RF Engineering
Introduction to the *Smith* Chart

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Outline and Objectives

- **Signals and reflections on transmission-lines**
  - Reflections effects of pulse signals on transmission-lines due to characteristic-termination impedance mismatch
  - Standing waves on transmission-lines for continuous wave (CW) sinusoidal signals, definition of the reflection coefficient
  - Relation between reflection coefficient, standing wave ration and return loss
- **The Smith chart**
  - Refresher: Visualization of a complex impedance in the frequency domain
  - Definition of the Smith chart, mapping the complex impedance / admittance plane with the complex reflection coefficient
  - Basic facts and important points on the Smith chart
  - Examples for a RL and RC series circuit, and for a transmission-line terminated with a RL series circuit.
  - Operation of a $\lambda/4$transformer based on a transmission-line as a (normalized) impedance inverter
- **Smith-chart software demo**

Transmission-lines

- **TEM: coaxial cable, PCB stripline, micro-stripline, co-planar waveguide, etc.**
  - **but also, TE / TM: waveguides (rectangular, circular, elliptical)**
- **Transport RF energy (EM waves) from a RF source to a load.**
- **Physical length ℓ becomes relevant for**

$$\ell \gtrsim \frac{\lambda_g}{10}, \qquad \text{with guide wavelength: } \lambda_g = \frac{v_p}{f}$$

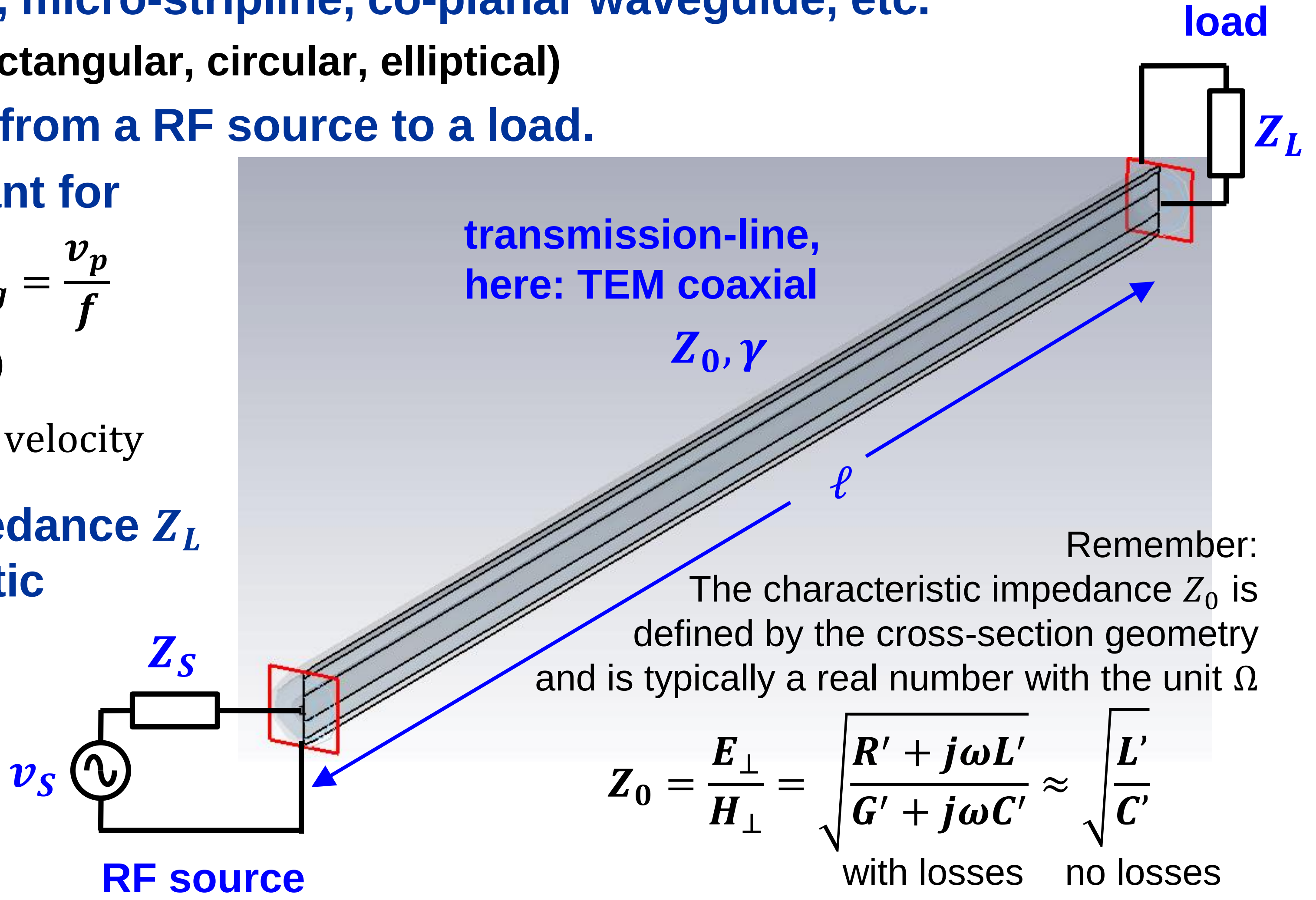
f: operating frequency (max.)

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{c}{\sqrt{\epsilon_r}} \text{ propagation velocity}$$

- **Reflections occur if the load impedance Z_L is not matched to the characteristic impedance Z_0 of the TL**

$Z_L = Z_0 \Rightarrow \Gamma = 0$: no reflections ☺

$Z_L \neq Z_0 \Rightarrow \Gamma \neq 0$: reflections! ☹

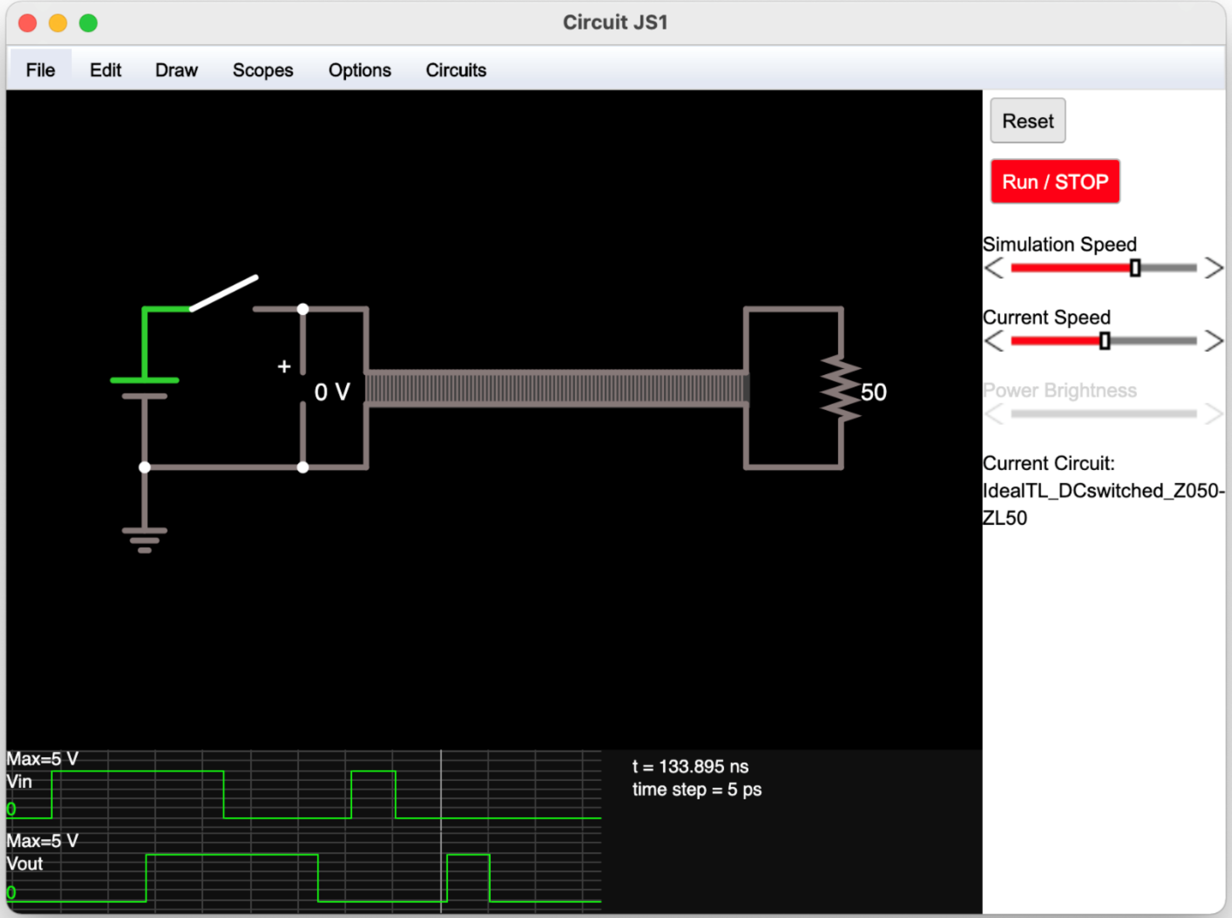


Remember:
The characteristic impedance Z_0 is defined by the cross-section geometry and is typically a real number with the unit Ω

$$Z_0 = \frac{E_\perp}{H_\perp} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \approx \sqrt{\frac{L'}{C'}}$$

