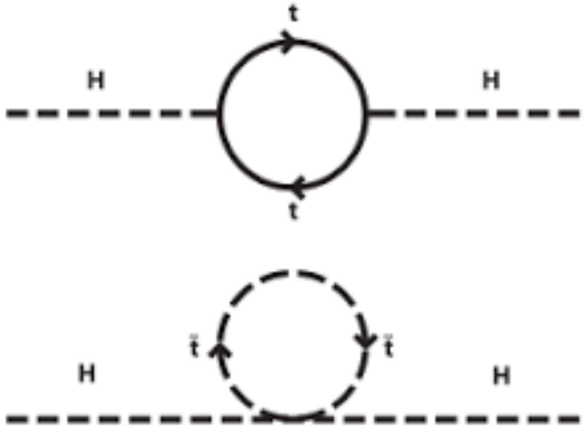
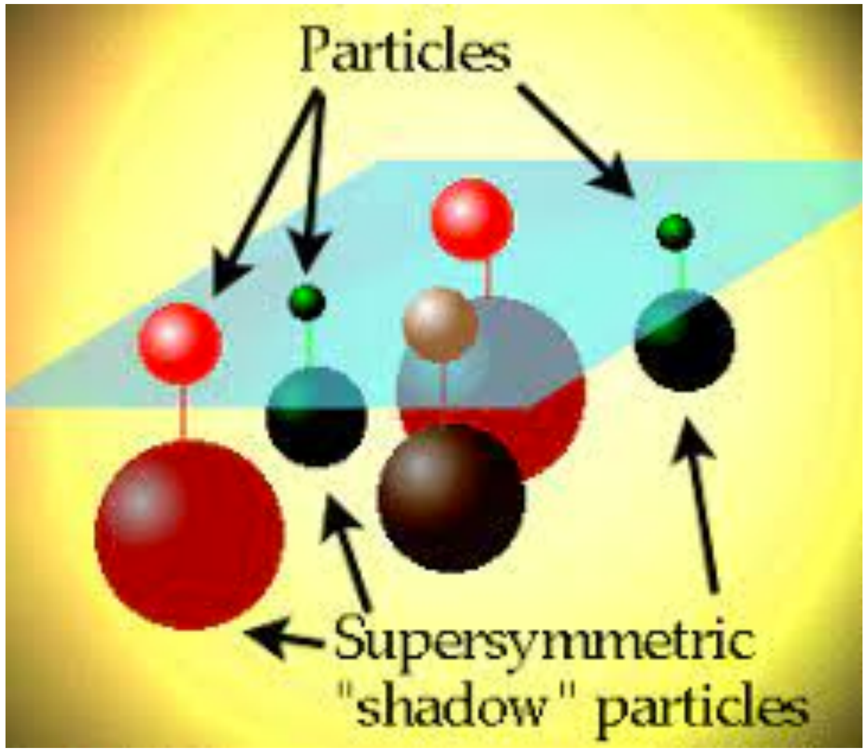
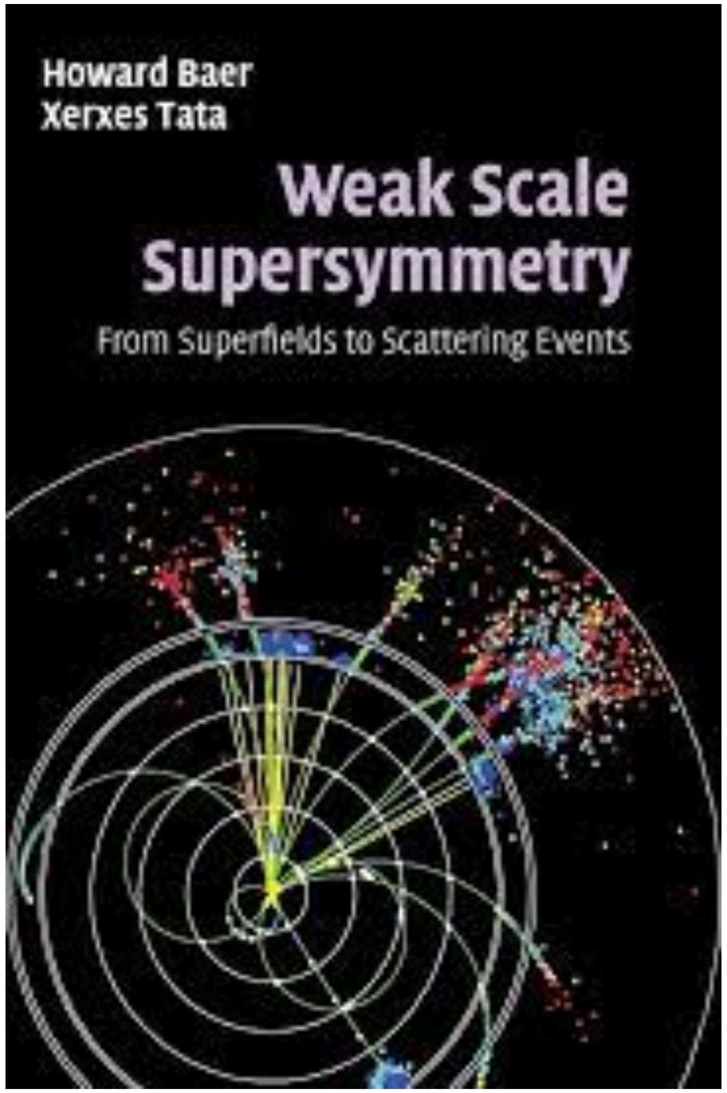
Supersymmetry phenomenology (including dark matter) 2023

Howard Baer
University of Oklahoma

preSUSY mtg,
July 12, 2023



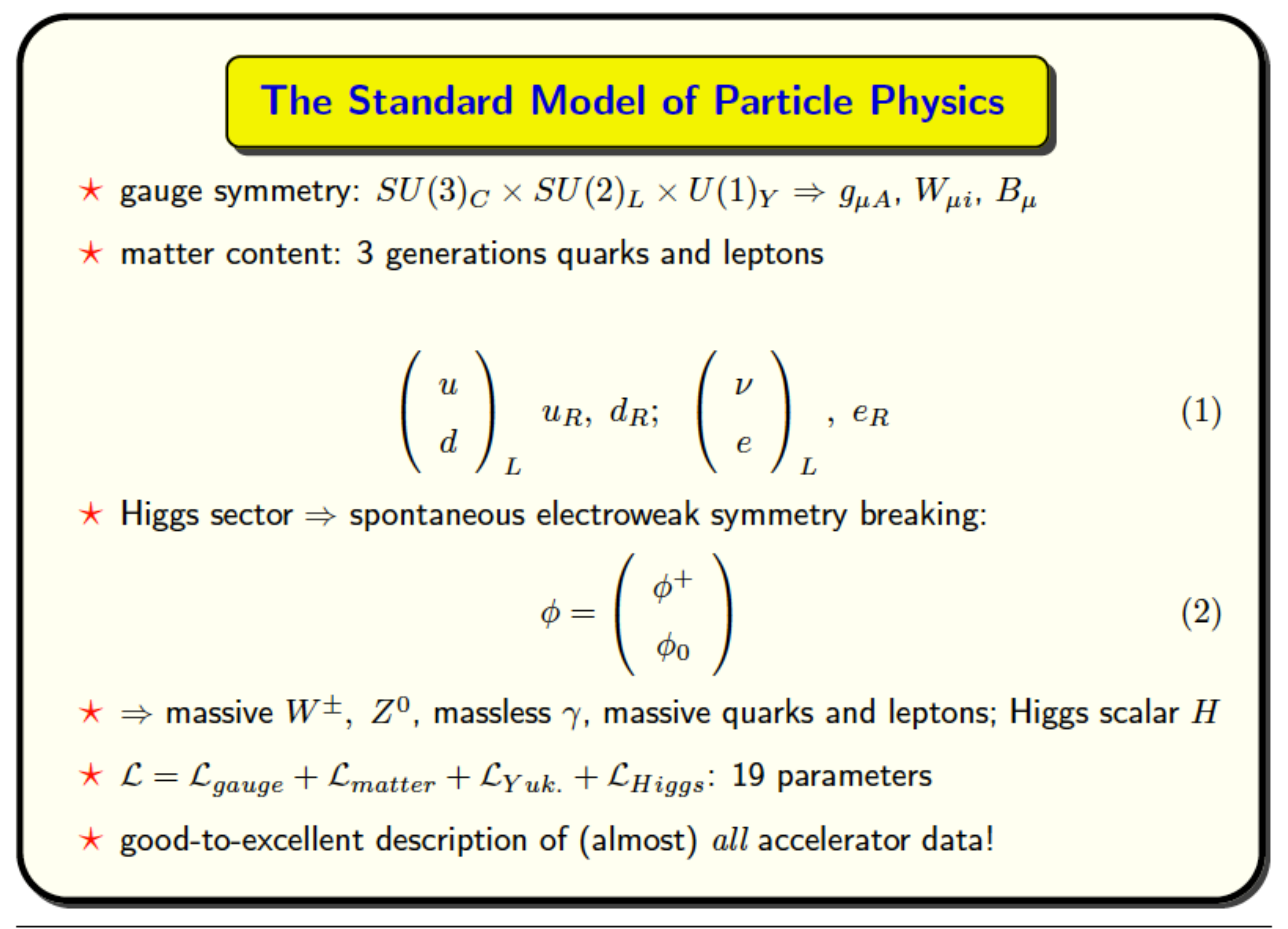




Howard Baer
Xerxes Tata

Weak Scale Supersymmetry

From Superfields to Scattering Events



The Standard Model of Particle Physics

★ gauge symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow g_{\mu A},\ W_{\mu i},\ B_\mu$

★ matter content: 3 generations quarks and leptons

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R,\ d_R; \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L,\ e_R \tag{1}$$

★ Higgs sector ⇒ spontaneous electroweak symmetry breaking:

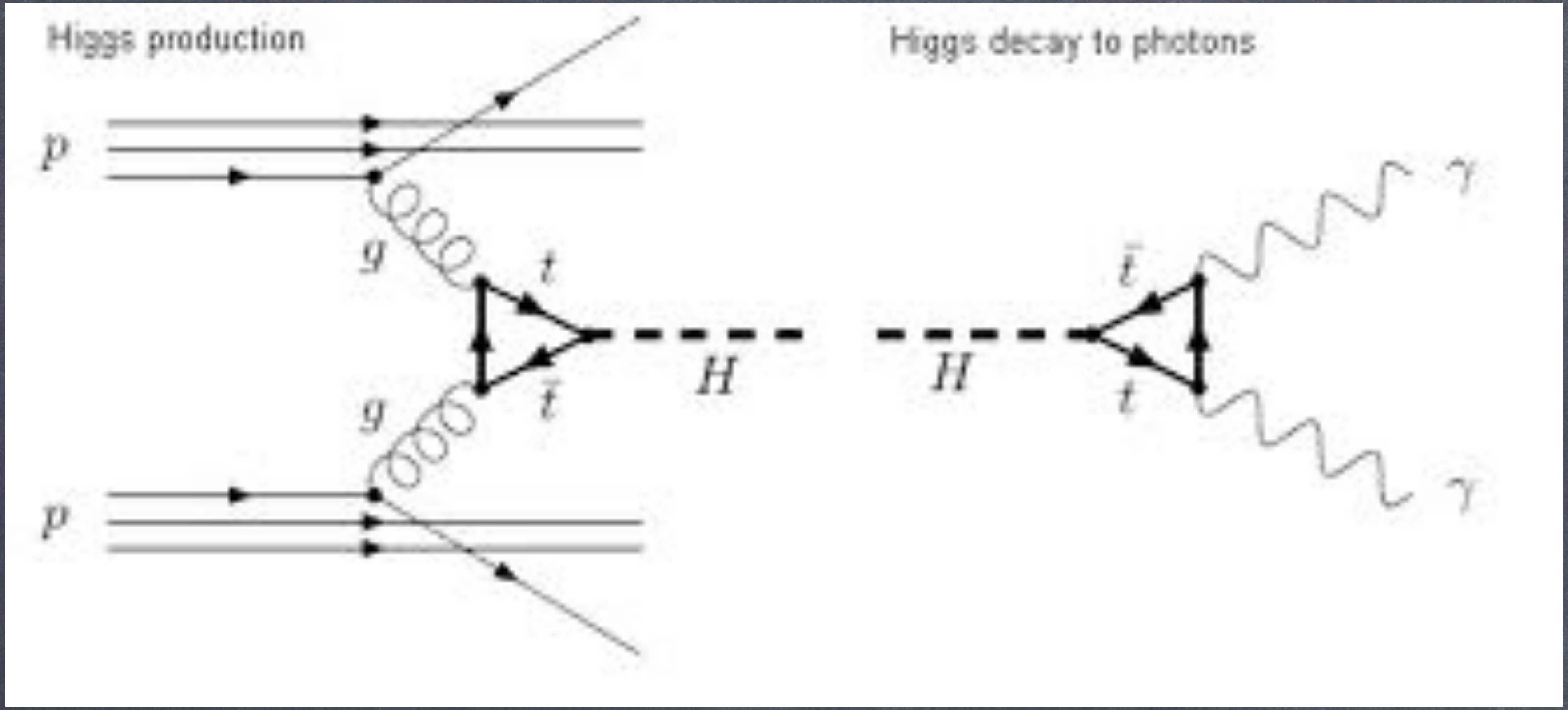
$$\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \tag{2}$$

★ ⇒ massive $W^\pm$, Z^0, massless γ, massive quarks and leptons; Higgs scalar H

★ $\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_{Yuk.} + \mathcal{L}_{Higgs}$: 19 parameters

★ good-to-excellent description of (almost) *all* accelerator data!

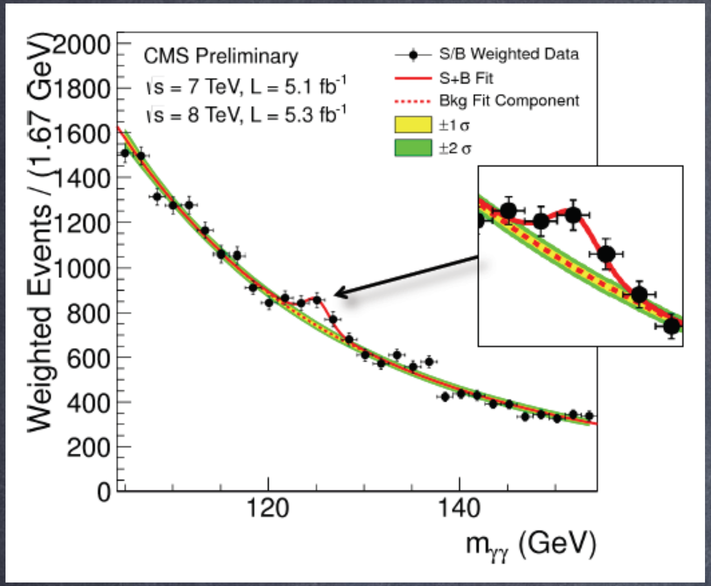
Last particle to be accounted for: the spin-0 Higgs boson!

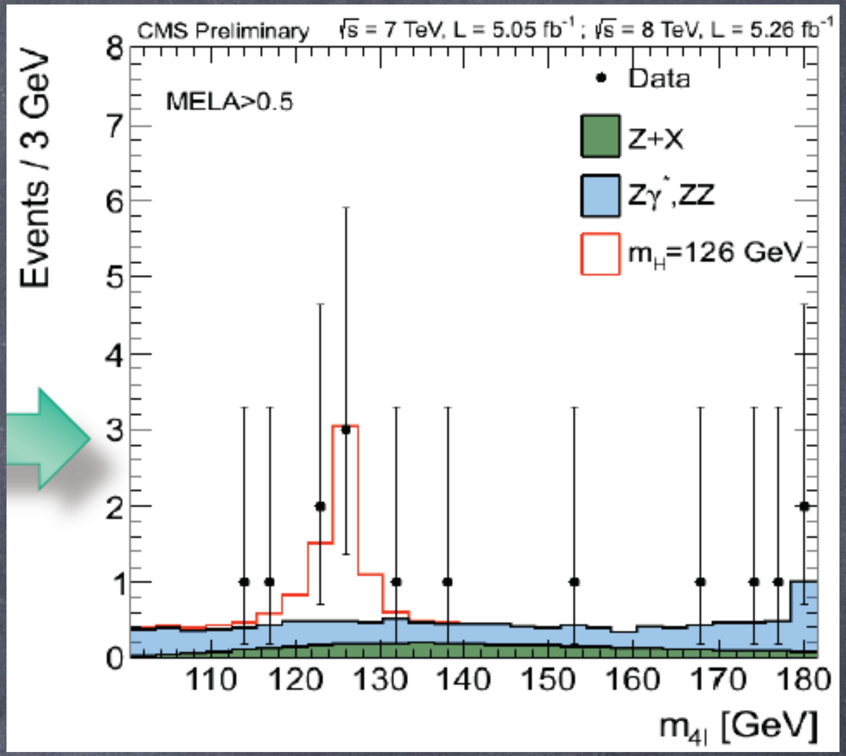


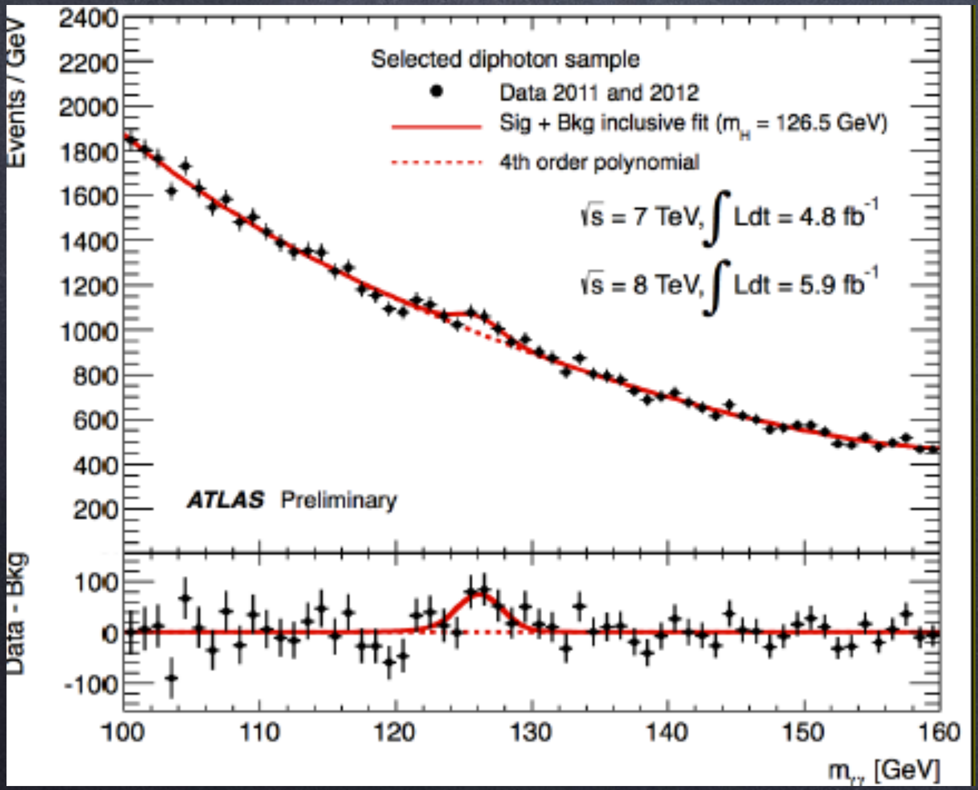
Can produce at LHC via gluon fusion;
mass reconstruction via decay
to 2 photons or 4 leptons

LHC Higgs discovery: July 4, 2012!

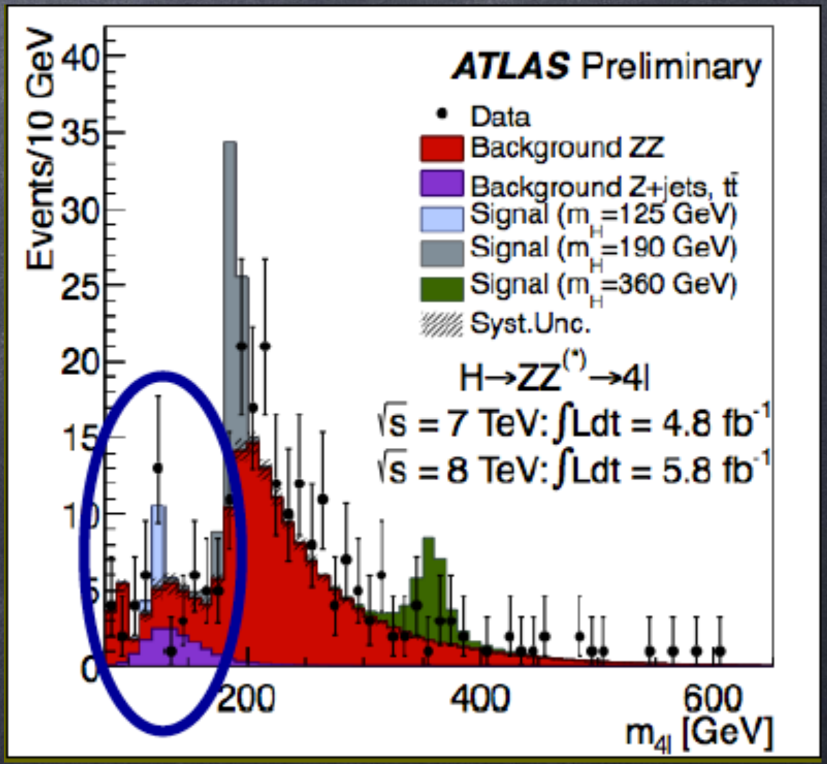
$$m_h \sim 125 \text{ GeV}$$







2013 Nobel



Excess of events also reported from CDF/D0

Standard Model is regarded as an ``effective field theory” valid at energy scales <~1 TeV

- Higgs mass instability
- strong CP problem
- cosmological constant/ dark energy
- unification with gravity
- origin of generations
- dark matter
- baryogenesis

The first three of these have to do with naturalness and fine-tuning

Introduce notion of practical naturalness:

HB, Barger, Savoy: arXiv:1509.02929

An observable $\mathcal{O}$ is *natural* if all contributions to $\mathcal{O}$ are $<\sim \mathcal{O}$

- *e.g* if $\mathcal{O} = a + b - c$, and if $a \gg \mathcal{O}$, then some *independent* contribution such as b would have to be fine-tuned to large opposite-sign value such as to maintain $\mathcal{O}$ at its measured value.
- Such a fine-tuning is regarded as unnatural and implausible, and indicative of some missing element in the theory (see Weinberg, Title page).
- A pit-fall occurs if $\mathcal{O} = a + b - b + c$ where $b \to large$, *i.e.* contributions are *dependent*: **combine dependent terms before evaluating fine-tuning!**

here is a pie baking metaphor:

1 kg pie= .2 kg(sugar)+.3 kg(flour)+.1 kg(water)+.5 kg(apples)
-.1 kg(evaporation)

Voila! It is very natural!

An unnatural recipe:

1 kg(pie)=.2 kg(sugar)+.3 kg(flour)+.5 kg(apples)+
10^4 kg(water)-10^4 kg(evaporation)

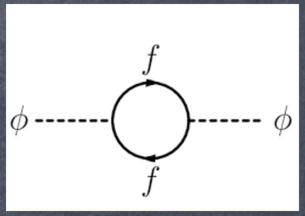
mathematically, it is possible-
but success seems highly implausible:
it is fine-tuned and hence
unnatural

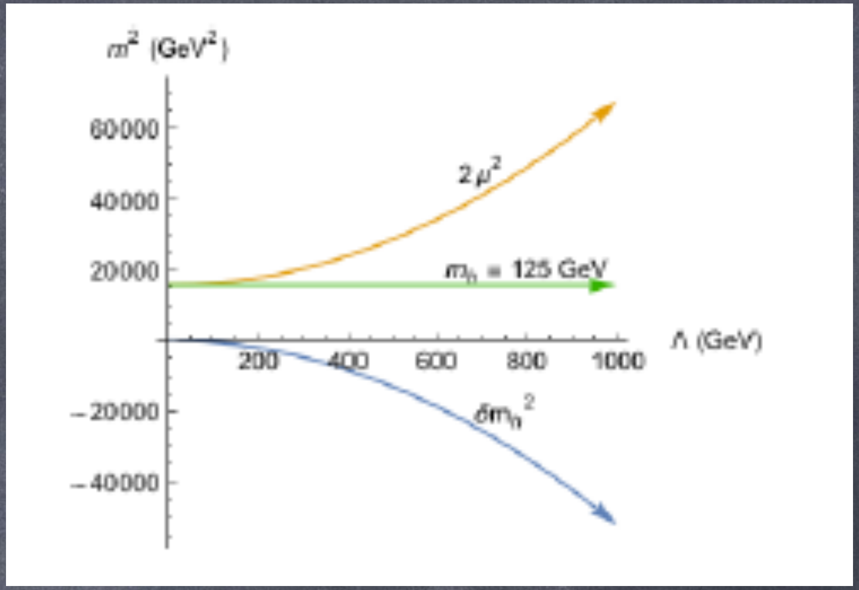
How the Higgs boson is like an apple pie

Biggest conundrum of SM: why is Higgs mass so small?

1. There is a lowest order mass term

2. Quantum corrections diverge quadratically with energy scale of new physics





$$m^2_{H_{SM}} = 2\mu^2 + \delta m^2_{H_{SM}}$$

$$\delta m^2_{H_{SM}} \simeq \frac{3}{4\pi^2}\left(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2\theta_W} + \lambda\right)\Lambda^2$$

3.To avoid the pathology of fine-tuning, SM must be valid only to Lambda~1 TeV

4. Need theory which is free of quadratic divergences to extend e.g. to GUT scale

Higgs mass (hierarchy) problem (SM):

$$V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$m_h^2 \simeq 2\mu^2 + \delta m_h^2$$

$$\delta m_h^2 \simeq \frac{3}{4\pi^2} \left(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8 \cos^2 \theta_W} + \lambda \right) \Lambda^2$$

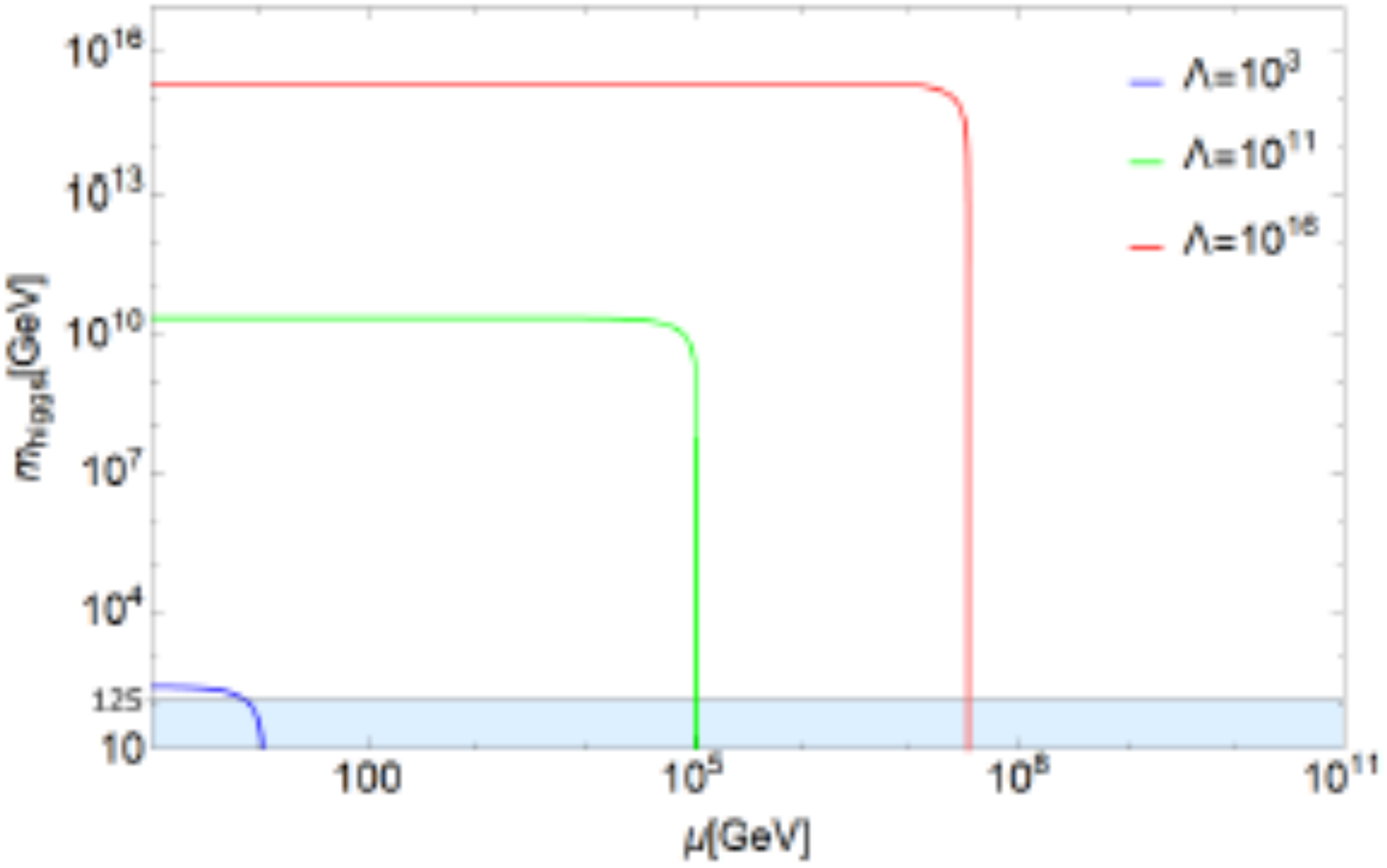
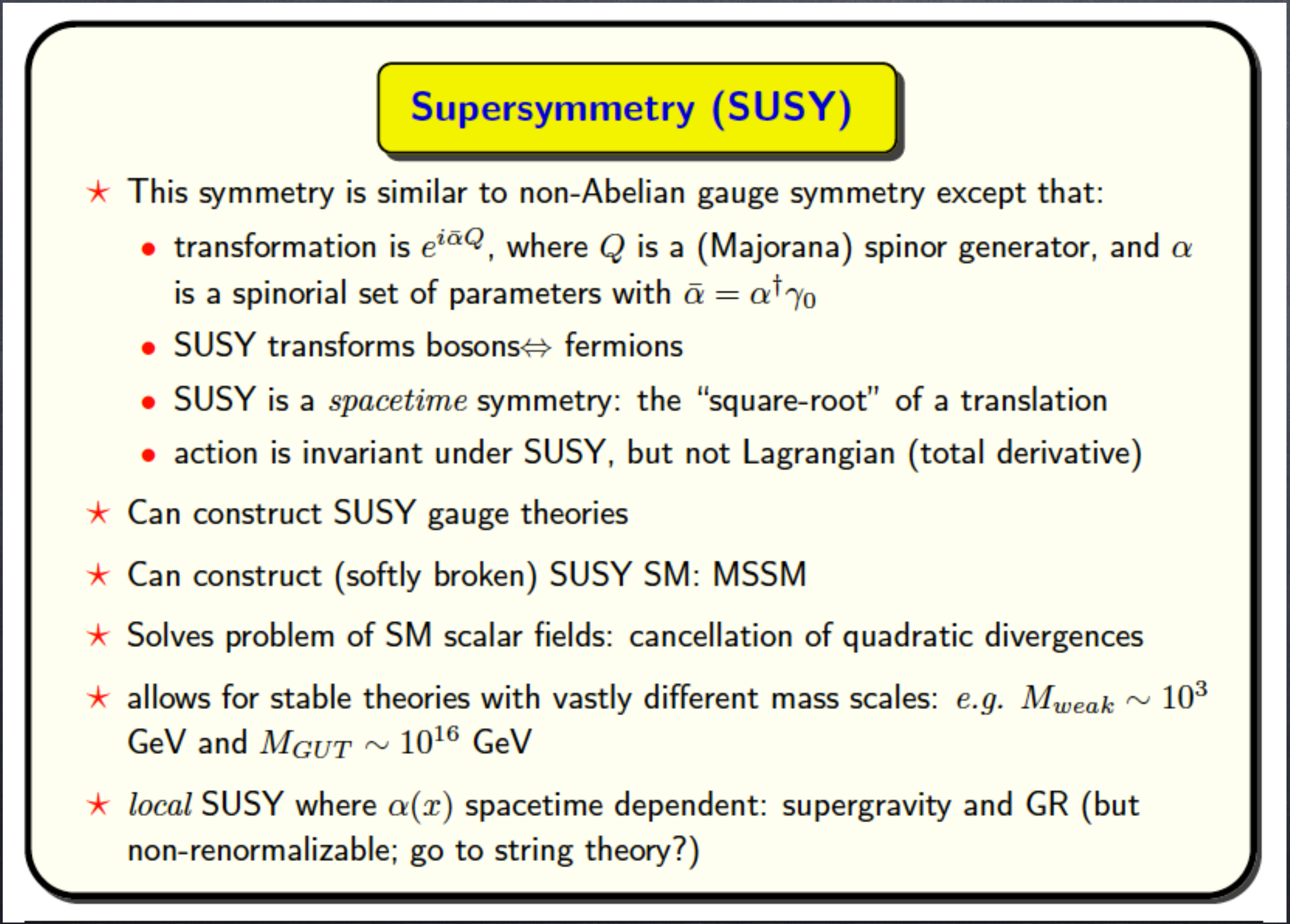


Figure 2: Value of $m_h(SM)$ versus SM μ parameter for theory cut-off values $\Lambda_{SM} = 10^3$, 10^{11} and 10^{16} GeV.

