

Computing one-loop corrections to quarkonium production impact-factors with Lipatov's EFT

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I. Quarkonium production at high energy

In collaboration with Jean-Philippe Lansberg and Melih Ozcelik.
Based on [JHEP 05 \(2022\) 083](#); [hep-ph/2306.02425](#) and ongoing work

Perturbative instability of quarkonium total cross sections

Inclusive η_c -hadroproduction (CSM)

[Mangano *et al.*, '97, ..., Lansberg, Ozcelik, '20]

$$p+p \rightarrow c\bar{c} \left[{}^1S_0^{[1]} \right] + X, \text{ LO: } g(p_1)+g(p_2) \rightarrow c\bar{c} \left[{}^1S_0^{[1]} \right],$$

$$\sigma(\sqrt{s_{pp}}) = f_i(x_1, \mu_F) \otimes f_j(x_2, \mu_F) \otimes \hat{\sigma}(z),$$

where $z = \frac{M^2}{\hat{s}}$ with $\hat{s} = (p_1 + p_2)^2$.

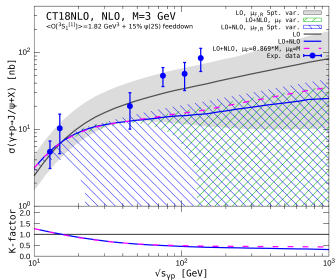
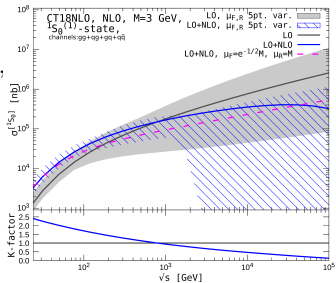
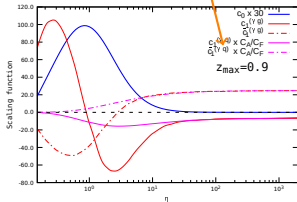
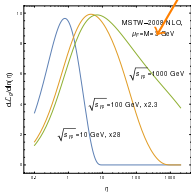
Inclusive J/ψ -photoproduction (CSM)

[Krämer, '96, ..., Colpani Serri *et al.*, '21]

$$\gamma+p \rightarrow c\bar{c} \left[{}^3S_1^{[1]} \right] + X, \text{ LO: } \gamma(q)+g(p_1) \rightarrow c\bar{c} \left[{}^3S_1^{[1]} \right] + g,$$

$$\sigma(\sqrt{s_{\gamma p}}) = f_i(x_1, \mu_F) \otimes \hat{\sigma}(\eta),$$

where $\eta = \frac{\hat{s}-M^2}{M^2}$ with $\hat{s} = (q + p_1)^2$, $z = \frac{pP}{qP}$.



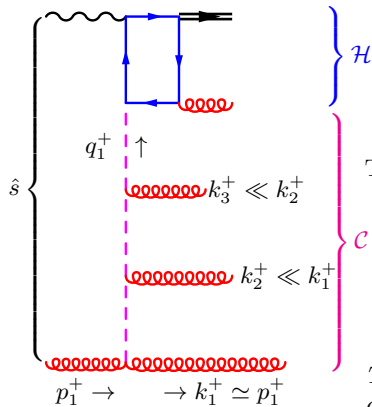
High-Energy Factorization (J/ψ photoproduction)

The **LLA** ($\sum_n \alpha_s^n \ln^{n-1}$) formalism [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann,

'91, '94]

Physical picture in the **LLA** for photoproduction:

The LLA in $\ln \frac{1}{\xi} = \ln \frac{p_1^+}{q_1^+} \sim \ln(1 + \eta)$:



$$\hat{\sigma}_{\text{HEF}}^{\ln(1/\xi)}(\eta) \propto \int_{1/z}^{1+\eta} \frac{dy}{y} \int_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{y}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \mathcal{H}(y, \mathbf{q}_{T1}^2),$$

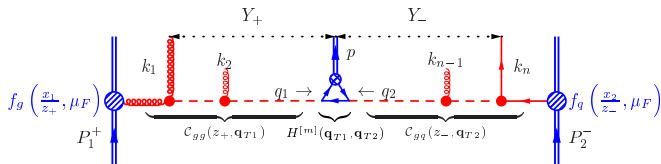
The **strict LLA** in $\ln(1 + \eta) = \ln \frac{\hat{s}}{M^2}$:

$$\hat{\sigma}_{\text{HEF}}^{\ln(1+\eta)}(\eta) \propto \int_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{1}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \int_{1/z}^\infty \frac{dy}{y} \mathcal{H}(y, \mathbf{q}_{T1}^2).$$

The LLA ($\ln(1/\xi)$) contains some ($N..$)NLLA contributions relative to the LLA ($\ln(1 + \eta)$).

The coefficient function \mathcal{H} has been calculated at LO [Kniehl, Vasin, Saleev, '06] and decreases as $1/y^2$ for $y \gg 1$.

High-Energy Factorization (η_c hadroproduction)



Small parameter: $z = \frac{M^2}{\hat{s}}$, LLA in $\alpha_s^n \ln^{n-1} \frac{1}{z}$:

$$\hat{\sigma}_{ij}^{[m], \text{HEF}}(z, \mu_F, \mu_R) = \int_{-\infty}^{\infty} d\eta \int_0^{\infty} d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2 C_{gi} \left(\frac{M_T}{M} \sqrt{z} e^\eta, \mathbf{q}_{T1}^2, \mu_F, \mu_R \right) \\ \times C_{gj} \left(\frac{M_T}{M} \sqrt{z} e^{-\eta}, \mathbf{q}_{T2}^2, \mu_F, \mu_R \right) \int_0^{2\pi} \frac{d\phi}{2} \frac{H^{[m]}(\mathbf{q}_{T1}^2, \mathbf{q}_{T2}^2, \phi)}{M_T^4}$$

The coefficient functions $H^{[m]}$ are known at LO in α_s [Hagler *et.al.*, 2000; Kniehl, Vasin, Saleev 2006] for $m = {}^1S_0^{(1,8)}, {}^3P_J^{(1,8)}, {}^3S_1^{(8)}$.

The $H^{[m]}$ is a tree-level “squared matrix element” of the $2 \rightarrow 1$ -type process:

$$R_+(\mathbf{q}_{T1}, q_1^+) + R_-(\mathbf{q}_{T2}, q_2^-) \rightarrow c\bar{c}[m].$$

LLA evolution w.r.t. $\ln 1/\xi$

In the LL($\ln 1/\xi$)-approximation, the $Y = \ln 1/\xi$ -evolution equation for *collinearly un-subtracted* $\tilde{\mathcal{C}}$ -factor has the form:

$$\tilde{\mathcal{C}}(\xi, \mathbf{q}_T) = \delta(1 - \xi)\delta(\mathbf{q}_T^2) + \hat{\alpha}_s \int_{\xi}^1 \frac{dz}{z} \int d^{2-2\epsilon} \mathbf{k}_T K(\mathbf{k}_T^2, \mathbf{q}_T^2) \tilde{\mathcal{C}}\left(\frac{\xi}{z}, \mathbf{q}_T - \mathbf{k}_T\right)$$

with $\hat{\alpha}_s = \alpha_s C_A / \pi$ and

$$K(\mathbf{k}_T^2, \mathbf{q}_T^2) = \frac{1}{\pi(2\pi)^{-2\epsilon} \mathbf{k}_T^2} + \delta^{(2-2\epsilon)}(\mathbf{k}_T) 2\omega_g(\mathbf{q}_T^2),$$

where $\omega_g(\mathbf{q}_T^2)$ – 1-loop Regge trajectory of a gluon. It is convenient to go from (z, \mathbf{q}_T) -space to (N, \mathbf{x}_T) -space:

