

# From factorization to proton tomography: direct determination of the rapidity anomalous dimension.



Low-x Workshop 2023

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Based on the recent work:

Phys.Rev.D 106 (2022) 9, L091501

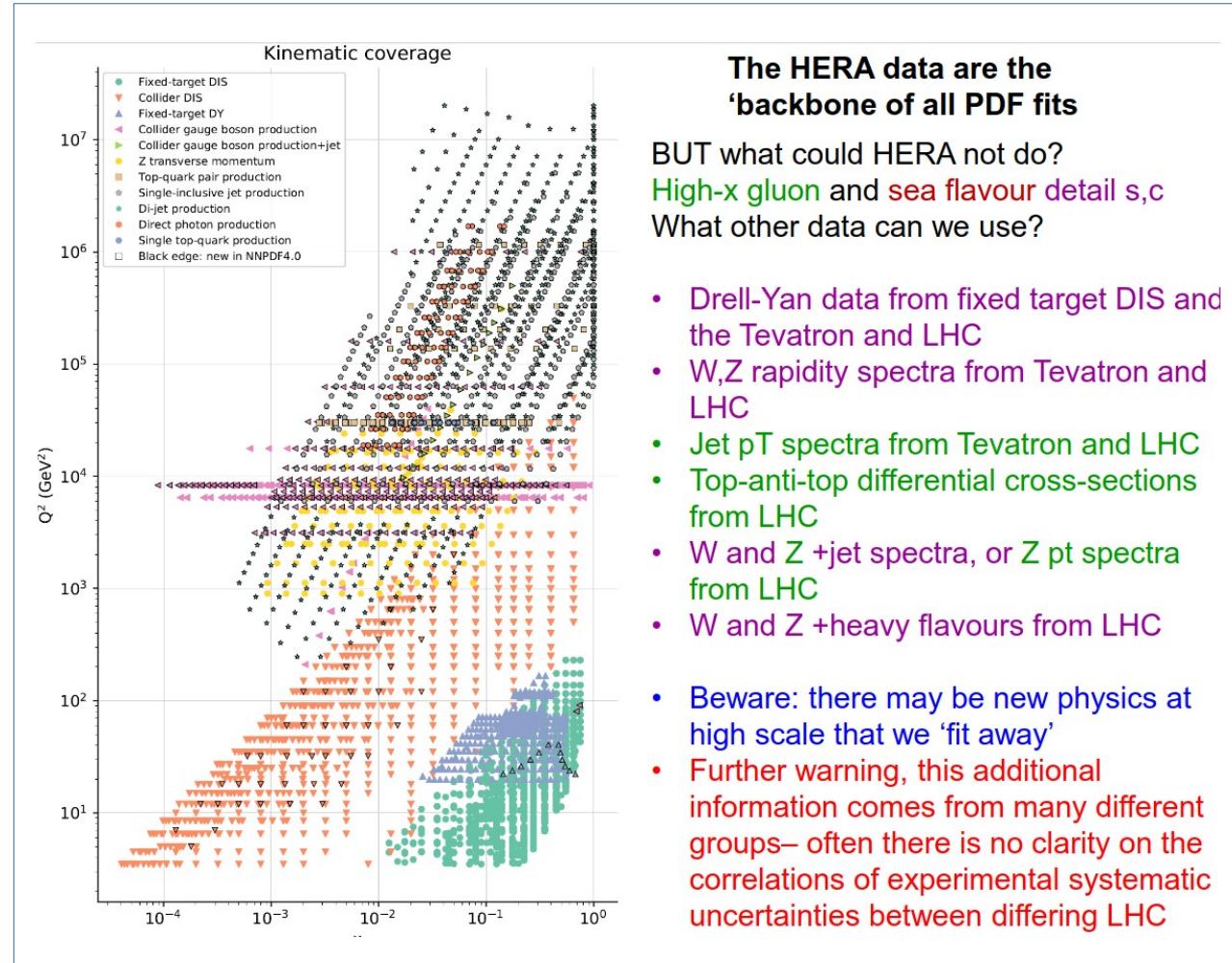
arXiv:2307.06704

<https://www.desy.de/f/students/2022/reports/David.Gutierrez.pdf>