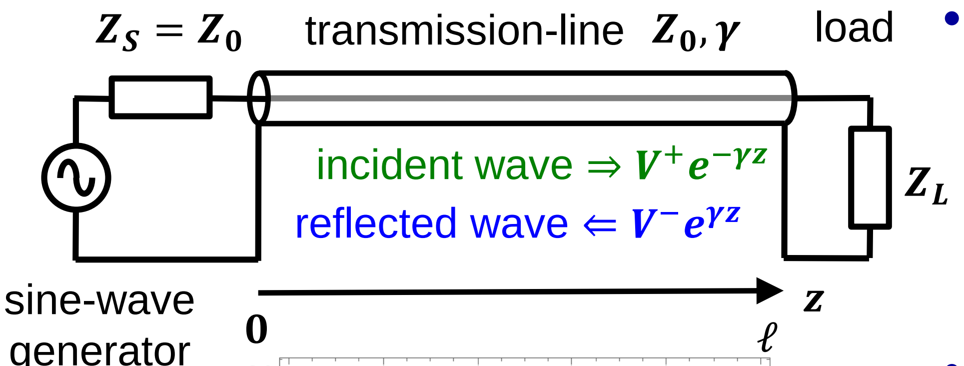
with losses — no losses

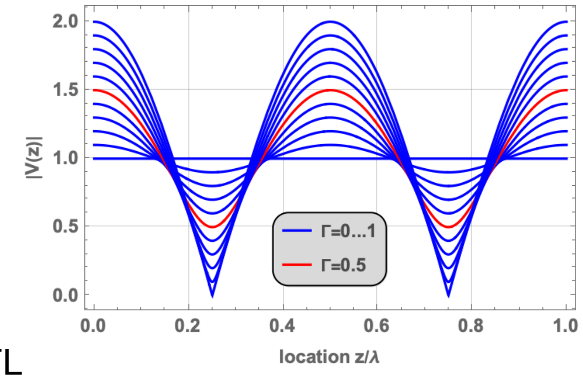
TL: Signal visualization in time-domain



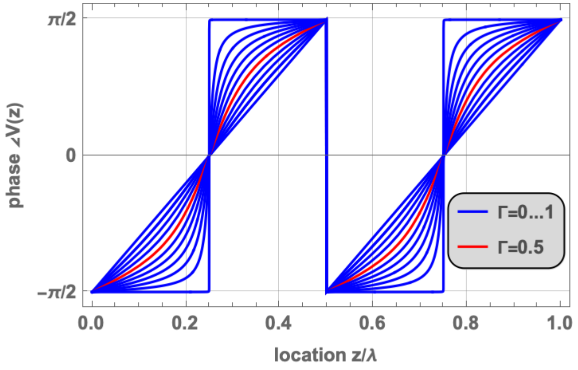
- **Circuit simulator applet: https://www.falstad.com/circuit/**
  - **Load file: *IdealTL_DCswitched_Z050-RL.txt***
    - ➢ Change the load resistor value: *RL = 50, 100, 25 Ω*
    - ➢ Operate the switch and observe the signals at the beginning, and at the end of the transmission-line.
  - **Load file: *IdealTL_pulsed_Z050-RL.txt***
    - ➢ Change the load resistor value: *RL = 50, 100, 25 Ω*
    - ➢ Observe the signal waveforms! Can you predict the values?!
      - (Press *Run/STOP* and hover with the mouse over the waveform)

TL: Operating with sinusoidal signals (FD)





here:
$V(z)$ for a lossless TL
$(\gamma = j\beta)$

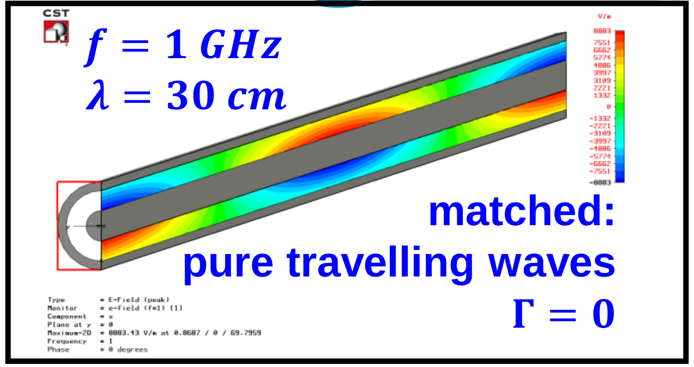


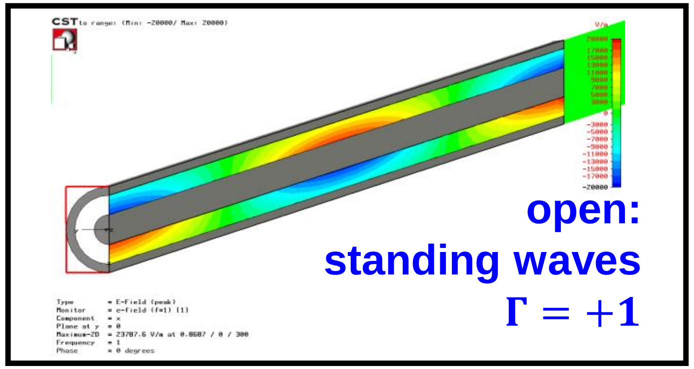
- **For a single frequency, continues wave (CW) signal on a transmission-line**
  - **Superposition of forward a, E^{inc}, V^+ and backward b, E^{refl}, V^- traveling waves**
  - **⇒ standing waves**
- **Reflection coefficient Γ**

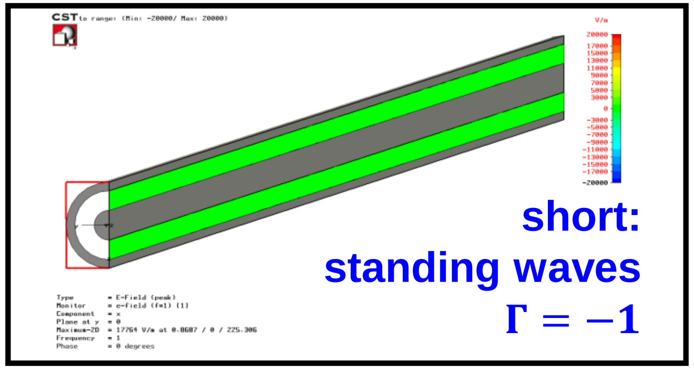
$$\Gamma = \frac{b}{a} = \frac{E^{refl}}{E^{inc}} = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

waves E-fields voltages impedances

  - **Γ is related to the impedance ratio: Z_L / Z_0**
    - The load impedance Z_L can be complex
  - **Γ is a complex number**
- **Reflections also originate due to discontinuities of the TL along z**
  - **Same Z_0 does not prevent reflections! EM-field effect!**







TL: Voltage Standing Wave Ratio (VSWR)

- **The voltage standing wave ratio (VSWR) expresses the ratio between the maximum and minimum voltage of a standing wave along a transmission-line**

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|a|+|b|}{|a|-|b|} = \frac{1+|\Gamma|}{1-|\Gamma|} = \left|\frac{Z_L}{Z_0}\right|$$

with: $\begin{cases} |V_{max}| = |V^+| + |V^-| \\ |V_{min}| = |V^+| - |V^-| \end{cases}$

symbols: incident (forward) wave: a, X^+
reflected (backward) wave: b, X^-

  - **The VSWR is a function of the frequency.**
  - **The generalized standing wave ratio (SWR), e.g., expressed as power ratio is less popular**

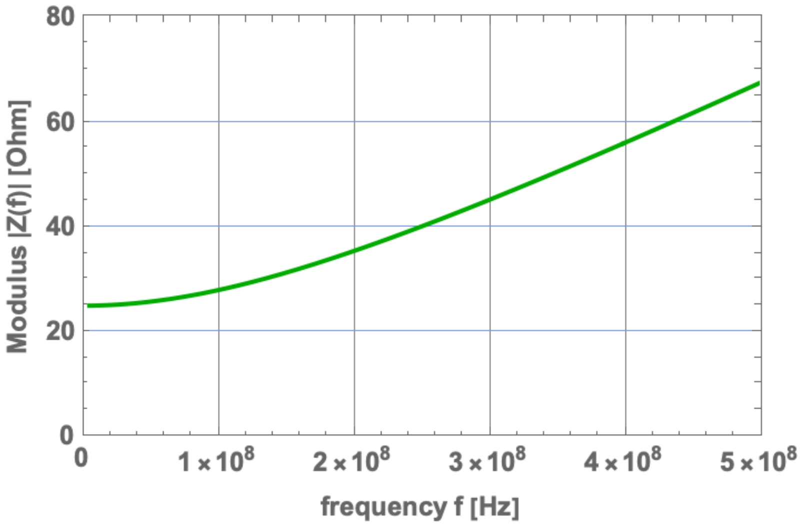
$$SWR = \frac{1+\sqrt{P^-/P^+}}{1-\sqrt{P^-/P^+}}$$

- **The return loss (RL) is another way to express reflection effects**

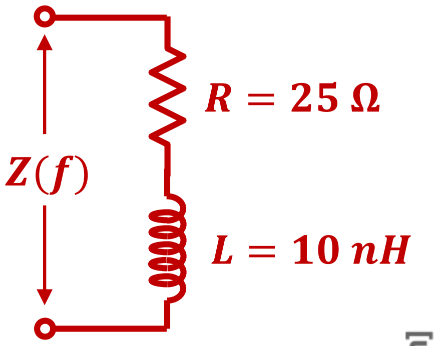
$$RL\,[dB] = 10\log_{10}\frac{P^+}{P^-} = -20\log_{10}|\Gamma|$$

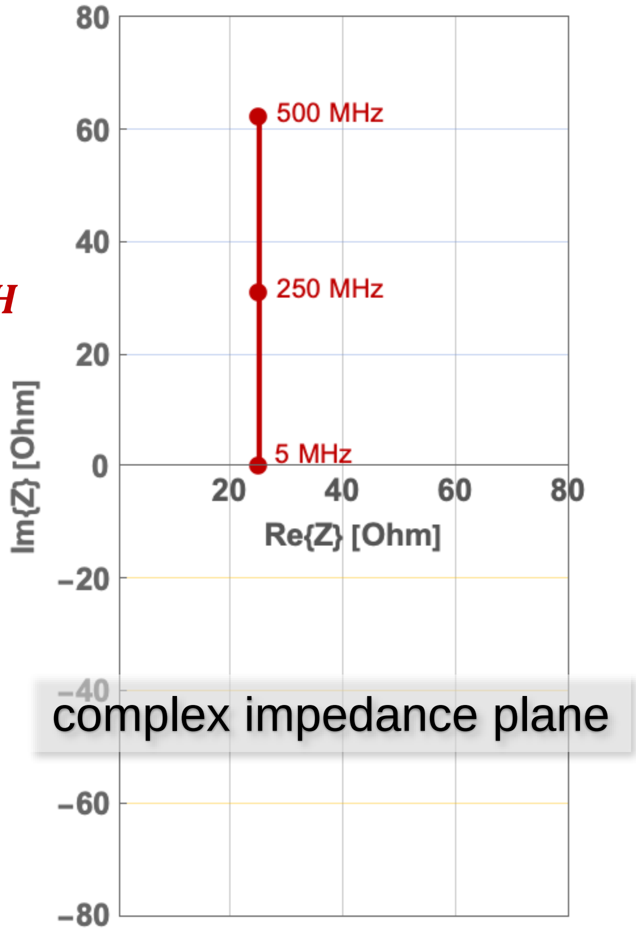
Γ	$VSWR = Z_L/Z_0$	Return Loss [dB]	Refl. Power $\lvert\Gamma\rvert^2$	Inc. Power $1-\lvert\Gamma\rvert^2$
0.0	1.00	∞	0.00	1.00
0.1	1.22	20.0	0.01	0.99
0.2	1.50	14.0	0.04	0.96
0.3	1.87	10.5	0.09	0.91
0.4	2.33	8.0	0.16	0.84
0.5	3.00	6.0	0.25	0.75
0.6	4.00	4.4	0.36	0.64
0.7	5.67	3.1	0.49	0.51
0.8	9.00	1.9	0.64	0.36
0.9	19.00	0.9	0.81	0.19
1.0	∞	0	1.00	0.00