Hardly plausible that SM is valid much beyond the TeV scale



Supersymmetry (SUSY)

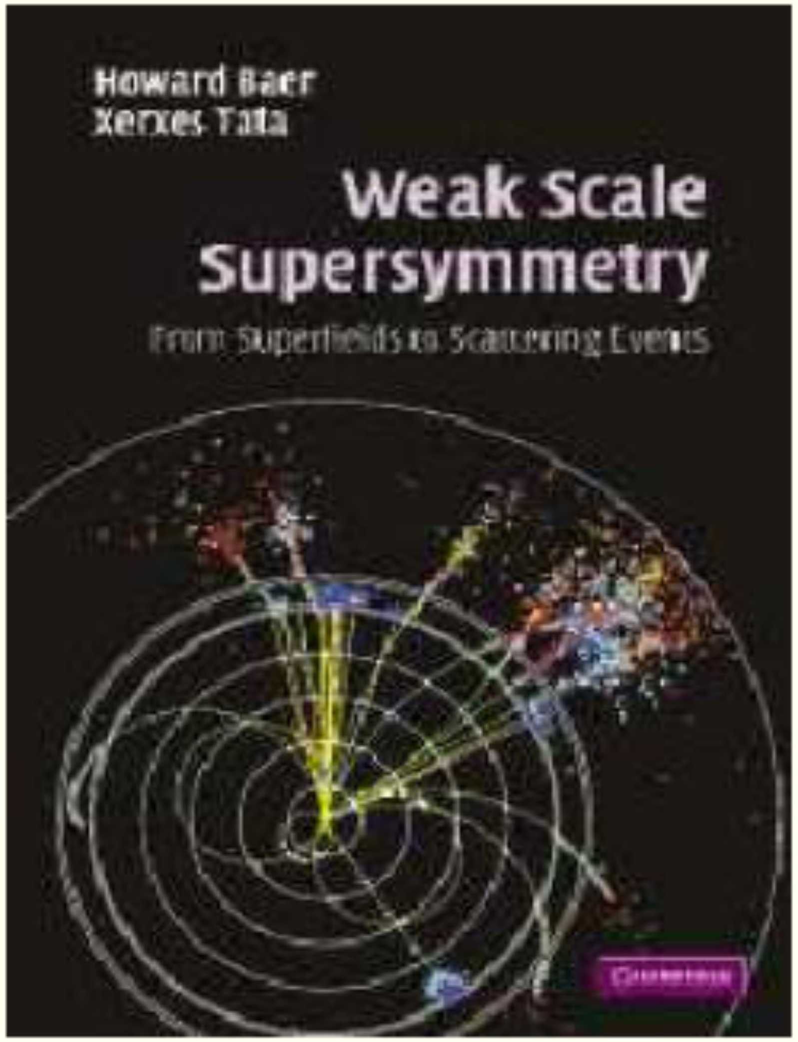
- ⋆ This symmetry is similar to non-Abelian gauge symmetry except that:
  - transformation is $e^{i\bar{\alpha}Q}$, where Q is a (Majorana) spinor generator, and α is a spinorial set of parameters with $\bar{\alpha} = \alpha^\dagger \gamma_0$
  - SUSY transforms bosons⇔ fermions
  - SUSY is a *spacetime* symmetry: the "square-root" of a translation
  - action is invariant under SUSY, but not Lagrangian (total derivative)
- ⋆ Can construct SUSY gauge theories
- ⋆ Can construct (softly broken) SUSY SM: MSSM
- ⋆ Solves problem of SM scalar fields: cancellation of quadratic divergences
- ⋆ allows for stable theories with vastly different mass scales: $e.g.\ M_{weak} \sim 10^3$ GeV and $M_{GUT} \sim 10^{16}$ GeV
- ⋆ *local* SUSY where $\alpha(x)$ spacetime dependent: supergravity and GR (but non-renormalizable; go to string theory?)

Weak Scale Supersymmetry

HB and X. Tata

Spring, 2006; Cambridge University Press

- Part 1: superfields/Lagrangians
  - 4-component spinor notation for exp'ts
  - master Lagrangian for SUSY gauge theories
- Part 2: models/implications
  - MSSM, SUGRA, GMSB, AMSB, $\cdots$
- Part 3: SUSY at colliders
  - production/decay/event generation
  - collider signatures
  - R-parity violation



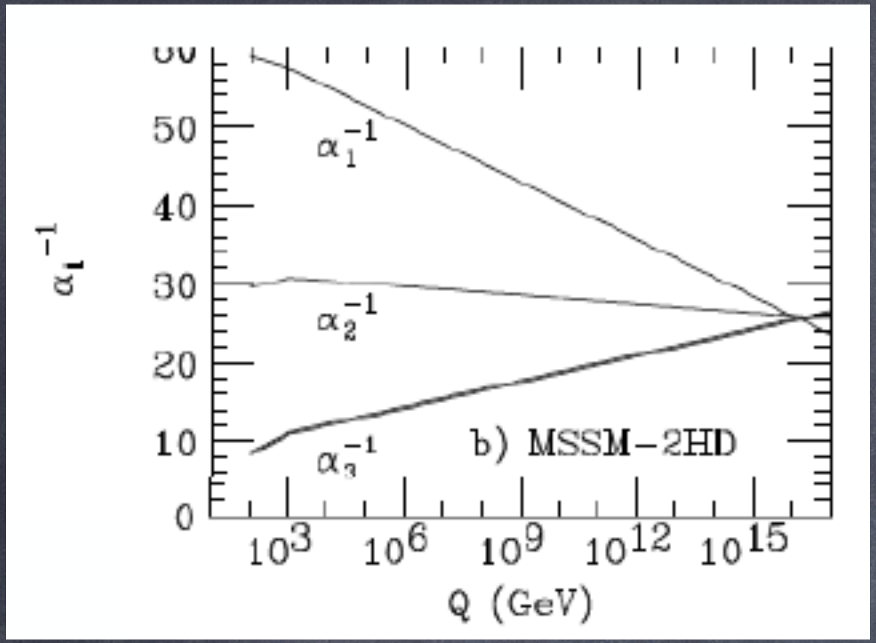
Minimal Supersymmetric Standard Model (MSSM)

- ⋆ Adopt gauge symmetry of Standard Model: $SU(3)_C \times SU(2)_L \times U(1)_Y$
  - gauge boson plus spin $\frac{1}{2}$ gaugino $\in$ gauge superfield
- ⋆ SM fermions $\in$ chiral scalar superfields: $\Rightarrow$ scalar partner for each SM fermion helicity state
  - electron $\Leftrightarrow$ $\tilde{e}_L$ and $\tilde{e}_R$
- ⋆ *two* Higgs doublets to cancel triangle anomalies: H_u and H_d
- ⋆ add all admissible soft SUSY breaking terms
- ⋆ resultant Lagrangian has 124 parameters!
- ⋆ Lagrangian yields mass eigenstates, mixings, Feynman rules for scattering and decay processes
- ⋆ predictive model!

Physical states of MSSM:

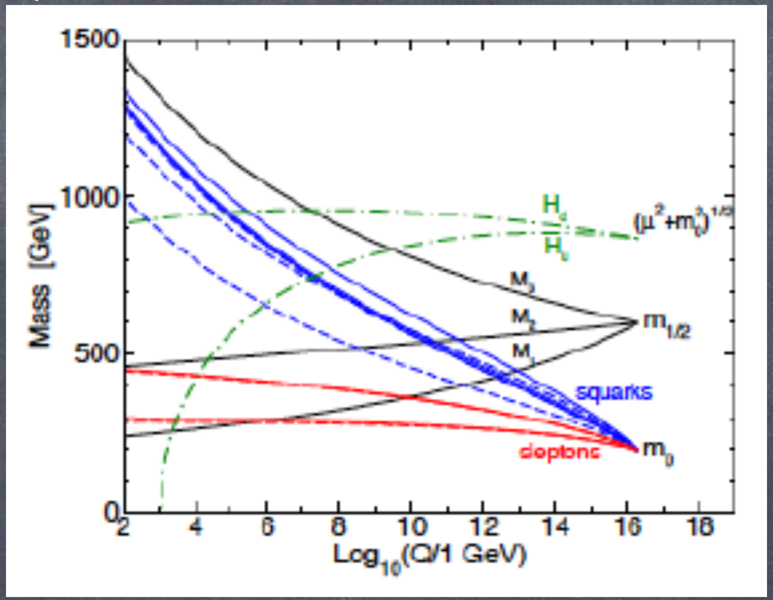
- ⋆ usual SM gauge bosons, quarks and leptons
- ⋆ gluino: $\tilde{g}$
- ⋆ bino, wino, neutral higgsinos⇒ neutralinos: $\widetilde{Z}_1, \widetilde{Z}_2, \widetilde{Z}_3, \widetilde{Z}_4$
- ⋆ charged wino, higgsino ⇒ charginos: $\widetilde{W}_1^{\pm}$, $\widetilde{W}_2^{\pm}$
- ⋆ squarks: $\tilde{u}_L$, $\tilde{u}_R$, $\tilde{d}_L$, $\tilde{d}_R, \cdots$, $\tilde{t}_1$, $\tilde{t}_2$
- ⋆ sleptons: $\tilde{e}_L$, $\tilde{e}_R$, $\tilde{\nu}_e, \cdots$, $\tilde{\tau}_1$, $\tilde{\tau}_2$, $\tilde{\nu}_\tau$
- ⋆ Higgs sector enlarged: h, H, A, $H^{\pm}$
- ⋆ a plethora of new states to be found at LHC/ILC?!

The MSSM is supported by virtual quantum effects!

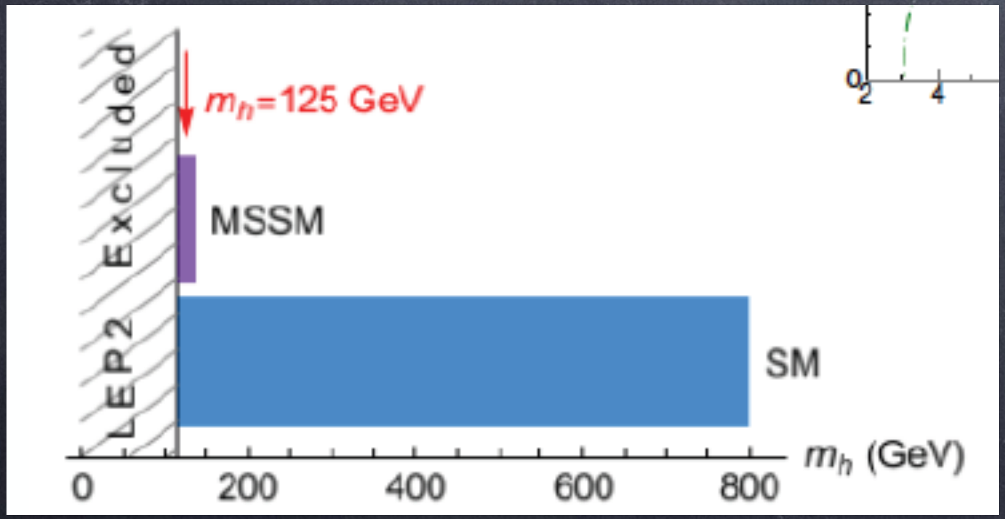


Unification of gauge couplings

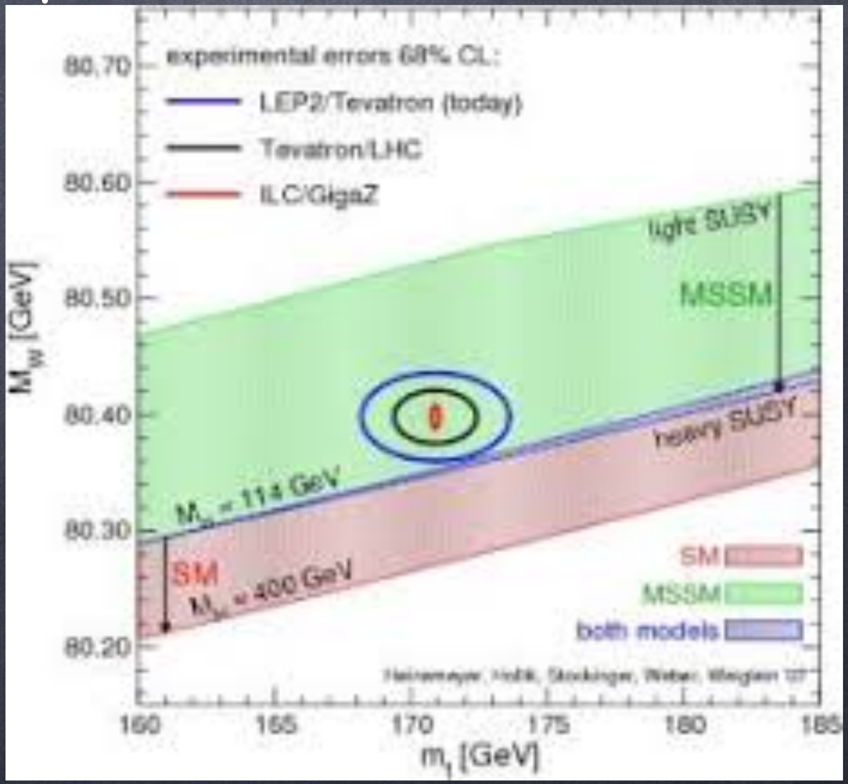
m(t)~150-200 GeV
required for radiative EWSB



m(h) just right



precision electroweak fits



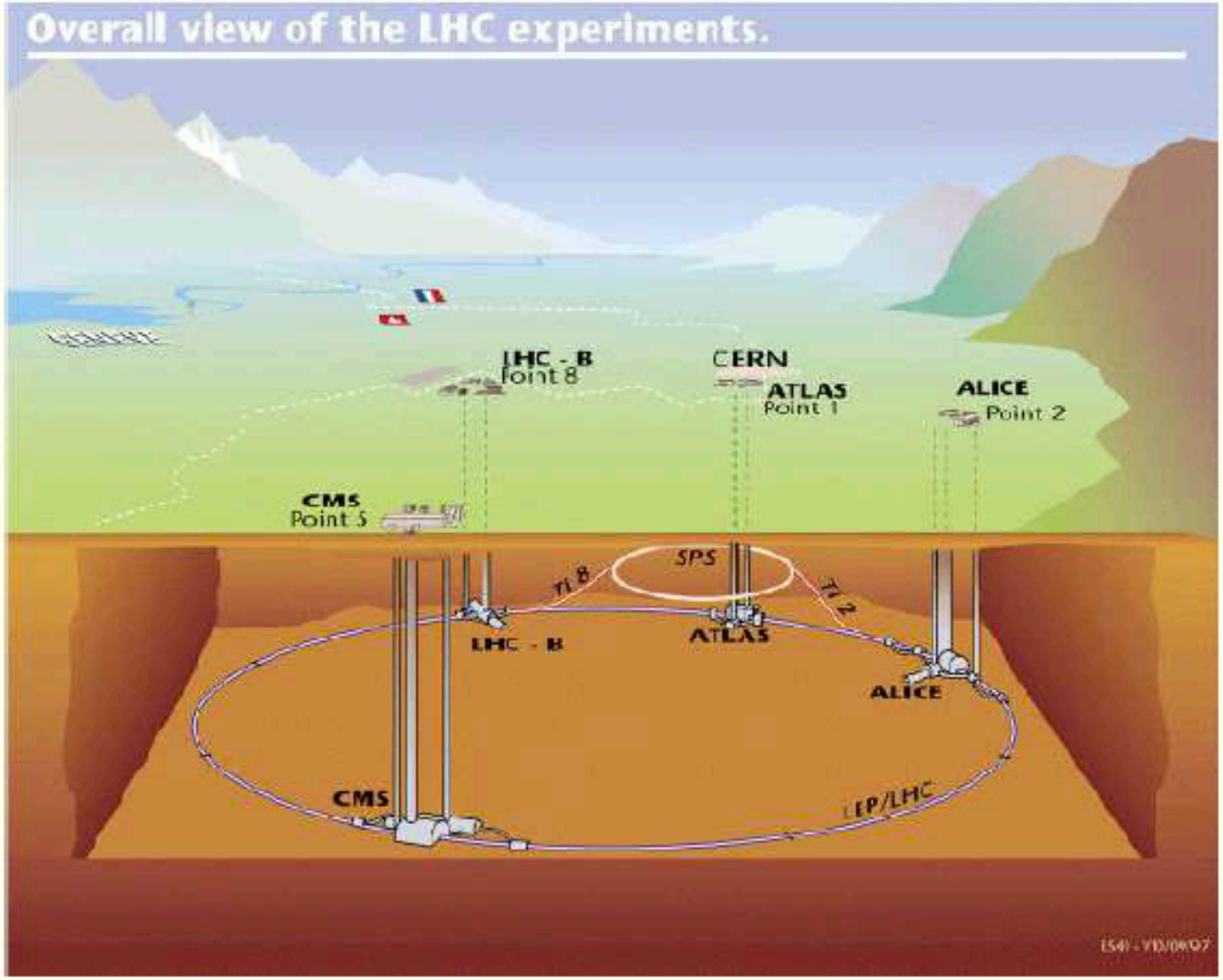
Radiative corrections have proven to be a reliable guide to new physics

Indirect: searches for virtual quantum effects

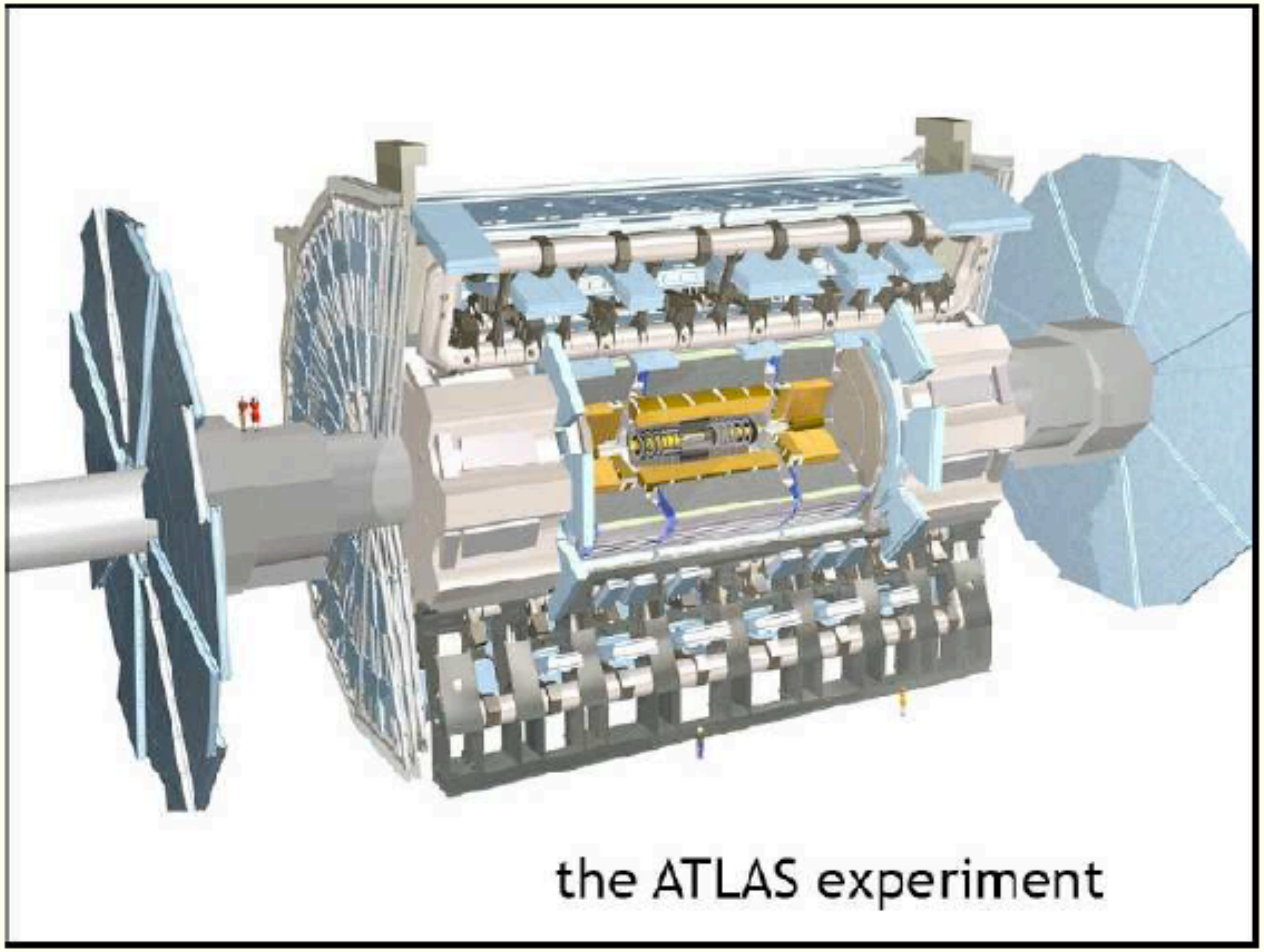
- $(g-2)_\mu$: at present, a 4σ deviation, but HVP contribution may not be reliable: sketchy data vs. lattice gauge (BMW, etc)
- R_K, R_{K^*} anomalies: LHCb, th/exp't now agree: arXiv:2212.09153
- R_D, R_{D^*} anomalies: also LHCb, th/exp't now agree: arXiv:2302.02886
- $b \to s\gamma$: th/exp't agree: $BF(b \to s\gamma \sim (3.4 \pm 0.17) \times 10^{-4}$
- $BF(B_s \to \mu^+\mu^-) = (3.09 \pm 0.45) \times 10^{-9}$: LHCb, th/exp't agree, CERN-EP-2021-133
- EDMe: $\left(\frac{5\ \mathrm{TeV}}{m_{\tilde{\ell}}}\right)^4 \frac{|\mu M_2|}{(1\ \mathrm{TeV})^2} \frac{\tan\beta}{10} \frac{\sin\theta_{CP}}{0.1} < \frac{d_e}{1.1\times 10^{-29}\ ecm}$
- EDMn: $\left(\frac{1.7\ \mathrm{TeV}}{m_{\tilde{q}}}\right)^4 \frac{|\mu M_2|}{(1\ \mathrm{TeV})^2} \frac{\tan\beta}{10} \frac{\sin\theta_{CP}}{0.1} < \frac{d_n}{1.8\times 10^{-26}\ ecm}$
- flavor: Δm_K, Δm_D, Δm_B, ϵ_K: need $m_{\tilde{f}} >\sim 5 - 50$ TeV (depending$\cdots$)

Each of these is solved or ameliorated by decoupling: TeV-scale SUSY

Layout of the LHC:two main detectors: Atlas and CMS

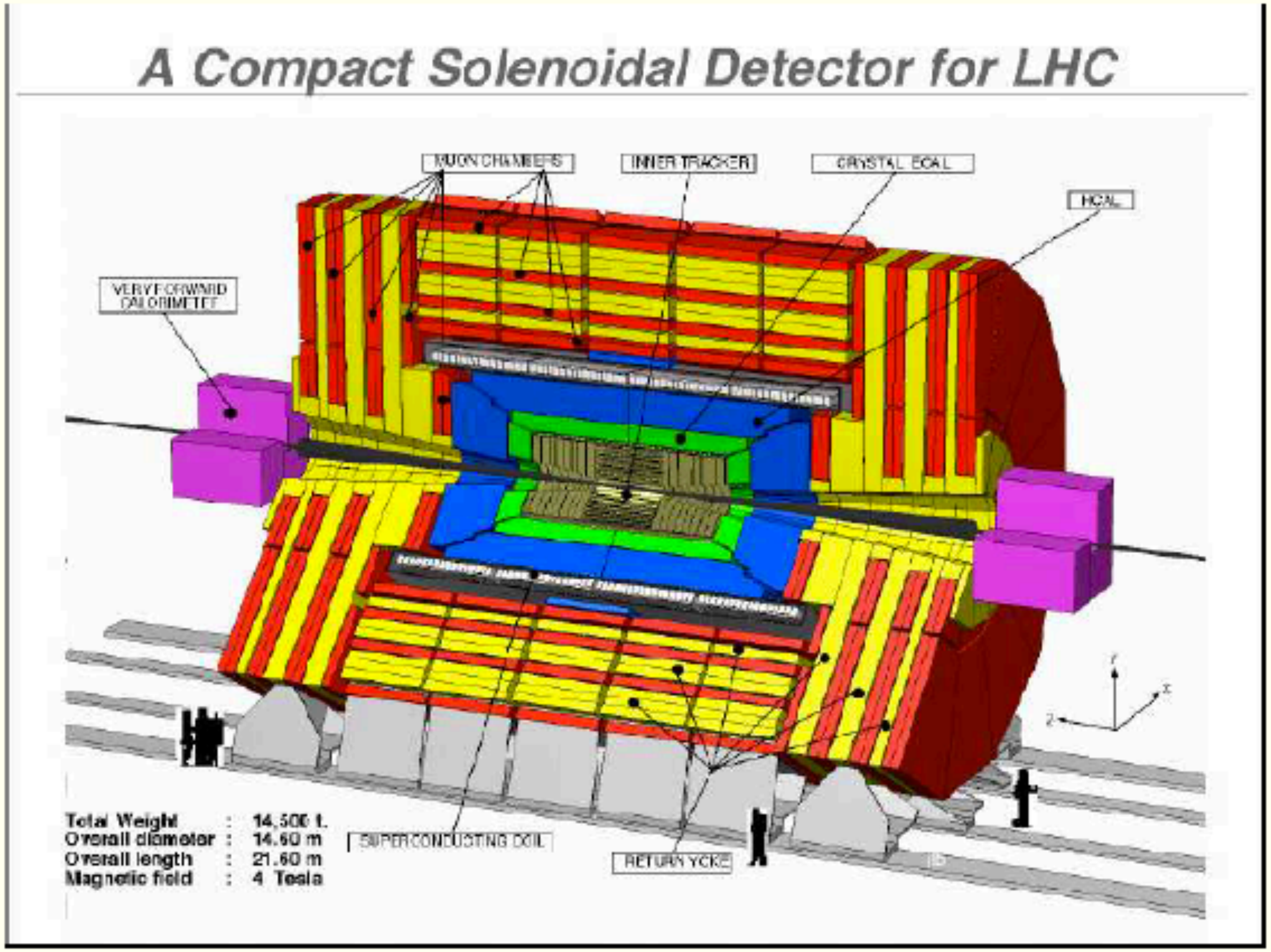


The Atlas detector



the ATLAS experiment

The CMS (Compact Muon Solenoid) detector



A Compact Solenoidal Detector for LHC

Total Weight : 14,500 t.
Overall diameter : 14.60 m
Overall length : 21.60 m
Magnetic field : 4 Tesla

Parton model of hadronic reactions

For a hadronic reaction,

$$A + B \to c + d + X,$$

where c and d are superpartners and X represents assorted hadronic debris, we have an associated subprocess reaction

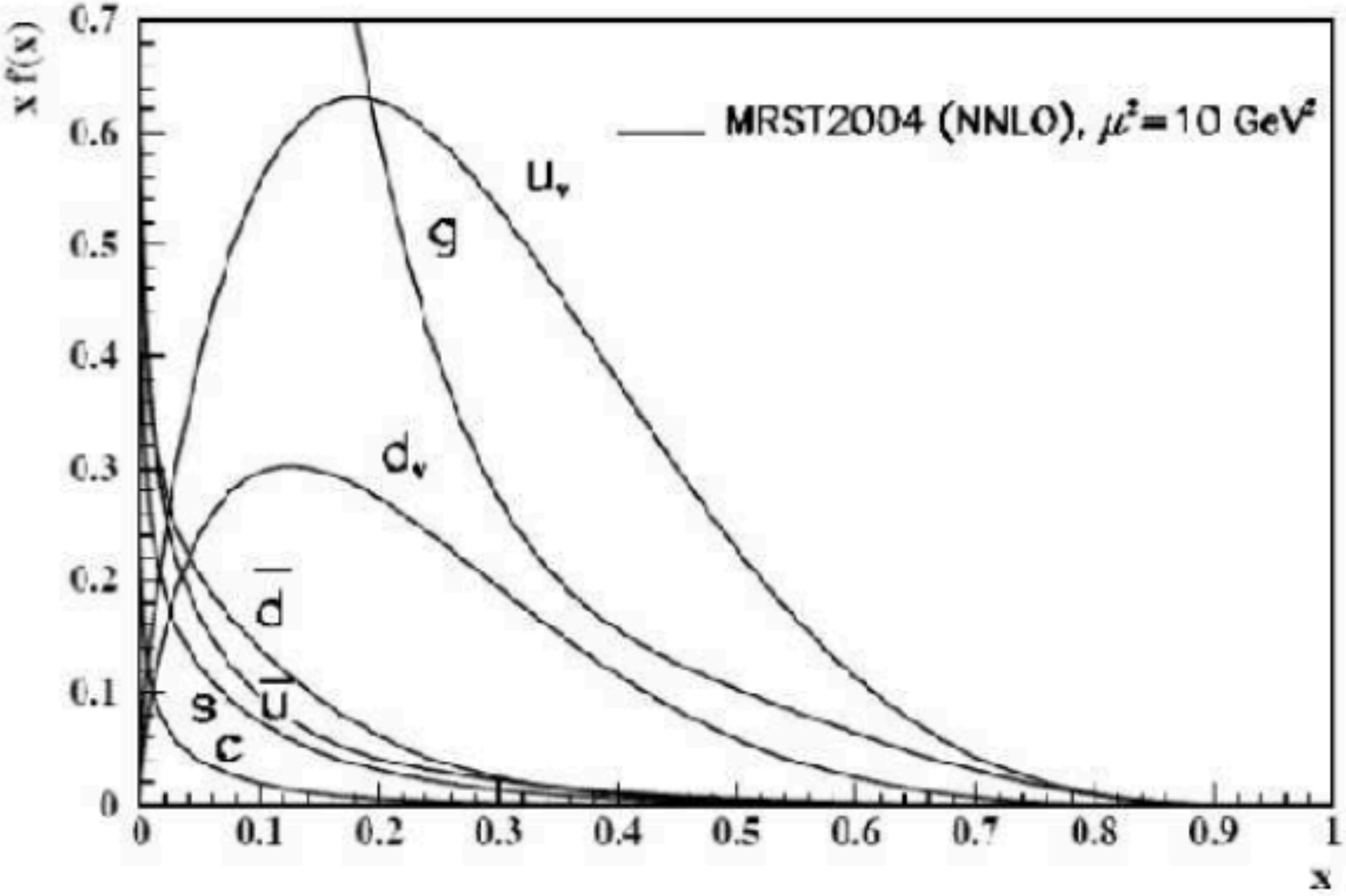
$$a + b \to c + d,$$

whose cross section can be computed using the Lagrangian for the MSSM. To obtain the final cross section, we must convolute the appropriate subprocess production cross section $d\hat{\sigma}$ with the parton distribution functions:

$$d\sigma(AB \to cdX) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b f_{a/A}(x_a, Q^2)\ f_{b/B}(x_b, Q^2)\ d\hat{\sigma}(ab \to cd).$$

where the sum extends over all initial partons a, b whose collisions produce the final state $c + d$.