# Motivation

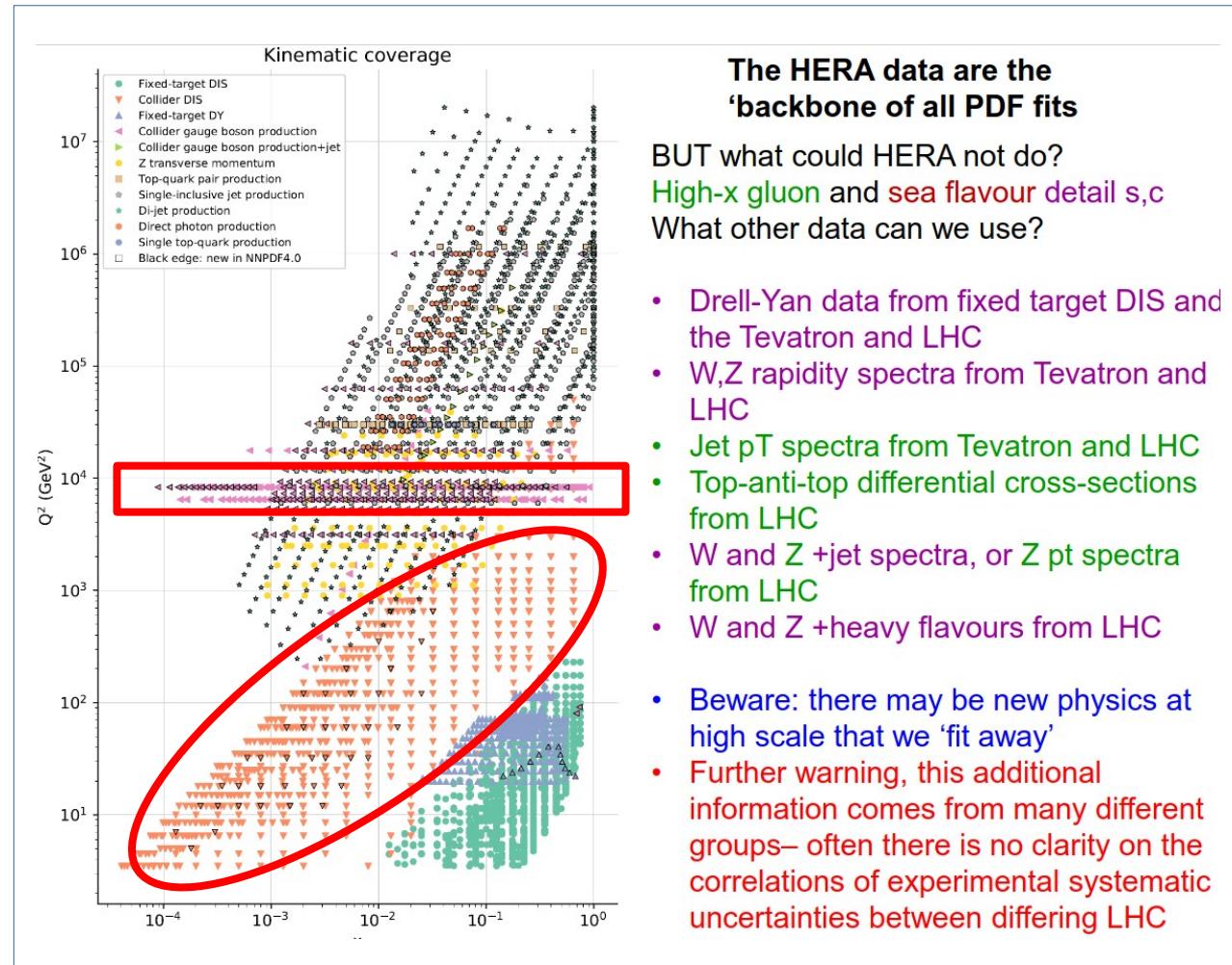
# Motivation: factorization

recall Amanda's talk



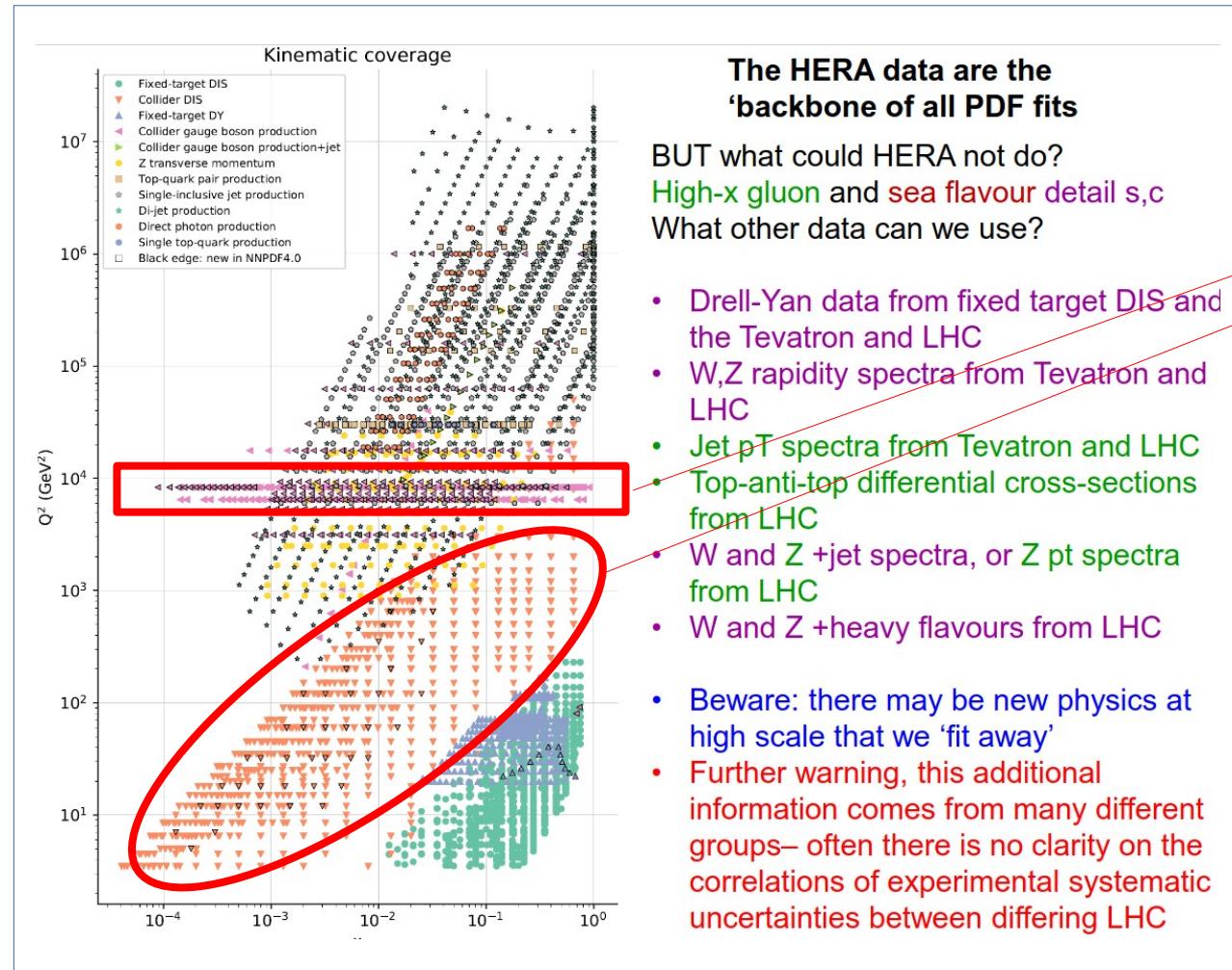
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recall Amanda's talk



Proven factorization for sufficiently inclusive observables

This is a very small fraction of observables we measure

Up to what extend is this correct?