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = \int d^{2-2\epsilon} \mathbf{q}_T e^{i\mathbf{x}_T \mathbf{q}_T} \int_0^1 dx x^{N-1} \tilde{\mathcal{C}}(x, \mathbf{q}_T),$$

because:

▶ Mellin convolutions over z turn into products: $\int \frac{dz}{z} \rightarrow \frac{1}{N}$

▶ Large logs map to poles at $N = 0$: $\alpha_s^{k+1} \ln^k \frac{1}{\xi} \rightarrow \frac{\alpha_s^{k+1}}{N^{k+1}}$

▶ All *collinear divergences* are contained inside \mathcal{C} in \mathbf{x}_T -space.

Exact LL solution and the DLA

In (N, \mathbf{q}_T) -space, subtracted \mathcal{C} , which resums all terms $\propto (\hat{\alpha}_s/N)^n$ (complete LLA) has the form [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann, '91, '94]:

$$\mathcal{C}(N, \mathbf{q}_T, \mu_F) = R(\gamma_{gg}(N, \alpha_s)) \frac{\gamma_{gg}(N, \alpha_s)}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_{gg}(N, \alpha_s)},$$

where $\gamma_{gg}(N, \alpha_s)$ is the solution of [Jaroszewicz, '82]:

$$\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}(N, \alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma),$$

where $\psi(\gamma) = d \ln \Gamma(\gamma) / d\gamma$ - Euler's ψ -function. The first few terms:

$$\gamma_{gg}(N, \alpha_s) = \underbrace{\frac{\hat{\alpha}_s}{N}}_{\text{DLA [Blümlein, '95]}} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \dots$$

LLA

$$\frac{\hat{\alpha}_s}{N} \leftrightarrow P_{gg}(z \rightarrow 0) = \frac{2CA}{z} + \dots$$

The function $R(\gamma)$ is

$$R(\gamma_{gg}(N, \alpha_s)) = 1 + O(\alpha_s^3).$$

Fixed-order asymptotics from HEF

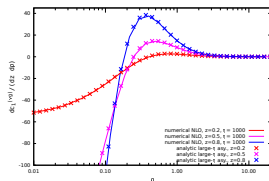
When expanded up to $O(\alpha_s)$ the HEF resummation should predict the $\hat{s} \gg M^2$ asymptotics of the CF coefficient function $\hat{\sigma}$

For the $g + g \rightarrow c\bar{c} [^1S_0^{(1)}, ^3P_0^{(1)}, ^3P_2^{(1)}]$ the NLO and NNLO ($\alpha_s^2 \ln(1/z)$) terms in $\hat{\sigma}$ are predicted [M.N., Lansberg, Ozcelik '22]:

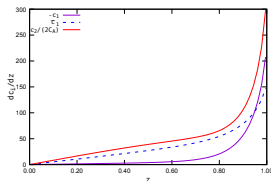
State	$A_0^{[m]}$	$A_1^{[m]}$	$A_2^{[m]}$	$B_2^{[m]}$
1S_0	1	-1	$\frac{\pi^2}{6}$	$\frac{\pi^2}{6}$
3S_1	0	1	0	$\frac{\pi^2}{6}$
3P_0	1	$-\frac{43}{27}$	$\frac{\pi^2}{6} + \frac{2}{3}$	$\frac{\pi^2}{6} + \frac{40}{27}$
3P_1	0	$\frac{5}{54}$	$-\frac{1}{9}$	$-\frac{2}{9}$
3P_2	1	$-\frac{53}{36}$	$\frac{\pi^2}{6} + \frac{1}{2}$	$\frac{\pi^2}{6} + \frac{11}{9}$

$$\hat{\sigma}_{gg}^{[m]}(z \rightarrow 0) = \sigma_{\text{LO}}^{[m]} \left\{ A_0^{[m]} \delta(1-z) + \frac{\alpha_s}{\pi} 2C_A \left[A_1^{[m]} + A_0^{[m]} \ln \frac{M^2}{\mu_F^2} \right] + \left(\frac{\alpha_s}{\pi} \right)^2 \ln \frac{1}{z} \cdot C_A^2 \left[2A_2^{[m]} + B_2^{[m]} \right] + 4A_1^{[m]} \ln \frac{M^2}{\mu_F^2} + 2A_0^{[m]} \ln^2 \frac{M^2}{\mu_F^2} \right\} + O(\alpha_s^3)$$

For the $\gamma + g \rightarrow c\bar{c} [^3S_1^{(1)}] + g$ we have computed $\eta \rightarrow \infty$ limit of the z and $\rho = \mathbf{p}_T^2/M^2$ -differential NLO “scaling functions” in closed analytic form,



and obtained numerical results for NNLO “scaling function” c_2 in front of $\alpha_s \ln(1+\eta)$.



Inverse Error Weighting (InEW) matching

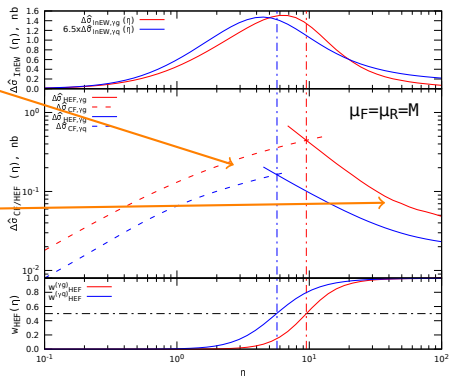
Development of an idea from [Echevarria *et al.*, 18'] :

$$\hat{\sigma}(\eta) = w_{\text{CF}}(\eta)\hat{\sigma}_{\text{CF}}(\eta) + (1 - w_{\text{CF}}(\eta))\hat{\sigma}_{\text{HEF}}(\eta),$$

the weights are determined through the estimates of “errors”:

$$w_{\text{CF}}(\eta) = \frac{\Delta\hat{\sigma}_{\text{CF}}^{-2}(\eta)}{\Delta\hat{\sigma}_{\text{CF}}^{-2}(\eta) + \Delta\hat{\sigma}_{\text{HEF}}^{-2}(\eta)}, \quad w_{\text{HEF}}(\eta) = 1 - w_{\text{CF}}(\eta).$$

- ▶ $\Delta\hat{\sigma}_{\text{CF}}(\eta)$ is due to **missing higher orders and large logarithms**, it can be estimated from the α_s expansion of $\hat{\sigma}_{\text{HEF}}(\eta)$.
- ▶ $\Delta\hat{\sigma}_{\text{HEF}}(\eta)$ is (mostly) due to **missing power corrections in $1/\eta$** : $\Delta\hat{\sigma}_{\text{HEF}}(\eta) \sim A\eta^{-\alpha_{\text{HEF}}}$. We determine A and α_{HEF} from behaviour of $\hat{\sigma}_{\text{CF}}(\eta) - \hat{\sigma}_{\text{CF}}(\infty)$ at $\eta \gg 1$.