Parton Distribution Functions (PDFs)



Calculating subprocess cross sections/decay rates in QFT

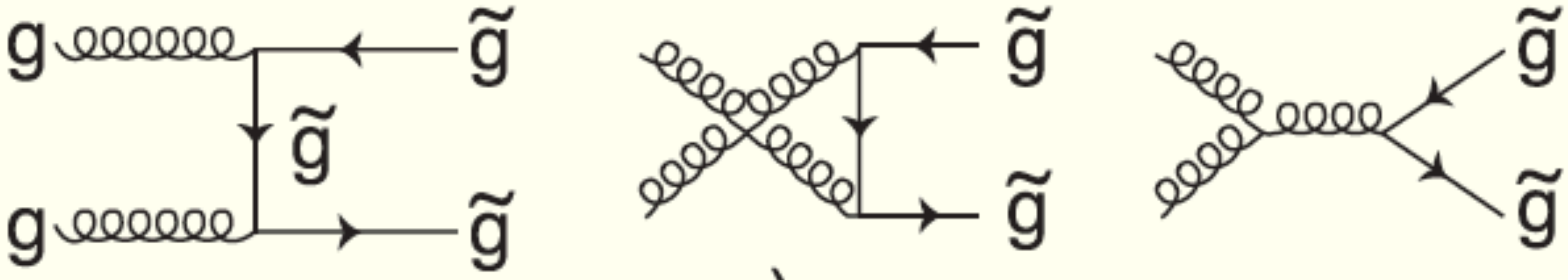
- The fundamental calculable object in QM is the *amplitude* $\mathcal{M}$ for a process to occur
- A pictorial representation of $\mathcal{M}$ is given by a *Feynman diagram*
- Feynman rules for many theories can be found in standard texts: *e.g.* Peskin& Schroeder, *Introduction to Quantum Field Theory*
- In the MSSM, an additional complication occurs due to presence of *Majorana* spinors
- Methods for handling these given *e.g.* in *Weak Scale Supersymmetry* (HB, X. Tata), or book by M. Drees, Godbole& Roy
- total amplitude $\mathcal{M}$ is sum of all different ways a process can occur
- $\mathcal{M}$ is a complex number; $|\mathcal{M}|^2$ gives probability
- must normalize and sum (integrate) over all momentum configurations to gain cross section, usually in *femtobarns*:

Calculating subprocess cross sections/decay rates in QFT

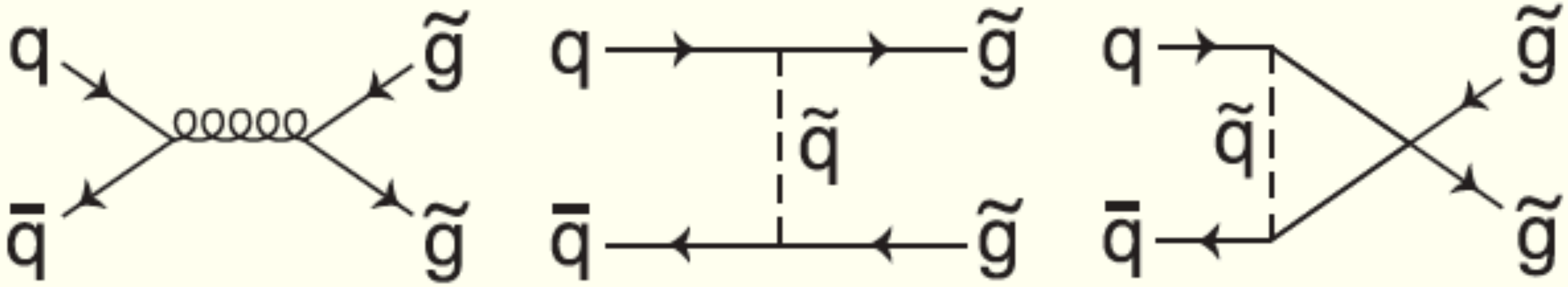
$$d\hat{\sigma} = \frac{1}{2\hat{s}}\frac{1}{(2\pi)^2}\int \frac{d^3p_c}{2E_c}\frac{d^3p_d}{2E_d}\delta^4(p_a+p_b-p_c-p_d)\cdot F_{\mathrm{color}}F_{\mathrm{spin}}\sum|\mathcal{M}|^2,$$

- Must sum (integrate) over all final state momentum configurations
- May be done analytically for simple processes *e.g.* $2 \to 2$
- Usually done using Monte Carlo method for $n \geq 3$
- Monte Carlo well suited for adding on particle decays so one has really $2 \to n$ processes where n can be very large
- Convolution of subprocess cross section with PDFs must be done numerically, since PDFs distributed as *subroutines*

Gluino pair production

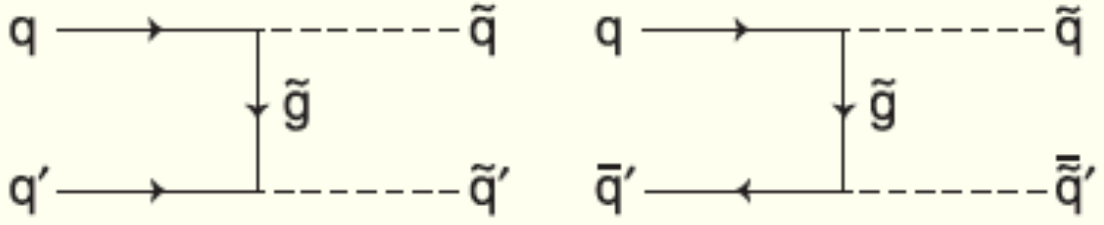


a)



b)

Squark pair production



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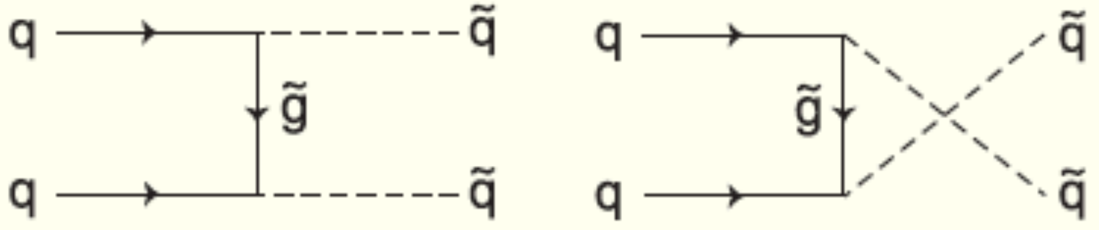
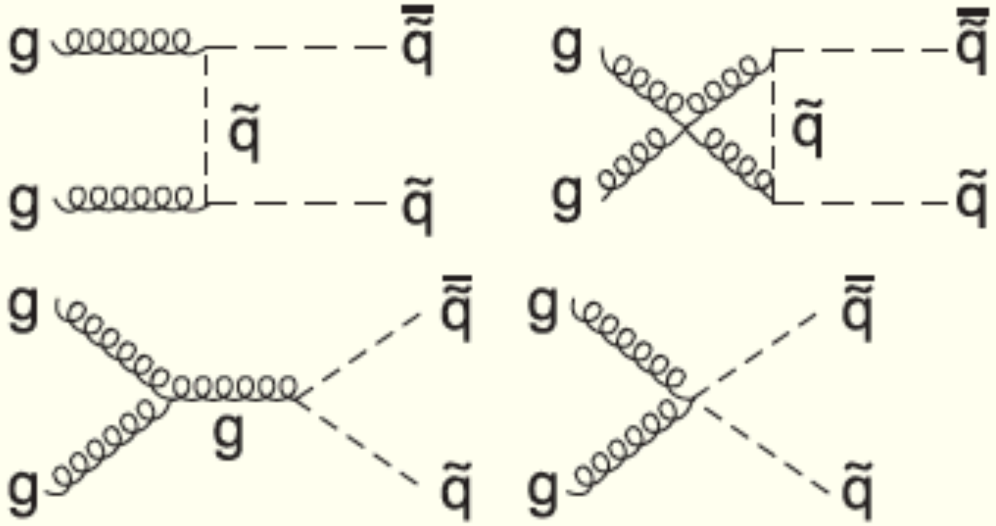
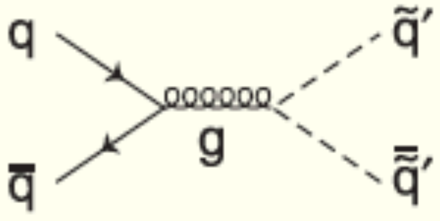


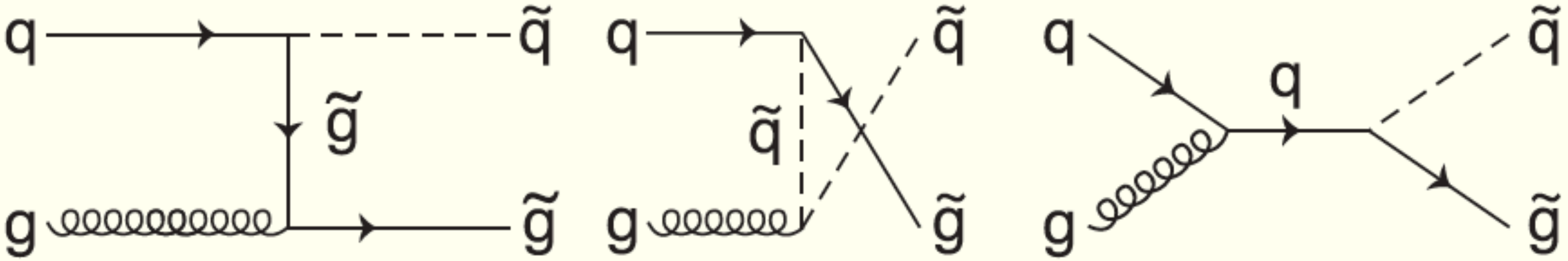
b)



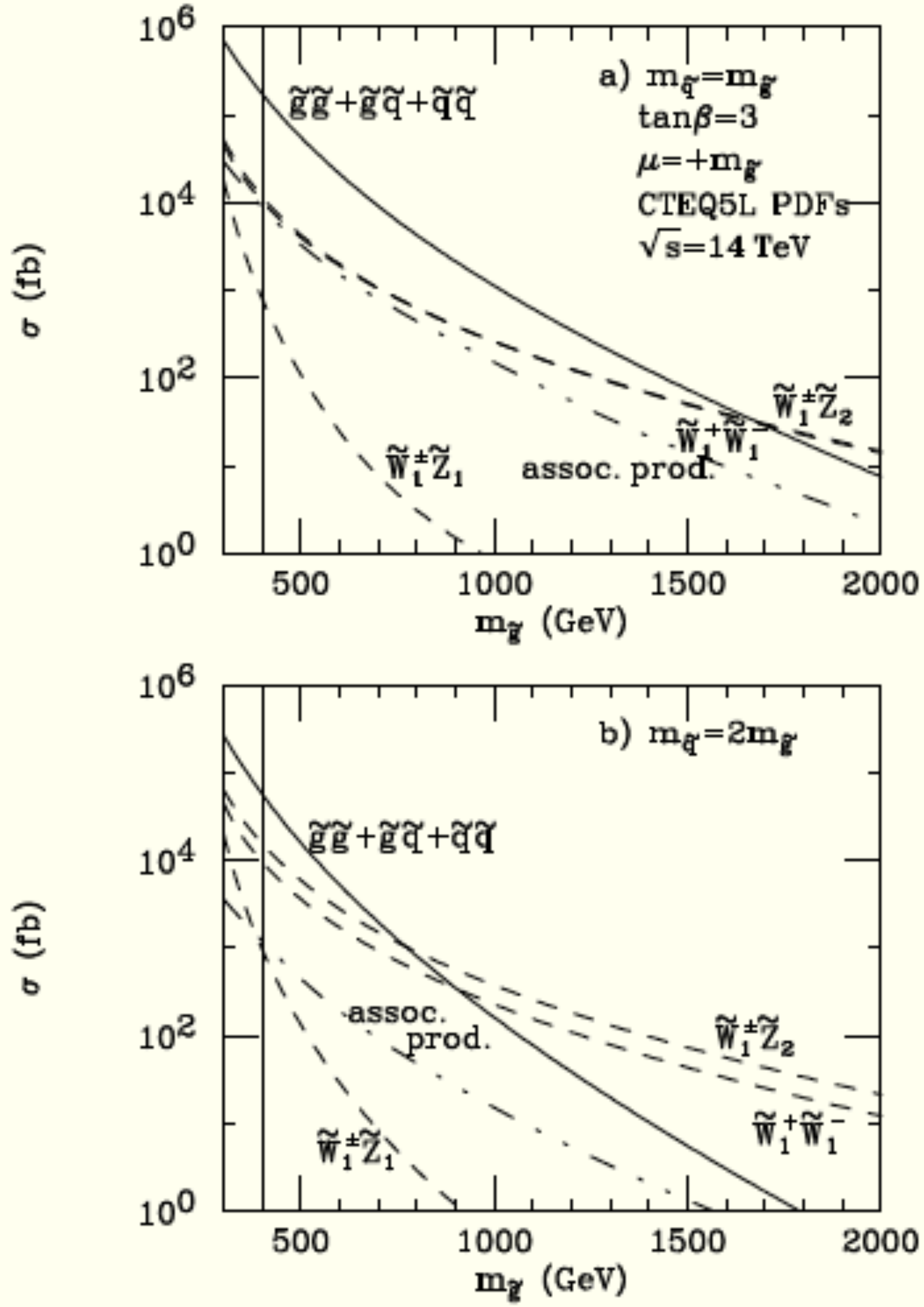


c)

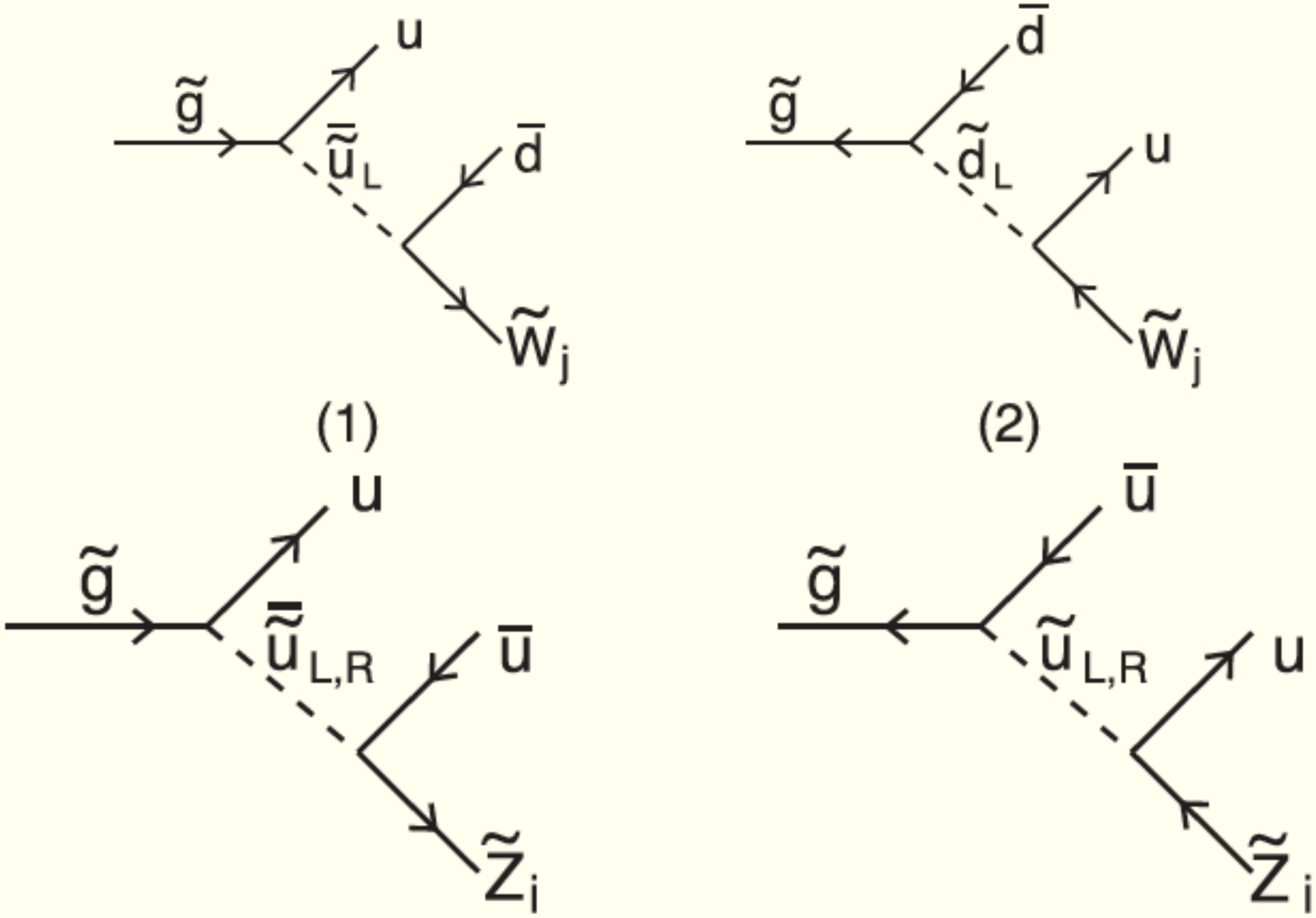
Gluino-squark associated production



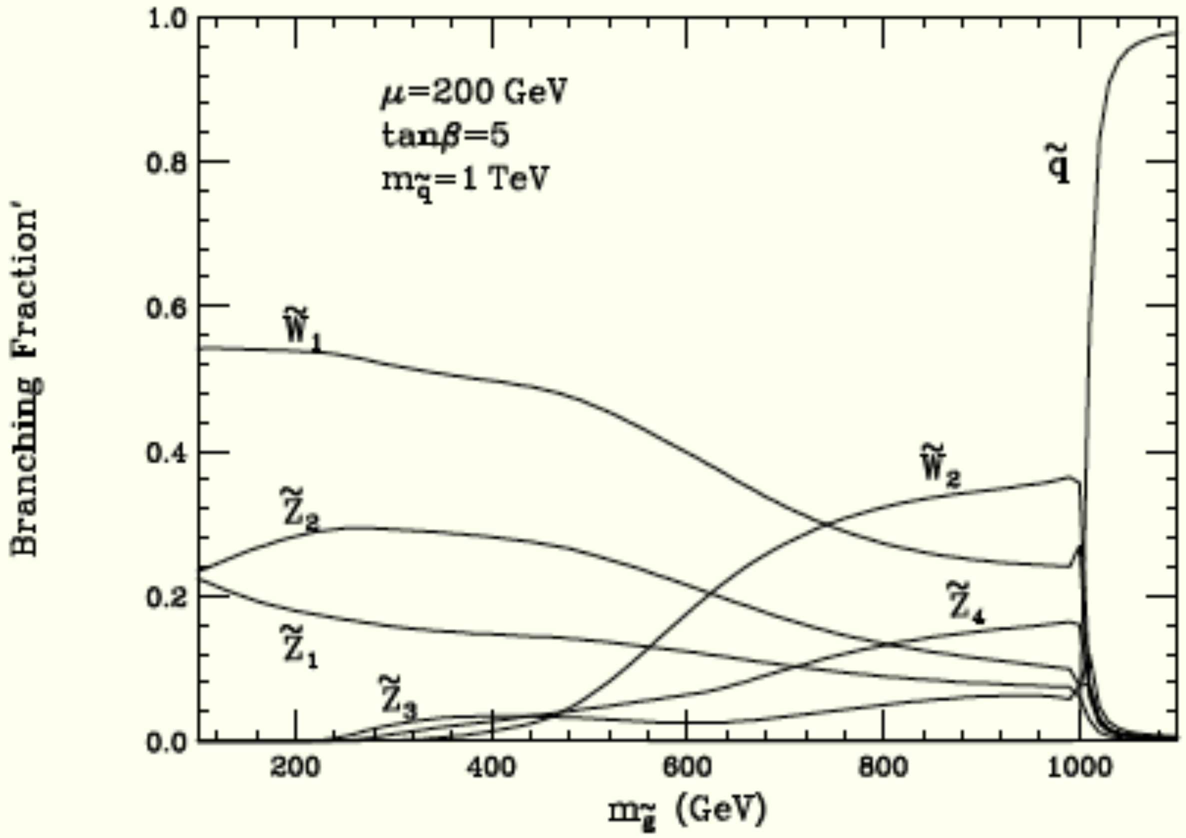
Production at LHC

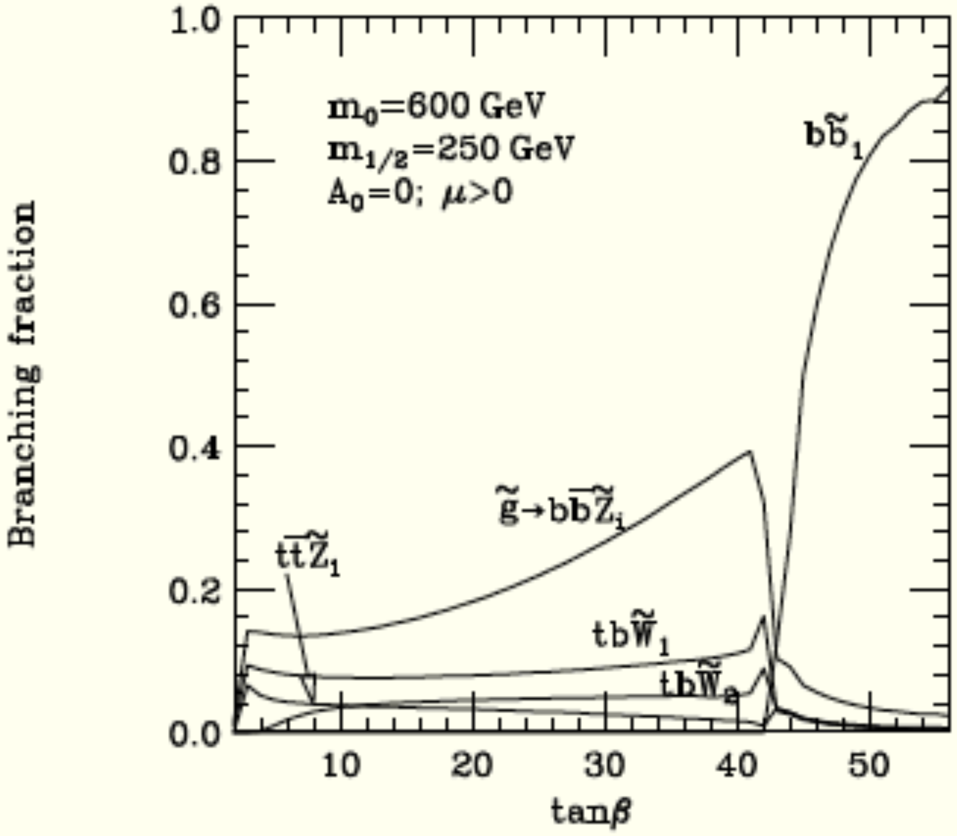


Gluino decays: $\tilde{g} \to q\tilde{q}$ or 3-body



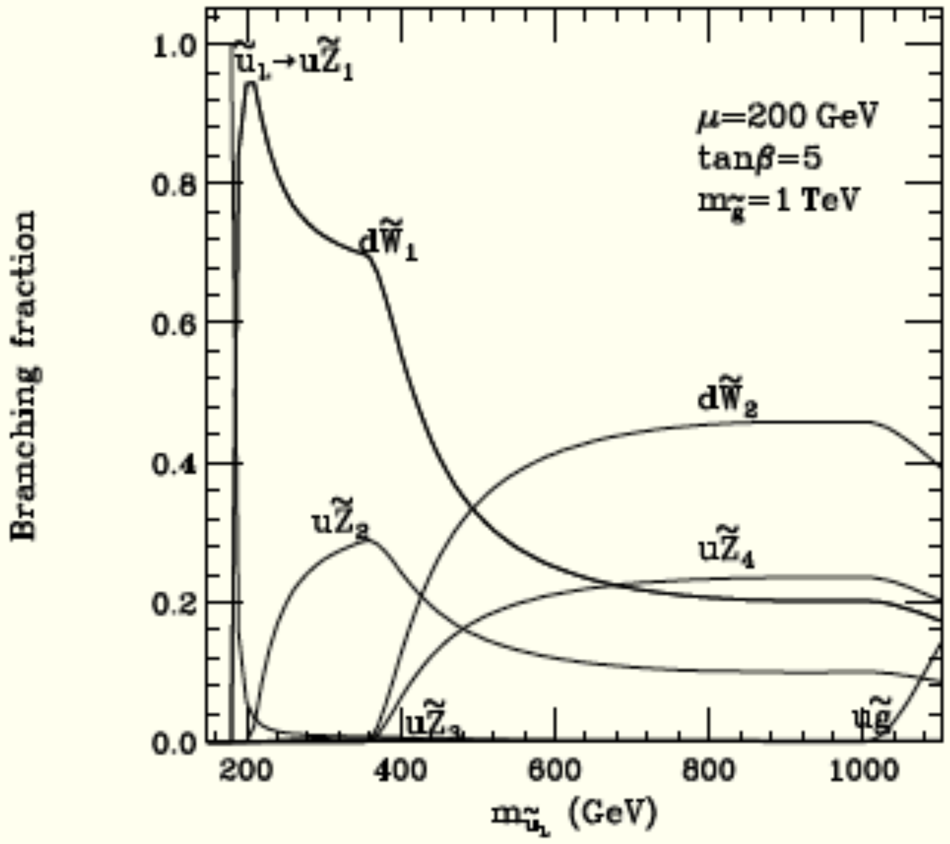
Gluino decays: branching fractions

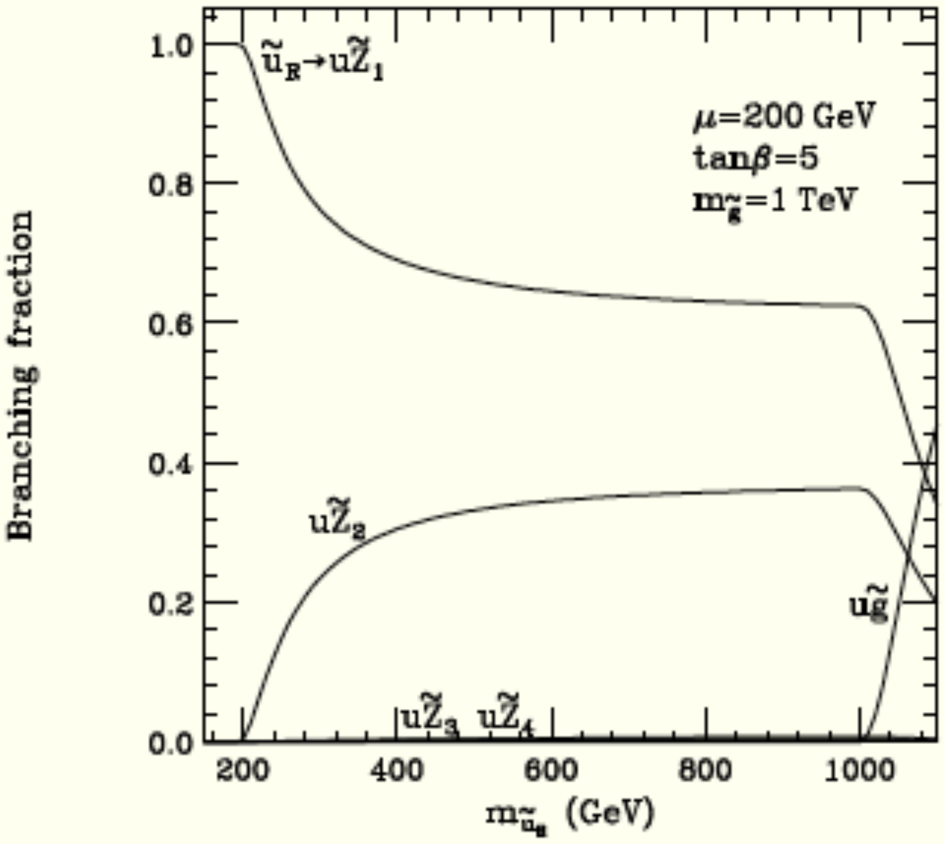




Squark decays

$$\begin{aligned}
\tilde{u}_L &\rightarrow u\widetilde{Z}_i,\ d\widetilde{W}_j^+,\ u\tilde{g}, \\
\tilde{d}_L &\rightarrow d\widetilde{Z}_i,\ u\widetilde{W}_j^-,\ d\tilde{g}, \\
\tilde{u}_R &\rightarrow u\widetilde{Z}_i,\ u\tilde{g}, \\
\tilde{d}_R &\rightarrow d\widetilde{Z}_i,\ d\tilde{g}.
\end{aligned}$$