Are we fitting away breaking effects, NP?

## Motivation: factorization

Imagine the case in which factorization dictates:

$$\sigma_1 = H_1 \times \text{PDF}(x \text{ range}; \mu_1)$$

$$\sigma_2 = H_2 \times \text{PDF}(x \text{ range}; \mu_2)$$

And now assume it also dictates:

$$\text{PDF}(x \text{ range}; \mu_2) = \text{PDF}(x \text{ range}; \mu_1) \times R$$

It would meant we could construct the observable:

$$R \propto \sigma_2 / \sigma_1$$

- ▶ R would depend neither on the scale choices, x range, PDF, nor on the process → clear way of testing factorization

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RAD hidden in here

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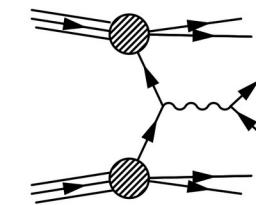
- ▶ R would depend neither on the scale choices, x range, PDF, nor on the process → clear way of testing factorization

**This case scenario exists in Drell Yan, at low transverse momentum**

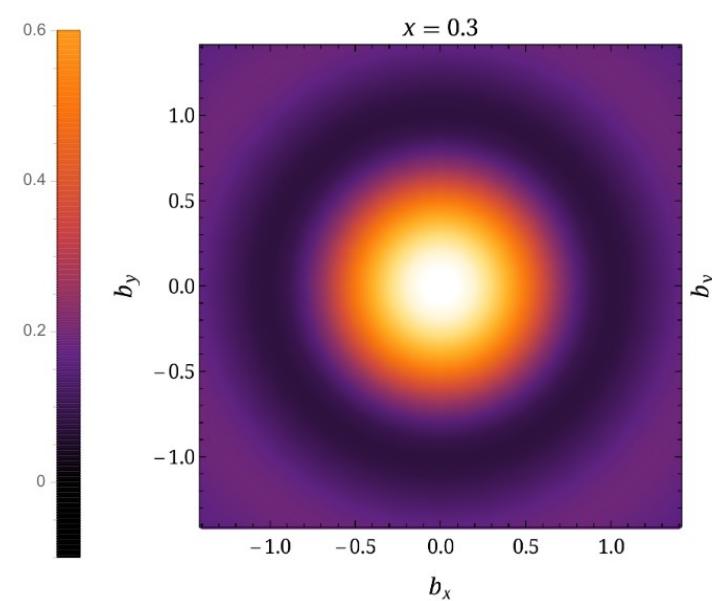
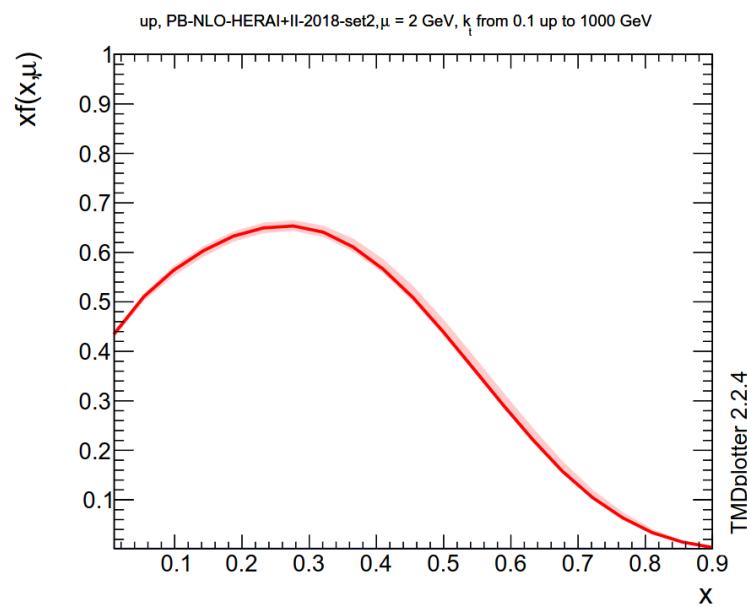
# Motivation: nucleon tomography

- ▶ Modern factorization theorems separate the hadron structure from the low distance hard scattering

$$\frac{d\sigma}{dp_T} = \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{i(b p_T)} C\left(\frac{Q}{\mu}\right) F_1(x_1, b; \mu, \zeta) F_2(x_2, b; \mu, \zeta)$$



- ▶ More complete description, going **beyond the simplest 1D parton structure**



# Motivation: nucleon tomography

- ▶ A new function emerges and it **dictates the evolution of the parton distributions**:

$$\frac{df_{q,h}(x, b; \mu, \zeta)}{d \ln \mu^2} = \frac{\gamma_F(\mu, \zeta)}{2} f_{q,h}(x, b; \mu, \zeta),$$

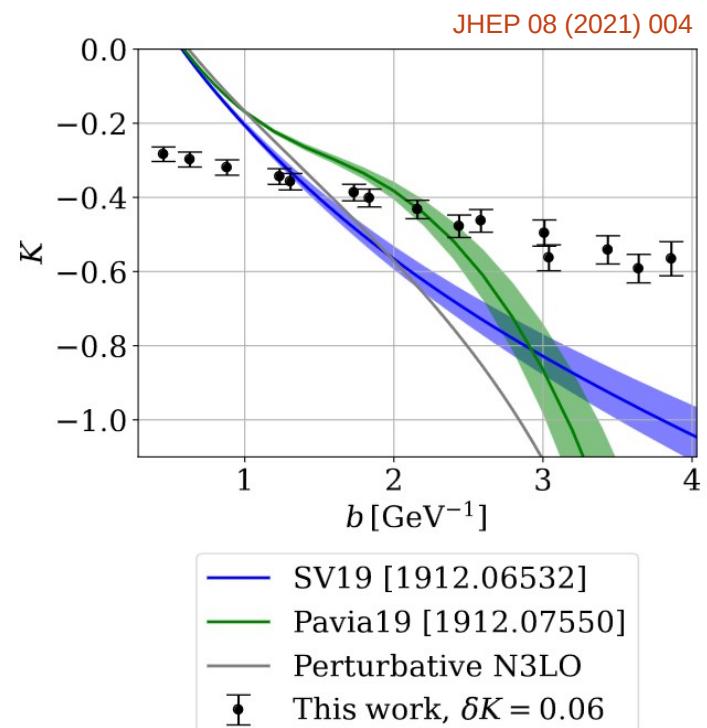
$$\frac{df_{q,h}(x, b; \mu, \zeta)}{d \ln \zeta} = -\boxed{\mathcal{D}(b, \mu)} f_{q,h}(x, b; \mu, \zeta).$$