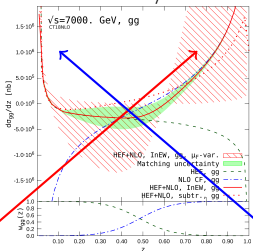


Matching with NLO

The HEF is valid in the **leading-power** in M^2/\hat{s} , so for $\hat{s} \sim M^2$ we match it with NLO CF by the *Inverse-Error Weighting Method* [Echevarria et al., 18'].

η_c -hadroproduction,

$$z = M^2/\hat{s}:$$

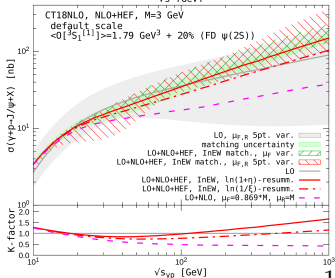
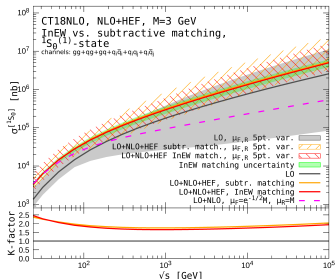
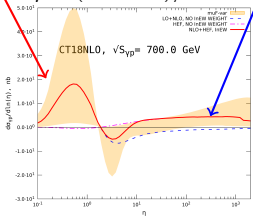


NLO

HEF

J/ψ -photoproduction,

$$\eta = (\hat{s} - M^2)/M^2:$$

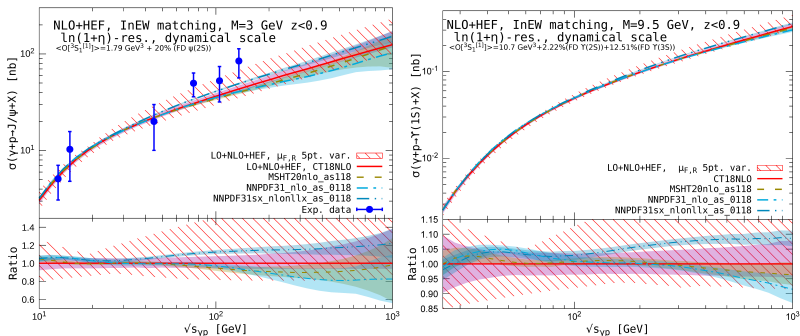


Vector quarkonium photoproduction: dynamical scale

Matched results for J/ψ photoproduction can be further improved by noticing that in the LO process:

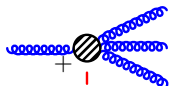
$$\gamma(q) + g(p_1) \rightarrow Q\bar{Q} \left[{}^3S_1^{[1]} \right] + g,$$

the emitted gluon can not be soft, so that $\langle \hat{s} \rangle_{\text{LO}}$ ($\sim 25 \text{ GeV}^2$ at high $\sqrt{s_{\gamma p}}$ for J/ψ) rather than M^2 can be taken as a default value of μ_F^2 and μ_R^2 :



II. Beyond DLA: one-loop corrections to quarkonium impact-factors

The Gauge-Invariant EFT for Multi-Regge processes in QCD



- ▶ Reggeized gluon fields R_{\pm} carry $(k_{\pm}, \mathbf{k}_T, k_{\mp} = 0)$: $\partial_{\mp} R_{\pm} = 0$.

- ▶ **Induced interactions** of particles and Reggeons [Lipatov '95, '97; Bondarenko, Zubkov '18]:

$$L = \frac{i}{g_s} \text{tr} \left[R_+ \partial_{\perp}^2 \partial_- \left(W[A_-] - W^{\dagger}[A_-] \right) + (+ \leftrightarrow -) \right],$$

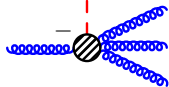
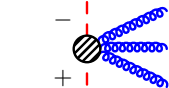
$$\text{with } W_{x_{\mp}}[x_{\pm}, \mathbf{x}_T, A_{\pm}] = P \exp \left[\frac{-ig_s}{2} \int_{-\infty}^{x_{\mp}} dx'_{\mp} A_{\pm}(x_{\pm}, x'_{\mp}, \mathbf{x}_T) \right] = (1 + ig_s \partial_{\pm}^{-1} A_{\pm})^{-1}.$$

- ▶ Expansion of the Wilson line generates **induced vertices**:

$$\text{tr} \left[R_+ \partial_{\perp}^2 A_- + (-ig_s) (\partial_{\perp}^2 R_+) (A_- \partial_-^{-1} A_-) + (-ig_s)^2 (\partial_{\perp}^2 R_+) (A_- \partial_-^{-1} A_- \partial_-^{-1} A_-) + O(g_s^3) + (+ \leftrightarrow -) \right].$$

- ▶ The *Eikonal propagators* $\partial_{\pm}^{-1} \rightarrow -i/(k^{\pm})$ lead to **rapidity divergences**, which are regularized by tilting the Wilson lines from the light-cone [Hentschinski, Sabio Vera, Chachamis *et. al.*, '12-'13; M.N. '19]:

$$n_{\pm}^{\mu} \rightarrow \tilde{n}_{\pm}^{\mu} = n_{\pm}^{\mu} + r n_{\mp}^{\mu}, \quad r \ll 1 : \tilde{k}^{\pm} = \tilde{n}^{\pm} k.$$



Prescription for the Eikonal poles

The interpretation of Eikonal poles $\partial_{\pm}^{-1} \rightarrow -i/(k^{\pm})$ comes from the

Hermitian form of the Lagrangian [Lipatov '97; Bondarenko, Zubkov '18]:

$$iR_+ \partial_{\perp}^2 \partial_- (W[A_-] - W^\dagger[A_-]) / g_s.$$

- ▶ For the Rgg vertex leads to the PV-prescription for the pole:

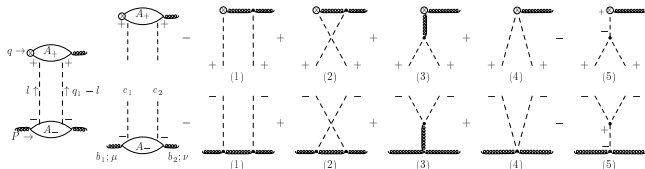
$$\frac{1}{[\tilde{k}^{\pm}]} = \frac{1}{2} \left(\frac{1}{\tilde{k}^{\pm} + i\epsilon} + \frac{1}{\tilde{k}^{\pm} - i\epsilon} \right),$$

for $Rggg \dots$ vertices – more complicated, mixes colour and kinematics.