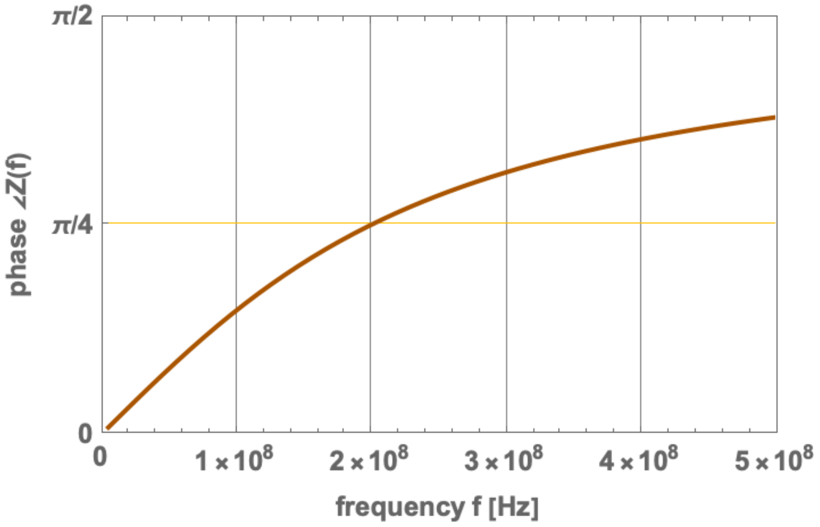
Reminder: The complex Z-Plane

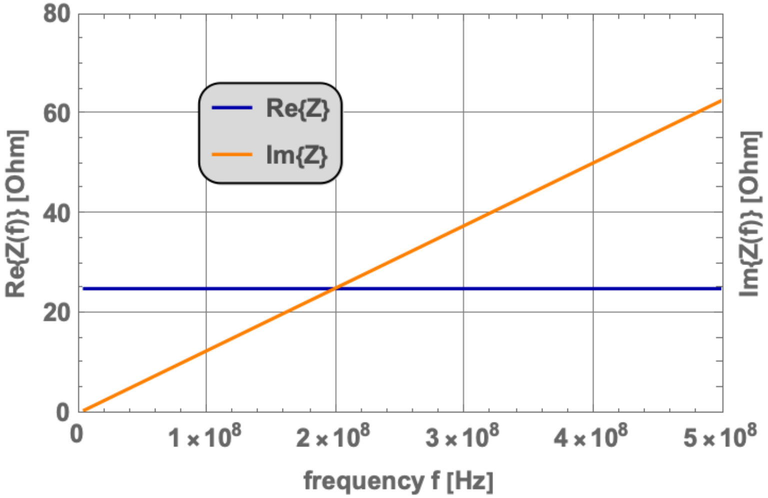


- **Different ways to visualize $Z(f)$**
  - magnitude / phase
  - real / imaginary
  - complex Z-plane
    - Plot in the complex plane with f as parameter



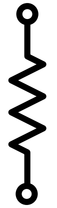




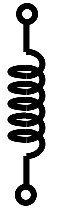


Reminder: Circuit Vocabulary

conductor, with:
conductance G $[S]$



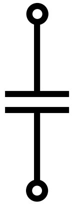
resistor, with:
resistance R $[\Omega]$



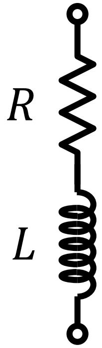
inductor, with:
inductance L $[H]$
reactance $X_L = \omega L$ $[\Omega]$

suceptance $B_L = 1/\omega L$ $[S]$

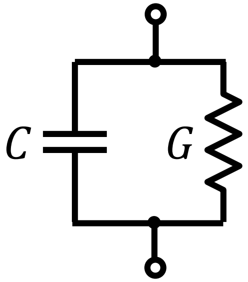
capacitor, with:
capacitance C $[F]$
susceptance $B_C = \omega C$ $[S]$



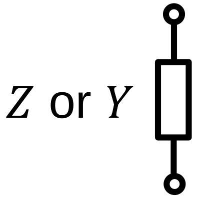
reactance $X_C = -1/\omega C$ $[\Omega]$



complex impedance example:
$Z = R + j\omega L$ $[\Omega]$



complex admittance example:
$Y = G + j\omega C$ $[S]$

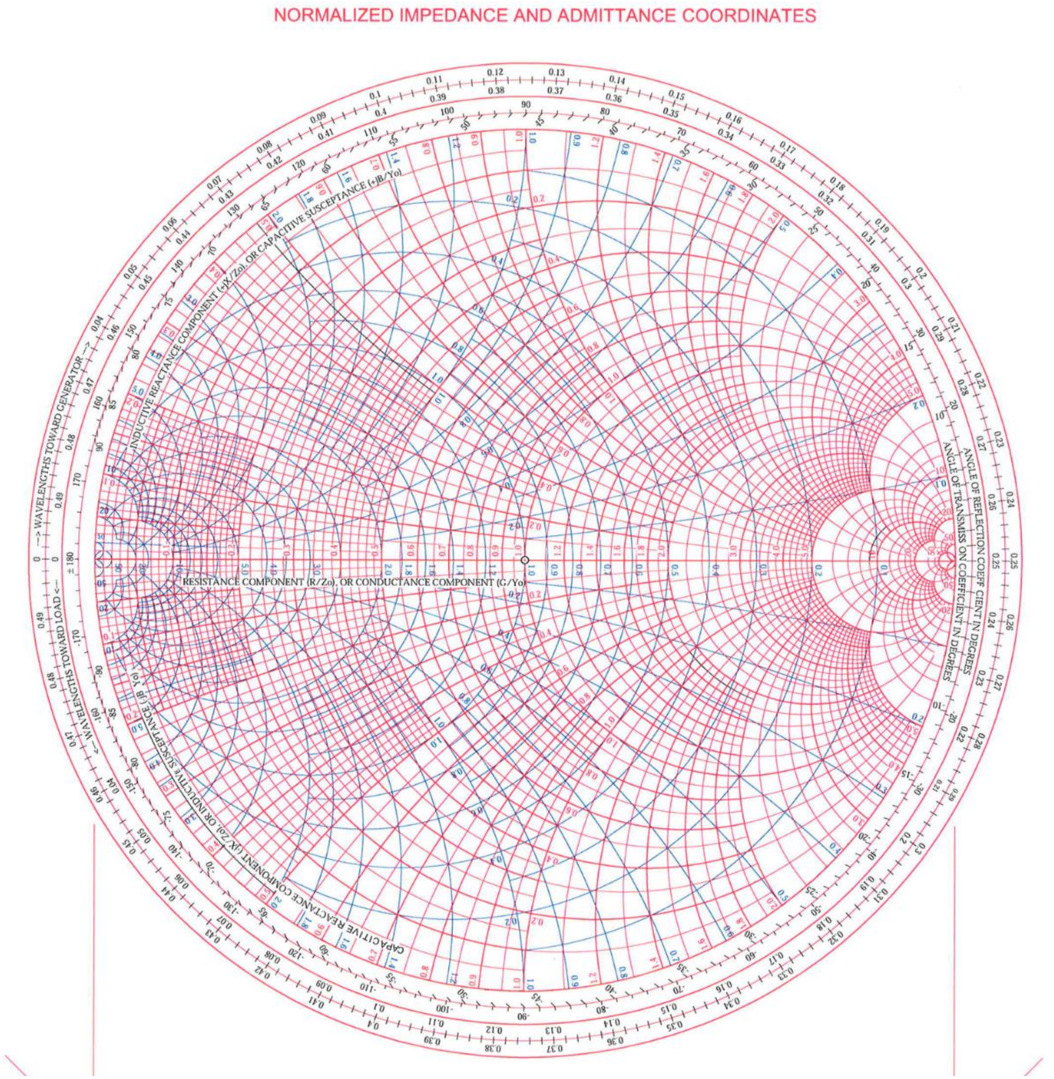


complex impedance Z
or
complex admittance Y

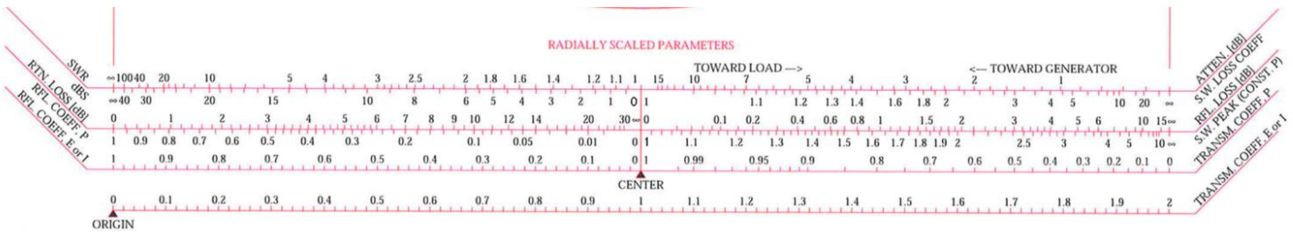
- **Resistance, impedance, reactance are inverse proportional to conductance, susceptance, admittance**

$$R = \frac{1}{G} \qquad Z = \frac{1}{Y} \qquad X = \frac{-1}{B}$$

The *Smith* Chart (1)



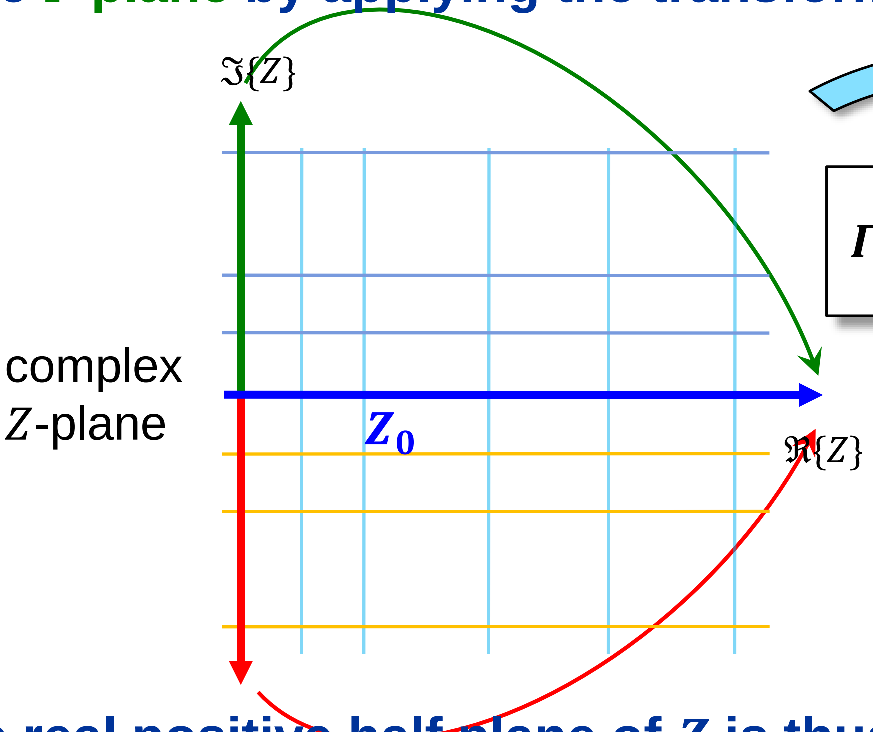
- **The *Smith* chart is a calculation tool, divided in 2 parts:**
  - **A transformation of the complex, normalized impedance z and admittance y planes on a circle.**
  - **A set of “rulers” below, for additional computations**
    - VSWR, return and reflection loss, etc.



