-(1-z)k)^2}{(q-zk)^2} - \frac{(l-k)^2}{q^2}\right);$$

$$J_2(q,k,l_1,l_2) = \frac{1}{4}\Bigg[\frac{l_1^2}{(q-k)^2(q-k+l_1)^2} + \frac{(k-l_1)^2}{(q-k)^2(q-l_1)^2}$$
$$+ \frac{l_2^2}{(q-k)^2(q-k+l_2)^2} + \frac{(k-l_2)^2}{(q-k)^2(q-l_2)^2} - \frac{1}{2}\bigg(\frac{(l_1-l_2)^2}{(q-l_1)^2(q-l_2)^2}$$
$$+ \frac{(k-l_1-l_2)^2}{(q-k+l_1)^2(q-l_2)^2} + \frac{(k-l_1-l_2)^2}{(q-k+l_2)^2(q-l_1)^2} + \frac{(l_1-l_2)^2}{(q-k+l_1)^2(q-k+l_2)^2}\bigg)\Bigg].$$

LL approximation: Non forward gluon Green function

The GGF is given by the Mellin transform of the function f_ω which is the solution of the BFKL equation. The solution of the non forward BFKL equation is more naturally expressed in the impact parameter space.

$$G(\mathbf{k},\mathbf{k}',\mathbf{q},Y) = \int_{-i\inf}^{+i\inf} \frac{d\omega}{2\pi i} e^{Y\omega} f_\omega(\mathbf{k},\mathbf{k}',\mathbf{q})$$

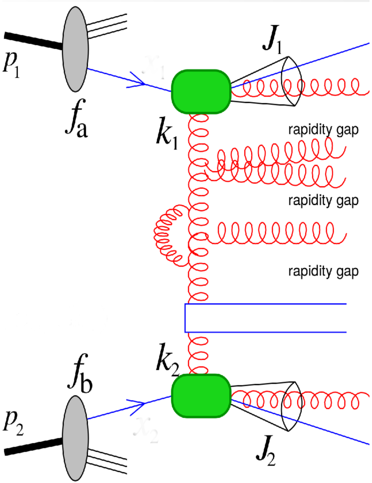
$$f_\omega(\rho_1,\rho_2,\rho_1',\rho_2') = \frac{1}{(2\pi)^6} \sum_{n=-\inf}^{+\inf} \int_{-\inf}^{+inf} d\nu \frac{R_{n\nu}}{\omega-\omega(n,\nu)} E^*_{n\nu}(\rho_1',\rho_2') E_{n\nu}(\rho_1,\rho_2)$$

$$E_{n\nu}(\rho_1,\rho_2) = \underbrace{\left(\frac{\rho_1-\rho_2}{\rho_1\rho_2}\right)^h \left(\frac{\rho_1^*-\rho_2^*}{\rho_1^*\rho_2^*}\right)^{\bar h}}_{\text{Lipatov term}} \underbrace{-\left(\frac{1}{\rho_2}\right)^h \left(\frac{1}{\rho_2^*}\right)^{\bar h} - \left(\frac{-1}{\rho_1}\right)^h \left(\frac{-1}{\rho_1^*}\right)^{\bar h}}_{\text{Mueller-Tang correction}}$$

$E_{n\nu}$ are the eigenfunctions in the impact parameter space.
The GGF in momentum space is recovered applying a Fourier transformation to the eigenfunctions.

$$\tilde E_{n\nu}(\mathbf{k},\mathbf{q}) = \int \frac{d^2 r_1 d^2 r_2}{(2\pi)^4} E_{n\nu}(\rho_1,\rho_2) e^{i(\mathbf{k}\cdot\mathbf{r}_1+(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}_2)}$$

Mueller Navelet jets at NLL



At NLL the approximation is refined including the terms $\propto \alpha_s^n \log^{(n-1)}(\frac{s}{-t})$.

- Larger variety of Feynman diagrams give rise to a much more complex iterating structure
- LL order diagrams evaluated in a broader kinematic domain
  Up to two partons are close in rapidity (Quasi-MRK).

$$y'_1 \gg y_1 \gg \cdots \gg y_i \simeq y_{i+1} \gg \cdots \gg y_n \gg y'_2$$

The jet vertex gets its part of the radiative corrections

$$V(\mathbf{k}_J, x_j, \mathbf{k}) = V^{(0)}(\mathbf{k}_J, x_j, \mathbf{k}) + \alpha_s V^{(1)}(\mathbf{k}_J, x_j, \mathbf{k})$$

- NL corrections to the jet vertex calculated by Bartels, Colferai and Vacca (BCV).
- QMRK $\rightarrow$ up to **two** outgoing parton per vertex