- ▶ Enforces $(-)$ -signature of 1R exchange. Important for obtaining correct 2-loop Regge trajectory [Hentchinski, Sabio-vera, '13]

Two interpretations of $3R$ vertices: $R_{\pm} R_{\mp} R_{\mp}$

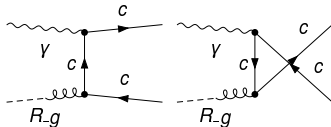
- ▶ As subtraction terms, leading to bootstrap [Hentchinski PhD-thesis; M.N. '19]:



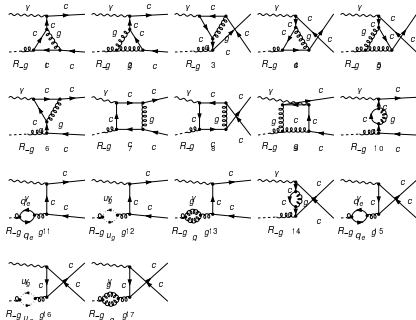
- ▶ If $\int \frac{dk^{\pm}}{[k^{\pm}]} = 0$, then $RRR, RRRRR, \dots$, vertices are **zero** (*Gribov's signature conservation rule*).

$$R\gamma \rightarrow c\bar{c} \left[{}^1S_0^{[8]} \right] @ 1 \text{ loop}$$

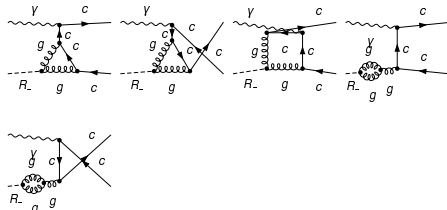
Interference with LO:



Rg -coupling diagrams:



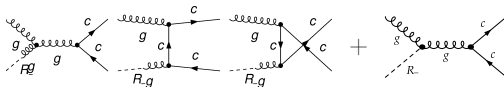
Induced Rgg coupling diagrams:



- ▶ Diagrams had been generated using custom `FeynArts` model-file, projector on the $c\bar{c} \left[{}^1S_0^{(8)} \right]$ -state is inserted
- ▶ heavy-quark momenta = $p_Q/2 \Rightarrow$ need to resolve linear dependence of quadratic denominators in some diagrams before IBP
- ▶ IBP reduction to master integrals has been performed using `FIRE`

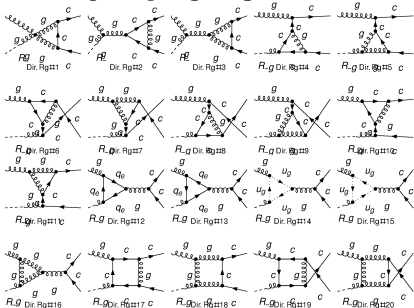
$Rg \rightarrow c\bar{c} [^1S_0^{[1]}]$ and $c\bar{c} [^3S_1^{[8]}]$ @ 1 loop

Interference with LO:

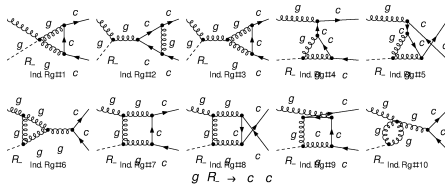


Induced Rg coupling diagrams:

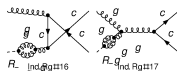
Some Rg -coupling diagrams:



$g R \rightarrow c c$



$g R \rightarrow c c$



and so on...

Rapidity divergences and regularization.

$$\begin{array}{c} p \downarrow \\ | \\ + \\ \text{---} \\ | \\ \text{---} \\ \text{---} \\ | \\ - \\ q \downarrow \end{array} = g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{(\mathbf{p}_T^2 (n_+ n_-))^2}{q^2 (p-q)^2 q^+ q^-}, \quad \int \frac{dq^+ dq^-}{q^+ q^-} = \int_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + \mathbf{q}_T^2}$$

the regularization by explicit cutoff in rapidity was originally proposed

[Lipatov, '95] ($q^\pm = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

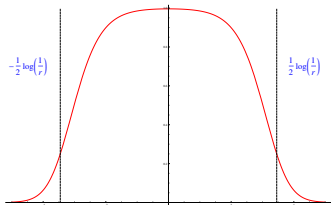
$$\delta_{ab} \mathbf{p}_T^2 \times C_A g_s^2 \underbrace{\int \frac{\mathbf{p}_T^2 d^{D-2} \mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

The square of regularized Lipatov vertex:

$$\Gamma_{+\mu} - \Gamma_{+\nu} - P^{\mu\nu} = \frac{16 \mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2}{\mathbf{k}_T^2} f(y),$$

$$\leftarrow f(y) = \frac{1}{(r e^{-y} + e^y)^2 (r e^y + e^{-y})^2},$$

$$\int_{-\infty}^{+\infty} dy f(y) = -1 - \log r + O(r)$$



Rapidity divergences at one loop

Only log-divergence $\sim \log r$ (Blue cells in the table) is related with Reggeization of particles in t -channel.

Integrals which **do not** have log-divergence may still contain the power-dependence on r :

- ▶ $r^{-\epsilon} \rightarrow 0$ for $r \rightarrow 0$ and $\epsilon < 0$.
- ▶ $r^{+\epsilon} \rightarrow \infty$ for $r \rightarrow 0$ and $\epsilon < 0$ – **weak-power divergence** (Pink cells in the table)
- ▶ $r^{-1+\epsilon} \rightarrow \infty$ – **power divergence.** (Red)

(# LC prop.) \ (# quadr. prop.)	1	2	3	4
1	$A_{[-]}$	$B_{[-]}$	$C_{[-]}$...
2	$A_{[+-]}$	$B_{[+-]}$	$C_{[+-]}$...
3

The **weak-power** and **power-divergences** cancel between Feynman diagrams describing one region in rapidity, so only log-divergences are left.

Scalar integrals with power RDs.

$$\text{Notation: } \left\{ \frac{\mu}{k} \right\}^{2\epsilon} = \frac{1}{2} \left[\left(\frac{\mu}{k-i\epsilon} \right)^{2\epsilon} + \left(\frac{\mu}{-k-i\epsilon} \right)^{2\epsilon} \right].$$

Tadpoles:

$$\begin{aligned} A_{[-]}(p) &= -\frac{\tilde{p}^- r^{-1+\epsilon}}{\cos(\pi\epsilon)} \frac{1}{2\epsilon(1-2\epsilon)} \left\{ \frac{\mu}{\tilde{p}^-} \right\}^{2\epsilon}, \\ A_{[- -]}(p) &= \frac{1}{\tilde{p}^-} A_{[-]}(p). \end{aligned}$$

Bubbles:

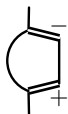
$$\begin{aligned} B_{[-]}(p) &= \frac{1}{p^- \epsilon^2} \left(\frac{\mu^2}{-p^2} \right)^\epsilon + \frac{1-2\epsilon}{\epsilon} \frac{r \cdot A_{[-]}(p)}{\tilde{p}_-^2} + \Delta B_{[-]}(-p^2, p_-) + O(r), \\ B_{[- -]}(p) &= \frac{2}{\tilde{p}_-} B_{[-]}(p), \end{aligned}$$

where:

$$\Delta B_{[-]}(-p^2, p_-) = -\frac{1}{p_-} \left(\frac{p_-^2 \mu^2}{(-p^2)^2} \right)^\epsilon \frac{\Gamma^2(1-2\epsilon)\Gamma(1+2\epsilon) \cdot r^{-\epsilon}}{2\epsilon^2 \Gamma^2(1-\epsilon)}.$$

Logarithmic RDs

- ▶ $[+-]$ -bubble in transverse kinematics $p^- = p^+ = 0$:



$$B_{[+-]}(\mathbf{p}_T) = \frac{1}{\mathbf{p}_T^2} \left(\frac{\mu^2}{\mathbf{p}_T^2} \right)^\epsilon \frac{i\pi + 2 \log r}{\epsilon},$$