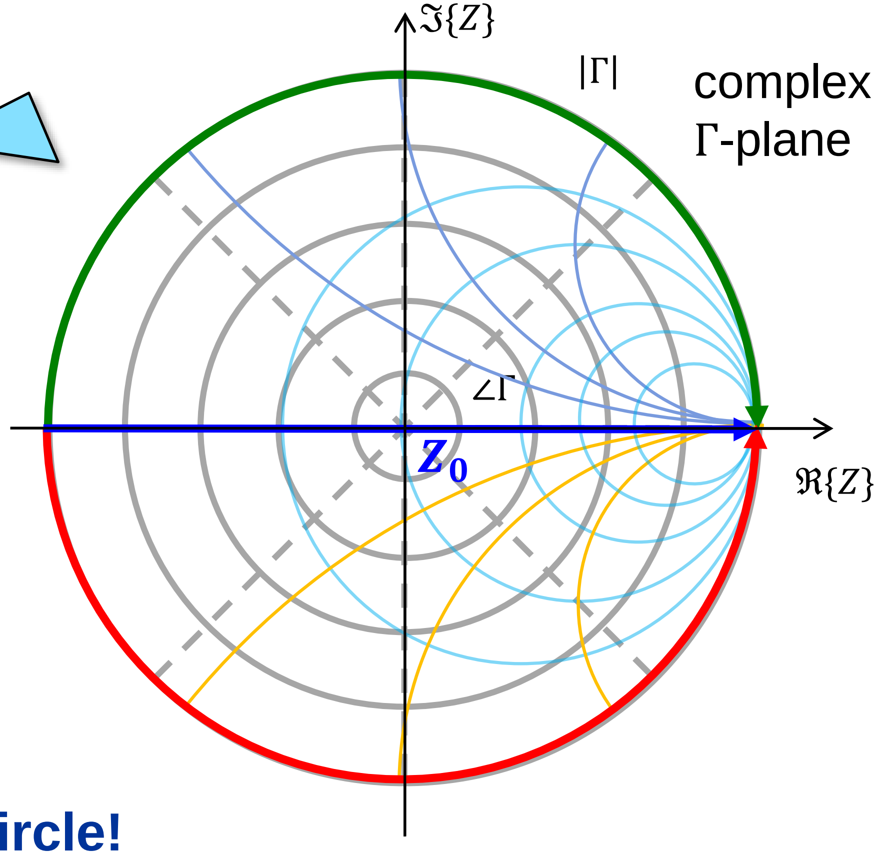
- **At a 1st look the Smith chart is quite overwhelming**
  - **In this introduction the focus is on the complex z-plane**

The *Smith* Chart (2)

- **The *Smith* chart (in impedance / admittance coordinates) represents the complex Γ-plane (in polar coordinates) within the unit circle.**
- **It is a conformal mapping of the complex Z-plane on the Γ-plane by applying the transformation:**



$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$



- **⇒ the real positive half plane of Z is thus transformed (*Möbius*) into the interior of the unit circle!**

The *Smith* Chart (3)

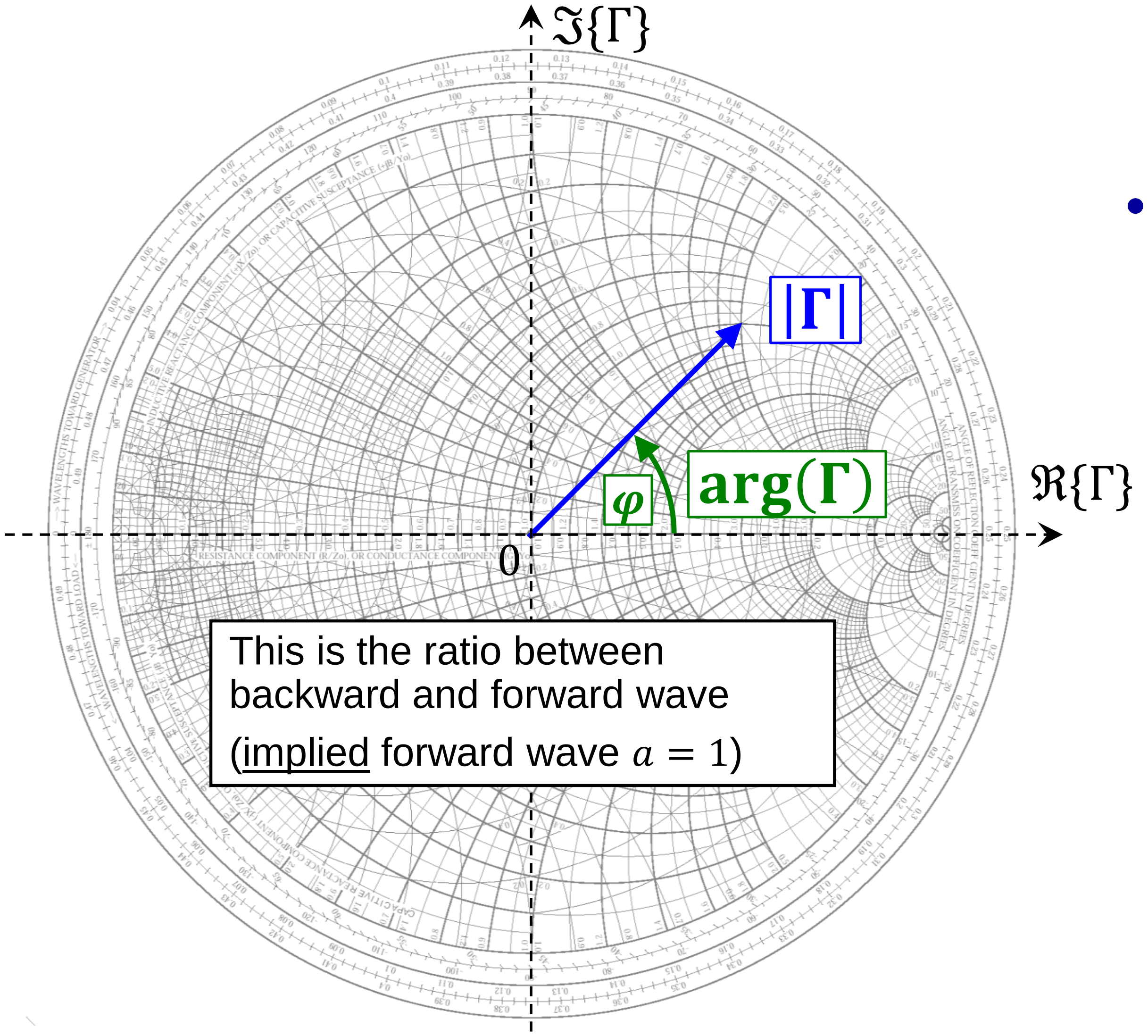
- The impedance Z is usually **normalized**: $$z = \frac{Z}{Z_0}$$

  to a reference impedance Z_0,
  typically, to the characteristic impedance of the coaxial cable transmission-lines used in RF / microwave engineering: $Z_0 = 50\ \Omega$.
- The normalized form of the transformation follows then as: $$\Gamma = \frac{z-1}{z+1} \quad \Rightarrow \quad \frac{Z}{Z_0} = z = \frac{1+\Gamma}{1-\Gamma}$$

- The Smith chart is a **parametric graph**
  - with the frequency f as parameter
  - and the normalized, complex impedance z and complex reflection coefficient Γ as variables
    - also, the normalized, complex admittance $y = 1/z$ is mapped and can be used as variable.

- In the past
  - The *Smith* chart was used as a calculation tool for impedance matching, e.g., antennas to transmitters or receivers, amplifier input / output stages, couplers of accelerating cavities, etc.
- At presence, the *Smith* chart is still popular
  - for visualization purposes, e.g., vector network analyzer (VNA) measurement of input / output impedances (display of the Sii scattering parameters)
  - for the optimization of the coupling between RF source and a cavity resonator.
  - …

The *Smith* Chart (4)



- **In the *Smith* chart, the complex reflection factor**

$$\Gamma = |\Gamma| e^{j\varphi} = \frac{b}{a}$$

is expressed in linear polar coordinates, representing the ratio of backward b vs. forward a traveling waves.

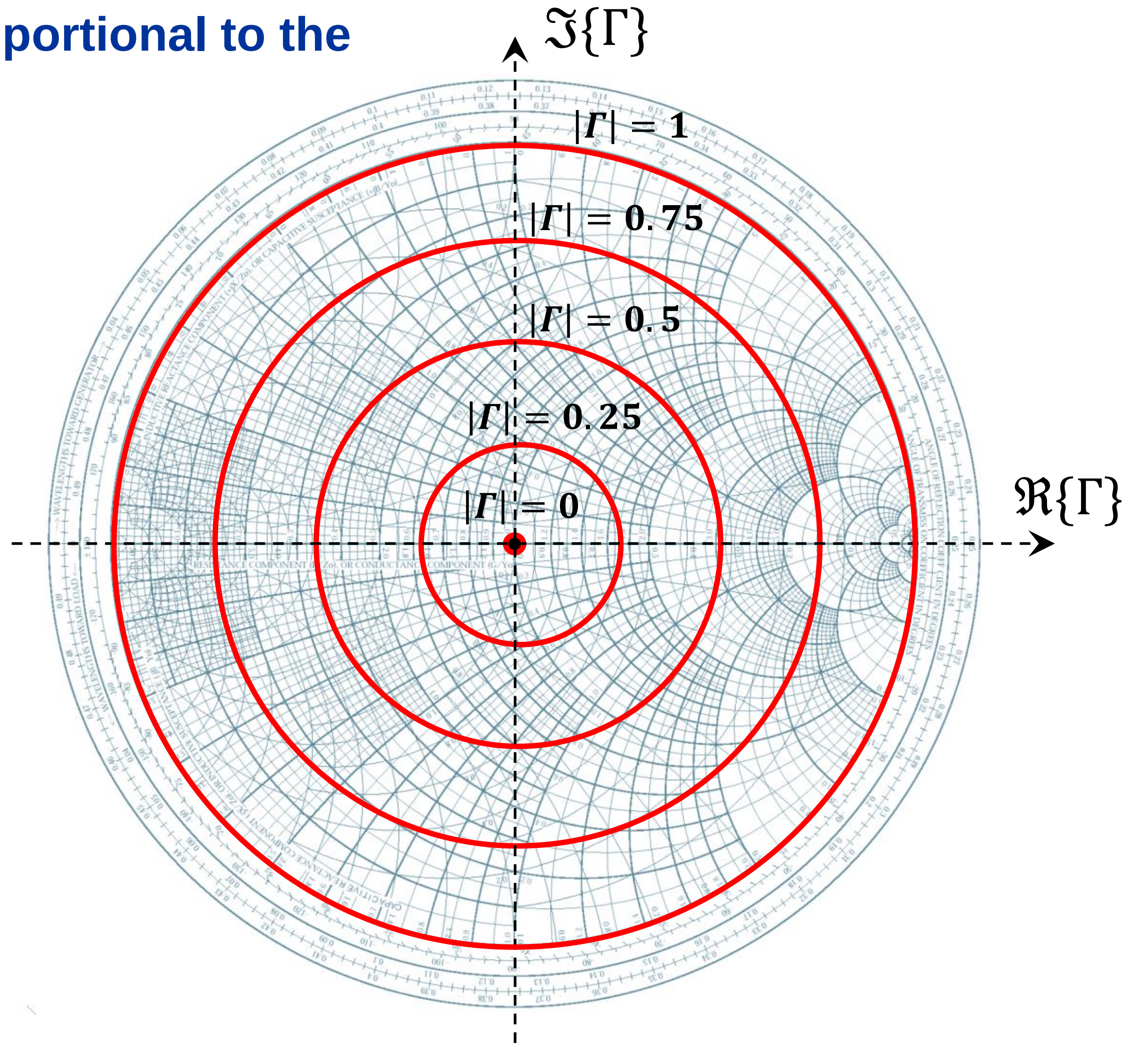
The *Smith* Chart (5)