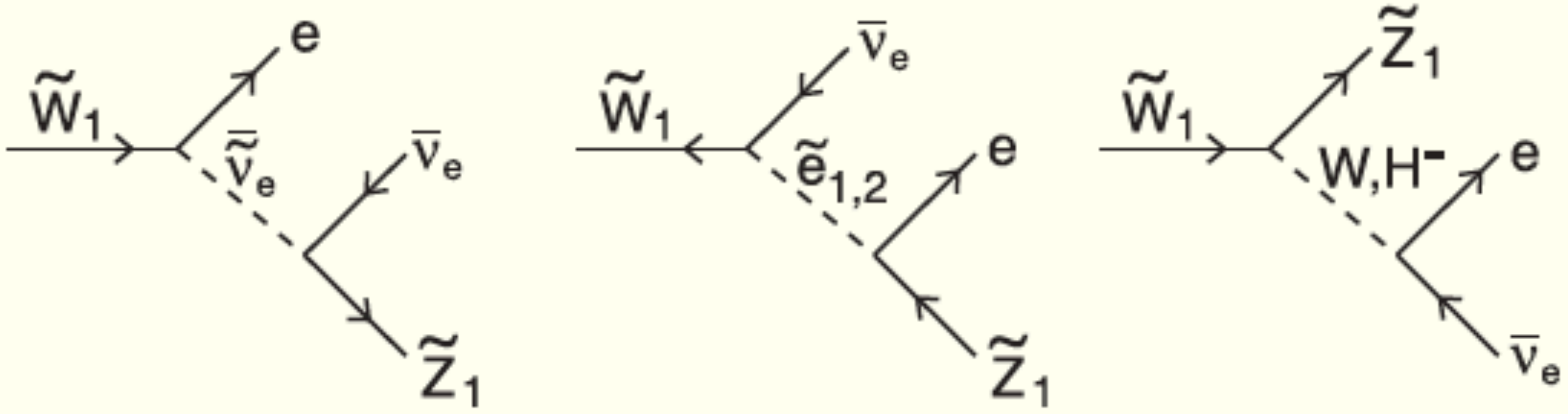


Chargino decays

$$\begin{array}{rcl} \widetilde{W}_j & \rightarrow & W\widetilde{Z}_i,\ H^-\widetilde{Z}_i, \\ & \rightarrow & \tilde{u}_L\bar{d},\ \bar{\tilde{d}}_L u,\ \tilde{c}_L\bar{s},\ \bar{\tilde{s}}_L c,\ \tilde{t}_{1,2}\bar{b},\ \tilde{b}_{1,2}t, \\ & \rightarrow & \tilde{\nu}_e\bar{e},\ \bar{\tilde{e}}_L\nu_e,\ \tilde{\nu}_\mu\bar{\mu},\ \bar{\tilde{\mu}}_L\nu_\mu,\ \tilde{\nu}_\tau\bar{\tau}, \bar{\tilde{\tau}}_{1,2}\nu_\tau,\ \text{and} \\ \widetilde{W}_2 & \rightarrow & Z\widetilde{W}_1,\ h\widetilde{W}_1,\ H\widetilde{W}_1 \text{ and } A\widetilde{W}_1. \end{array}$$

Charginos may decay to a lighter neutralino via

$$\widetilde{W}_j \rightarrow \widetilde{Z}_i + f\bar{f}' \ , \tag{1}$$

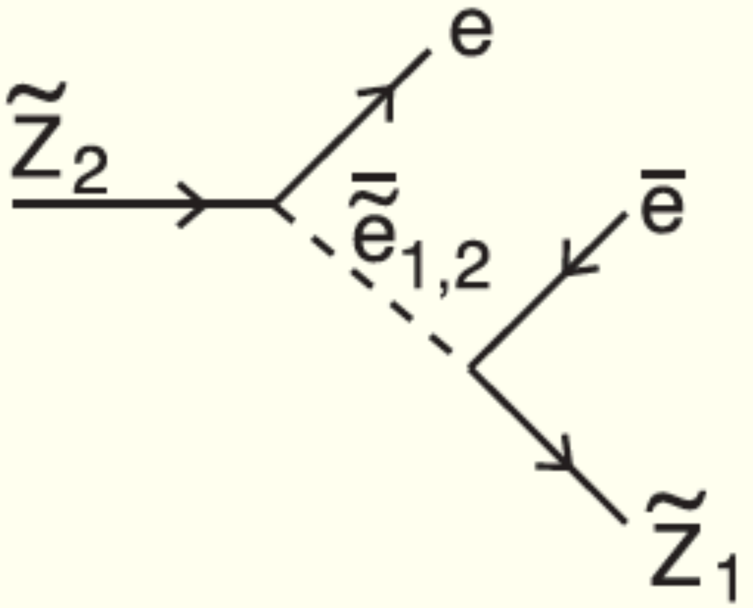


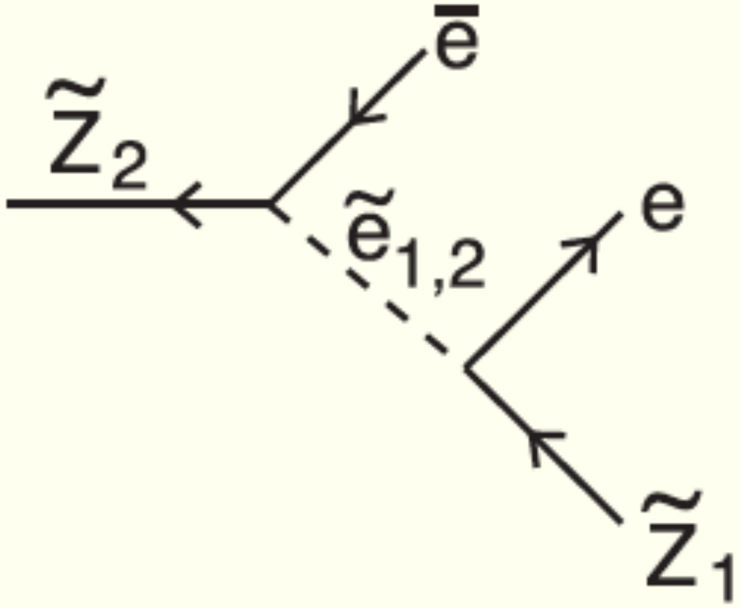
Neutralino decays

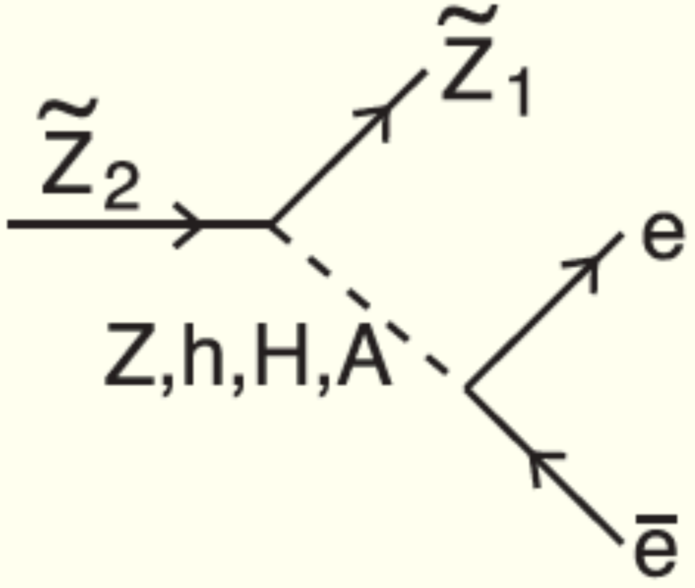
$$\begin{aligned}\widetilde{Z}_i &\to W\widetilde{W}_j,\ H^-\widetilde{W}_j,\ Z\widetilde{Z}_{i'},\ h\widetilde{Z}_{i'},\ H\widetilde{Z}_{i'},\ A\widetilde{Z}_{i'} \\ &\to \tilde{q}_{L,R}\bar{q},\ \bar{\tilde{q}}_{L,R}q,\ \tilde{\ell}_{L,R}\bar{\ell},\ \bar{\tilde{\ell}}_{L,R}\ell,\ \tilde{\nu}_\ell\bar{\nu}_\ell,\ \bar{\tilde{\nu}}_\ell\nu_\ell.\end{aligned}$$

If 2-body modes are closed,then the neutralino can decay via

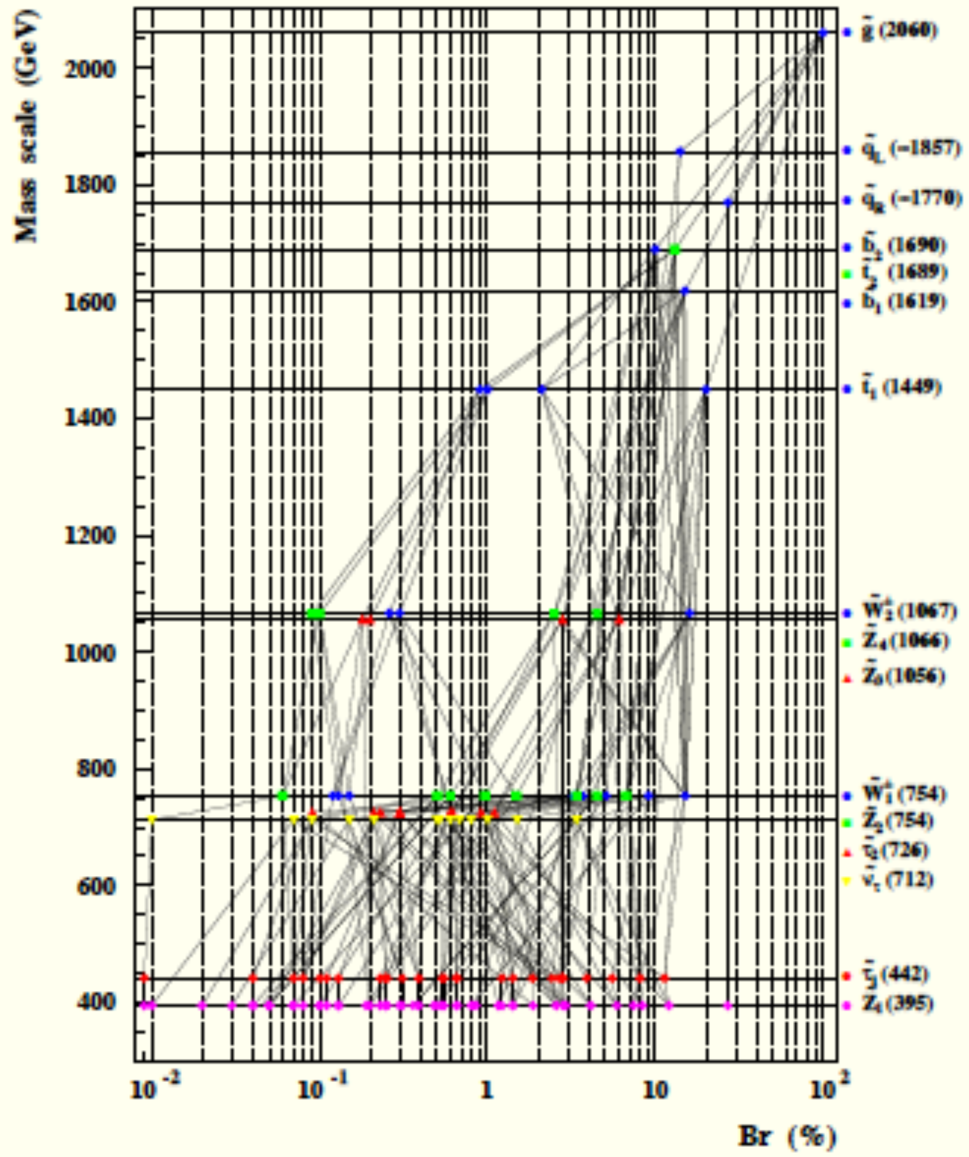
$$\widetilde{Z}_i \to \widetilde{Z}_{i'} + f\bar{f} \tag{2}$$







Sparticle cascade decays

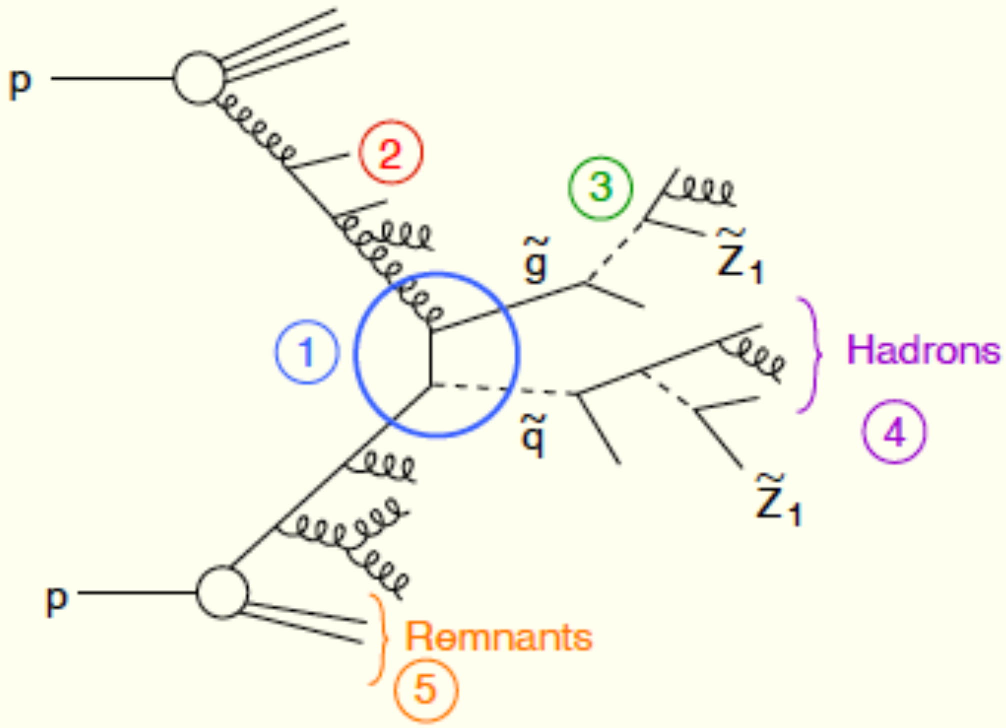


$\tilde{Z}_1$ qq	(27.0 %)	$\tilde{Z}_1$ τνWWWbb	(4.1 %)
$\tilde{Z}_1$ τνWbb	(12.1 %)	$\tilde{Z}_1$ ττbb	(2.9 %)
$\tilde{Z}_1$ ττWWbb	(8.4 %)	$\tilde{Z}_1$ ττqq	(2.9 %)
$\tilde{Z}_1$ WWbb	(7.4 %)	$\tilde{Z}_1$ τνZWbb	(2.8 %)
$\tilde{Z}_1$ τνqq	(5.9 %)	$\tilde{Z}_1$ τνhWbb	(2.6 %)

A realistic picture of what SUSY matter looks like at LHC

- ⋆ Counting different flavor states (which are potentially measurable), there are well over 1000 subprocess reactions expected at LHC from the MSSM
- ⋆ on average, each sparticle has 5-20 decay modes
- ⋆ rough estimate of distinct SUSY $2 \to n$ processes:
  - $\sim 1000 \times 10 \times 10 \sim 10^5$
  - this is actually a gross underestimate since each daughter of a produced sparticle has multiple decay modes, and so on...
- ⋆ the way forward: Monte Carlo program
  - calculate all prod'n cross sections: generate according to relative weights
  - calculate all branching fractions, and generate decays according to them
  - interface with parton shower, hadronization, underlying event
  - computer generated events should look something like what we would expect from the MSSM at the LHC

Event generation for sparticles



Event generation in LL - QCD

1) Hard scattering / convolution with PDFs

2) Intial / final state showers

3) Cascade decays

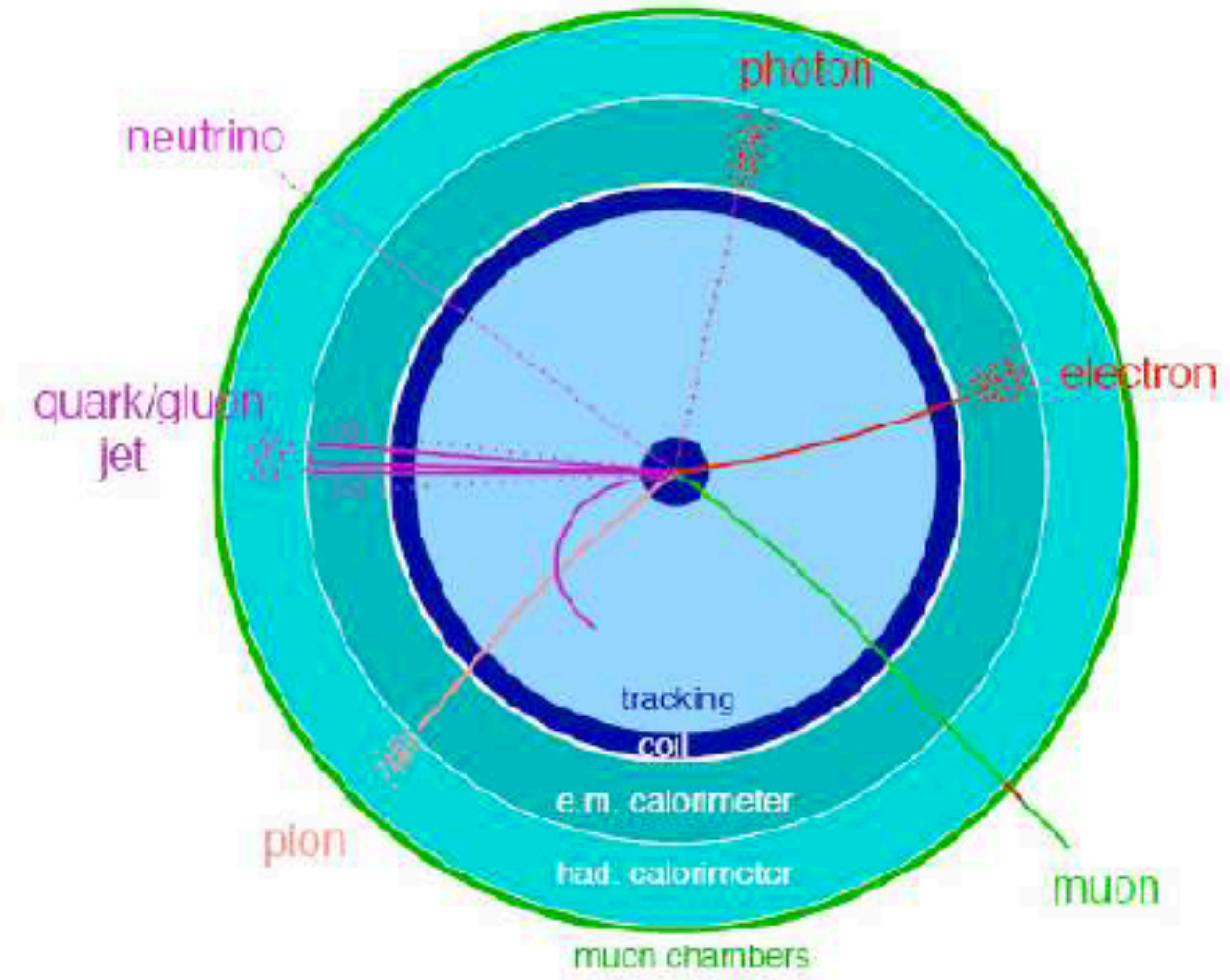
4) Hadronization

5) Beam remnants

Event generations for SUSY

- ⋆ Isajet (HB, Paige, Protopopsecu, Tata)
  - IH, FW-PS, n-cut Pomeron UE
- ⋆ Pythia (Sjöstrand, Lönnblad, Mrenna)
  - SH, FW-PS, multiple scatter UE, SUSY at low $\tan\beta$ only
- ⋆ Herwig (Marchesini, Webber, Seymour, Richardson,...)
  - CH, AO-PS, Phen. model UE, Isawig, Spin corr.!
- ⋆ SUSYGEN (Ghodbane, Katsanevas, Morawitz, Perez)
  - mainly for e^+e^-; interfaces to Pytha
- ⋆ SHERPA (Gleisberg, Hoche, krauss, Schalicke, Schumann, Winter)
  - $C++$ code for various $2 \to n$ processes
- ⋆ CompHEP, CalcHEP, Madgraph: for automatic Feynman diagram evaluation: interface via LHA

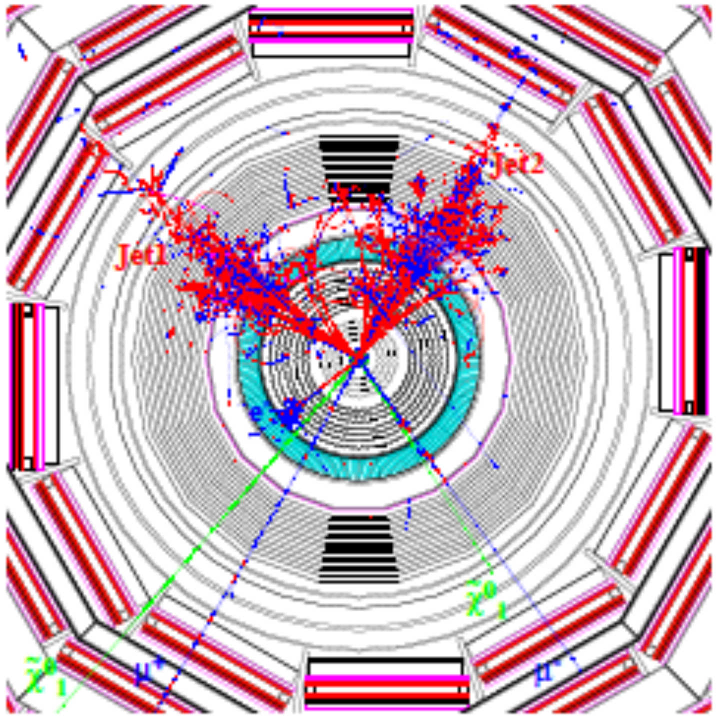
Briefly: particle interactions with detector



SUSY scattering event: Isajet simulation

SUSY event with 3 lepton + 2 Jets signature

$m_0 = 100$ GeV, $m_{1/2} = 300$ GeV, $\tan\beta = 2$, $A_0 = 0$, $\mu < 0$,
$m(\tilde{q}) = 686$ GeV, $m(\tilde{g}) = 766$ GeV, $m(\tilde{\chi}^0_2) = 257$ GeV,
$m(\tilde{\chi}^0_1) = 128$ GeV.



Leptons:	Jets:	Sparticles:
$p_t(\mu^+) = 55.2$ GeV	$E_t(\text{Jet1}) = 237$ GeV	$p_t(\tilde{\chi}^0_1) = 95.1$ GeV
$p_t(\mu^-) = 44.3$ GeV	$E_t(\text{Jet2}) = 339$ GeV	$p_t(\tilde{\chi}^0_1) = 190$ GeV
$p_t(e^-) = 43.9$ GeV		

Charged particles with $p_t > 2$ GeV, $|\eta| < 3$ are shown;
neutrons are not shown; no pile up events superimposed.

Search for SUSY at LHC: model dependent

- ⋆ GMSB
- ⋆ AMSB
  - MM-AMSB (mirage mediation)
  - hypercharged-AMSB (HCAMSB)
  - deflected AMSB
  - deflected mirage mediation
- ⋆ gravity-mediated models
  - mSUGRA or CMSSM
  - NUHM1, NUHM2
  - non-universal gaugino masses: MWDM, BWCA, LM3DM, HM2DM, ...
  - normal scalar mass hierarchy ($m_0(1,2) > m_0(3)$)
  - compressed SUSY
- ⋆ Split SUSY, pMSSM, NMSSM, $\cdots$

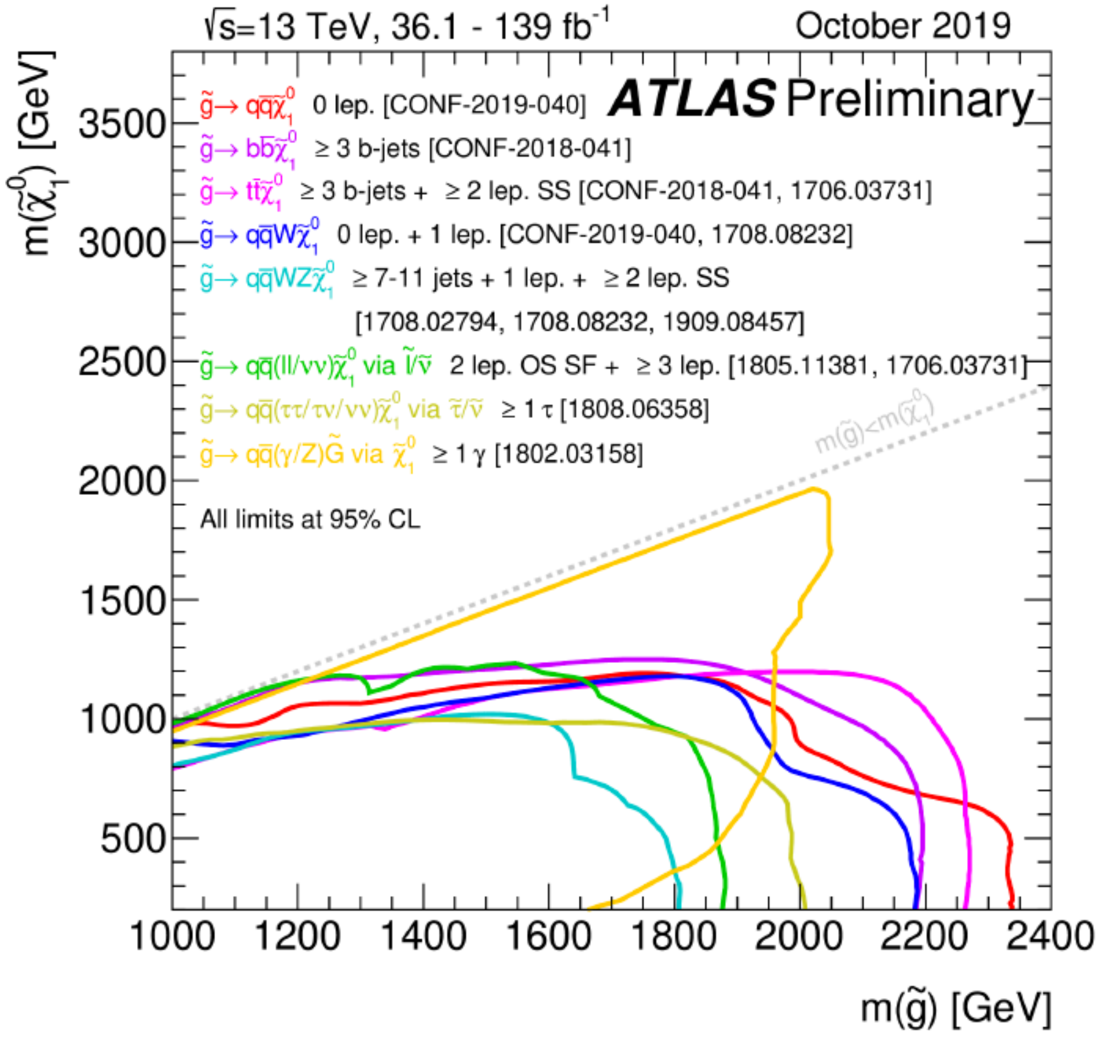
Sparticle/Higgs spectra codes

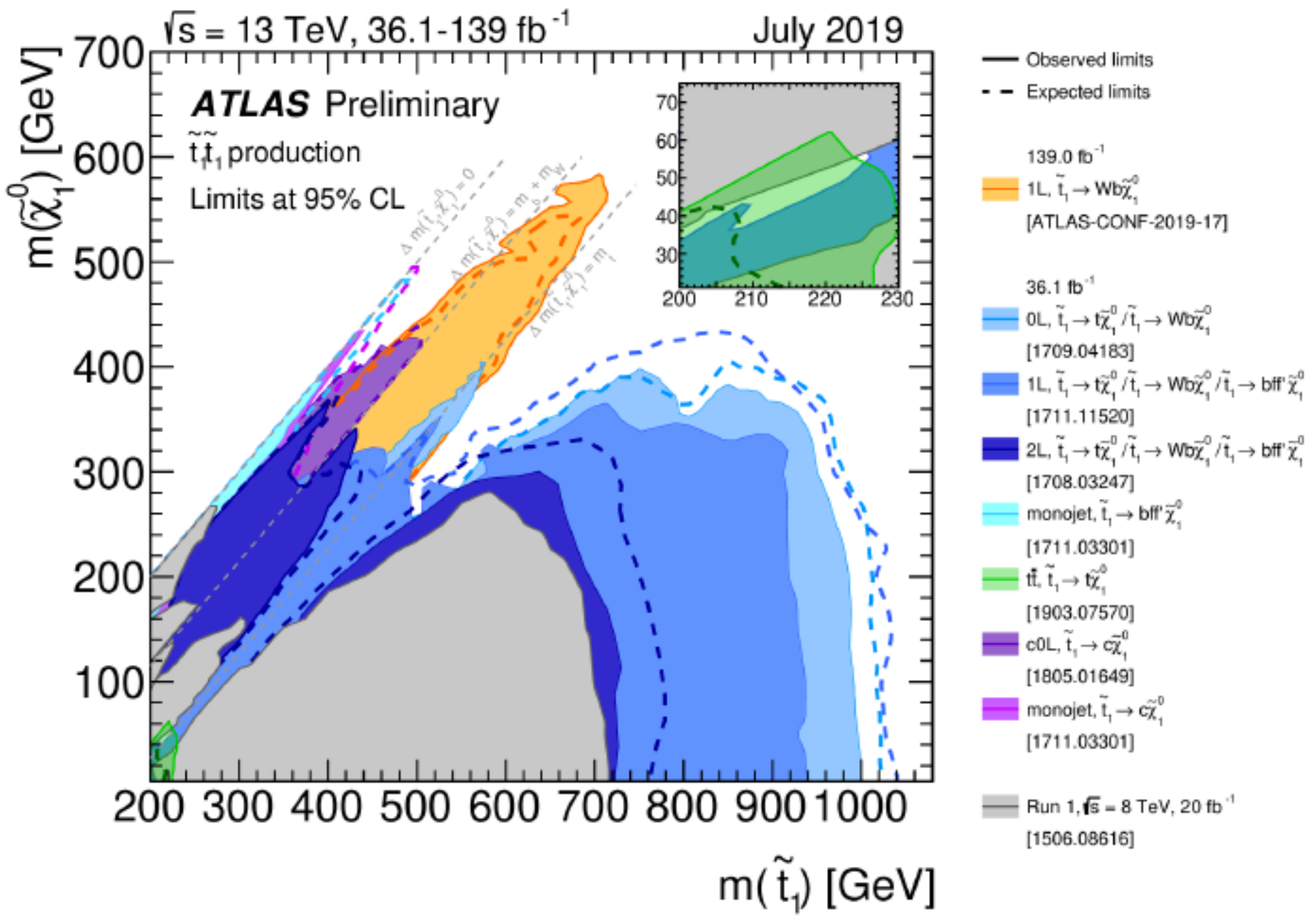
- Isasugra/Isajet (HB, Paige, Tata, 1994)
- SUSYHIT (Djouadi et al.,2002)
- SoftSUSY (Allanach, 2001)
- SPheno (Porod, 2003)
- FeynHiggs (Heinemeyer et al. 1998)
- etc. (Sarah, CPSuperH,...)