- ▶ $[+-]$ -bubble in $p^- = 0$ kinematics:

$$\begin{aligned} B_{[+-]}(\mathbf{p}_T, p^+) &= \frac{1}{\mathbf{p}_T^2} \left(\frac{\mu^2}{\mathbf{p}_T^2} \right)^\epsilon \frac{\Gamma^2(1 + \epsilon)\Gamma(2 + \epsilon) \sin(\pi\epsilon)}{\pi\epsilon^2} \\ &\times \left[i\pi + \log r - \log \frac{p_+^2}{\mathbf{p}_T^2} - \psi(1 + \epsilon) + \psi(1) \right] + O(r^{1/2}) \end{aligned}$$

- ▶ $[+-]$ -bubble in light-like kinematics $p^2 = 0$:

$$B_{[+-]}(\mathbf{p}_T^2, p^2 = 0) = \int \frac{[d^d l]}{l^2(l+p)^2[l^+][l^- + p^-]} = \frac{-2\Gamma(1 - \epsilon)\Gamma(1 + \epsilon)}{\mathbf{p}_T^2 \epsilon^2} \left(\frac{\mu^2}{\mathbf{p}_T^2} \right)^\epsilon.$$

Triangle integrals, logarithmic RD

Result for $Q^2 = 0$:

$$C_{[-]}(t_1, 0, q^-) = \frac{1}{q^- t_1} \left(\frac{\mu^2}{t_1} \right)^\epsilon \frac{1}{\epsilon} \left[\log r + i\pi - \log \frac{|q^-|^2}{t_1} - \psi(1 + \epsilon) - \psi(1) + 2\psi(-\epsilon) \right] + O(r^{1/2}),$$

coincides with the result of [G. Chachamis, *et. al.*, '12] .

Result for $Q^2 \neq 0$ [M.N., '19] :

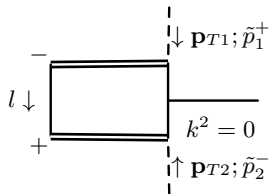
$$C_{[-]}(t_1, Q^2, q_-) = C_{[-]}(t_1, 0, q_-) + \left(\frac{\mu^2}{t_1} \right)^\epsilon \frac{I(Q^2/t_1)}{q_- t_1} - \frac{1}{t_1} \Delta B_{[-]}(Q^2, q_-),$$

where

$$\begin{aligned} I(X) &= -\frac{2X^{-\epsilon}}{\epsilon^2} - \frac{2}{\epsilon} \int_0^X \frac{(1-x^{-\epsilon})dx}{1-x} \\ &= -\frac{2X^{-\epsilon}}{\epsilon^2} + 2 \left[-\text{Li}_2(1-X) + \frac{\pi^2}{6} \right] + O(\epsilon). \end{aligned}$$

Triangle with two light-cone propagators

Usual one-loop Feynman integrals with more than 4 propagators are reducible to more simple integrals up to terms $O(\epsilon)$.



We apply method of [Bern, Dixon, Kosower, '92]. The $O(\epsilon)$ remnant is proportional to $(d-4)I^{(d+2)}$ and integral $I^{(6)}$ is finite.

The result in Euclidean region ($p_1^+ > 0$, $-p_2^- > 0$, $\mathbf{p}_{T1,2}^2 > 0$):

$$C_{[+-]}(\mathbf{p}_{T1}^2, \mathbf{p}_{T2}^2, p_1^+, -p_2^-) = \frac{(-1)}{2\mathbf{p}_{T1}^2 \mathbf{p}_{T2}^2 \mathbf{k}_T^2} \times$$

$$\{ \mathbf{p}_{T1}^2 (\mathbf{p}_{T2}^2 - \mathbf{p}_{T1}^2 + \mathbf{k}_T^2) [B_{[+-]}(\mathbf{p}_{T1}^2, p_1^+) + (-p_2^-) C_{[-]}(\mathbf{p}_{T1}^2, \mathbf{p}_{T2}^2, -p_2^-)]$$

$$+ \mathbf{p}_{T2}^2 (\mathbf{p}_{T1}^2 - \mathbf{p}_{T2}^2 + \mathbf{k}_T^2) [B_{[+-]}(\mathbf{p}_{T2}^2, -p_2^-) + p_1^+ C_{[+]}(\mathbf{p}_{T2}^2, \mathbf{p}_{T1}^2, p_1^+)]$$

$$- \mathbf{k}_T^2 (\mathbf{p}_{T1}^2 + \mathbf{p}_{T2}^2 - \mathbf{k}_T^2) B_{[+-]}(\mathbf{k}_T^2, k^2 = 0) \},$$

where $\mathbf{k}_T^2 = p_1^+ (-p_2^-)$.

The **log r**-divergence cancels within square brackets, as expected.

Integrals with massive internal lines

In presence of the linear denominator the massive propagator can be converted to the massless one:

$$\frac{1}{((\tilde{n}_+ l) + k_+)(l^2 - m^2)} = \frac{1}{((\tilde{n}_+ l) + k_+)(l + \kappa \tilde{n}_+)^2} + \frac{2\kappa \left[\cancel{((\tilde{n}_+ l) + k_+)} + \frac{m^2 + \tilde{n}_+^2 + \kappa^2}{2\kappa} \right]}{\cancel{((\tilde{n}_+ l) + k_+)}(l + \kappa \tilde{n}_+)^2(l^2 - m^2)}$$

\Rightarrow all the masses can be moved to integrals with **only quadratic propagators**. New **massless** scalar integrals with RDs arise ($k^2 = 0$, $p^2 = 4m^2$, $p = q + q_1$, $q^2 = 0$):

$$\begin{aligned} B_{[+]}(-k, k - q) &= \int \frac{d^D l}{[\tilde{l}^+](l - k)^2(l + k - q)^2}, \\ C_{[+]}(0, -k, k - q) &= \int \frac{d^D l}{[\tilde{l}^+]l^2(l - k)^2(l + k - q)^2}, \\ B_{[+]}(p, k) &= \int \frac{d^D l}{[\tilde{l}^+](l + p)^2(l + k)^2}, \\ C_{[+]}(p, k, q_1) &= \int \frac{d^D l}{[\tilde{l}^+](l + p)^2(l + k)^2(l + q_1)^2}, \end{aligned}$$

but they have the same complexity as already encountered ones.