- The distance from the center of the directly proportional to the magnitude of the reflection factor $|\Gamma|$ and permits an easy visualization of the matching performance.
  - In particular, the perimeter of the diagram represents total reflection: $|\Gamma| = 1$.
  - (power dissipated in the load) = (forward power) – (reflected power)

$$\mathbf{P} = |\boldsymbol{a}|^2 - |\boldsymbol{b}|^2$$
$$= |\boldsymbol{a}|^2\left(1 - |\Gamma|^2\right)$$

available source power

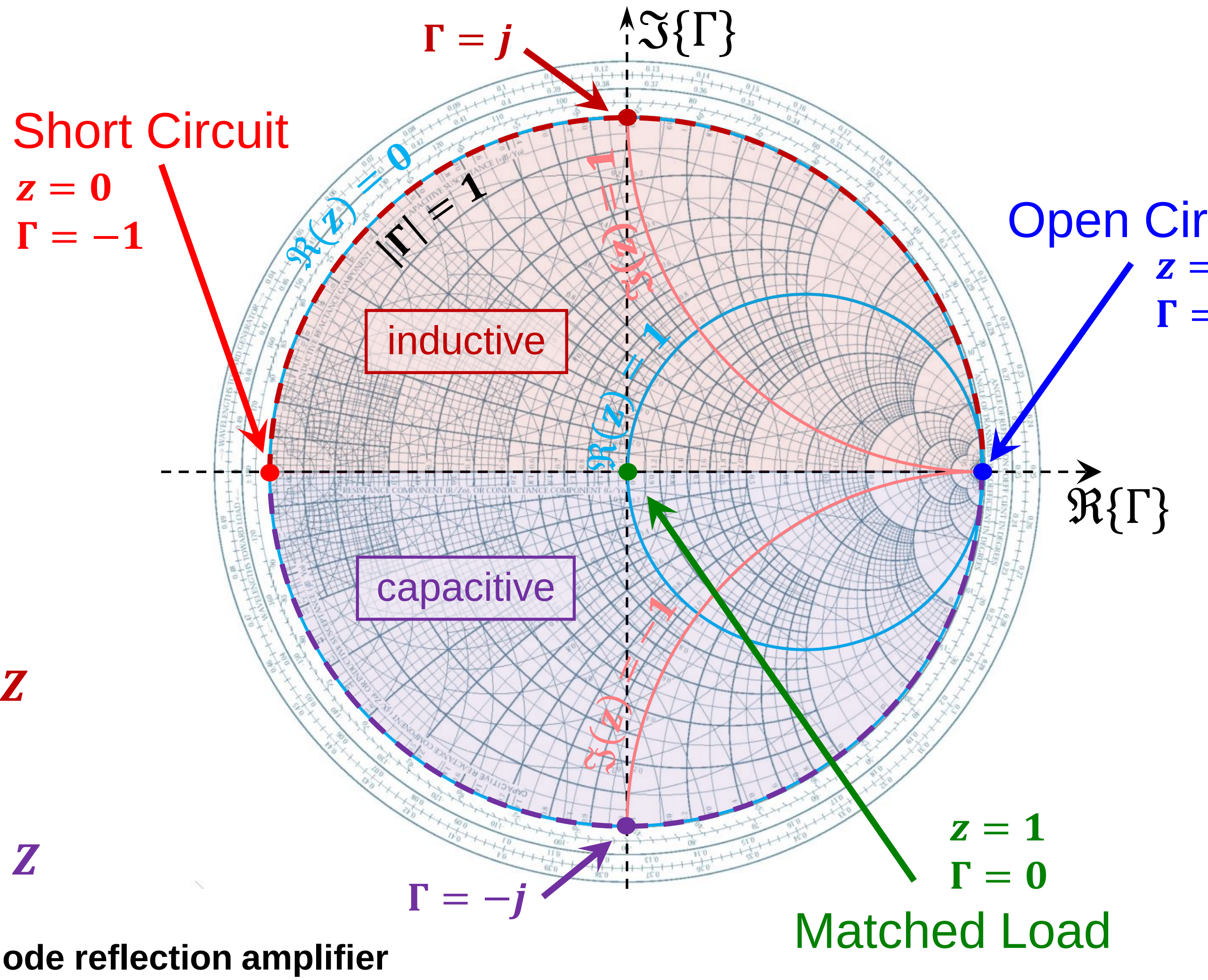
mismatch losses



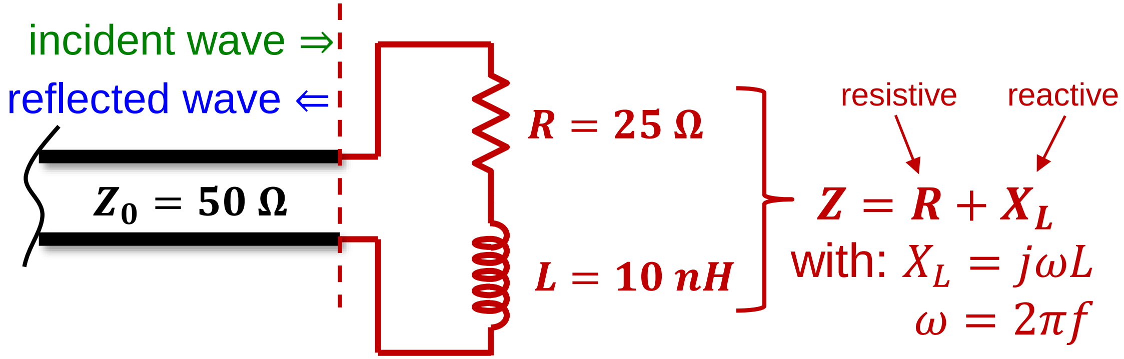
The *Smith* Chart – “Important Points”

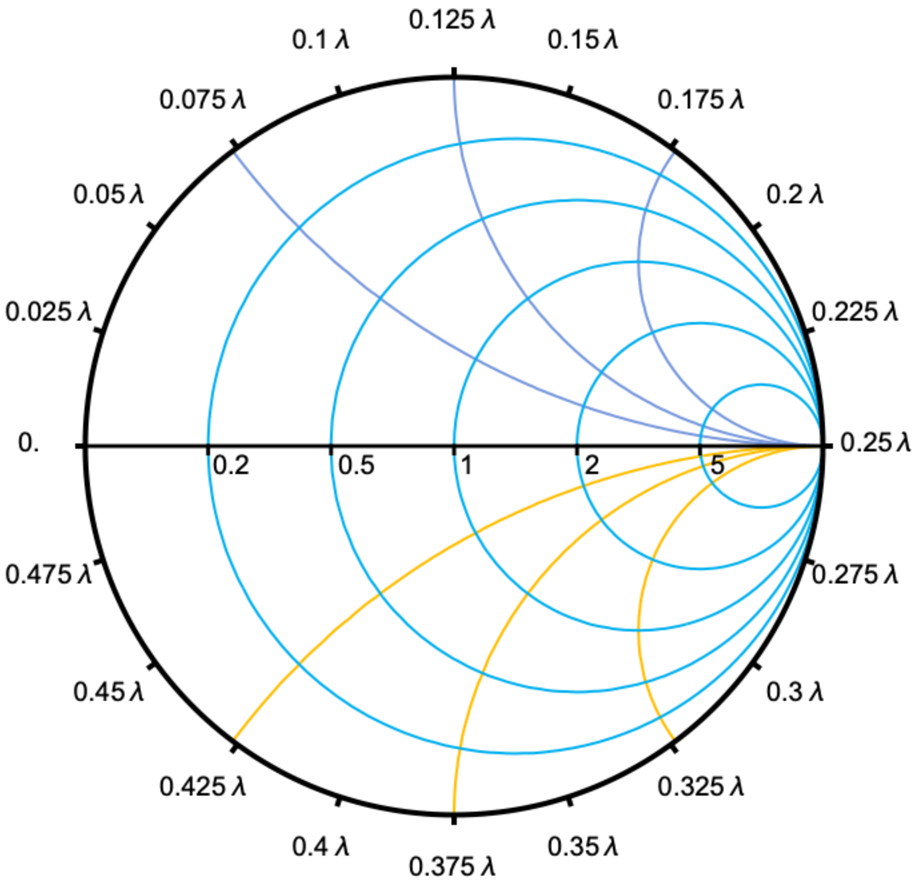
Important Points:

- **Short Circuit**
  $\Gamma = -1, z = 0$
- **Open Circuit**
  $\Gamma = +1, z \to \infty$
- **Matched Load**
  $\Gamma = 0, z = 1$
- **On the circle $|\Gamma| = 1$:**
  lossless element
- **Upper half:**
  ”inductive” =
  positive imaginary part of Z
- **Lower half:**
  ”capacitive” =
  negative imaginary part of Z
  – **Outside the circle, $\Gamma > 1$:**
  active element, e.g., tunnel diode reflection amplifier



The *Smith* Chart – Basic Example (1)