Some of these calculate decay tables as well

But where are the sparticles?

none seen so far at LHC



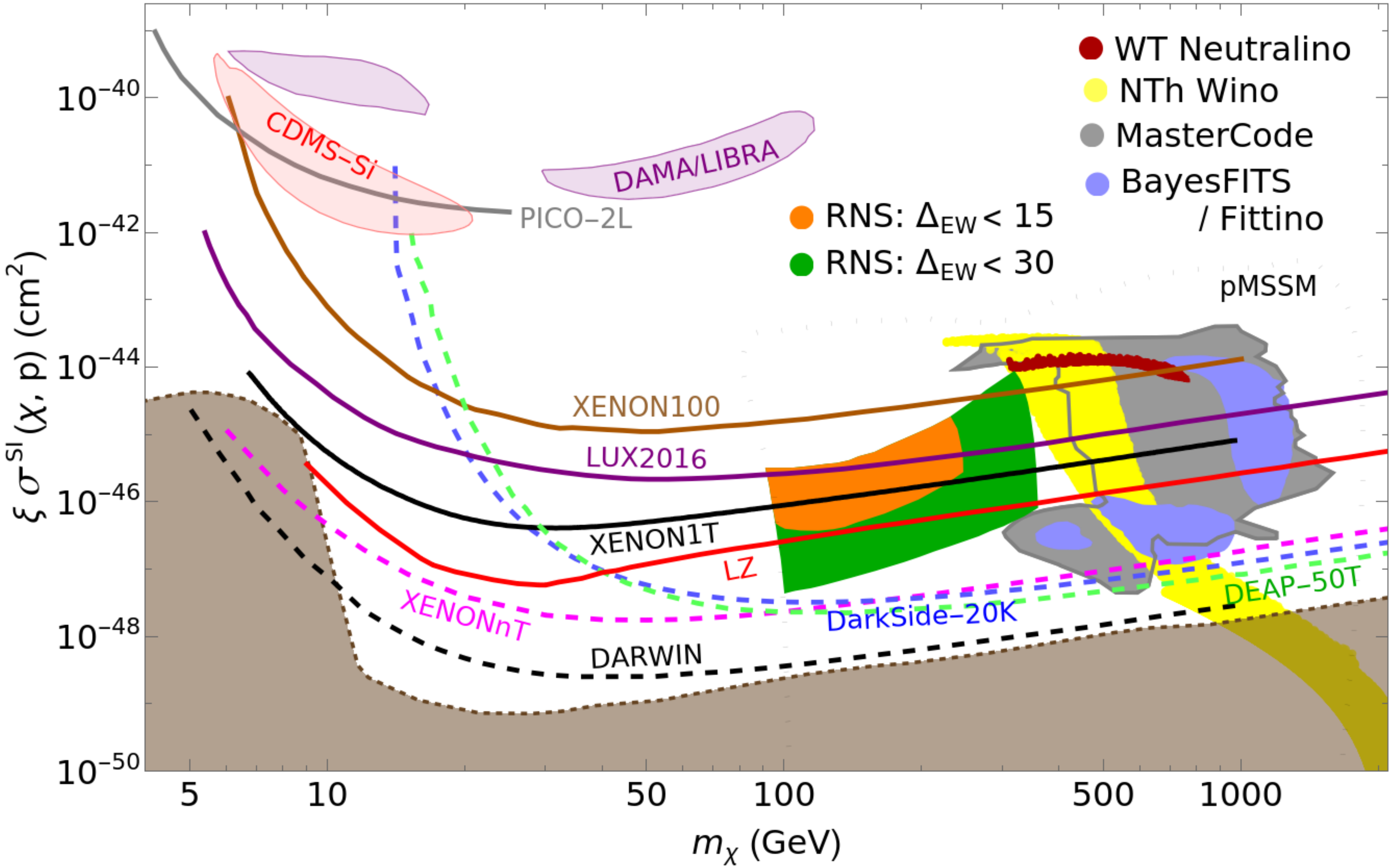


$$m_{\tilde{g}} > 2.25 \text{ TeV}$$

$$m_{\tilde{t}_1} > 1.1 \text{ TeV}$$

Exp'ists moved analyses from SUSY models to simplified models (2011, SLAC); this makes analyses easier, but it is not how SUSY is expected to manifest itself at LHC

Where are the WIMPs?

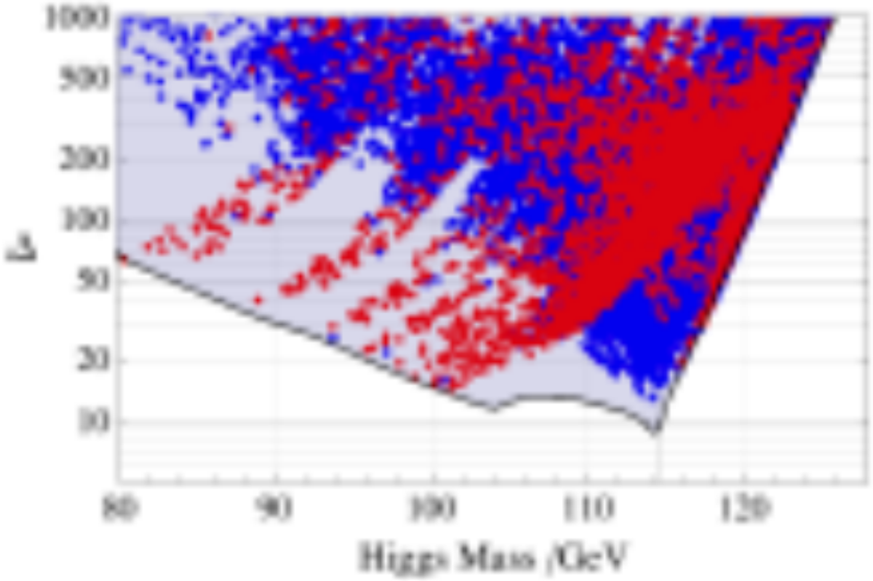


latest DD bounds from LZ2022: still no signal

These bounds appear in sharp conflict with EW ``naturalness''

	mass
gluino	400 GeV
uR	400 GeV
eR	350 GeV
chargino	100 GeV
neutralino	50 GeV

Barbieri-Giudice 10% bounds, 1987



Cassel, Ghilencea, Ross,2009

$$\Delta \to 1000$$

as $m_h \to 125$ GeV

0.1% tuning!?

Pardon this slide but I grew up in Wisconsin in the 1960s

``…settling the ultimate fate of naturalness is perhaps
the most profound theoretical question
of our time''

Arkani-Hamed et al.,
arXiv:1511.06495

``Given the magnitude of the stakes
involved,
it is vital to get a clear verdict
on naturalness from experiment''

This should be matched by theoretical scrutiny
of what we mean by naturalness

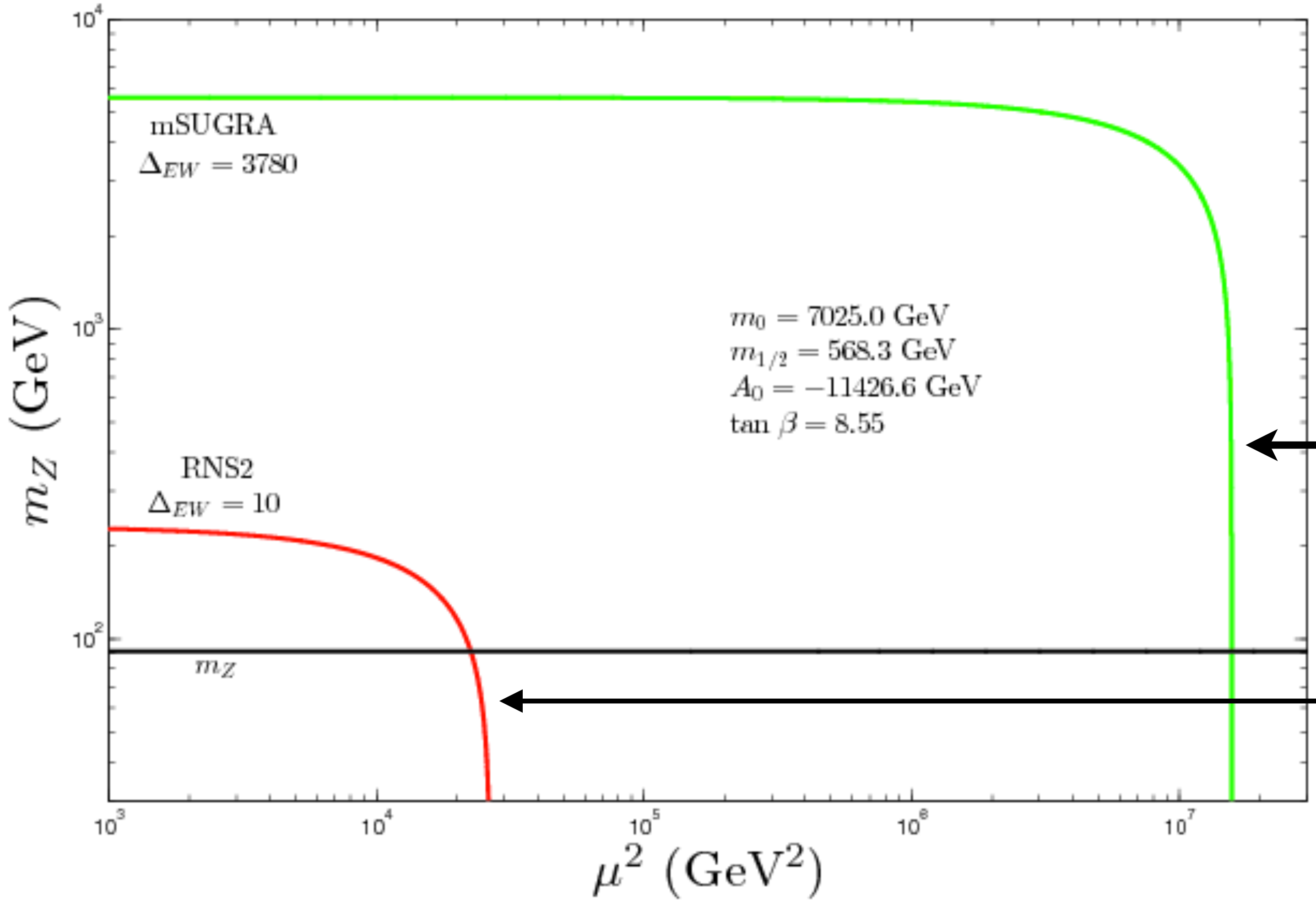
Next: simple electroweak fine-tuning in SUSY:
minimize Higgs potential in MSSM to relate
magnitude of weak scale m(Z) to SUSY Lagrangian;
dial value of mu so that Z mass comes out right:
everybody does it but it is hidden inside spectra codes
(Isajet, SuSpect, SoftSUSY, Spheno, SSARD)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2$$

SUSY breaking
Higgs mass

>40 loop
corrections

SUSY preserving
mu parameter



finetuning required

natural solution

#1: Simplest SUSY measure: Δ_{EW}

No large uncorrelated cancellations in m(Z) or m(h)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \sim -m_{H_u}^2 - \Sigma_u^u - \mu^2$$

$\Delta_{EW} \equiv max_i \left|C_i\right| / (m_Z^2/2)$ with $C_{H_u} = -m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)$ etc.

simple, direct, unambiguous interpretation:

- $|\mu| \sim m_Z \sim 100 - 200$ GeV
- $m_{H_u}^2$ should be driven to small negative values such that $-m_{H_u}^2 \sim 100 - 200$ GeV at the weak scale and
- that the radiative corrections are not too large: $\Sigma_u^u \lesssim 100 - 200$ GeV

CETUP*-12/002, FTPI-MINN-12/22, UMN-TH-3109/12, UH-511-1195-12

Radiative natural SUSY with a 125 GeV Higgs boson

Howard Baer,[1] Vernon Barger, Peisi Huang,[2] Azar Mustafayev,[3] and Xerxes Tata[4]

[1] *Dept. of Physics and Astronomy, University of Oklahoma, Norman, OK, 73019, USA*
[2] *Dept. of Physics, University of Wisconsin, Madison, WI 53706, USA*
[3] *W. I. Fine Institute for Theoretical Physics, University of Minnesota, Minneapolis, MN 55455, USA*

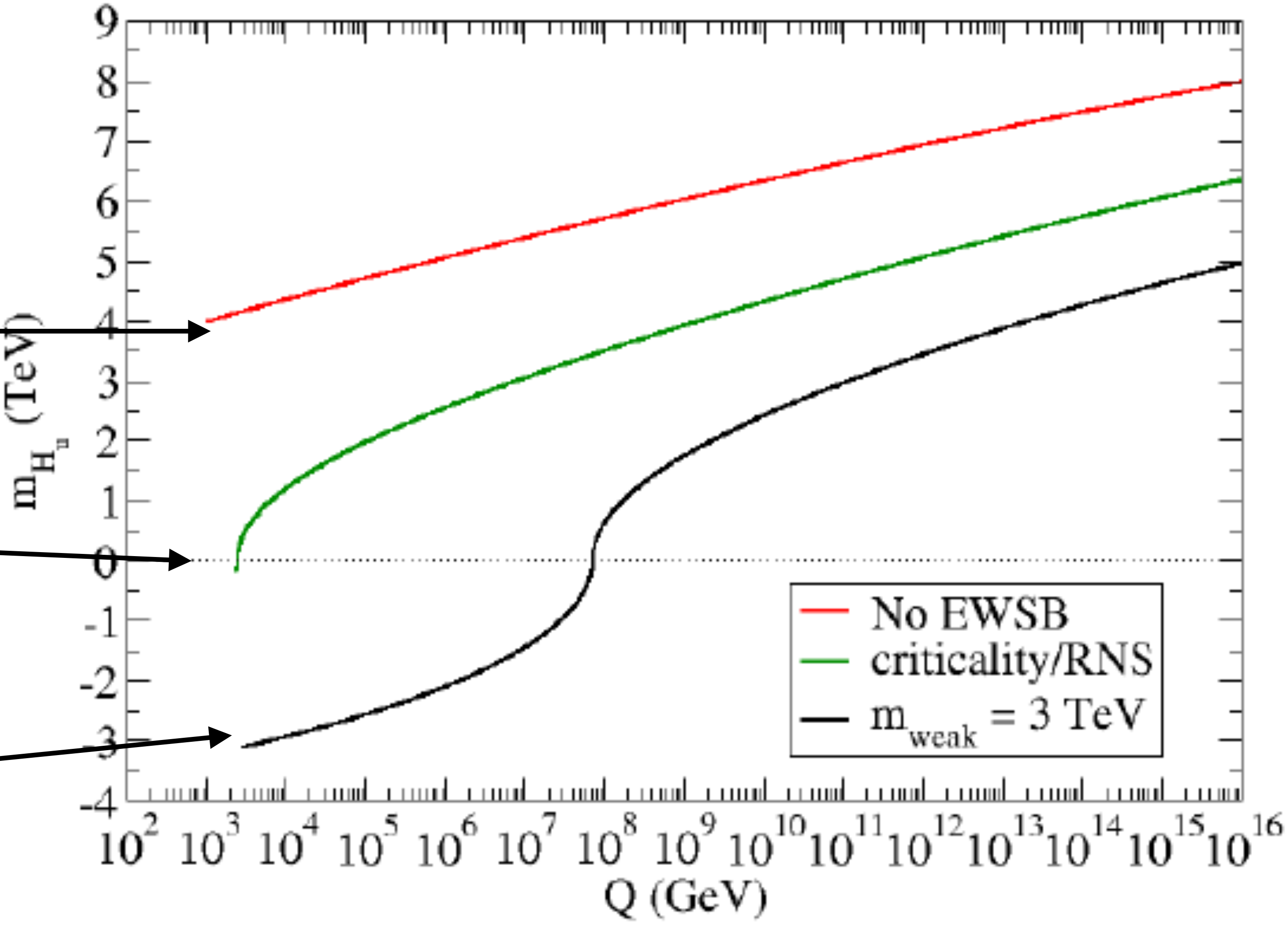
PRL109 (2012) 161802

radiative corrections drive $m_{H_u}^2$ from unnatural GUT scale values to naturalness at weak scale: radiatively-driven naturalness

EWS not broken

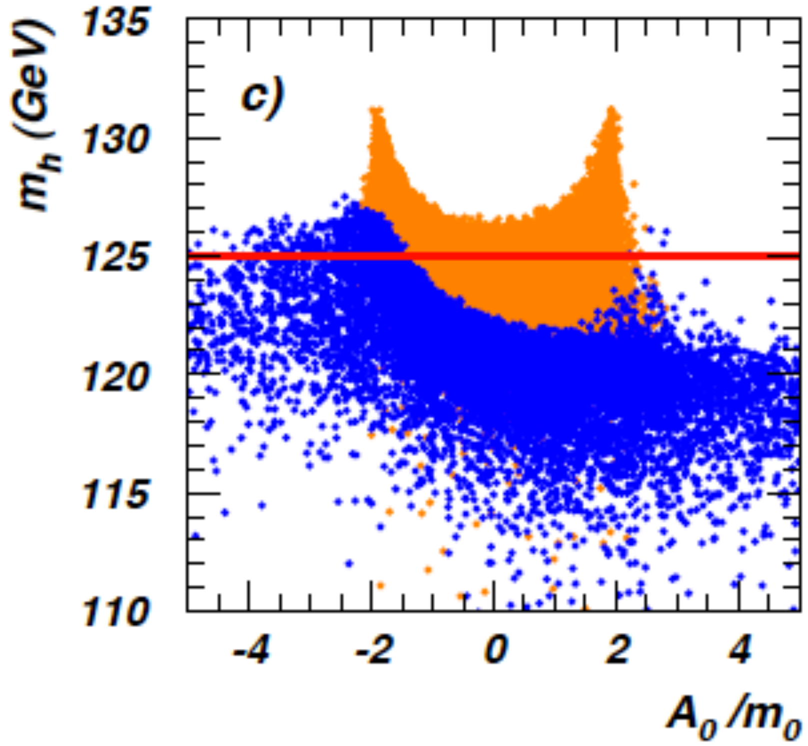
natural: EWS is barely broken

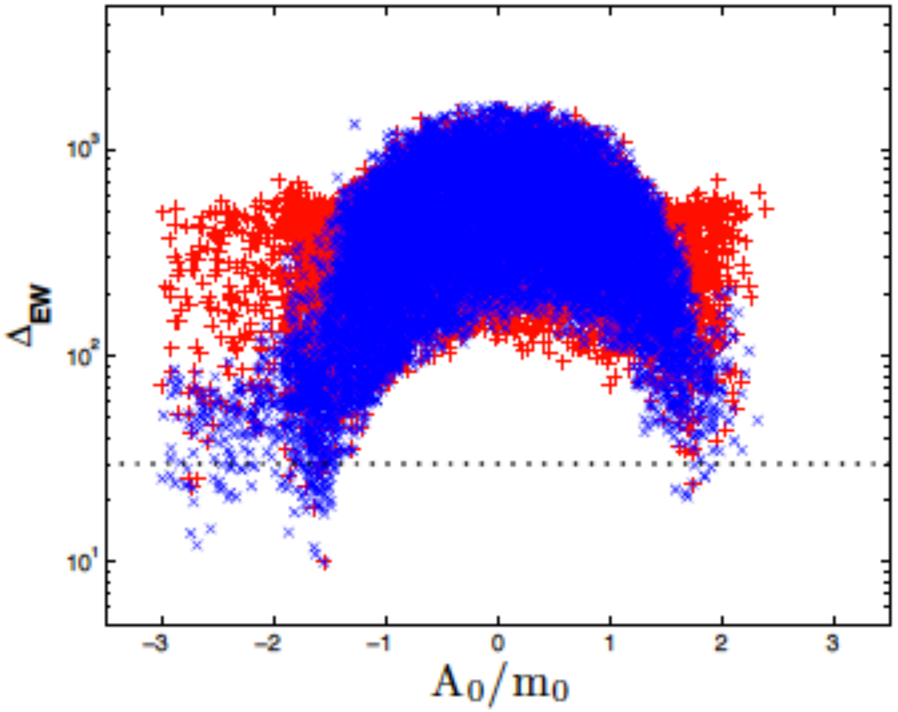
unnatural



Evolution of the soft SUSY breaking mass squared term $sign(m_{H_u}^2)\sqrt{|m_{H_u}^2|}$ vs. Q

Large value of A_t reduces $\Sigma_u^u(\tilde{t}_{1,2})$ contributions to Δ_{EW} while uplifting m_h to ~ 125 GeV





$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m^2_{\tilde{t}_{1,2}}) \left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8 g_Z^2 (\frac{1}{4} - \frac{2}{3} x_W) \Delta_t}{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}} \right]$$

$$\Delta_t = (m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R})/2 + M_Z^2 \cos 2\beta (\tfrac{1}{4} - \tfrac{2}{3} x_W)$$

$$F(m^2) = m^2 \left(\log \frac{m^2}{Q^2} - 1 \right) \qquad Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$$

#2: Higgs mass or large-log fine-tuning Δ_{HS}

It is tempting to pick out one-by-one
quantum fluctuations but must combine log divergences
before taking any limit

$$m_h^2 \simeq \mu^2 + m_{H_u}^2 + \delta m_{H_u}^2|_{rad}$$

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2}\left(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t\right) \qquad X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$$

neglect gauge pieces, S, mHu and running;
then we can integrate from m(SUSY) to Lambda

$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2}\left(m_{Q_3}^2 + m_{U_3}^2 + A_t^2\right)\ln(\Lambda/m_{SUSY})$$

$$\Delta_{HS} \sim \delta m_h^2/(m_h^2/2) < 10$$

$$m_{\tilde{t}_{1,2},\tilde{b}_1} < 500\ \mathrm{GeV}$$

$$m_{\tilde{g}} < 1.5\ \mathrm{TeV}$$

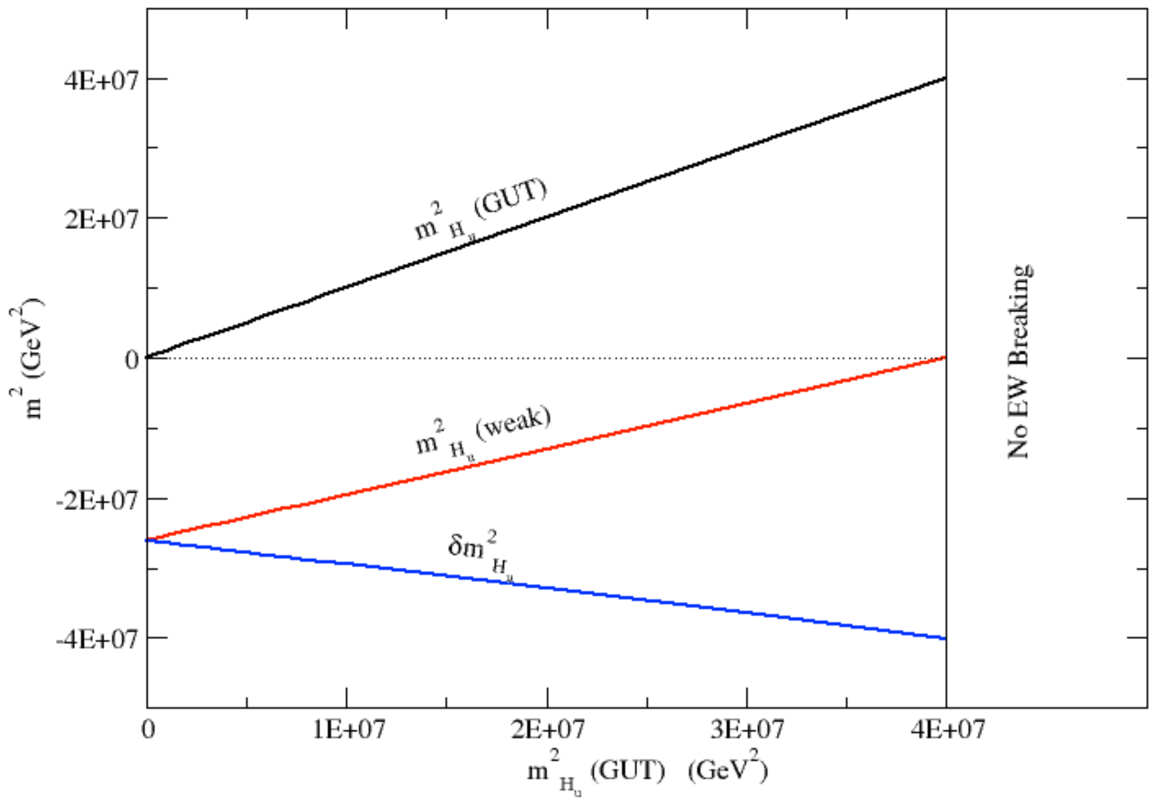
old natural SUSY then A_t can't be too big

What's wrong with this argument?

In zeal for simplicity, have made several simplifications: most egregious is that one sets m(Hu)^2=0 at beginning to simplify

$$m_{H_u}^2(\Lambda) \text{ and } \delta m_{H_u}^2 \text{ are } \textit{not} \text{ independent!}$$

violates prime directive!



The larger $m_{H_u}^2(\Lambda)$ becomes, then the larger becomes the cancelling correction!