- ▶ It is a **self-contained object with new non-perturbative information**

Phys. Rev. Lett. 125, 192002 (2020)

- ▶ Exclusively sensitive to **QCD vacuum**

- ▶ RAD has been **studied extensively**
- ▶ Yet, only QCD function which is **largely unknown**



# Motivation: nucleon tomography

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- ▶ It is a **self-contained object with new non-perturbative information**

Phys. Rev. Lett. 125, 192002 (2020)

- ▶ Exclusively sensitive to **QCD vacuum**
- ▶ **A direct measurement of RAD would imply:**
  - Stringent test of **factorization** and **universality** of the TMDs
  - **Higher precision imaging** of hadrons
  - Higher precision for measurements, e.g **W mass**
  - Input to probe parton **spin-orbit correlations**
  - information on **confinement** and **hadronization**

# Novel method to determine RAD

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- ▶ Start from the factorization formula:

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{2\pi}{9} \frac{\alpha_{\text{em}}^2(Q)}{s Q^2} W_{f_1 f_2}(x_1, x_2, Q, q_T),$$

- ▶ Apply the inverse Hankel transform:

$$\Sigma(y, Q; b) = \frac{2\pi}{9} \frac{\alpha_{\text{em}}^2(Q)}{s Q^2} |C_V(Q)|^2 \sum_q e_q^2 f_{q_1}(x_1, b; Q, Q^2) f_{q_2}(x_2, b; Q, Q^2)$$

- ▶ Evolve the parton distribution to a reference scale:

$$\Sigma(y, Q; b) = \frac{2\pi}{9} \frac{\alpha_{\text{em}}^2(Q)}{s Q^2} |C_V(Q)|^2 e^{2\Delta(b; Q \rightarrow (\mu_0, \zeta_0))} \sum_q e_q^2 f_{q_1}(x_1, b; \mu_0, \zeta_0) f_{q_2}(x_2, b; \mu_0, \zeta_0)$$

the target

disposable

- ▶ Build ratios of the cross sections at different scales:

$$\frac{\Sigma_1}{\Sigma_2} = \frac{\frac{\alpha_{\text{em}}^2(Q_1)}{s_1 Q_1^2} |C_V(Q_1)|^2 e^{2\Delta(b; Q_1 \rightarrow (\mu_0, \zeta_0))}}{\frac{\alpha_{\text{em}}^2(Q_2)}{s_2 Q_2^2} |C_V(Q_2)|^2 e^{2\Delta(b; Q_2 \rightarrow (\mu_0, \zeta_0))}} \frac{\sum_q e_q^2 \cancel{f_{q_1}(x_1, b; \mu_0, \zeta_0)} \cancel{f_{q_2}(x_2, b; \mu_0, \zeta_0)}}{\sum_q e_q^2 \cancel{f_{q_1}(x_1, b; \mu_0, \zeta_0)} \cancel{f_{q_2}(x_2, b; \mu_0, \zeta_0)}}$$

# Novel method to determine RAD

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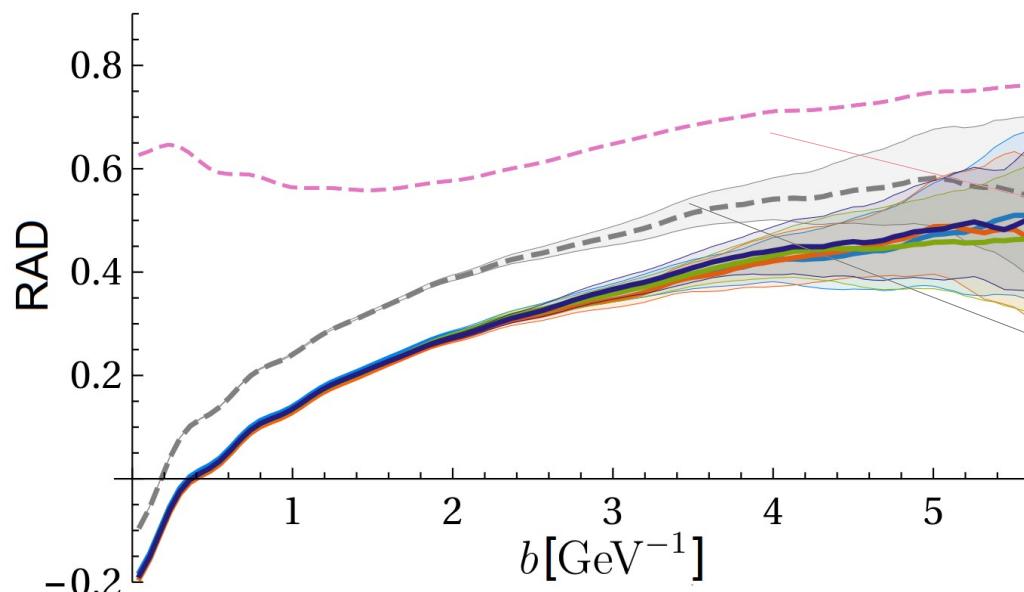
- We get the master formula:

$$\mathcal{D}(b, \mu_0) = \frac{\ln\left(\frac{\Sigma_1}{\Sigma_2}\right) - \ln Z(Q_1, Q_2) - 2\Delta_R(Q_1, Q_2; \mu_0)}{4 \ln(Q_2/Q_1)} - 1$$