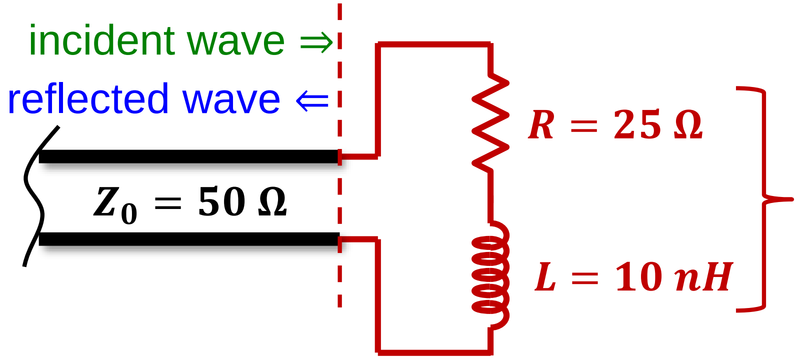




Complex impedance based on lumped element components

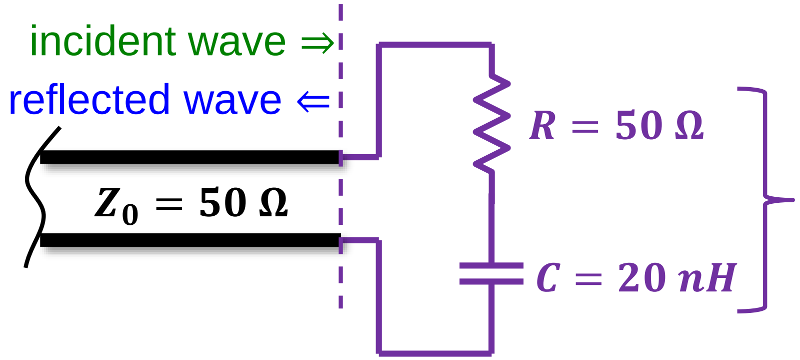
- **Calculate Z for a given frequency, e.g., $f = 50\ MHz$: $Z = (25 + j6.28)\ \Omega$**
- **Calculate the normalized impedance $z = Z/Z_0 = 0.5 + j0.126$**
  - **Locate z in the Smith chart**
  - **Retrieve $\Gamma = 0.34\angle 161° = 0.34e^{j2.81}$**
- **Repeat for other frequencies…**

The *Smith* Chart – Basic Example (2)

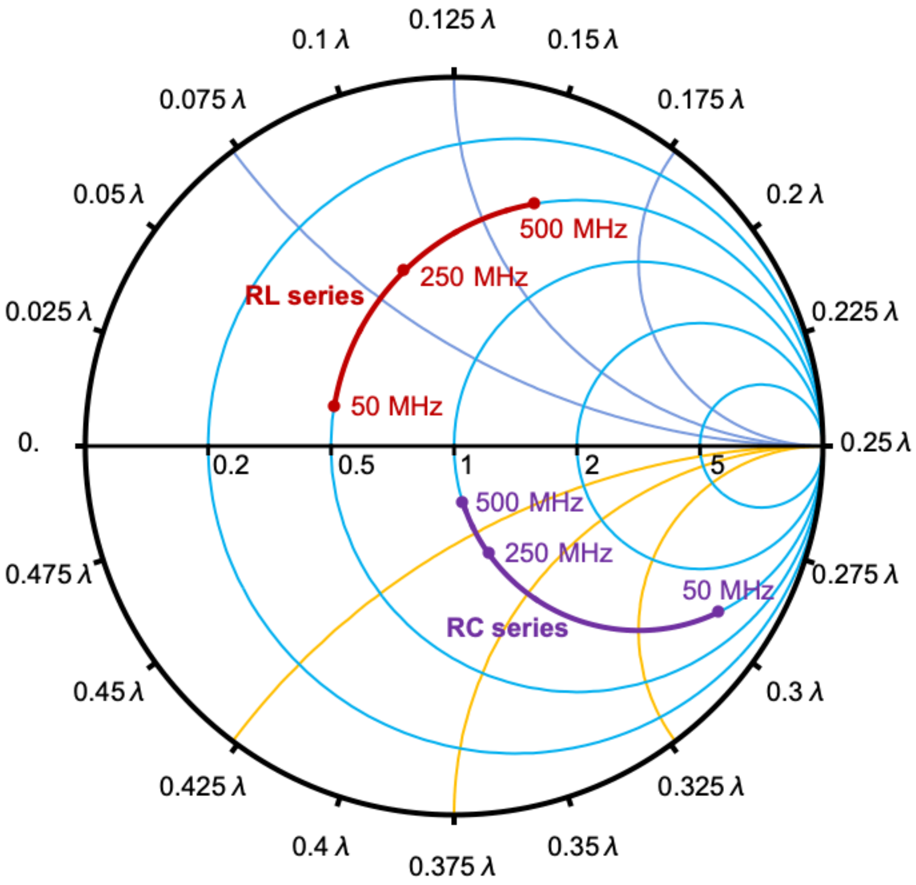


$Z = R + jX_L$
with: $X_L = \omega L$

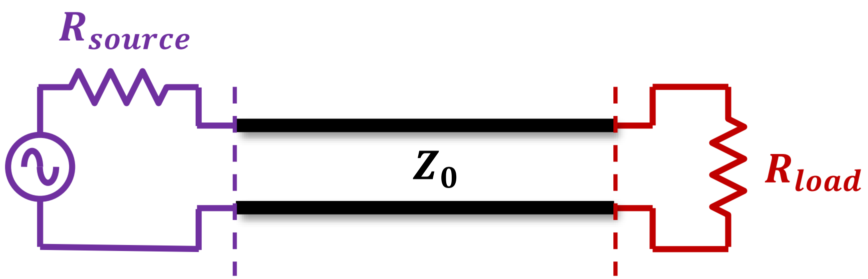
- **…and for different component values and circuit combinations**



$Z = R + jX_C$
with: $X_C = \frac{-1}{\omega C}$



Quiz (1)

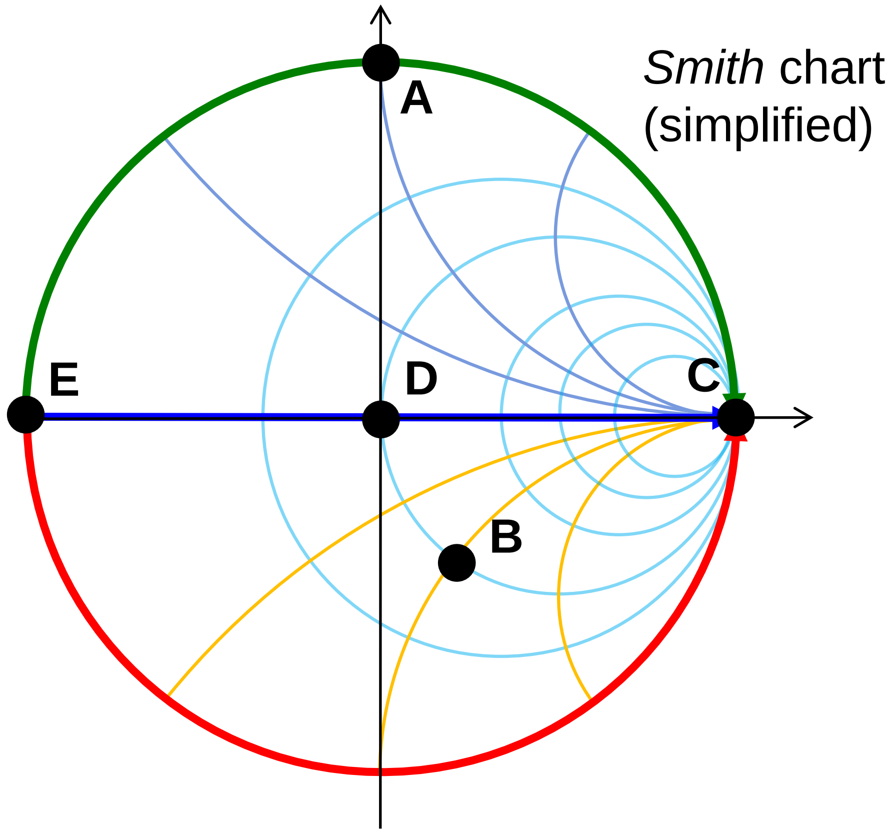


1. **When do no signal reflections occur at the end of a transmission-line?**
   - ○ $R_{source} = R_{load}$
   - ○ $R_{source} = Z_0$
   - ✗ $Z_0 = R_{load}$
   - ✗ $R_{source} = Z_0 = R_{load}$

2. **The Smith chart transforms onto the complex Gamma (reflection coefficient) plane within the unit circle.**

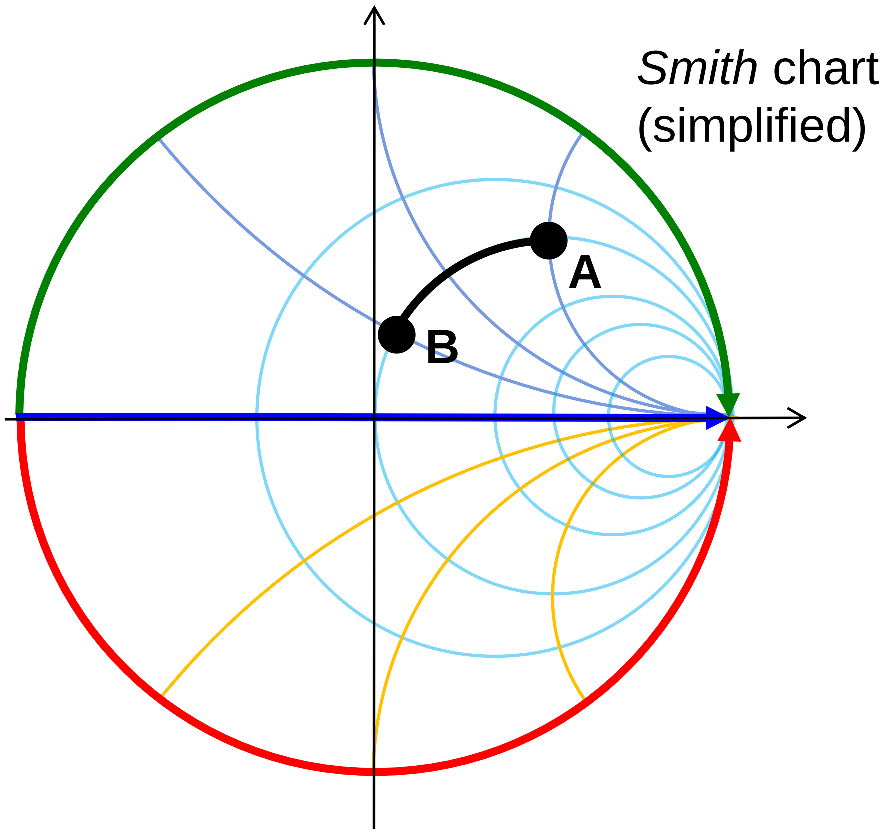
Quiz (2)

Prompts		Possible Answers
A. Point A		1. $\Gamma = +1, z \to \infty$
B. Point B		2. $\Gamma = -j$
C. Point C		3. $\Gamma = 0, z = 1$, match
D. Point D		4. Point in the capacitive half plane
E. Point E		5. $\Gamma = +j$
		6. $\Gamma = -1, z = 0$
		7. Point in the inductive half plane