HB, Barger, Savoy

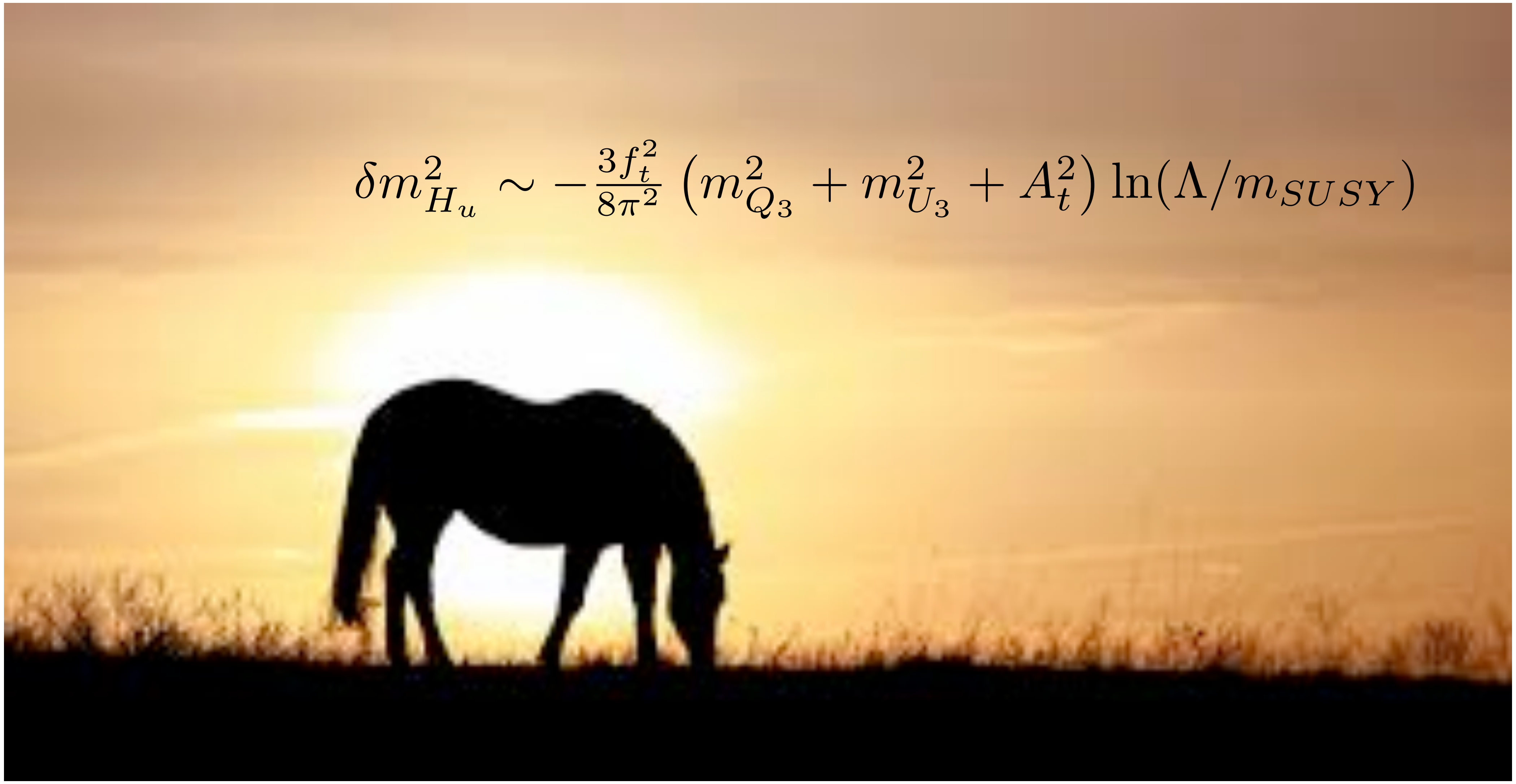
To fix: combine dependent terms:

$m_h^2 \simeq \mu^2 + \left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$ where now both μ^2 and $\left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$ are $\sim m_Z^2$

After re-grouping: $\Delta_{HS} \simeq \Delta_{EW}$

Instead of: the radiative correction $\delta m_{H_u}^2 \sim m_Z^2$
we now have: the radiatively-corrected $m_{H_u}^2 \sim m_Z^2$

Recommendation: put this horse out to pasture



$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2}\left(m_{Q_3}^2 + m_{U_3}^2 + A_t^2\right)\ln(\Lambda/m_{SUSY})$$

R.I.P.

sub-TeV 3rd generation squarks not required for naturalness

#3. What about EENZ/BG measure?

$$\Delta_{BG} = max_i \left| \frac{\partial \log m_Z^2}{\partial \log p_i} \right| = max_i \left| \frac{p_i}{m_Z^2} \frac{\partial m_Z^2}{\partial p_i} \right|$$

p_i are the theory parameters

applied to pMSSM, then $\Delta_{BG} \simeq \Delta_{EW}$

apply to high (e.g. GUT) scale parameters

$$\begin{aligned} m_Z^2 \simeq\ & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\ & +0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ & -0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\ & -1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ & +0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ & +0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ & +0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned}$$

applied to most parameters,

Δ_{BG} large, looks fine-tuned for *e.g.* $m_{\tilde{t}_1} \sim 1$ TeV

$\Delta_{BG}(Q_3) \simeq 0.73\frac{1000^2}{91.2^2} \sim 100$

#3. What about EENZ/BG measure?

$$\Delta_{BG} = max_i \left| \frac{\partial \log m_Z^2}{\partial \log p_i} \right| = max_i \left| \frac{p_i}{m_Z^2} \frac{\partial m_Z^2}{\partial p_i} \right|$$

applied to pMSSM, then $\Delta_{BG} \simeq \Delta_{EW}$

What if we apply to high (e.g. GUT) scale parameters ?

$$\begin{aligned}
m_Z^2 \simeq\ & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\
& +0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\
& -0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\
& -1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\
& +0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\
& +0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\
& +0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2,
\end{aligned}$$

Feng, Matchev, Moroi

For correlated scalar masses $\equiv m_0$,
scalar contribution collapses:
what looks fine-tuned isn't: *focus point SUSY*
multi-TeV scalars are *natural*

Even with FP, still
fine-tuned on m(gluino) :(

But wait! in more complete models, soft terms not independent

violates prime directive!

e.g. in SUGRA, for well-specified hidden sector,
each soft term calculated as multiple of m(3/2);
soft terms must be combined!

e.g. dilaton-dominated SUSY breaking: $m_0^2 = m_{3/2}^2$ with $m_{1/2} = -A_0 = \sqrt{3} m_{3/2}$

in general:

$$\begin{aligned} m_{H_u}^2 &= a_{H_u} \cdot m_{3/2}^2, \\ m_{Q_3}^2 &= a_{Q_3} \cdot m_{3/2}^2, \\ A_t &= a_{A_t} \cdot m_{3/2}, \\ M_i &= a_i \cdot m_{3/2}, \\ & \cdots . \end{aligned}$$

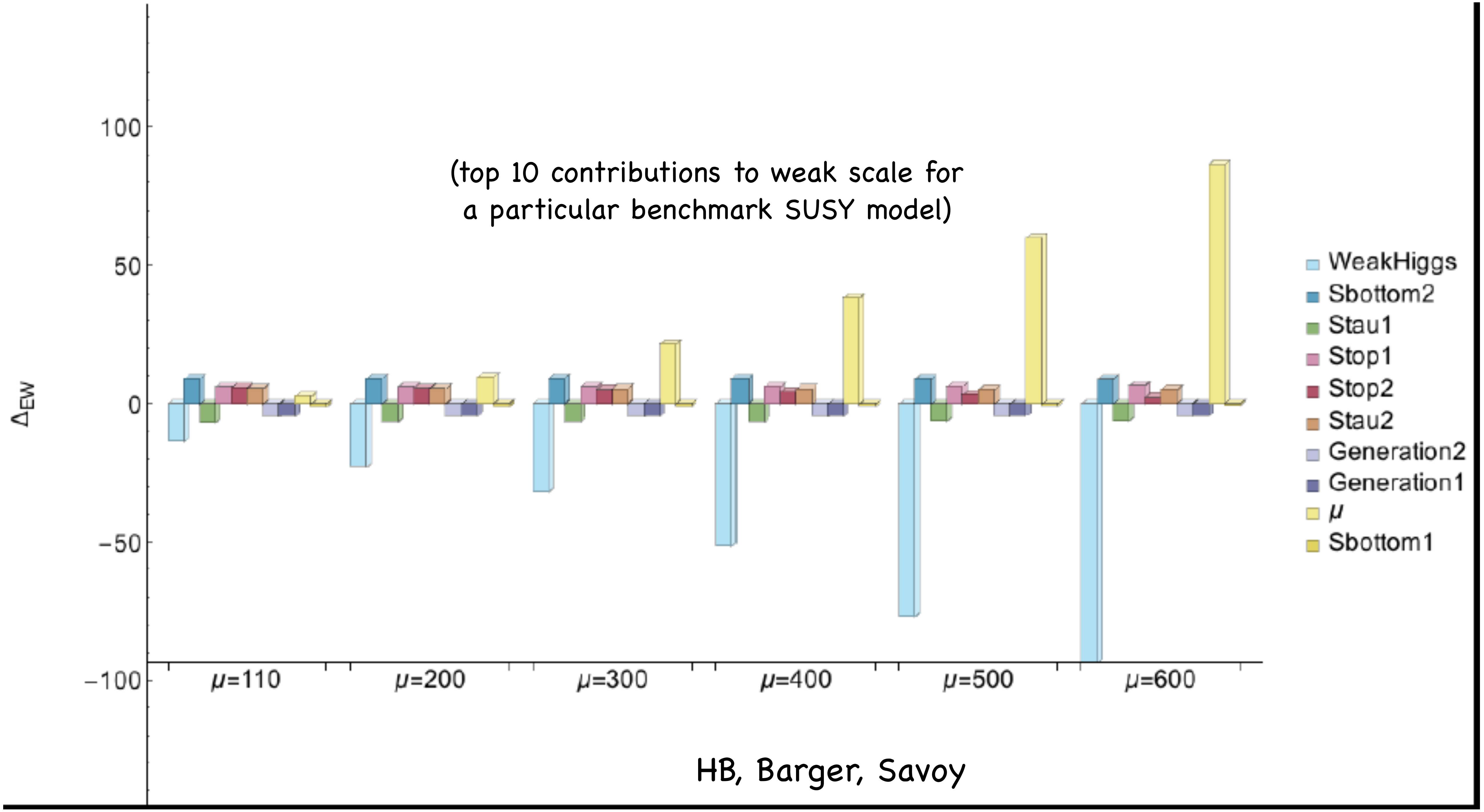
since μ hardly runs, then

$$\boxed{\begin{aligned} m_Z^2 &\simeq -2\mu^2 + a \cdot m_{3/2}^2 \\ &\simeq -2\mu^2 - 2m_{H_u}^2(weak) \end{aligned}}$$

$$m_{H_u}^2(weak) \sim -(100-200)^2 \ \mathrm{GeV}^2 \sim -a \cdot m_{3/2}^2/2$$

using μ^2 and $m_{3/2}^2$ as fundamental,
then $\Delta_{BG} \simeq \Delta_{EW}$ even using high scale parameters!

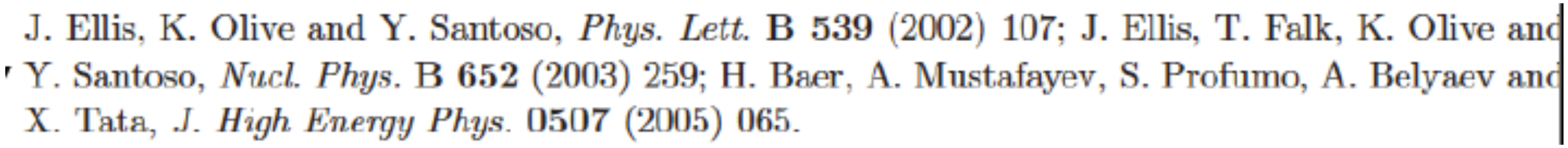
How much is too much fine-tuning?


(top 10 contributions to weak scale for a particular benchmark SUSY model)

HB, Barger, Savoy

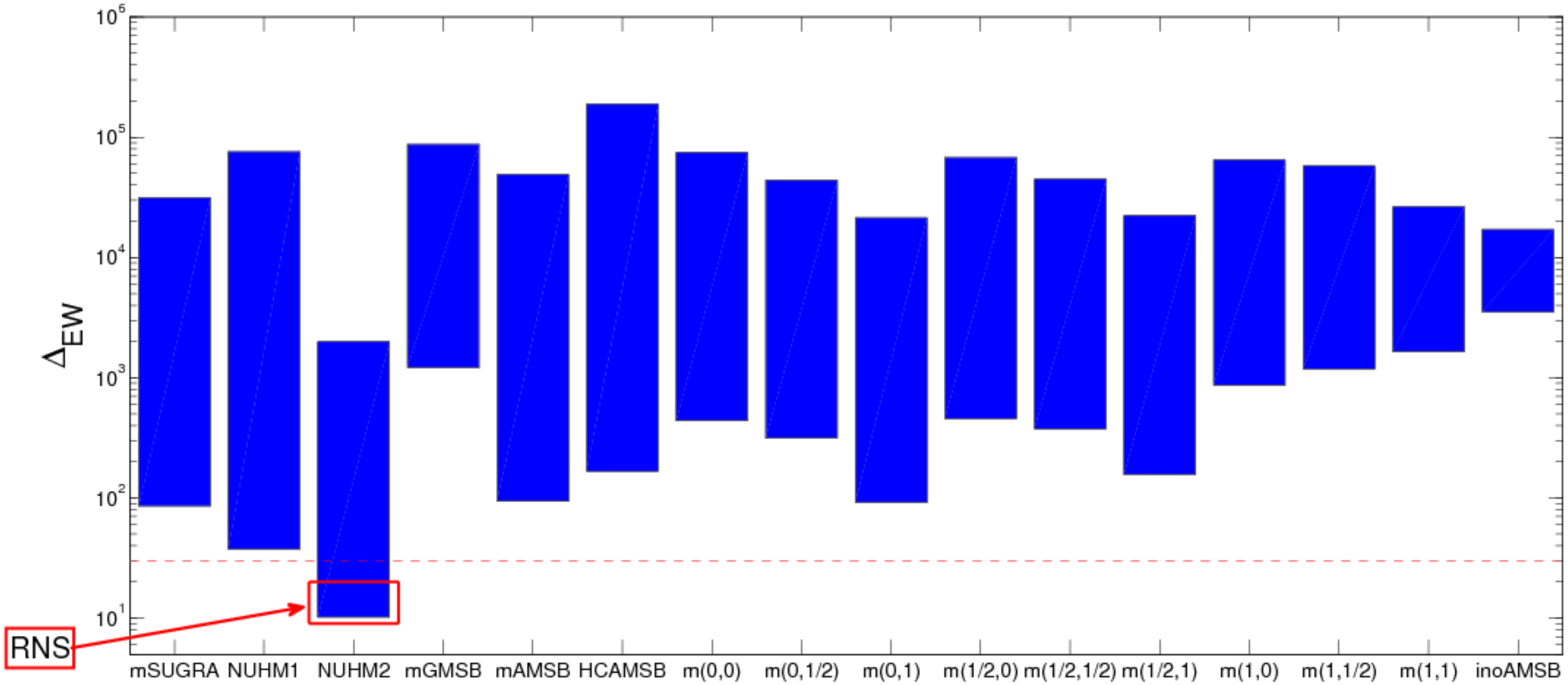
Visually, large fine-tuning has already developed by $\mu \sim 350$ or $\Delta_{EW} \sim 30$

Δ_{EW} is highly selective: most constrained models are ruled out except NUHM2 and its generalizations:



J. Ellis, K. Olive and Y. Santoso, *Phys. Lett.* **B 539** (2002) 107; J. Ellis, T. Falk, K. Olive and Y. Santoso, *Nucl. Phys.* B **652** (2003) 259; H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, *J. High Energy Phys.* **0507** (2005) 065.

scan over p-space with m(h)=125.5+-2.5 GeV:



0.1%

1%

10%

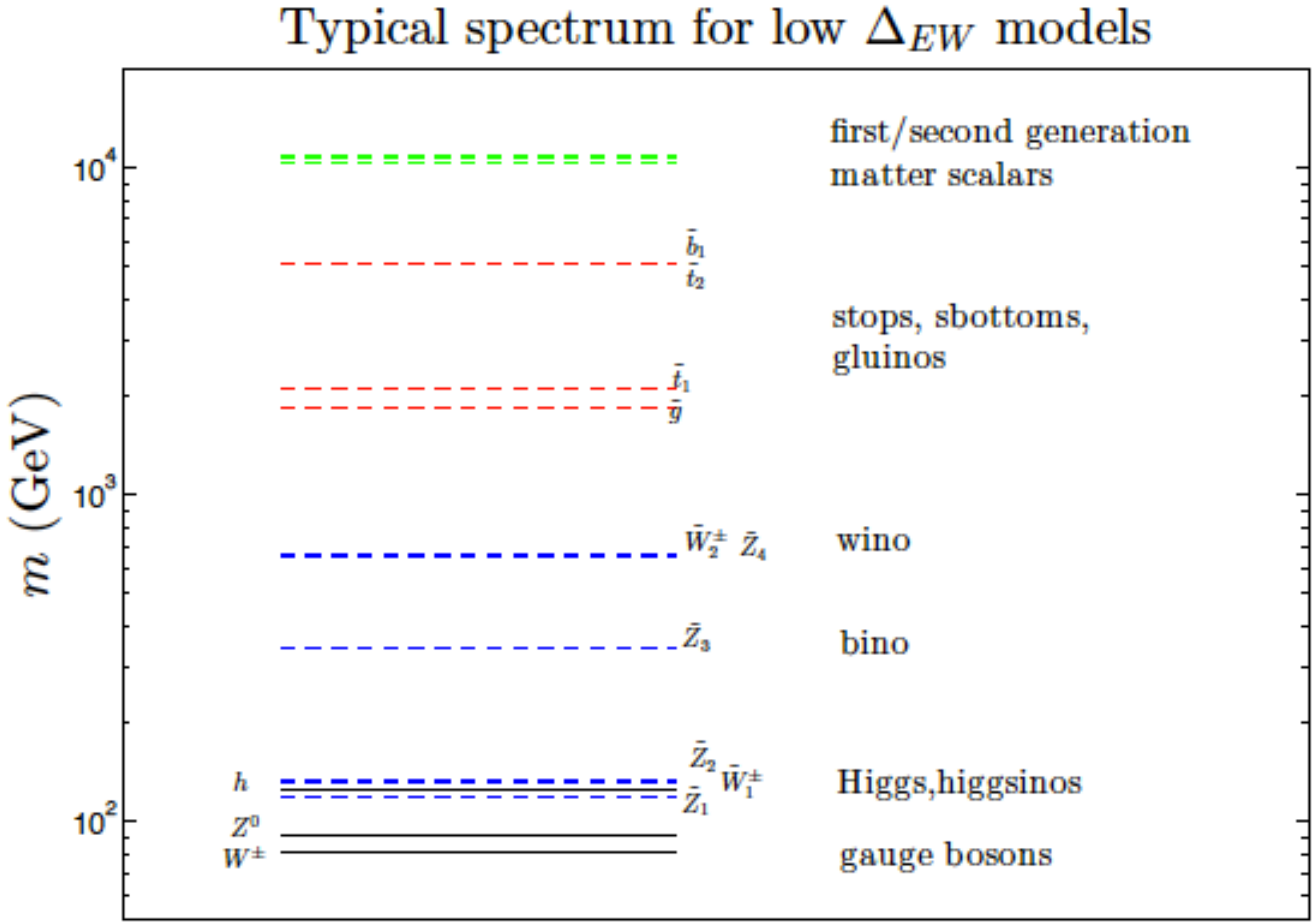
HB, Barger, Mickelson,Padeffke-Kirkland, PRD89 (2014) 115019

bounds from naturalness (3%)	BG/DG	Delta_EW
mu	350 GeV	350 GeV
gluino	400-600 GeV	6 TeV
t1	450 GeV	3 TeV
sq/sl	550-700 GeV	10-30 TeV

h(125) and LHC limits are perfectly compatible with 3-10% naturalness: no crisis!

Computer code
DEW4SLHA

- input SLHA file
- output DEW, DBG, DHS
- author: Dakotah Martinez



Typical spectrum for low Δ_{EW} models

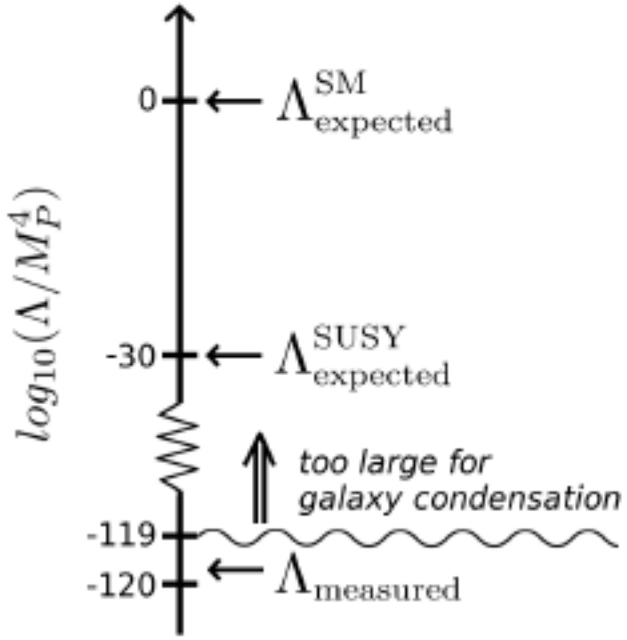
There is a Little Hierarchy, but it is no problem

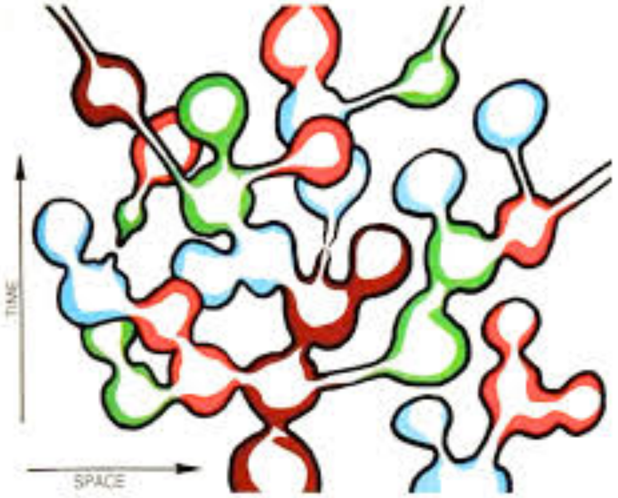
$\mu \ll m_{3/2}$

higgsinos likely the lightest superparticles!

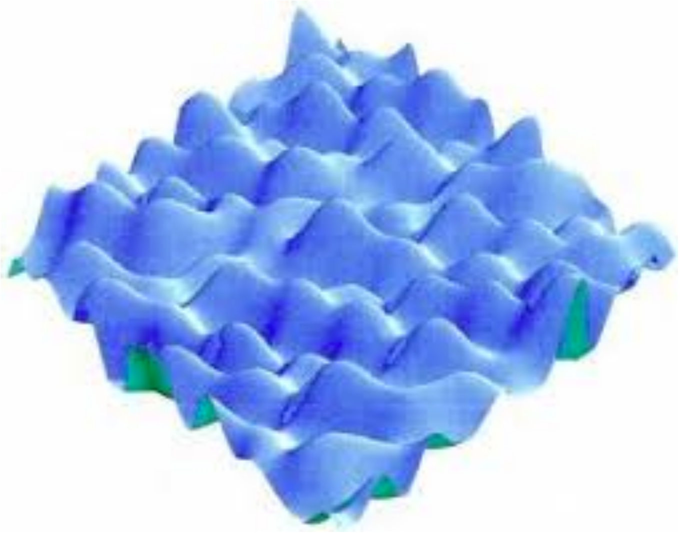
How does this all relate to string landscape?

It is sometimes invoked that maybe we should abandon naturalness: after all, isn't the cosmological constant (CC) fine-tuned?





eternally inflating multiverse



In the landscape with 10ˆ500 vacua with different CCs, then the tiny value of the CC may not be surprising since larger values would lead to runaway pocket universes where galaxies wouldn't condense- anthropics: no observers in such universes (Weinberg)

Bousso & Polchinski

The CC is as natural as possible subject to the condition that it leads to galaxy condensation

For some recent review material, see M. Douglas, The String Theory Landscape, 2018, Universe 5 (2019) 7, 176

Statistical analysis of SUSY breaking scale in IIB theory:

M. Douglas, hep-th/0405279

start with 10^500 string vacua states

- string theory landscape contains vast ensemble of N=1, d=4 SUGRA EFTs at high scales
- the EFTs contain the SM as weak scale EFT
- the EFTs contain visible sector +potentially large hidden sector
- visible sector contains MSSM plus extra gauge singlets (e.g. a PQ sector, RN neutrinos,...)
- SUGRA is broken spontaneously via superHiggs mechanism via either F- or D- terms or in general a combination

In fertile patch of vacua with MSSM as weak scale effective theory
but with no preferred SUSY breaking scale...

$$dP/d\mathcal{O} \sim f_{prior} \cdot f_{selection}$$

What is f(prior) for SUSY breaking scale?

In string theory, usually multiple (~10) hidden sectors
containing a variety of F- and D- breaking fields

For comparable <Fi> and <Dj> values, then expect

$$f_{prior} \sim m_{soft}^{2n_F+n_D-1}$$

Douglas ansatz
arXiv:0405279

Under single F-term
SUSY breaking,
expect linearly increasing
statistical selection
of soft terms

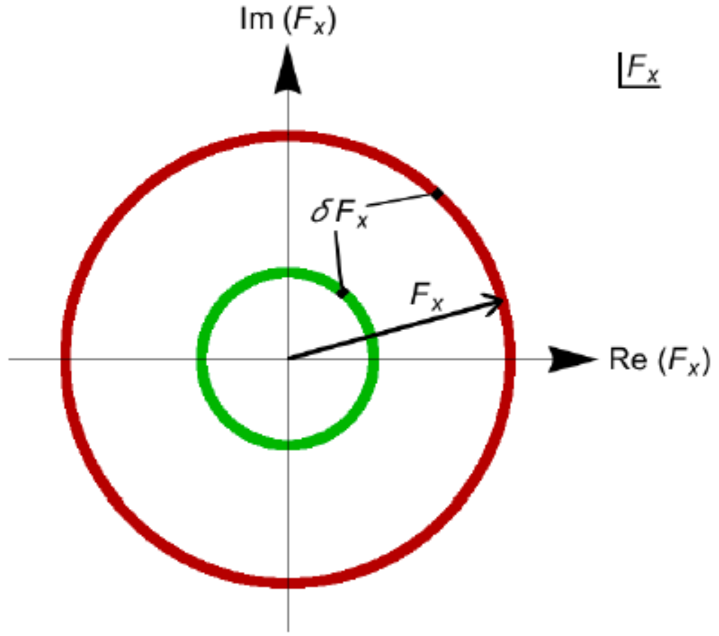


Figure 1: Annuli of the complex F_X plane giving rise to linearly increasing selection of soft SUSY breaking terms.

For uniform values of SUSY
breaking moduli, expect landscape to prefer
high scale of SUSY breaking!

What about f(selection)?

Originally, people adopted $f_{EWFT} \sim m_{weak}^2/m_{soft}^2$

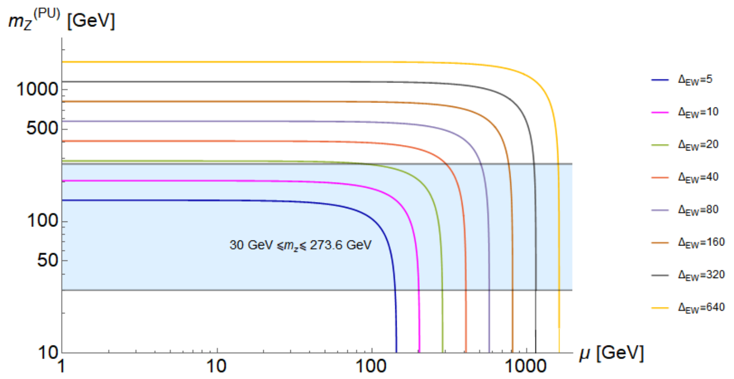
to penalize soft terms straying too far from weak scale

This doesn't work for variety of cases

- Too big soft terms can lead to CCB minima: must veto such vacua
- Bigger m(Hu)^2 leads to more natural value at weak scale
- Bigger A(t) trilinear suppresses t1, t2 contribution to weak scale

$$\frac{(m_Z^{PU})^2}{2} = \frac{m_{H_d}^2+\Sigma_d^d-(m_{H_u}^2+\Sigma_u^u)\tan^2\beta}{\tan^2\beta-1} - \mu^2$$

Adopt mu value so no longer available for tuning; then mZ(PU).ne.91.2 GeV



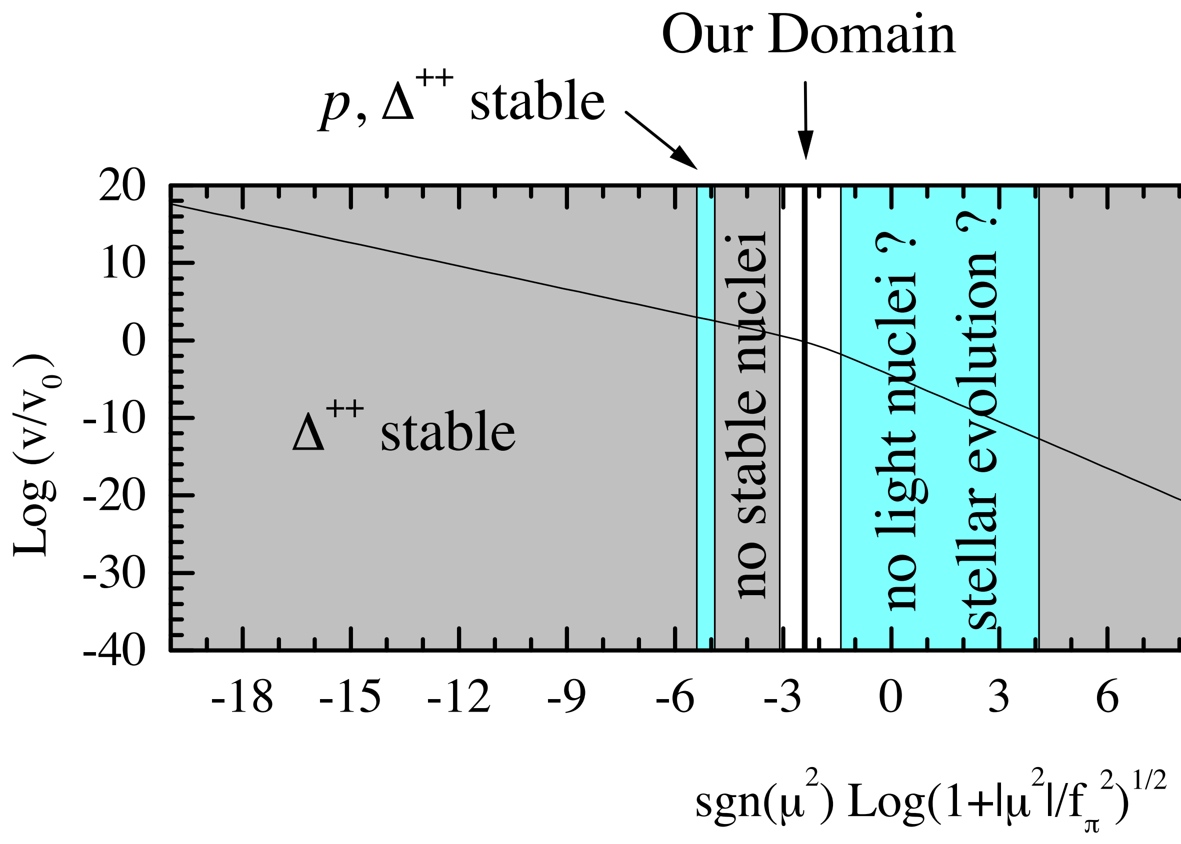
Then for statistically selected soft terms, m(weak) is output, not input

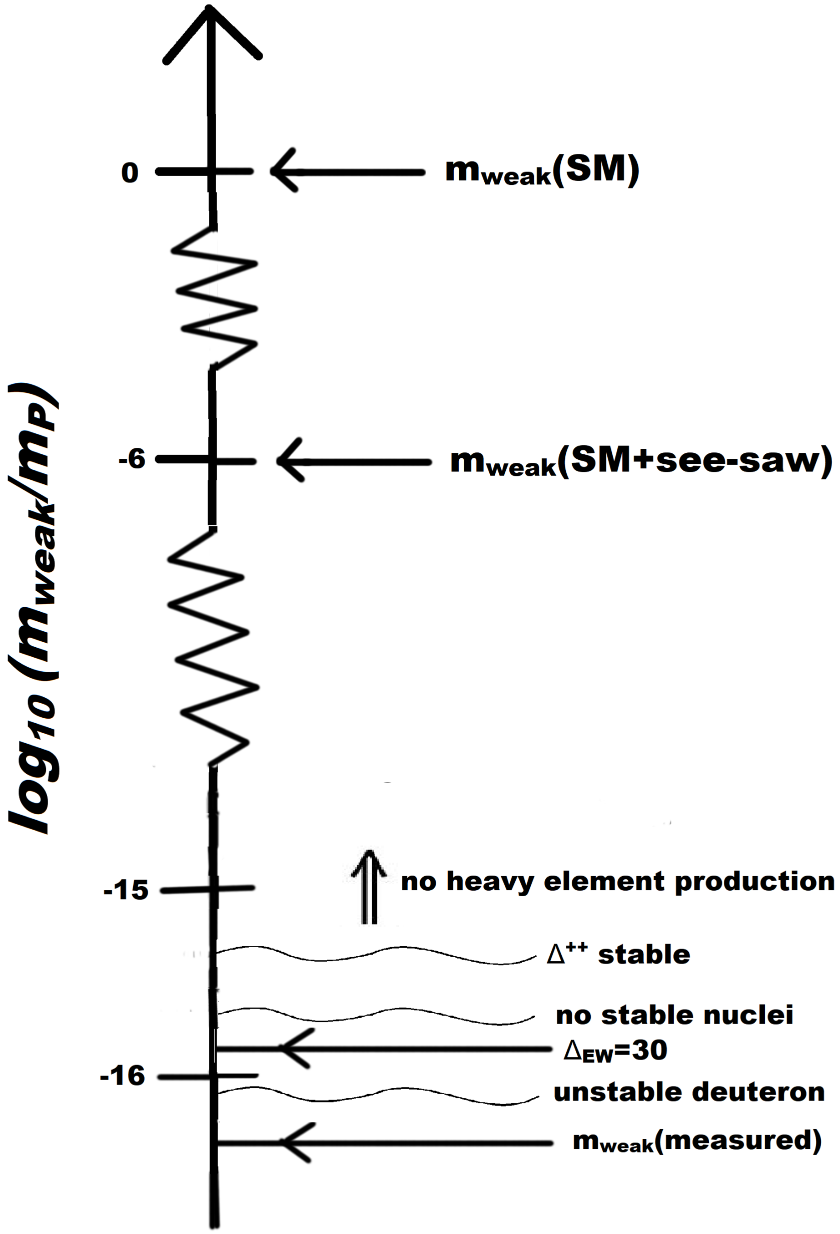
Must veto too large m(weak) values: nuclear physics screw up: no complex atoms
(Agrawal, Barr, Donoghue, Seckel, 1998)