**measurement**                                   **perturbative terms**

- Things to remember:

- No dependence on the chosen scales
- No dependence on process
- Cancellation of the longitudinal part



Clear test of  
factorization premise

Different x range

Different PDF

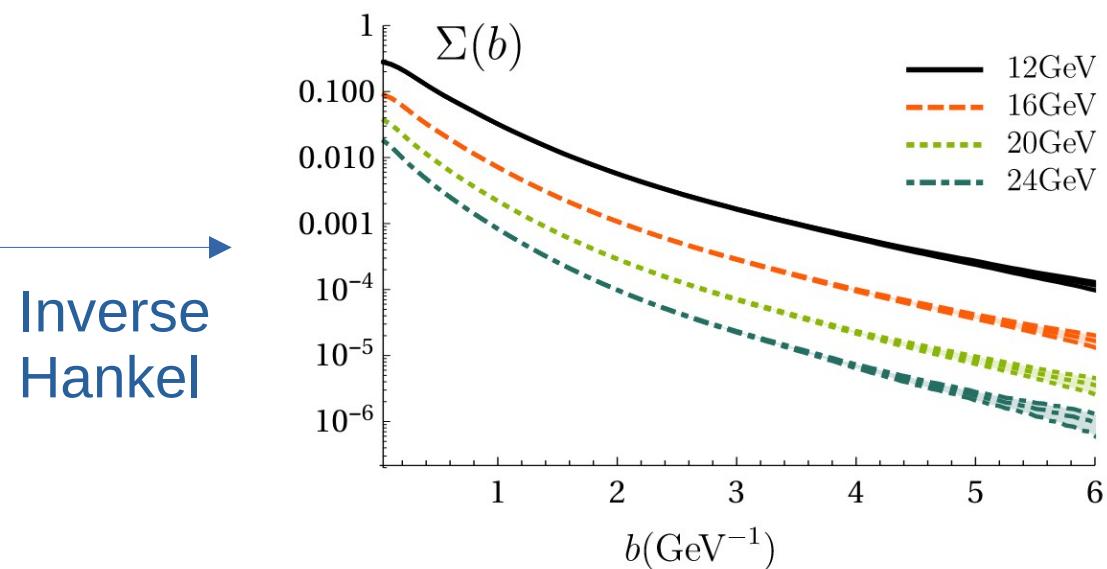
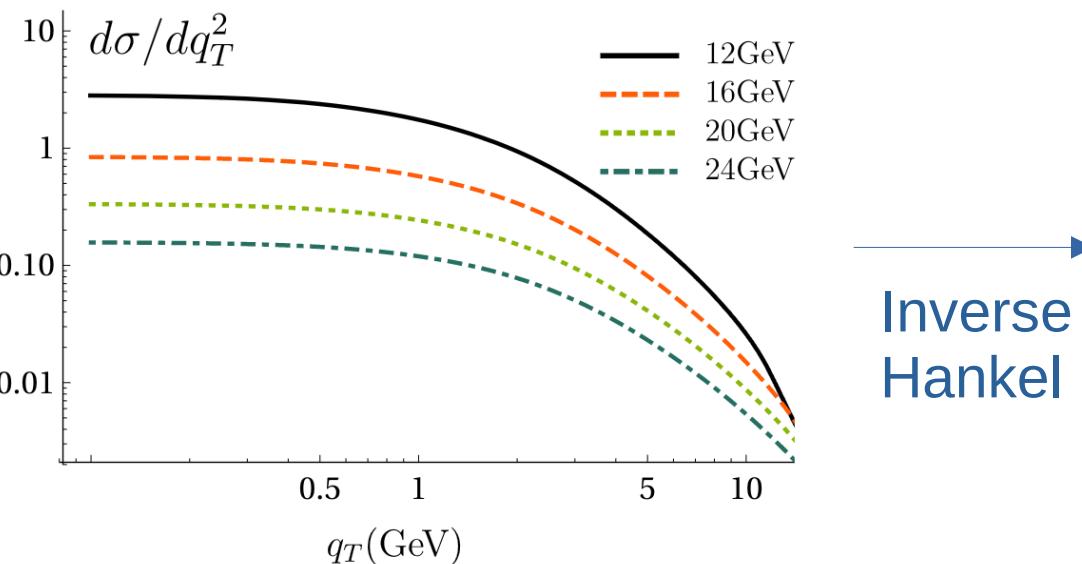
# Applying the method to simulated data