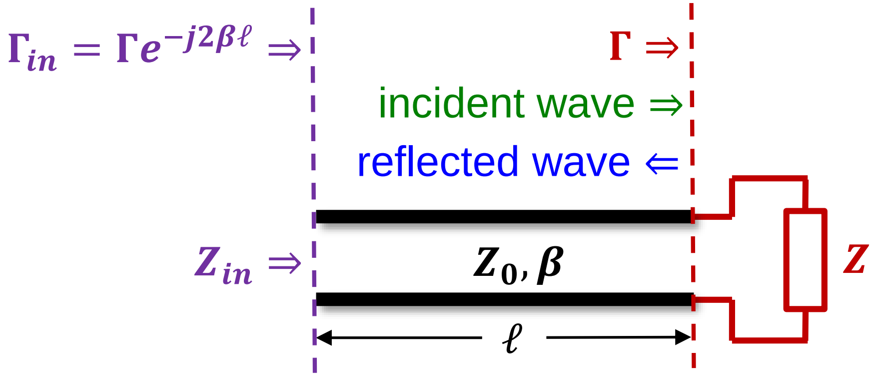


4. Trace with marker points in the simplified *Smith* chart for an RL series impedance

- ○ $f_B > f_A$
- ✗ $f_B < f_A$
- ○ There is no frequency f related to points A and B
- ○ $f_A = f_B$



The *Smith* Chart – TL Transformer (1)



- **S-parameter of a lossless transmission-line:**

$$S = \begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix}$$

backward transmission coefficient S12

forward transmission coefficient S21

  - **Phase delay (electrical length) of**

$$\beta\ell = \theta$$

with: $\beta = \frac{2\pi}{\lambda_g} = k$ **(wave number)**

- **The lossless transmission-line adds a phase delay of $2\beta\ell$, seen at its input, Γ_{in}, to the reflection coefficient Γ at its output:**

$$\Gamma_{in} = \Gamma e^{-j2\beta\ell}$$

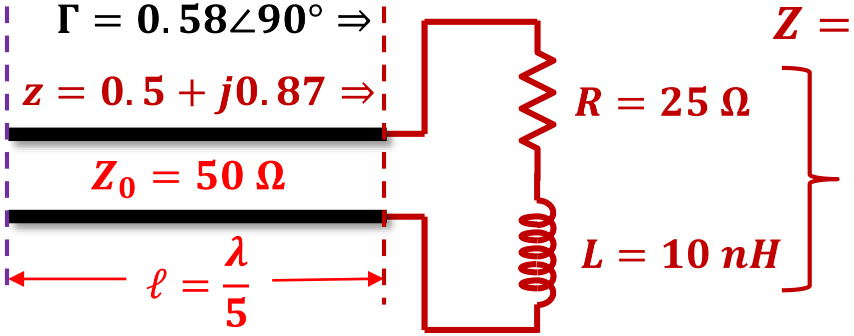
- **This results in a transformation of the impedance Z at the end of the line to a different impedance Z_{in} at the input of the line**
  - **The *Smith* chart offers an effective, simple graphical way to calculate this transmission-line based impedance transformation**

The *Smith* Chart – TL Transformer (2)

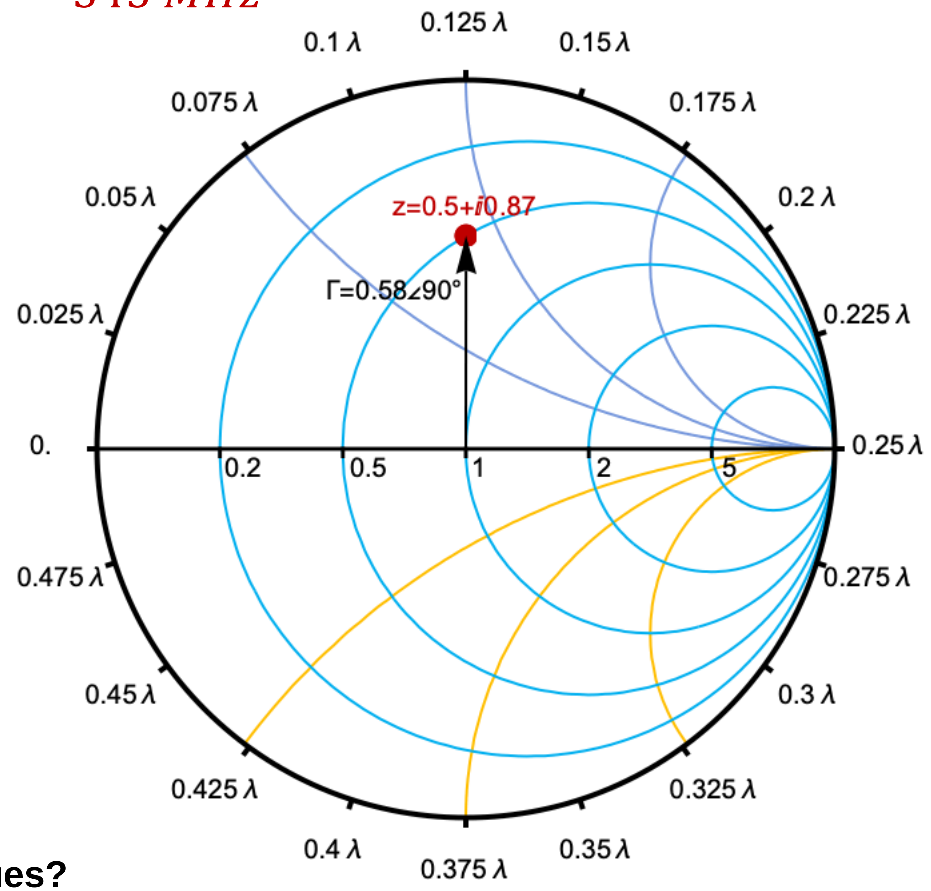
$\Gamma_{in} = 0.58\angle -54° \Rightarrow$ | $\Gamma = 0.58\angle 90° \Rightarrow$

$z_{in} = 1.02 - j1.43 \Rightarrow$ | $z = 0.5 + j0.87 \Rightarrow$

$Z_{in} = (50.8 - j71.3)\ \Omega \Rightarrow$



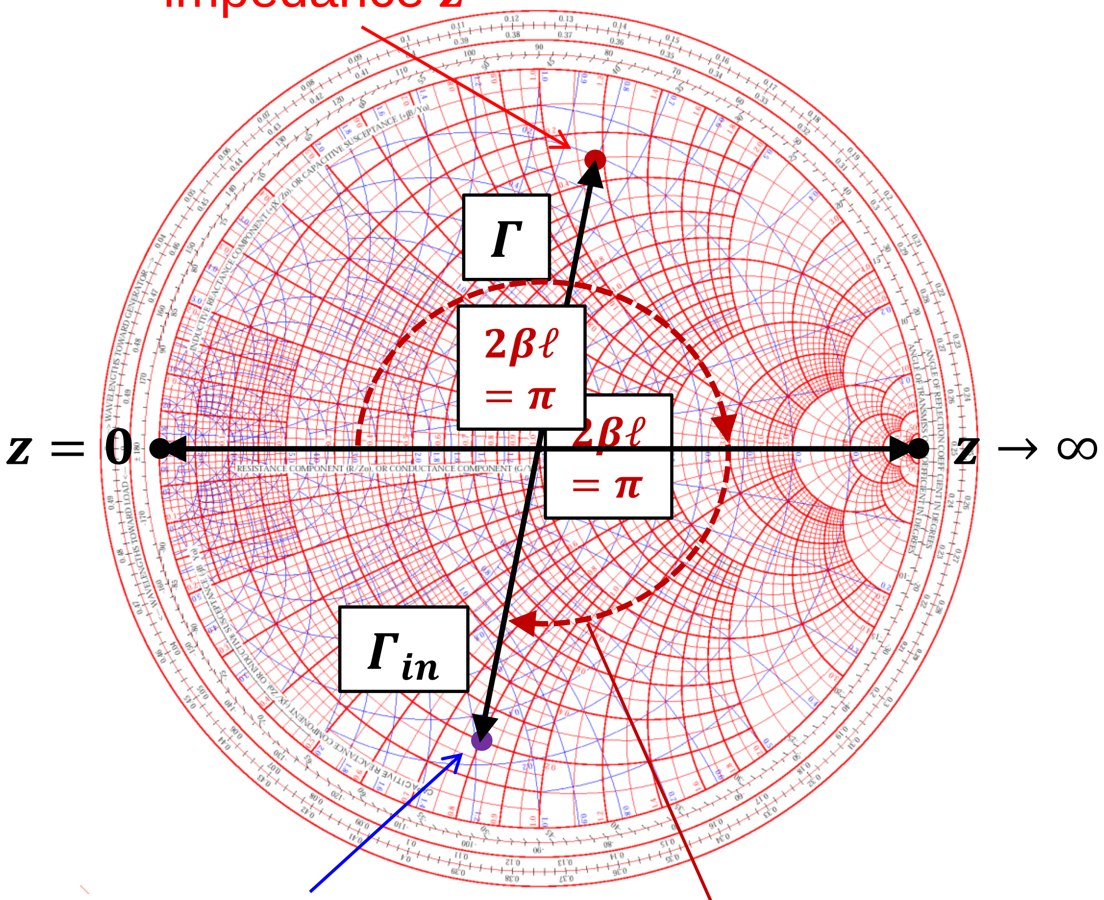
$Z = (25 + j43.4)\ \Omega$
$@\ f = 345\ MHz$



Based on the previous example

- **Calculate the normalized impedance z for $f = 345\ MHz$ and locate the point in the Smith chart.**
  - **Notice the corresponding value of Γ, and read the λ-length value on the rim**
- **Add $\ell = \lambda/5$ by rotating Γ by $2\beta\ell = 4\pi/5 \equiv 144°$**
  - **From $0.125\ \lambda$ to $0.325\ \lambda$**
    - The phase is subtracted, therefore clockwise rotation!
- **Notice the value of Γ_{in} and read the corresponding value of the normalized impedance z_{in}**
  - **Calculate the transformed impedance $Z_{in}@f = 345\ MHz$**
  - **What is the equivalent circuit, and what are the component values?**
    - …and what is the physical length ℓ of the transmission-line?
      - assuming a coaxial cable as transmission-line with a dielectric constant of $\epsilon_r = 2.1$

The *Smith* Chart – TL Transformer (3)



when adding a transmission-line to some terminating impedance we rotate clockwise through the *Smith*-Chart