Results: $R\gamma \rightarrow c\bar{c} [^1S_0^{[8]}]$ vs. $Rg \rightarrow c\bar{c} [^1S_0^{[1]}]$ @ 1 loop



Results for $2\Re \left[\frac{H_{1L} \times \text{LO}(\mathbf{q}_T) - (\text{On-shell mass CT})}{(\alpha_s/(2\pi))H_{LO}(\mathbf{q}_T)} \right]$:

$$^1S_0^{[8]} : \left(\frac{\mu^2}{\mathbf{q}_T^2} \right)^\epsilon \frac{1}{\epsilon} \left[N_c \left(\ln \frac{\mathbf{q}_T^2}{M^2} + \ln \frac{q_-^2}{\mathbf{q}_T^2 r} + \frac{19}{6} \right) - \frac{2n_F}{3} - \frac{3}{2N_c} \right] + F_{1S_0^{[8]}}(\mathbf{q}_T^2/M^2)$$

$$^1S_0^{[1]} : \left(\frac{\mu^2}{\mathbf{q}_T^2} \right)^\epsilon \left\{ -\frac{N_c}{\epsilon^2} + \frac{1}{\epsilon} \left[N_c \left(\ln \frac{q_-^2}{\mathbf{q}_T^2 r} + \frac{25}{6} \right) - \frac{2n_F}{3} - \frac{3}{2N_c} \right] \right\} + F_{1S_0^{[1]}}(\mathbf{q}_T^2/M^2)$$

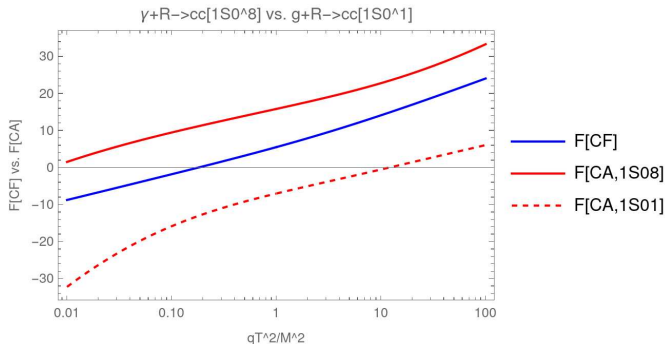
$$F_{1S_0^{[1]}}(\tau) = -\frac{10}{9}n_F + \Re[C_F F_{1S_0^{[1]}}^{(C_F)}(\tau) + C_A F_{1S_0^{[1]}}^{(C_A)}(\tau)],$$

$$F_{1S_0^{[1]}}^{(C_F)}(\tau) = F_{1S_0^{[8]}}^{(C_F)}(\tau),$$

while $F_{1S_0^{[1]}}^{(C_A)}(\tau) \neq F_{1S_0^{[8]}}^{(C_A)}(\tau)$.

The C_F coefficient

$$\begin{aligned}
 F_{1S_0^{[8]}}^{(C_F)}(\tau) &= \frac{\mathcal{L}_2 + \mathcal{L}_\tau(1 - 2\tau)}{\tau + 1} \\
 &+ \frac{1}{6(\tau + 1)(2\tau + 1)^2} \{144L_1\tau^2 + 144L_1\tau + 36L_1 - 16\pi^2\tau^3 - 72\tau^3 + 72\tau^3 \log(2) \\
 &- 156\tau^2 + 12\tau^2 \log^2(2\tau + 1) + 168\tau^2 \log(2) - 24(3\tau^2 + 5\tau + 2)\tau \log(\tau + 1) \\
 &+ 12\pi^2\tau - 108\tau + 12\tau \log^2(2\tau + 1) + 3\log^2(2\tau + 1) + 132\tau \log(2) \\
 &+ 18(\tau + 1)(2\tau + 1)^2 \log(\tau) + 4\pi^2 - 24 + 36 \log(2)\}
 \end{aligned}$$



The C_A coefficient for $R\gamma \rightarrow c\bar{c} \left[{}^1S_0^{[8]} \right]$

$$\begin{aligned}
F_{1S_0^{[8]}}^{(C_A)}(\tau) &= \frac{1}{2(\tau-1)(\tau+1)^3} \{ (\tau+1)^2 (-4\mathcal{L}_4(\tau^2-1) + \mathcal{L}_2(\tau+1)(2\tau+1) + \mathcal{L}_7\tau(2\tau-3) + \mathcal{L}_7) \\
&\quad + 2\mathcal{L}_6(\tau(\tau((\tau-4)\tau-6)-4)+1) \} \\
+ &\frac{1}{36(\tau-1)(\tau+1)^3(2\tau+1)} \{ -216L_1\tau^4 - 324L_1\tau^3 + 108L_1\tau^2 + 324L_1\tau + 108L_1 \\
&\quad + 120\pi^2\tau^5 + 608\tau^5 - 36\tau^5 \log^2(\tau+1) + 36\tau^5 \log^2(2\tau+1) - 36\tau^5 \log^2(2) \\
&\quad - 72\tau^5 \log(2) \log(\tau+1) + 216\tau^5 \log(\tau+1) + 72\tau^5 \log(2) + 228\pi^2\tau^4 + 1520\tau^4 \\
&\quad - 306\tau^4 \log^2(\tau+1) + 144\tau^4 \log^2(2\tau+1) - 306\tau^4 \log^2(2) \\
&\quad + 252\tau^4 \log(2) \log(\tau+1) + 432\tau^4 \log(\tau+1) + 360\tau^4 \log(2) + 84\pi^2\tau^3 + 608\tau^3 \\
&\quad - 360\tau^3 \log^2(\tau+1) + 225\tau^3 \log^2(2\tau+1) - 360\tau^3 \log^2(2) + 576\tau^3 \log(2) \log(\tau+1) \\
&\quad + 72\tau^3 \log(\tau+1) + 72\tau^3 \log(2) - 120\pi^2\tau^2 - 1216\tau^2 - 108\tau^2 \log^2(\tau+1) \\
&\quad + 171\tau^2 \log^2(2\tau+1) - 108\tau^2 \log^2(2) + 504\tau^2 \log(2) \log(\tau+1) - 360\tau^2 \log(\tau+1) \\
&\quad - 360\tau^2 \log(2) - 72(\tau+1)^3 (2\tau^2 - \tau - 1) \log(\tau-1) (\log(2) - \log(\tau+1)) \\
&\quad + 36(2\tau+1) \log(\tau) [-\tau^4 + \tau^4 \log(8) - 6\tau^2 \log(2) + (-\tau^3 + 4\tau^2 + 6\tau + 4) \tau \log(\tau+1) \\
&\quad - 8\tau \log(2) - \log(2\tau+2) + 1] - 18 (2\tau^5 + 17\tau^4 + 20\tau^3 + 6\tau^2 - 6\tau - 3) \log^2(\tau) \\
&\quad - 84\pi^2\tau - 1216\tau + 108\tau \log^2(\tau+1) + 63\tau \log^2(2\tau+1) + 108\tau \log^2(2) \\
&\quad + 54 \log^2(\tau+1) + 9 \log^2(2\tau+1) + 72\tau \log(2) \log(\tau+1) - 288\tau \log(\tau+1) \\
&\quad - 144\tau \log(2) - 36 \log(2) \log(\tau+1) - 72 \log(\tau+1) - 12\pi^2 - 304 + 54 \log^2(2) \}
\end{aligned}$$