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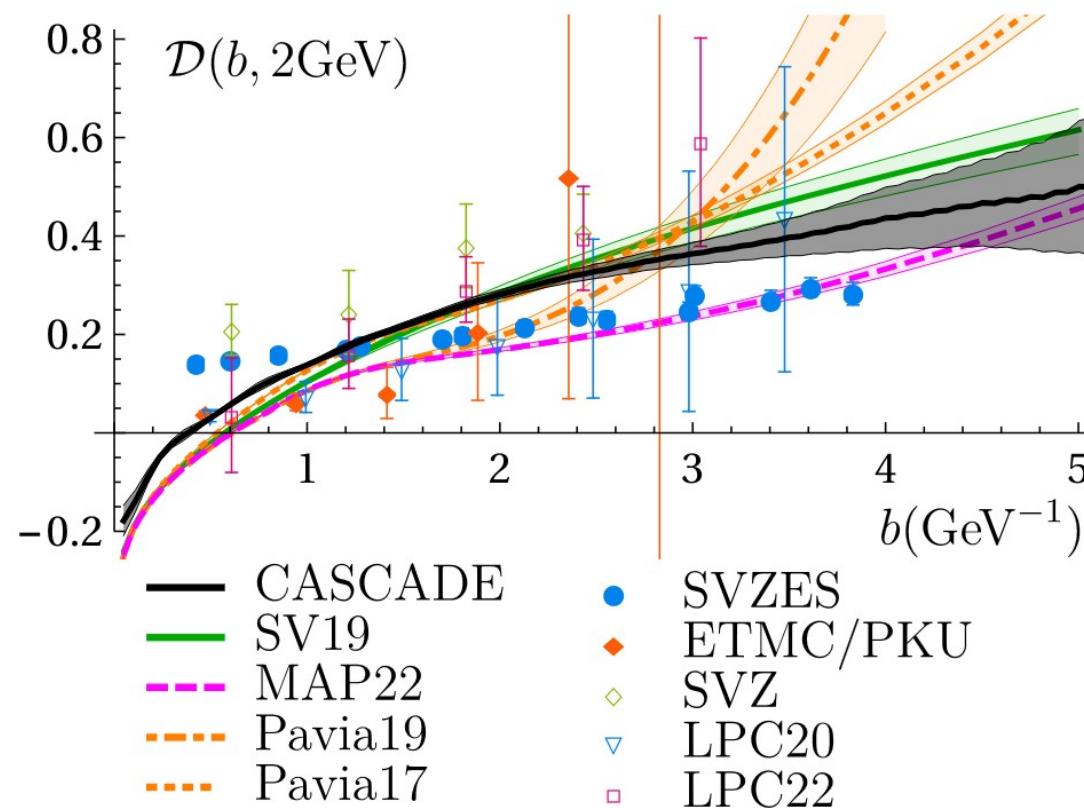
- ▶ Master formula can be used with data, provided:
  - small  $q_T$  and  $Q$  bin sizes
  - choices of  $y$ ,  $Q$  and center-of-mass energy ensure same  $x$  range
  - $Q$  below  $Z$  peak
- ▶ Simulation using the CASCADE MC generator:



# Applying the method to simulated data

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- ▶ All properties of RAD, like universality, are observed for the PB approach
- ▶ This non-trivially supports both factorization and PB approaches, sets the stage for a comparison
- ▶ The method can be applied to the experimental data!



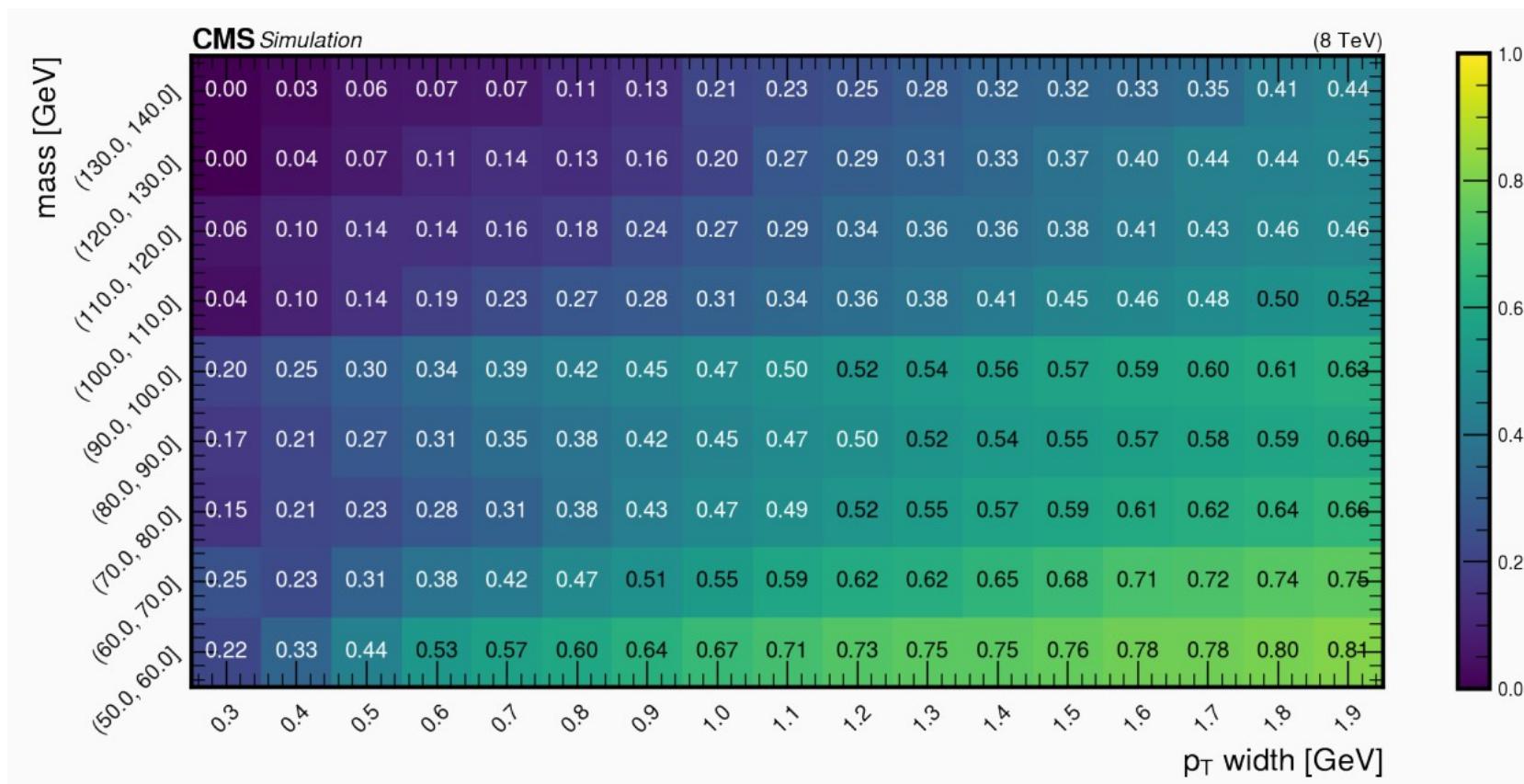
# Applying the method to **experimental** data

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# Applying the method to experimental data