- **A transmission-line of length** $\ell = {}^{\lambda}/_{4} \equiv \beta\ell = \dfrac{\pi}{2}$

  transforms a reflection Γ at the end of the line to its input as

  $$\Gamma_{in} = \Gamma\, e^{-j2\beta\ell} = \Gamma\, e^{-j\pi} = -\Gamma$$

- **This results the unitless, normalized impedance z at the end of the line to be transformed into:**

  $$z_{in} = {}^1/_z$$

  at the beginning of the line

- **A short circuit at the end of the $\lambda/4$-transformer is transformed to an open, and vice versa**
  - **This is the principle of the $\lambda/4$-resonator.**

Final Remarks

- **The *Smith* chart is a special type of a parametric plot for the complex impedance or admittance, mapped to the complex reflection coefficient.**
- **The use of the *Smith* chart can be viewed in two ways:**
  - **As a calculation and impedance matching tool as originally envisioned.**
    - However, today this will happen rather rarely!
    - Please notice: All the examples presented are based on the paper-style *Smith* chart, which is always based on an unitless, normalized impedance $z = Z/Z_0$
  - **As a visualization tool for the complex impedance, along with the reflection coefficient**
    - Still very popular and useful for displaying and analyzing S_{ii} on a vector network analyzer, also used in data-sheets, and RF simulation and education software.
    - Here the *Smith* chart utilizes the actual complex impedance Z in units of Ω! Markers on the parametric trace give all relevant information, including the element values of a selected equivalent circuit.
- **Old, but excellent information on transmission-lines and standing waves:**
  - https://www.youtube.com/watch?v=I9m2w4DgeVk
  - https://www.youtube.com/watch?v=DovunOxlY1k&t=38s
- ***Smith* chart education software (only for MS-Windows):**
  - https://www.fritz.dellsperger.net/smith.html

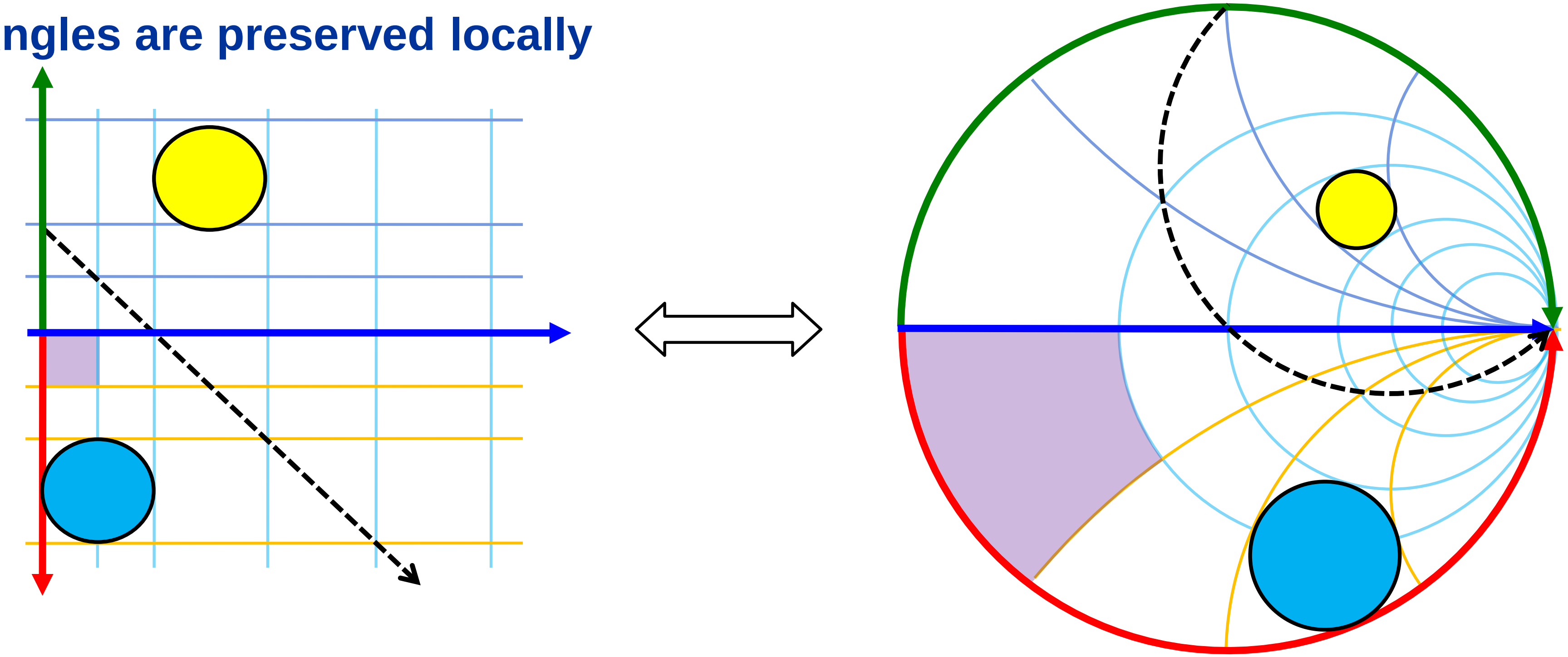
Backup Slides

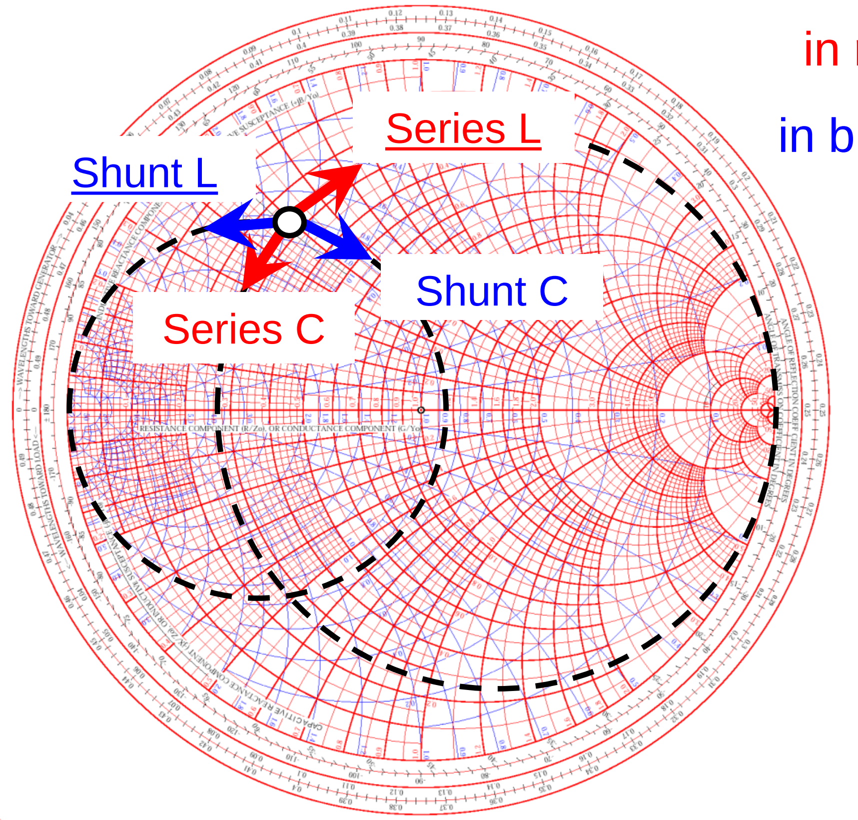
Navigation in the *Smith* Chart (1)

This is a "bilinear" transformation with the following properties:

- **Generalized circles are transformed into generalized circles**
  - **circle → circle**
  - **straight line → circle**
  - **circle → straight line**
  - **straight line → straight line**
- **Angles are preserved locally**

- a straight line is equivalent to a circle with infinite radius
- a circle is defined by 3 points
- a straight line is defined by 2 points



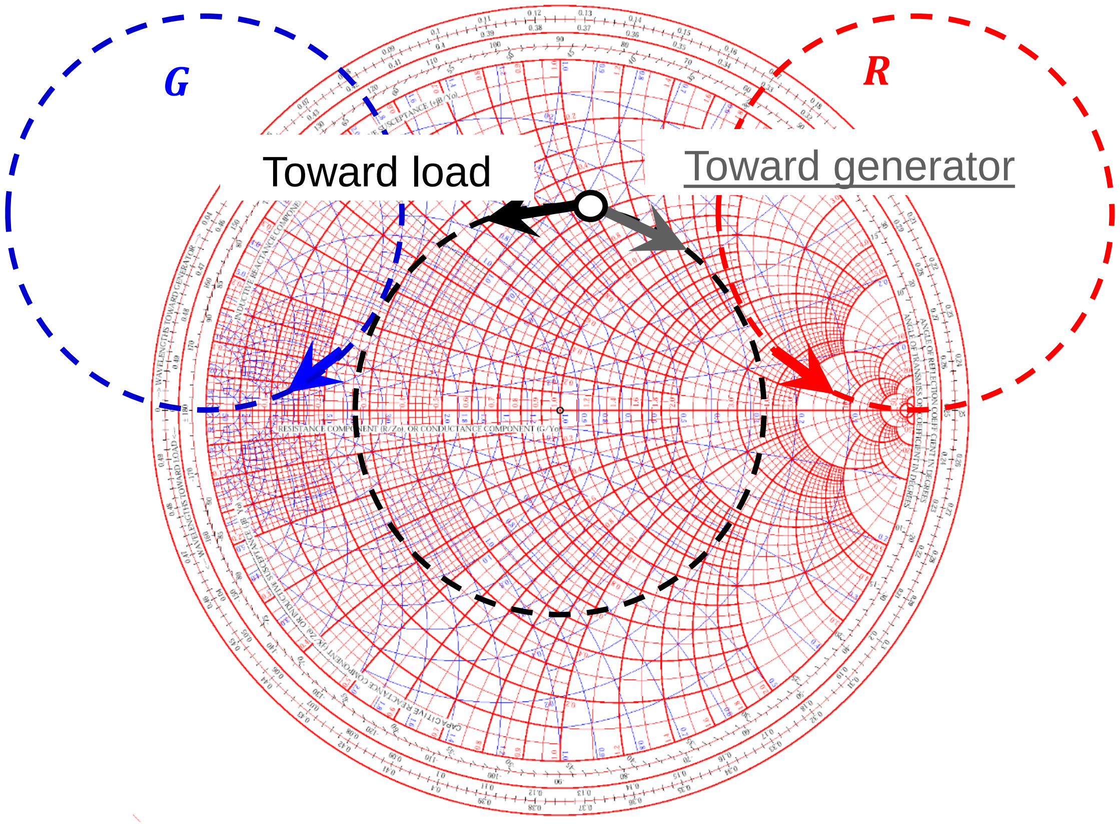


in red: impedance plane ($= \boldsymbol{z}$)

in blue: admittance plane ($= \boldsymbol{y}$)

	Up	**Down**
Red circles	**Series L**	**Series C**
Blue circles	**Shunt L**	**Shunt C**

Navigation in the *Smith* Chart (3)



Red arcs	Resistance R
Blue arcs	Conductance G
Con-centric circle	Transmission line going Toward load Toward generator