Factor four deviation of weak scale from measured value => $\Delta_{EW} < 30$

Agrawal, Barr, Donoghue, Seckel result (1998):

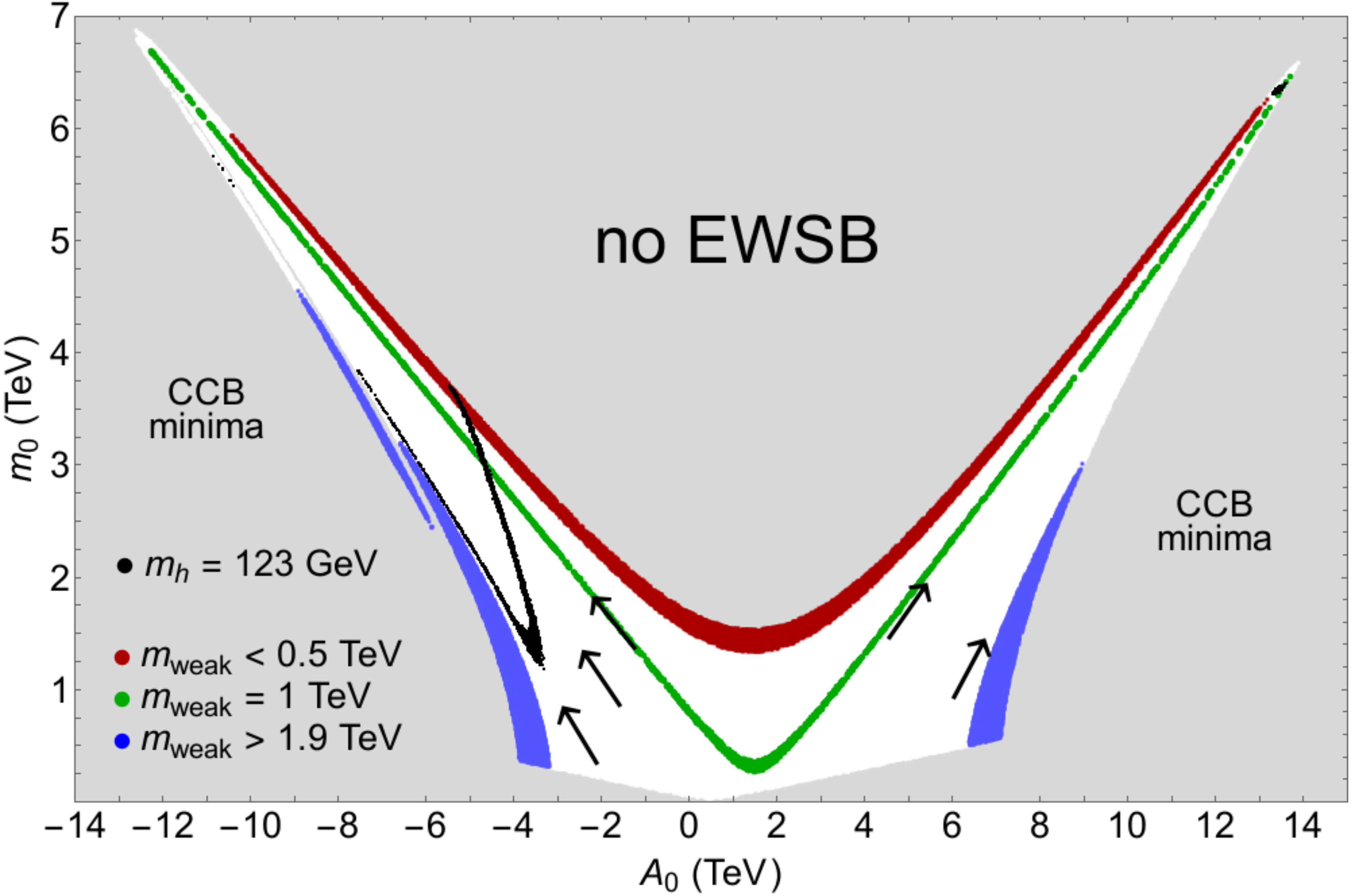
pocket-universe value of weak scale
cannot deviate by more than
factor 2-5 from its measured value
lest disasters occur in nuclear physics: no nuclei, no atoms
(violates atomic principle)





Veto pocket universes with CCB minima or minima leading to weak scale a (conservative) factor four greater than our value m(W,Z,h)~100 GeV

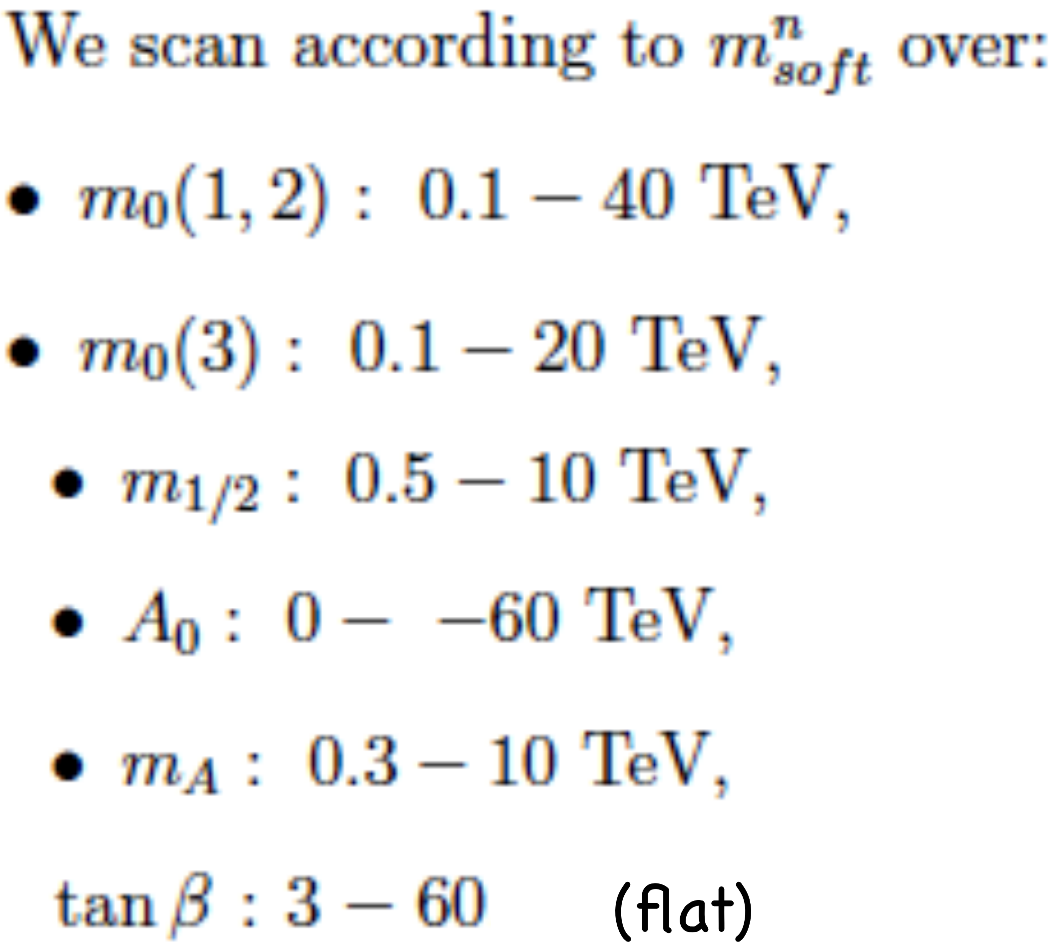
$$m_{H_u} = 1.3 m_0$$



statistical draw to large soft terms balanced by
anthropic draw toward red (m(weak)~100 GeV):
then m(Higgs)~125 GeV and natural SUSY spectrum!

HB, Barger, Savoy, Serce, PLB758 (2016) 113

Recent work: place on more quantitative footing: scan soft SUSY breaking parameters in NUHM3 model as m(soft)^n along with f(EWFT) penalty



We scan according to m_{soft}^n over:

- $m_0(1,2):\ 0.1-40$ TeV,
- $m_0(3):\ 0.1-20$ TeV,
- $m_{1/2}:\ 0.5-10$ TeV,
- $A_0:\ 0-\ -60$ TeV,
- $m_A:\ 0.3-10$ TeV,

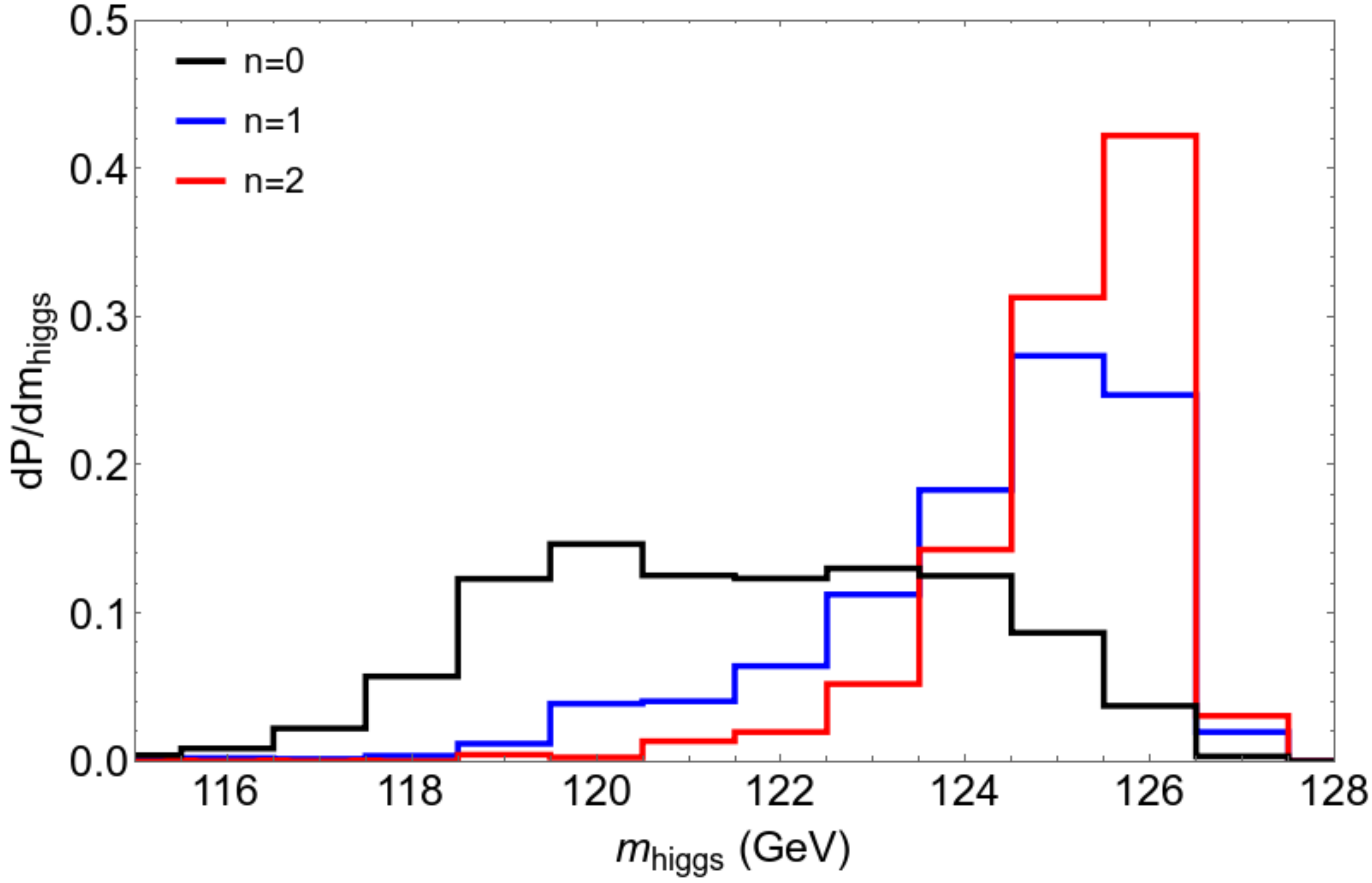
$\tan\beta:3-60$ (flat)

mu=150 GeV (fixed)

HB, Barger, Serce, Sinha, JHEP1803 (2018) 002

Making the picture more quantitative:

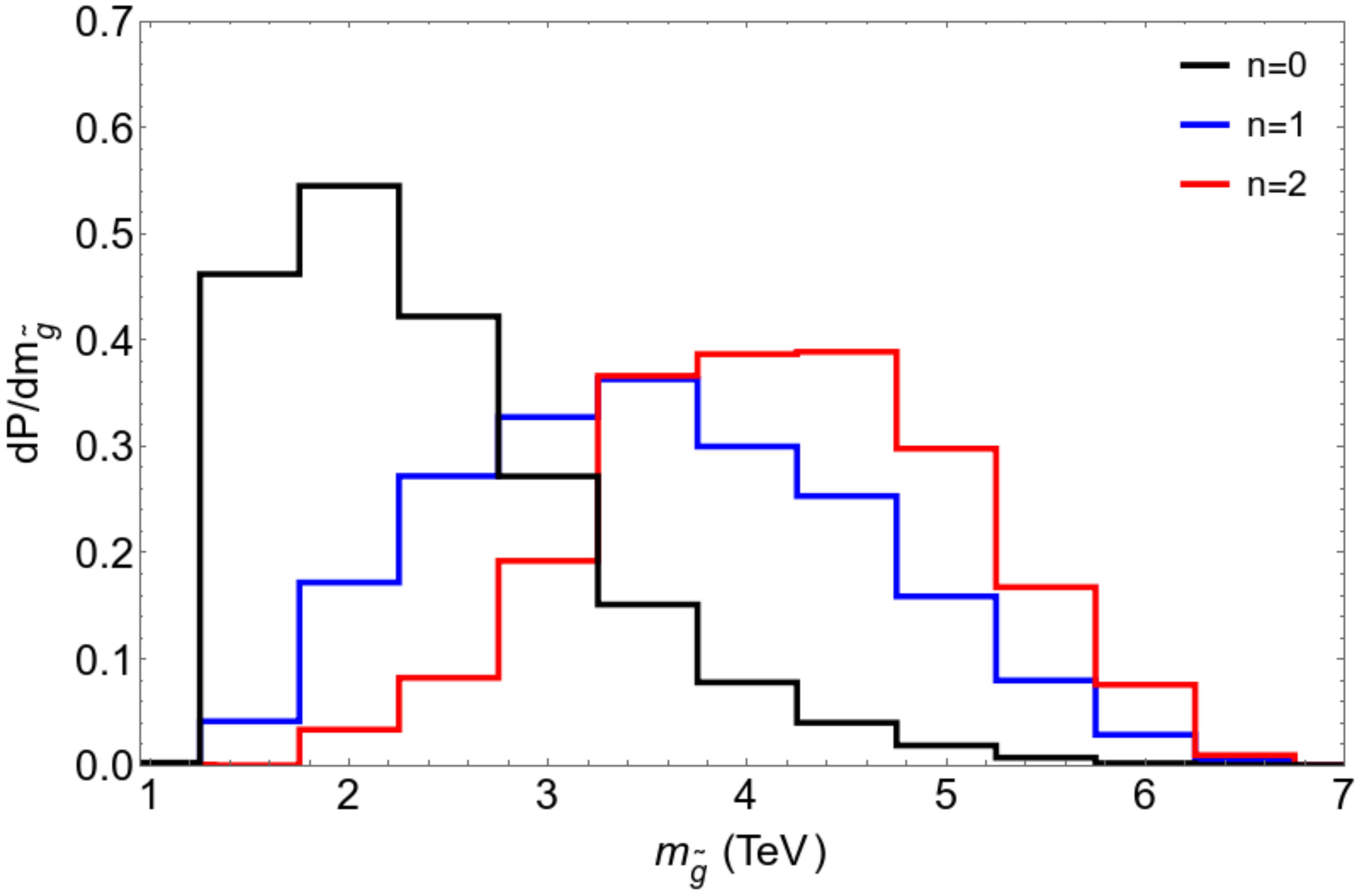
$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} dm^2_{hidden}$$



m(h)~125 most favored for n=1,2

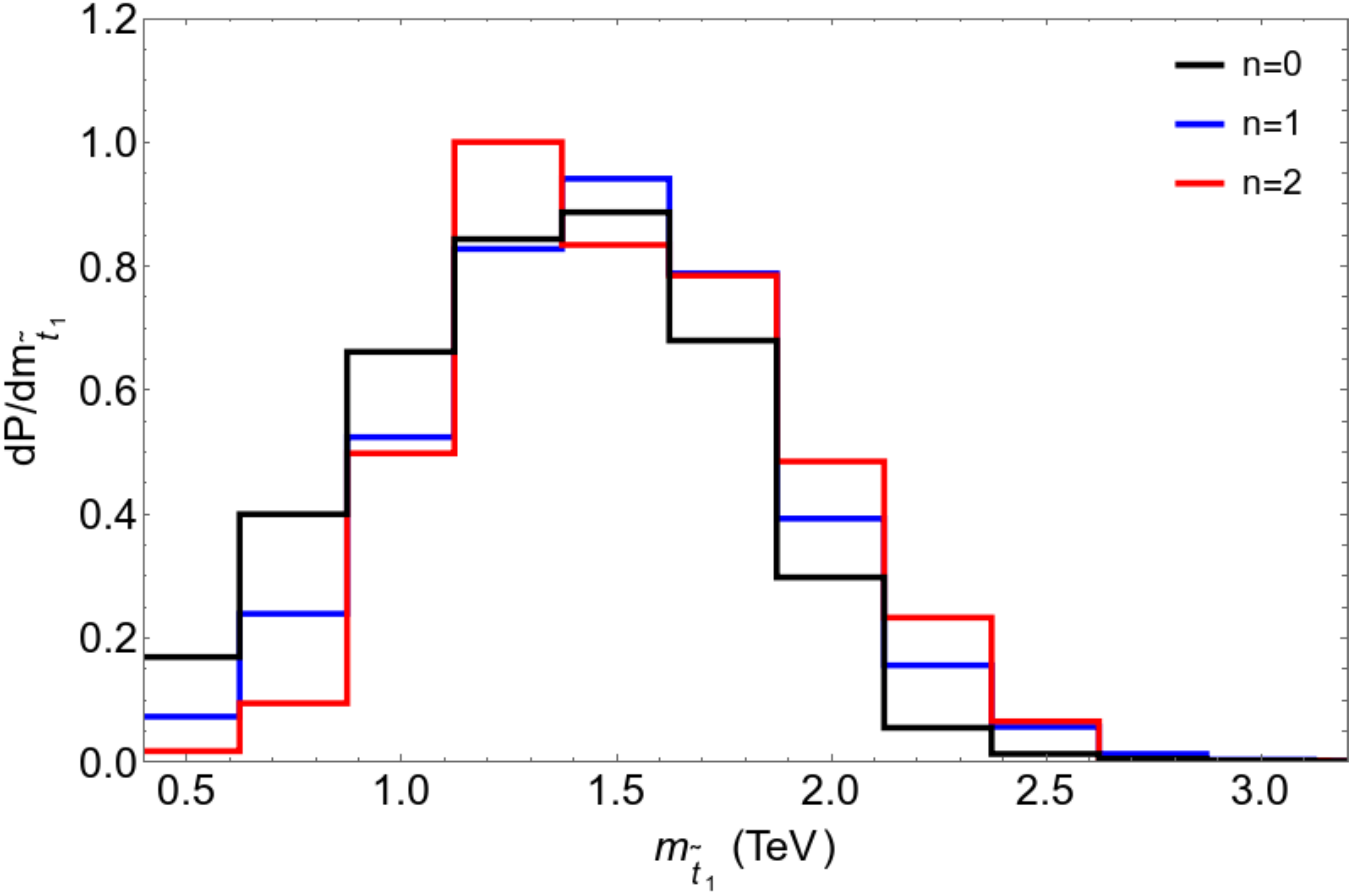
HB,Barger, Serce, Sinha

What is corresponding distribution for gluino mass?



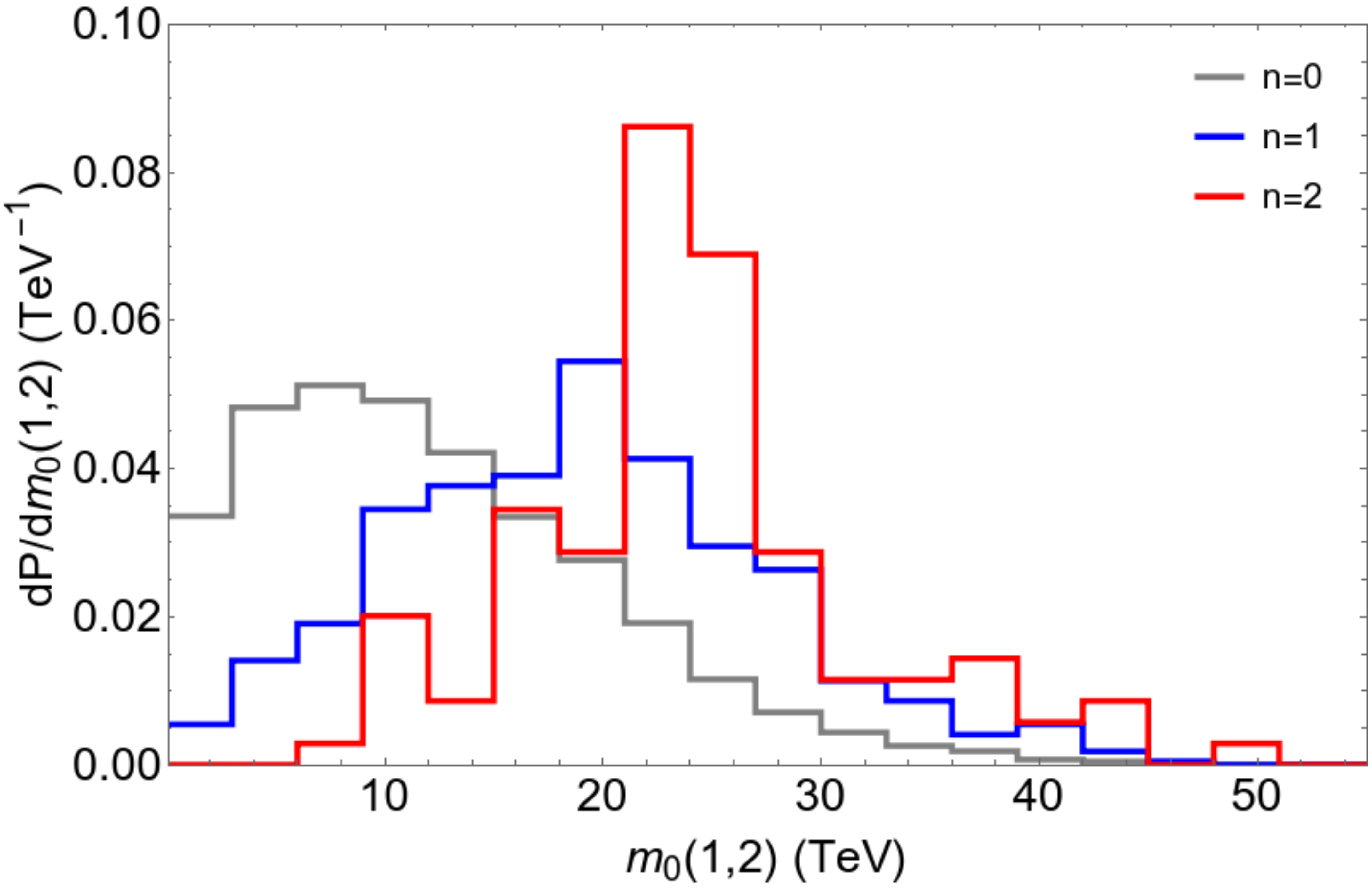
gluino typically beyond LHC 14 reach
(need higher energy hadron collider)

and top-squark mass m(t1)?



m(t1) typically beyond present LHC reach

first/second generation sfermions pulled to 10-30 TeV thus softening any SUSY flavor/CP problems



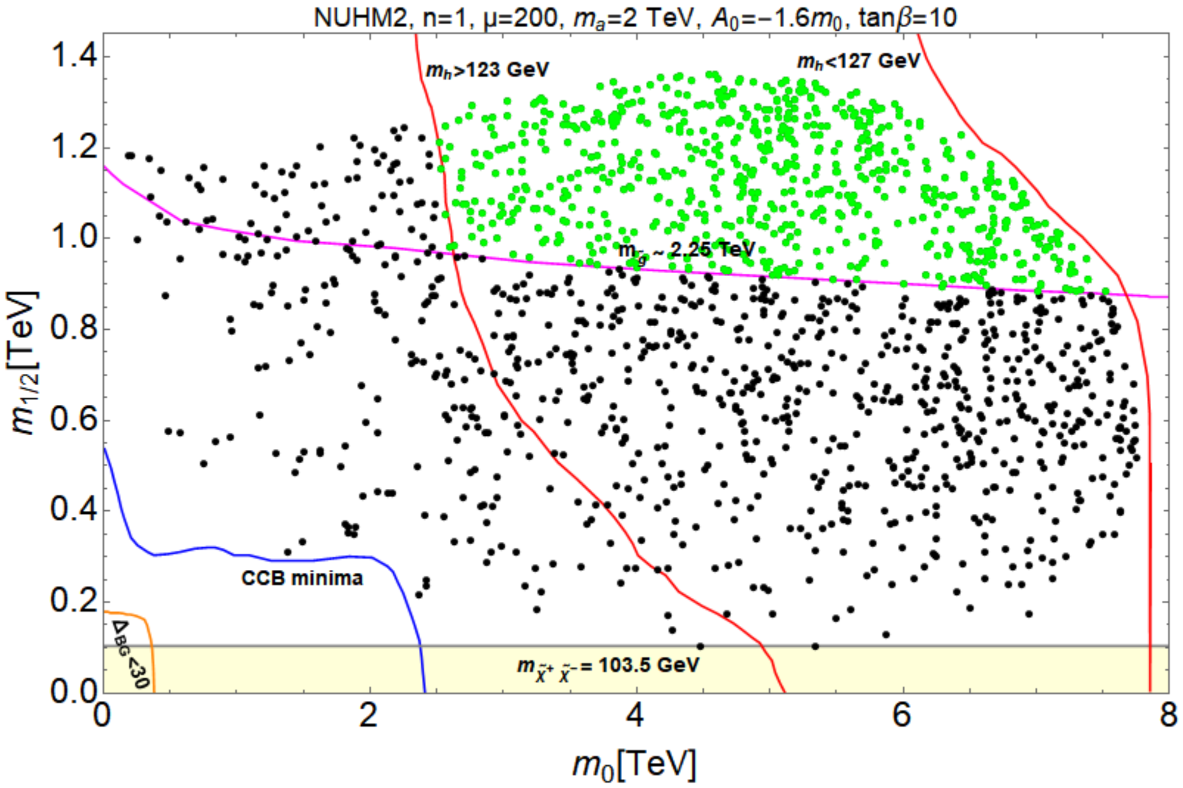
Stringy naturalness: higher density of points are more stringy natural!

conventional natural: favor low m0, mhf

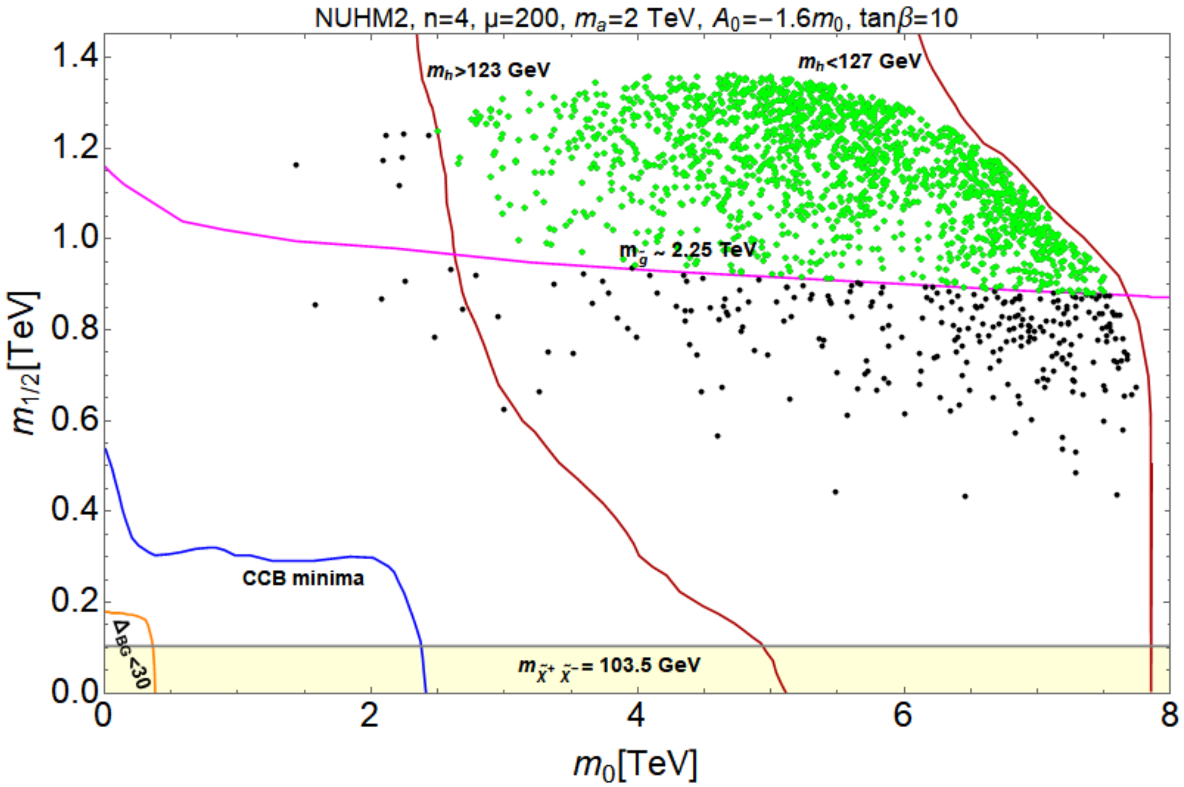
stringy naturalness: favor high m0, mhf so long as m(weak)~100 GeV

HB, Barger, Salam, arXiv:1906.07741

Living dangerously: Arkani-Hamed, Dimopoulos, Kachru, hep-ph/0501082



$$m(soft)^1$$



$$m(soft)^4$$

Under stringy naturalness, a 3 TeV gluino is
more natural than a 300 GeV gluino!