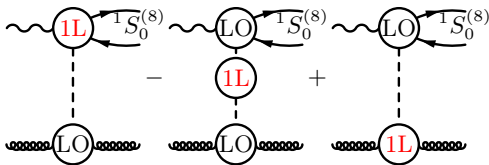
The C_A coefficient for $Rg \rightarrow c\bar{c} \left[{}^1S_0^{[1]} \right]$

$$\begin{aligned}
F_{1S_0^{[1]}}^{(C_A)}(\tau) &= \frac{1}{(\tau-1)(\tau+1)^3} \{ 2\mathcal{L}_1(\tau^2 + \tau - 2)(\tau+1)^3 + \tau [2\mathcal{L}_5(\tau(\tau+1)(\tau^2-2) + 1) \\
&\quad - \mathcal{L}_7(\tau^2 + \tau - 1) - (\mathcal{L}_2(\tau+2)(\tau+1)^2) + \mathcal{L}_6(\tau(\tau(6 - (\tau-4)\tau) + 4) - 1)] \\
&\quad + 2\mathcal{L}_3(\tau-1)(\tau+1)^3 + 2\mathcal{L}_5 + \mathcal{L}_7 \} \\
- &\frac{1}{18(\tau-1)(\tau+1)^3} \{ 6\pi^2\tau^5 - 36\tau^5 \log(2) \log(\tau+1) + 36\tau^5 \log(\tau+1) \log(\tau+2) + 63\pi^2\tau^4 \\
&\quad - 98\tau^4 - 63\tau^4 \log^2(\tau+1) + 9\tau^4 \log^2(2\tau+1) - 63\tau^4 \log^2(2) + 54\tau^4 \log(2) \log(\tau+1) \\
&\quad - 36\tau^4 \log(\tau+1) + 36\tau^4 \log(\tau+1) \log(\tau+2) + 36\tau^4 \log(2) + 138\pi^2\tau^3 - 196\tau^3 \\
&\quad - 72\tau^3 \log^2(\tau+1) + 36\tau^3 \log^2(2\tau+1) - 72\tau^3 \log^2(2) + 144\tau^3 \log(2) \log(\tau+1) \\
&\quad - 36\tau^3 \log(\tau+1) - 72\tau^3 \log(\tau+1) \log(\tau+2) - 36\tau^3 \log(2) + 18\pi^2\tau^2 \\
&\quad - 18\tau^2 \log^2(\tau+1) + 45\tau^2 \log^2(2\tau+1) - 18\tau^2 \log^2(2) + 108\tau^2 \log(2) \log(\tau+1) \\
&\quad + 36\tau^2 \log(\tau+1) - 72\tau^2 \log(\tau+1) \log(\tau+2) - 36\tau^2 \log(2) \\
&\quad - 18(4\tau^4 + 5\tau^3 + \tau^2 - 3\tau - 1) \log^2(\tau) + 18 \log(\tau) [\tau^5 \log(2) - \tau^4(\log(4) - 2) \\
&\quad - \tau^3 \log(4) - 2\tau^2(1 + \log(4)) - (\tau^4 - 4\tau^3 - 6\tau^2 - 4\tau + 1) \tau \log(\tau+1) - \tau \log(8) - \log(4)] \\
&\quad - 120\pi^2\tau + 196\tau + 36\tau \log^2(\tau+1) + 18\tau \log^2(2\tau+1) + 36\tau \log^2(2) + 9 \log^2(\tau+1) \\
&\quad - 36\tau \log(2) \log(\tau+1) + 36\tau \log(\tau+1) + 36\tau \log(\tau+1) \log(\tau+2) + 36\tau \log(2) \\
&\quad - 36(\tau-1)(\tau+1)^3 \log(\tau-1)(\log(2) - \log(\tau+1)) - 18 \log(2) \log(\tau+1) \\
&\quad + 36 \log(\tau+1) \log(\tau+2) - 69\pi^2 + 98 + 9 \log^2(2) \}
\end{aligned}$$

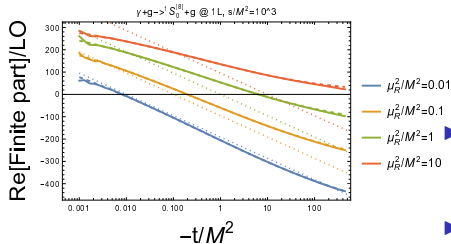
$$\begin{aligned}
L_1 &= \sqrt{\tau(1+\tau)} \ln \left[1 + 2\tau + 2\sqrt{\tau(1+\tau)} \right], \\
\mathcal{L}_1 &= \text{Li}_2 \left(\frac{1}{\tau} + 1 \right) \\
\mathcal{L}_2 &= \text{Li}_2 \left(\frac{1}{-2\tau - 1} \right) \\
\mathcal{L}_3 &= \text{Li}_2 \left(\frac{1}{\tau} \right) + \text{Li}_2 \left(\frac{\tau - 1}{\tau + 1} \right) - \text{Li}_2 \left(\frac{\tau + 1}{2\tau} \right) + \frac{\text{Li}_2 \left(\frac{1}{4} \right)}{2} + \text{Li}_2(-2) \\
\mathcal{L}_4 &= \text{Li}_2 \left(1 + \frac{1}{\tau} \right) + \text{Li}_2 \left(\frac{1}{\tau} \right) + \text{Li}_2 \left(\frac{\tau - 1}{\tau + 1} \right) - \text{Li}_2 \left(\frac{\tau + 1}{2\tau} \right) + \frac{\text{Li}_2 \left(\frac{1}{4} \right)}{2} + \text{Li}_2(-2) \\
\mathcal{L}_5 &= \text{Li}_2 \left(-\frac{1}{\tau + 1} \right) - \text{Li}_2(\tau + 2) + \frac{1}{2} \text{Li}_2 \left(\frac{2\tau + 1}{2\tau + 2} \right) \\
\mathcal{L}_6 &= -\text{Li}_2 \left(-\frac{2\tau + 1}{\tau^2} \right) + \text{Li}_2 \left(-\frac{-2\tau^2 + \tau + 1}{2\tau^2} \right) + \text{Li}_2 \left(\frac{1}{2} - \frac{\tau}{2} \right) + \text{Li}_2 \left(-\frac{1}{\tau} \right) \\
&\quad - \text{Li}_2 \left(\frac{\tau - 1}{2\tau} \right) - \text{Li}_2(-\tau) + \text{Li}_2 \left(\frac{1 - \tau}{\tau + 1} \right) \\
\mathcal{L}_7 &= \text{Li}_2(-2\tau - 1) - \text{Li}_2 \left(\frac{2\sqrt{\tau}}{\sqrt{\tau} - \sqrt{\tau + 1}} \right) - \text{Li}_2 \left(\frac{2\sqrt{\tau}}{\sqrt{\tau} + \sqrt{\tau + 1}} \right)
\end{aligned}$$

$R\gamma \rightarrow c\bar{c} \left[{}^1S_0^{(8)} \right]$ @ 1 loop, cross-check

In the combination of 1-loop results in the EFT:



the $\ln r$ cancels and it should reproduce the the Regge limit ($s \gg -t$) of the real part of the $2 \rightarrow 2$ 1-loop QCD amplitude:



Solid lines – QCD, dashed lines – EFT, dotted

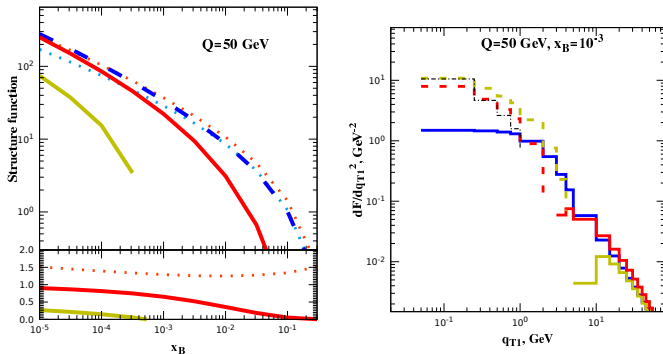
lines – $-2C_A \ln(-t/\mu_R^2) \ln(s/M^2)$

$$\gamma + g \rightarrow c\bar{c} \left[{}^1S_0^{(8)} \right] + g.$$

- ▶ The $2 \rightarrow 2$ QCD 1-loop amplitude can be computed numerically using **FormCalc** (with some tricks, due to Coulomb divergence)
- ▶ The Regge limit of $1/\epsilon$ divergent part agrees with the EFT result
- ▶ For the finite part agreement within few % is reached, need to push to higher s