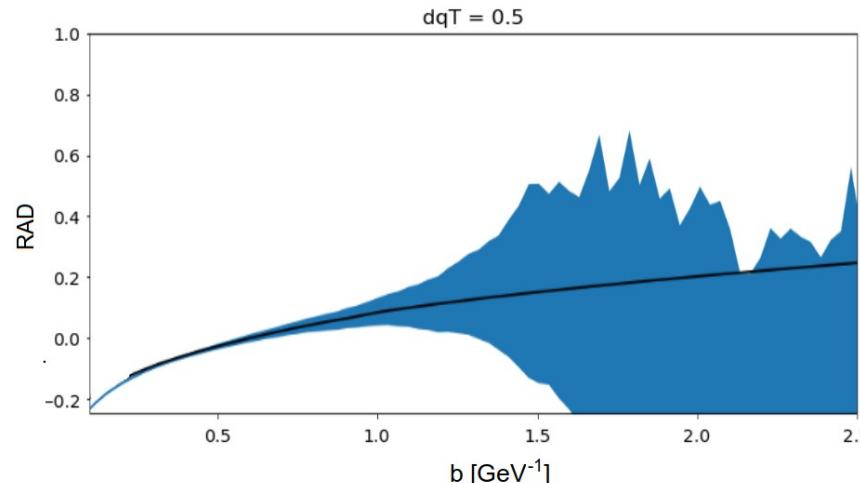
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- ▶ CMS provides excellent muon capabilities
- ▶ High quality data at 7, 8, 13, 13.5 TeV
- ▶ Feasibility studies on the di-muon resolution show promising results:



# Applying the method to experimental data

- ▶ As an example  $Q_1, Q_2 = 28, 46 \text{ GeV}$
- ▶ Small  $q_T$  bin size ensure sensitivity up to around  $b = 1.5$



- ▶ Adding statistical and  $dQ$  uncertainties:

At $b = 0.75$ [0% stat.]					
$dQ$	0.5	1.0	1.5	2.0	2.5
10.0	0.17	0.35	0.49	0.59	0.60
9.0	0.16	0.27	0.41	0.51	0.57
8.0	0.13	0.21	0.37	0.50	0.49
7.0	0.10	0.15	0.24	0.39	0.51
6.0	0.08	0.12	0.25	0.33	0.51
5.0	0.05	0.09	0.18	0.28	0.34
4.0	0.04	0.08	0.14	0.32	0.44
3.0	0.03	0.07	0.15	0.25	0.37
2.0	0.02	0.06	0.15	0.23	0.36
1.0	0.02	0.07	0.13	0.26	0.36

At $b = 0.75$ [2% stat.]					
$dQ$	0.5	1.0	1.5	2.0	2.5
10.0	0.20	0.29	0.52	0.57	0.66
9.0	0.15	0.27	0.43	0.66	0.57
8.0	0.13	0.21	0.32	0.47	0.54
7.0	0.09	0.18	0.24	0.40	0.43
6.0	0.07	0.14	0.23	0.39	0.45
5.0	0.06	0.10	0.18	0.35	0.43
4.0	0.04	0.09	0.17	0.27	0.38
3.0	0.04	0.08	0.16	0.26	0.35
2.0	0.03	0.08	0.14	0.25	0.39
1.0	0.03	0.07	0.14	0.26	0.34

At $b = 0.75$ [4% stat.]					
$dQ$	0.5	1.0	1.5	2.0	2.5
10.0	0.19	0.28	0.51	0.54	0.54
9.0	0.14	0.26	0.47	0.46	0.51
8.0	0.13	0.22	0.34	0.47	0.54
7.0	0.10	0.16	0.29	0.36	0.54
6.0	0.08	0.13	0.23	0.39	0.42
5.0	0.06	0.12	0.18	0.39	0.40
4.0	0.05	0.09	0.20	0.38	0.38
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1.0	0.04	0.07	0.17	0.27	0.35

- ▶ Statistical uncertainty mild, main uncertainty from  $q_T$  binning

# Applying the method to transform PB TMDs to CSS

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- ▶ This is a long standing problem

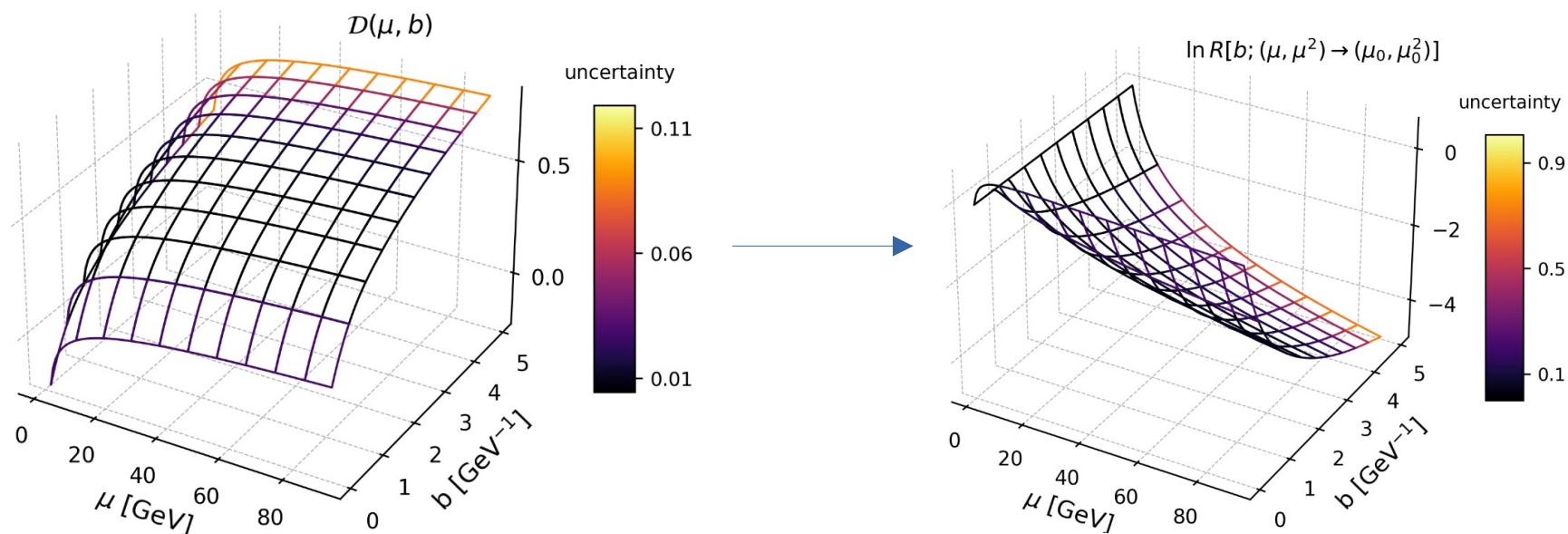
- ▶ Evolution of a TMD can be expressed as:

→ Evolution factor

$$F(x, b; \mu, \zeta) = R[b; (\mu, \zeta) \rightarrow (\mu_0, \zeta_0)] F(x, b)$$

- ▶  $R[b; (\mu, \mu^2) \rightarrow (\mu_0, \mu_0^2)] = \exp \left\{ - \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} (2\mathcal{D}(\mu', b) + \gamma_V(\mu')) \right\}$

- ▶ We use the method to determine RAD from DY in CASCADE:



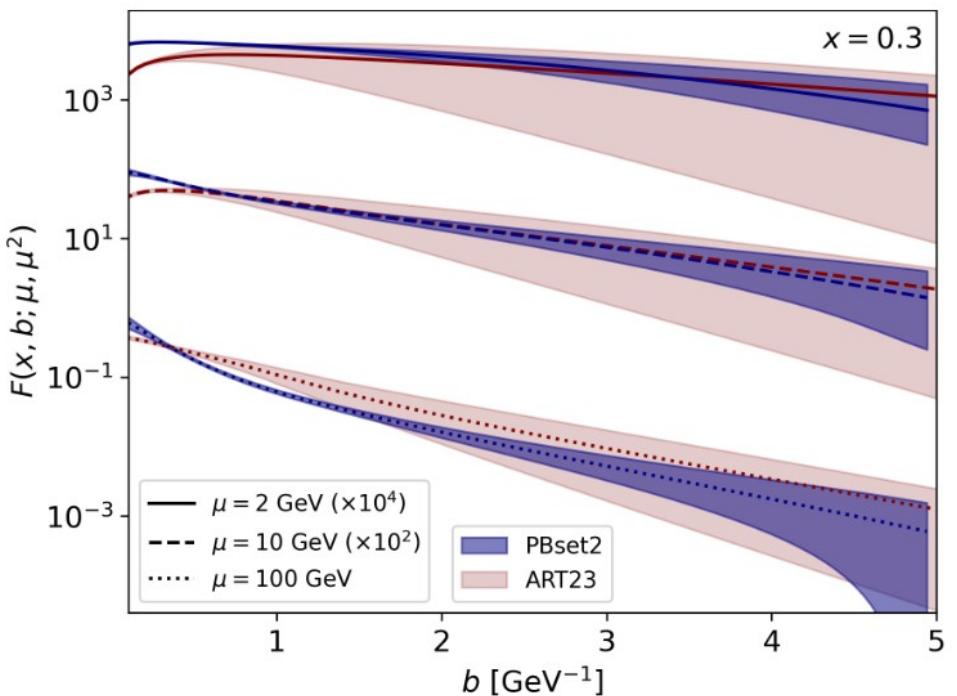
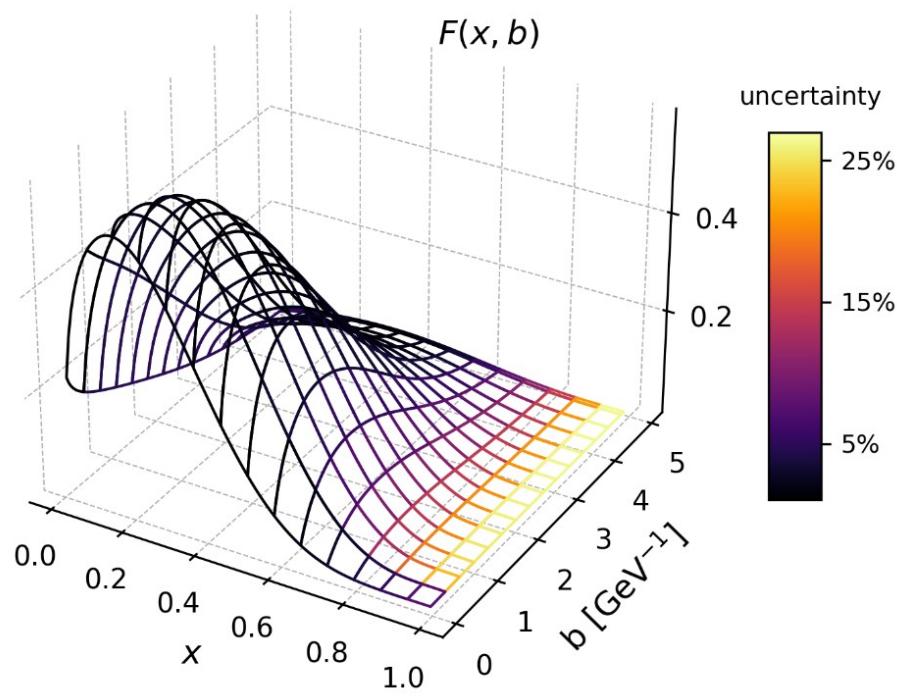
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→ Evolution factor

- ▶ Comparing PB TMD set2 to MAPP22



## Summary and conclusions

- ▶ Determination of RAD would be a **stringent test of factorization** and can have a deep impact on hadron 3D imaging
- ▶ Novel method to determine **RAD** was introduced
- ▶ Its application to simulated data from PB approach has **solved long standing problem of comparison** between factorization and PB
- ▶ Feasibility studies using CMS full simulated public data **have shown** promising results