“Monster logs” in the coefficient function

In standard k_T -factorisation (or CGC/saturation) computations, where the $\sigma = \Phi(x, \mathbf{q}_T^2) \otimes \mathcal{H}(y, \mathbf{q}_T^2)$ the appearance of $\alpha_s \ln^{1,2}(\mathbf{q}_T^2/\mu^2)$ for $\mathbf{q}_T^2 \ll \mu^2$ at NLO for \mathcal{H} is a serious problem. Example, Higgs-DIS in the $m_t \rightarrow \infty$ limit [M.N. '20]:



(look at yellow curves – standard MRK computation, red curves – computation with modified-MRK \simeq “kinematic constraint”)

$$\mathbf{q}_T^2 \ll Q^2 : \mathcal{H}(y, \mathbf{q}_T^2) \sim -\frac{\alpha_s C_A}{2\pi} \ln^2 \frac{\mathbf{q}_T^2}{Q^2} + (\text{single-log terms}).$$

The “Monster logs” at small \mathbf{q}_T are not scary for the matching computation

$$\hat{\sigma}_{\text{HEF}}(\eta) \propto \int_0^{1+\eta} \frac{dy}{y} \int_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C} \left(\frac{y}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R \right) \mathcal{H}(y, \mathbf{q}_{T1}^2).$$

At NLO for \mathcal{H} one typically encounters corrections $\propto \alpha_s \ln^n \frac{M^2}{\mathbf{q}_T^2}$ at $\mathbf{q}_T^2 \ll M^2$ with $n = 1, 2$. Let's study their effect in N -space (note that $\gamma_N = \hat{\alpha}_s/N$):

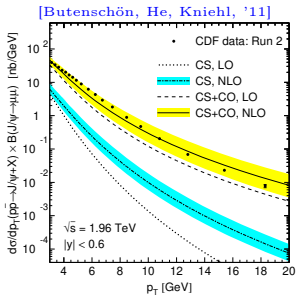
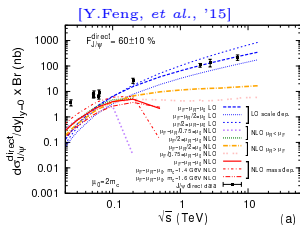
$$\begin{aligned} & \int_0^{\mu_F^2} d\mathbf{q}_T^2 \mathcal{C}_{\text{DLA}}(N, \mathbf{q}_T^2, \mu_F^2) \times \hat{\alpha}_s \ln^n \frac{\mu_F^2}{\mathbf{q}_T^2} = \hat{\alpha}_s \gamma_N \int_0^{\mu_F^2} \frac{d\mathbf{q}_T^2}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_N} \ln^n \frac{\mu_F^2}{\mathbf{q}_T^2} \\ & = \hat{\alpha}_s \frac{(-1)^n n!}{\gamma_N^n} = \begin{cases} -N & \text{for } n = 1 \\ \frac{2N^2}{\hat{\alpha}_s} & \text{for } n = 2 \end{cases} \xrightarrow{\text{Mellin transform}} \begin{cases} -\delta'(\eta) & \text{for } n = 1 \\ \frac{2}{\hat{\alpha}_s} \delta''(\eta) & \text{for } n = 2 \end{cases} \end{aligned}$$

So these contributions *do not belong to NLA in $\eta = (\hat{s} - M^2)/M^2 \gg 1$ and will be removed by the matching!*

Conclusions and outlook

- ▶ The perturbative instability of p_T -integrated quarkonium production cross sections at NLO comes from the region $\hat{s} \gg M^2$
- ▶ The problem can be solved via matching of NLO calculation at $\hat{s} \sim M^2$ and LLA HEF calculation at $\hat{s} \gg M^2$
- ▶ The LLA HEF has to be truncated down to DLA for resummation factors, to be consistent with NLO DGLAP evolution
- ▶ The inclusive η_c hadroproduction and J/ψ photoproduction have been considered as examples
- ▶ The *next-to-DLA* calculation is needed to further reduce scale-uncertainties. Both virtual and real corrections to HEF coefficient function can be computed within the *High-Energy EFT* formalism
- ▶ **The virtual corrections to $\gamma R \rightarrow c\bar{c}[^1S0^{[8]}]$ and $gR \rightarrow c\bar{c}[^1S0^{[1]}]$ IFs has been computed**
- ▶ The logarithms $\ln M^2/q_T^2$ for $q_T^2 \ll M^2$ in the NLO HEF coefficient function will not be a problem for the matching calculation!

There is a lot to do even at DLA+NLO!



Mini workshop on overlap between QCD resummations

- ▶ 3 day mini-workshop (14–17 Jan. 2024) in “Centre Paul Langevin” in Aussois (France), right after “**Quarkonia as tools 2024**”
- ▶ Indico:
<https://indico.cern.ch/event/1290502/>
- ▶ maxim DOT nefedov AT desy DOT de



Topics:

- ▶ Overlap between small- x and TMD resummations, rapidity divergences in both cases
- ▶ Problem of collinear/running coupling logarithms in BFKL
- ▶ BFKL beyond NLL approximation in $\mathcal{N} = 4$ SYM and QCD
- ▶ Threshold resummation
- ▶ From small to moderate x and back

Thank you for your attention!

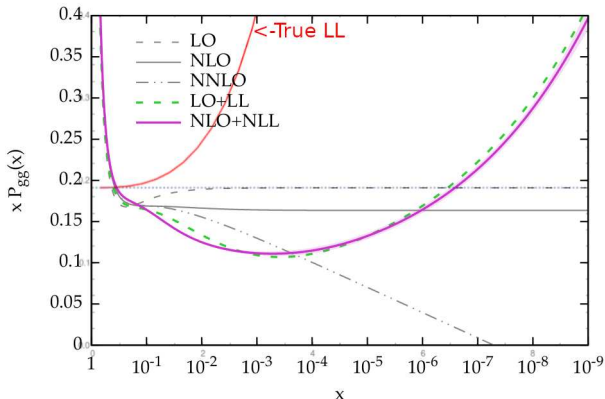
Backup: DGLAP P_{gg} at small z

$$\text{LO: } P_{gg}(z) = \frac{2CA}{z} + \dots \Leftrightarrow \gamma_N = \frac{\hat{\alpha}_s}{N}$$

Plot from [hep-ph/1607.02153](https://arxiv.org/abs/hep-ph/1607.02153) with my curve (in red) for the **strict LLA**:

$$\frac{\hat{\alpha}_s}{N} \chi_{LO}(\gamma_{gg}(N)) = 1 \Rightarrow \gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \dots$$

$$\alpha_s = 0.2, n_f = 4, Q_0 \overline{\text{MS}}$$



The “LO+LL” and “NLO+NLL” curves represent a form of matching between DGLAP and BFKL expansions, in a scheme by [Altarelli, Ball and Forte](#) which is more complicated than the **strict LL or NLL approximation**.

Effect of anomalous dimension beyond LO

Effect of taking **full LLA** for $\gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \dots$
together with NLO PDF.