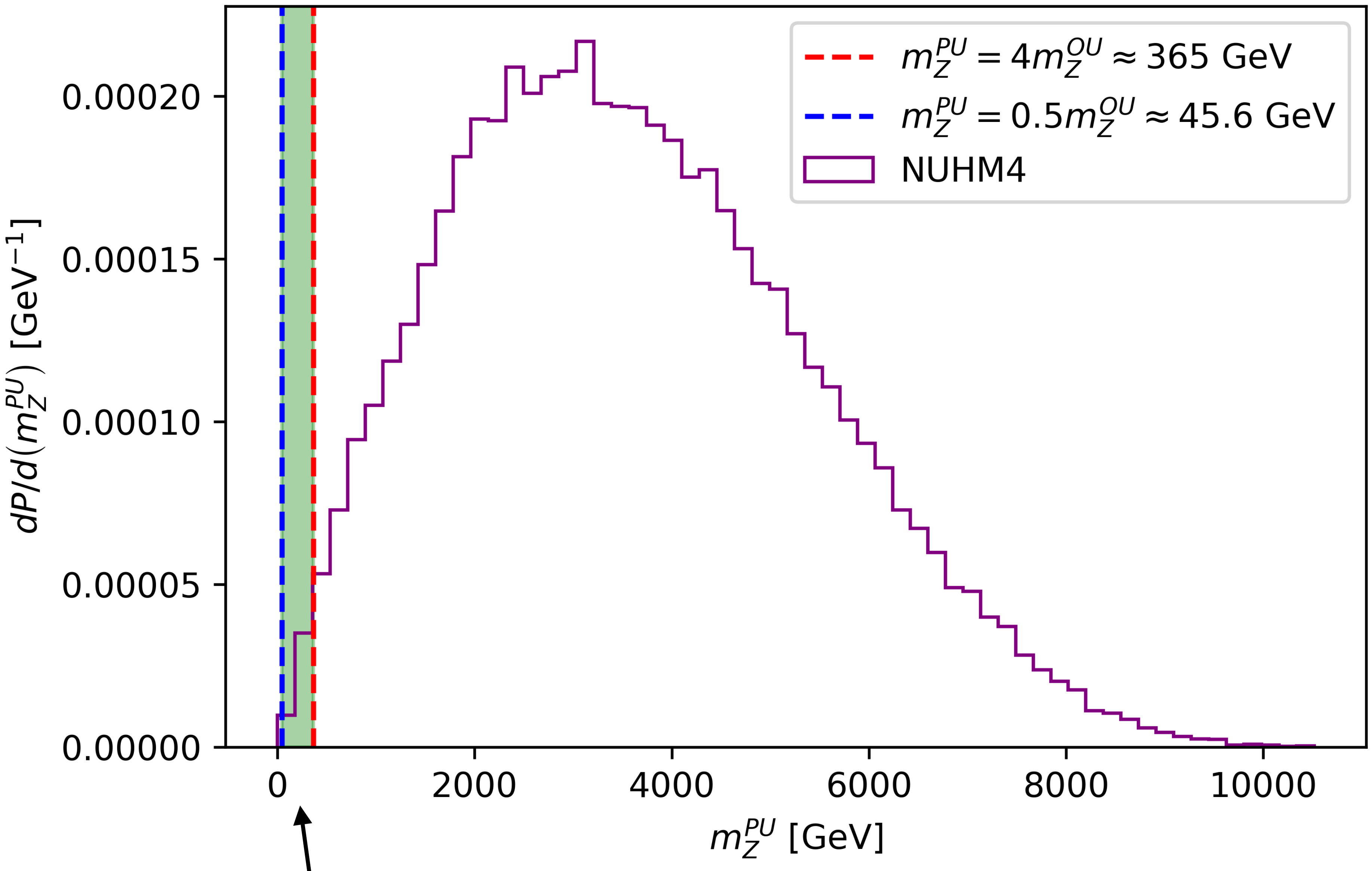
What sort of SUSY does one expect from the landscape?

This picture I believe is totally wrong!

string landscape:

1. statistical draw to large soft terms

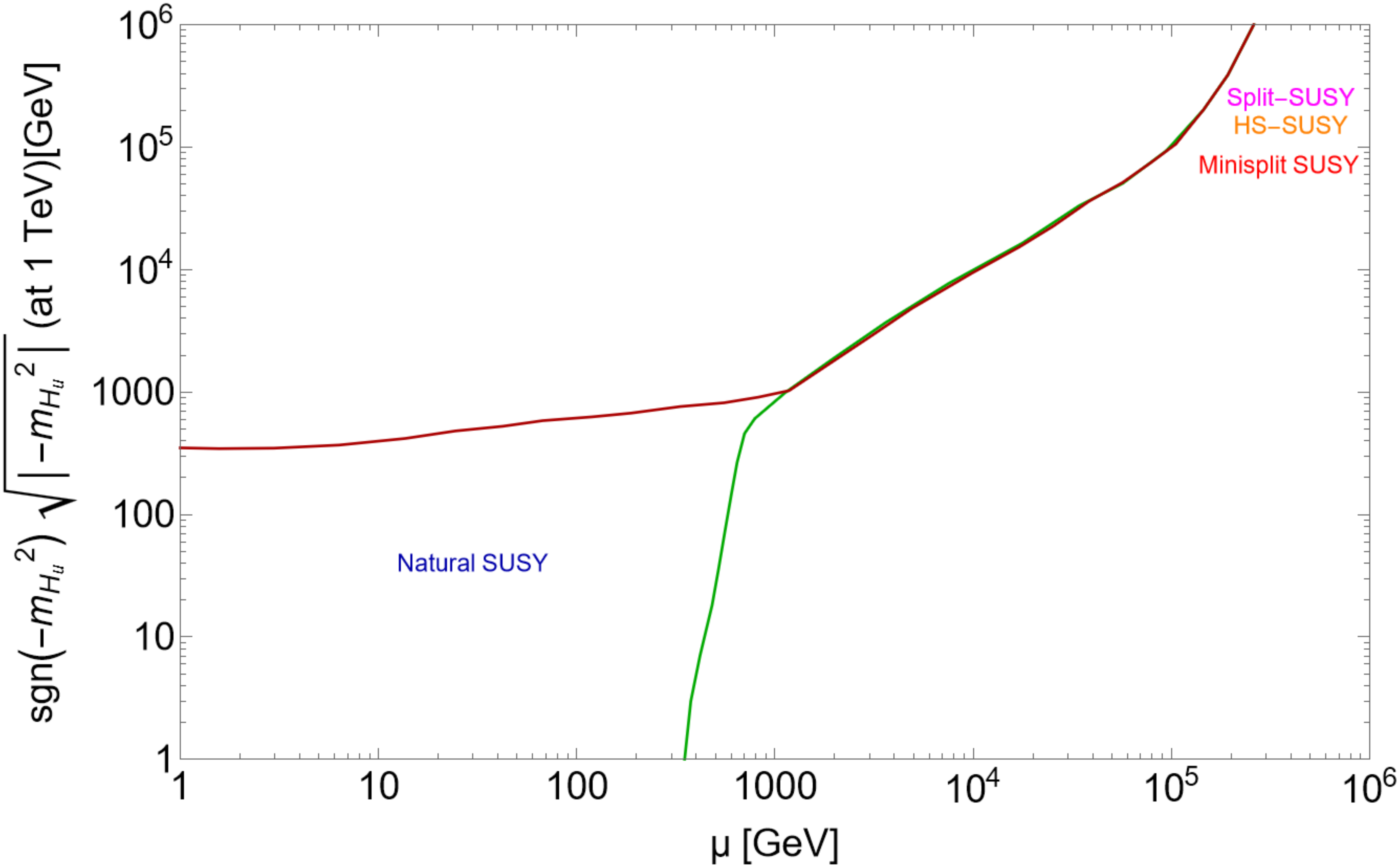
2. must lie within ABDS window



ABDS window

expected distribution of weak scale on landscape

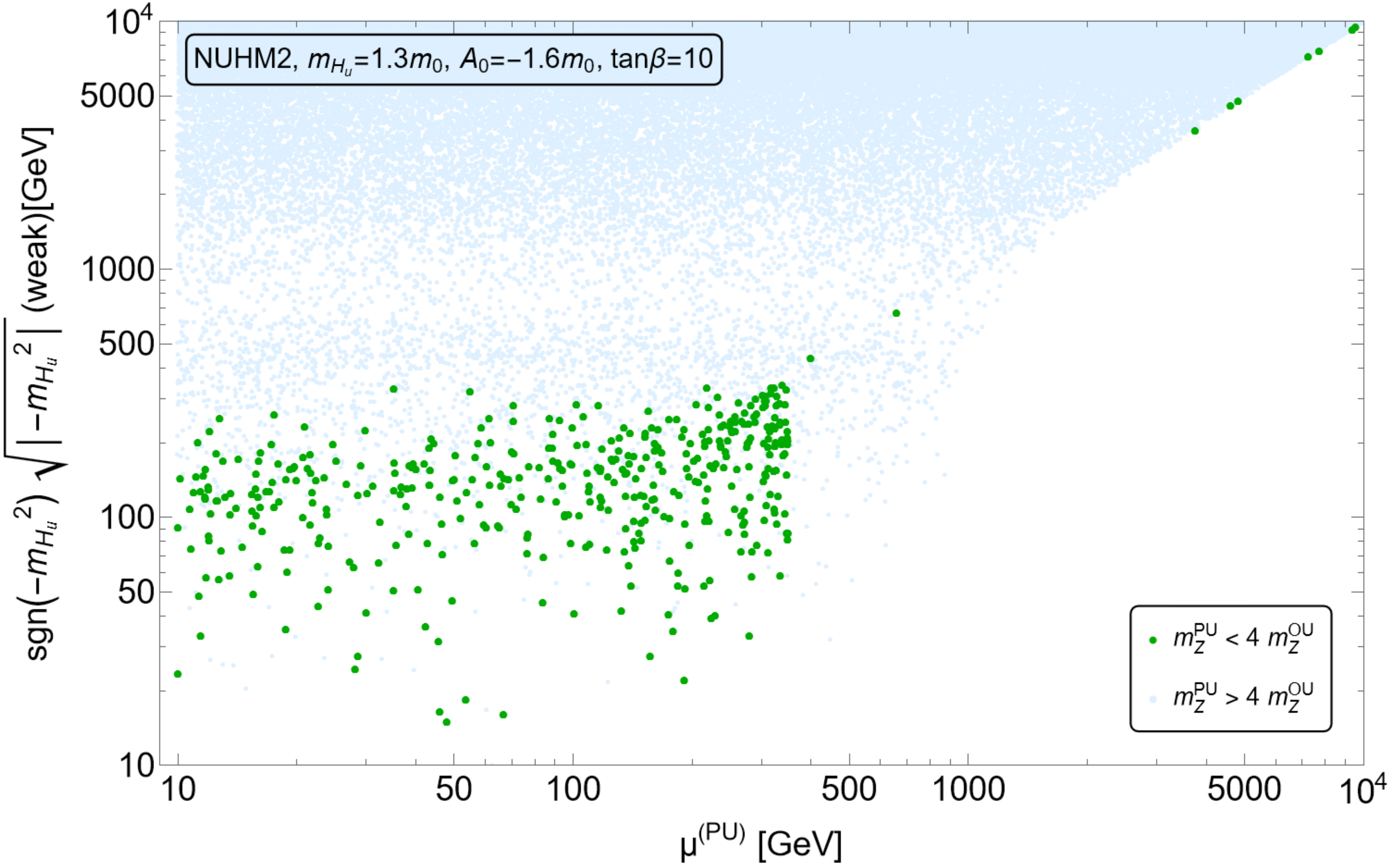
ABDS window in m(Hu)(weak) vs mu(weak) p-space



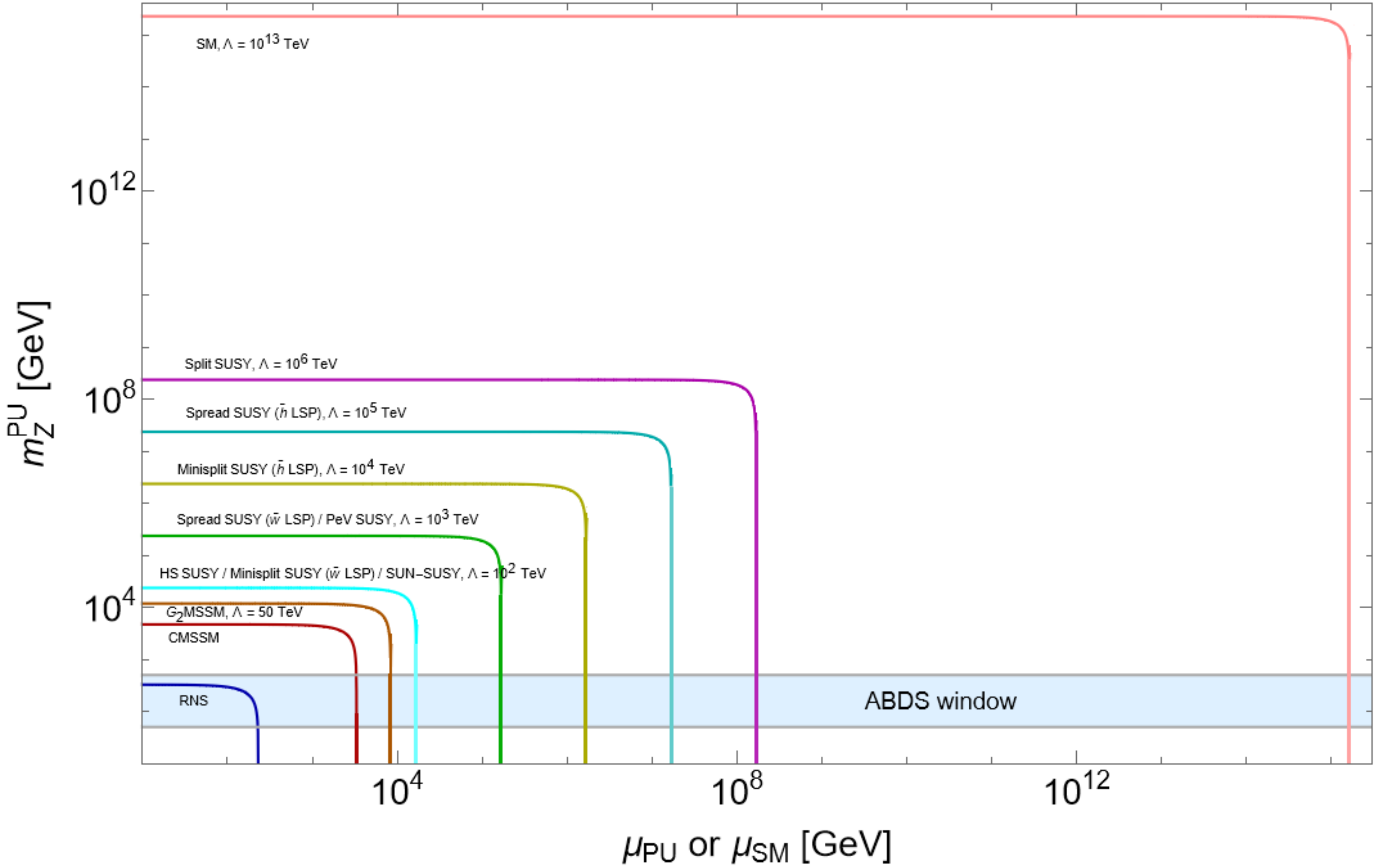
must lie between red and green curves: ABDS window

toy simulation of multiverse:

natural models favored over finetuned

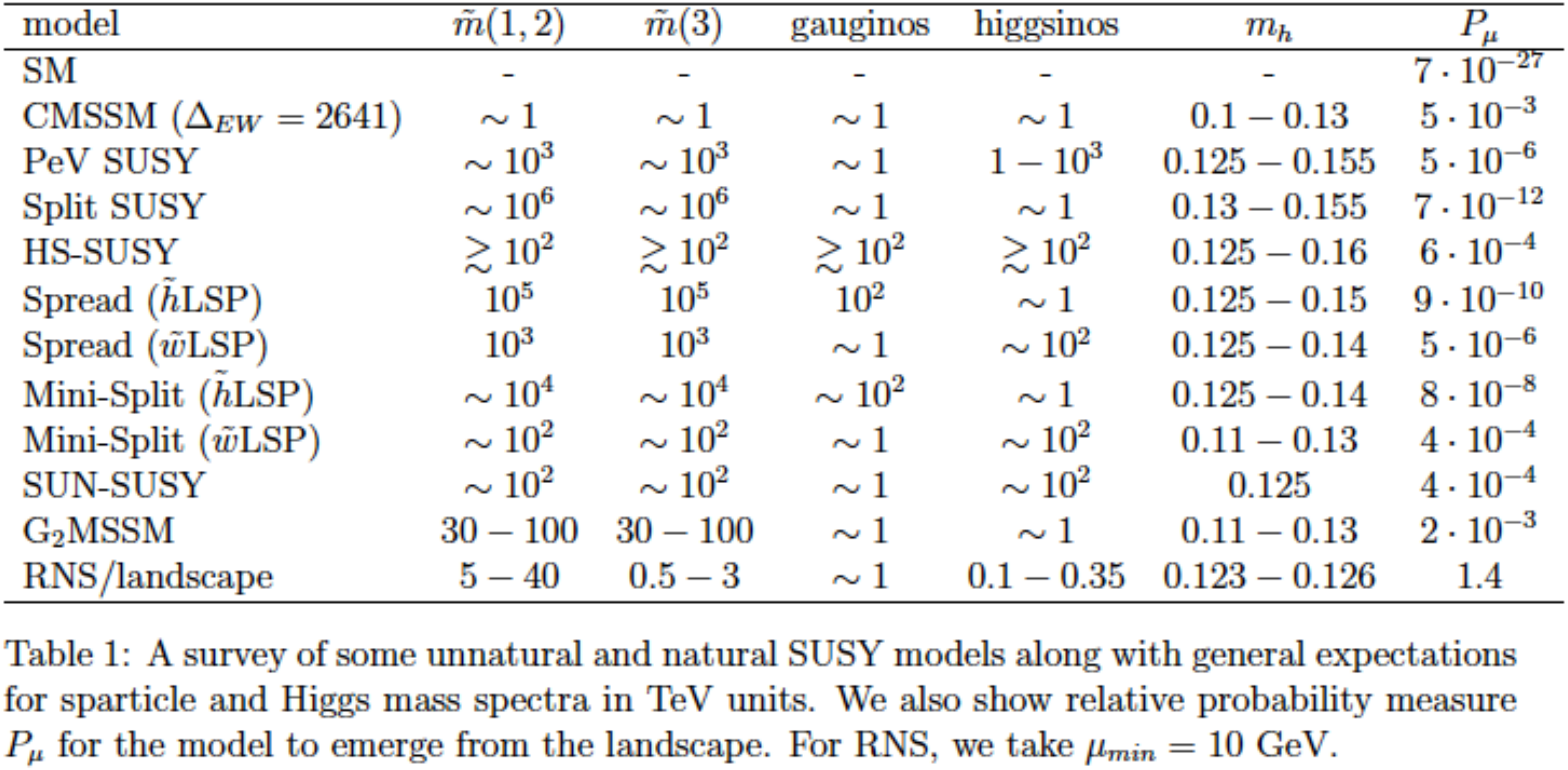


HB, Barger, Martinez,Salam: arXiv:2202.07046 a scheme to calculate relative probabilities for models from landscape:



shoot darts at bottom scale:

what is probability to lie within ABDS window?



model	$\tilde{m}(1,2)$	$\tilde{m}(3)$	gauginos	higgsinos	m_h	P_μ
SM	-	-	-	-	-	$7 \cdot 10^{-27}$
CMSSM ($\Delta_{EW} = 2641$)	~ 1	~ 1	~ 1	~ 1	$0.1 - 0.13$	$5 \cdot 10^{-3}$
PeV SUSY	$\sim 10^3$	$\sim 10^3$	~ 1	$1 - 10^3$	$0.125 - 0.155$	$5 \cdot 10^{-6}$
Split SUSY	$\sim 10^6$	$\sim 10^6$	~ 1	~ 1	$0.13 - 0.155$	$7 \cdot 10^{-12}$
HS-SUSY	$\gtrsim 10^2$	$\gtrsim 10^2$	$\gtrsim 10^2$	$\gtrsim 10^2$	$0.125 - 0.16$	$6 \cdot 10^{-4}$
Spread ($\tilde{h}$LSP)	10^5	10^5	10^2	~ 1	$0.125 - 0.15$	$9 \cdot 10^{-10}$
Spread ($\tilde{w}$LSP)	10^3	10^3	~ 1	$\sim 10^2$	$0.125 - 0.14$	$5 \cdot 10^{-6}$
Mini-Split ($\tilde{h}$LSP)	$\sim 10^4$	$\sim 10^4$	$\sim 10^2$	~ 1	$0.125 - 0.14$	$8 \cdot 10^{-8}$
Mini-Split ($\tilde{w}$LSP)	$\sim 10^2$	$\sim 10^2$	~ 1	$\sim 10^2$	$0.11 - 0.13$	$4 \cdot 10^{-4}$
SUN-SUSY	$\sim 10^2$	$\sim 10^2$	~ 1	$\sim 10^2$	0.125	$4 \cdot 10^{-4}$
G_2MSSM	$30 - 100$	$30 - 100$	~ 1	~ 1	$0.11 - 0.13$	$2 \cdot 10^{-3}$
RNS/landscape	$5 - 40$	$0.5 - 3$	~ 1	$0.1 - 0.35$	$0.123 - 0.126$	1.4

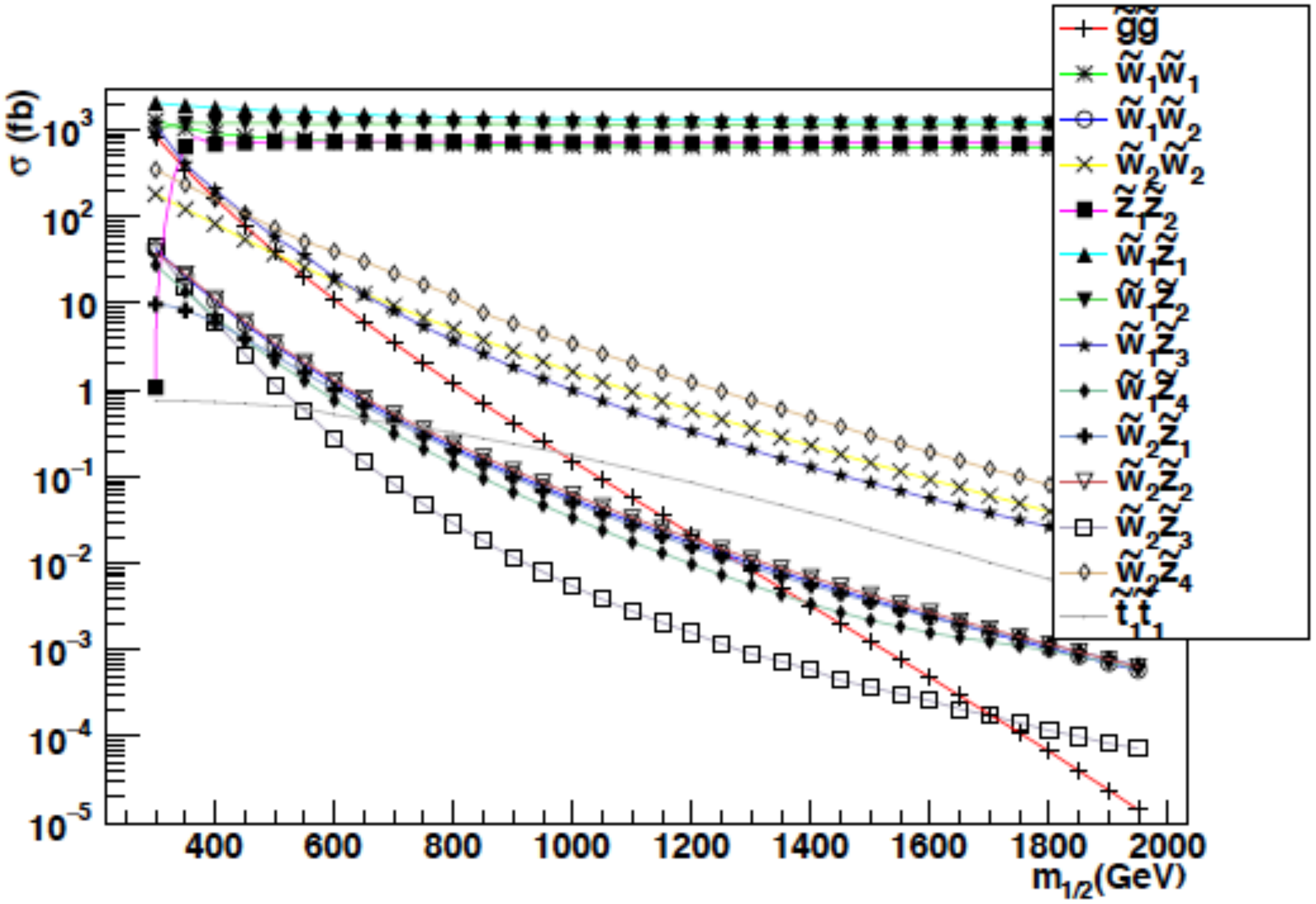
Table 1: A survey of some unnatural and natural SUSY models along with general expectations for sparticle and Higgs mass spectra in TeV units. We also show relative probability measure P_μ for the model to emerge from the landscape. For RNS, we take $\mu_{min} = 10$ GeV.

natural SUSY by far most likely
to emerge from landscape:
minimal tuning= greatest probability!

HB, Barger, Martinez,Salam:arXiv: 2206.14839

Prospects for discovering landscape/natural SUSY at LHC and ILC

Sparticle prod'n along RNS model-line at LHC14:



higgsinos

gauginos

gluinos

higgsino pair production dominant-but only soft visible energy release from higgsino decays

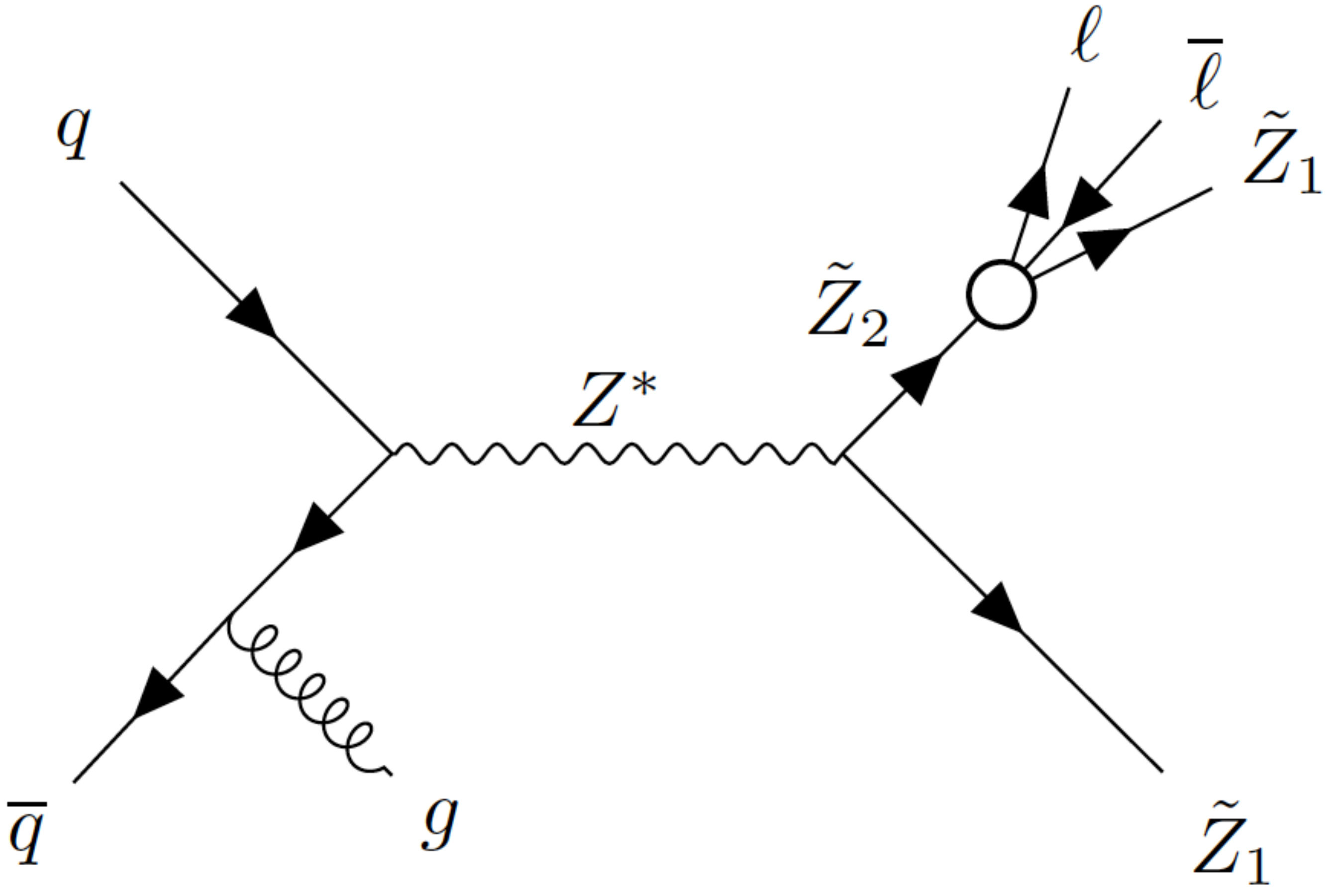
largest visible cross section: wino pairs

gluino pairs sharply dropping

HL-LHC best bet: higgsino pair production

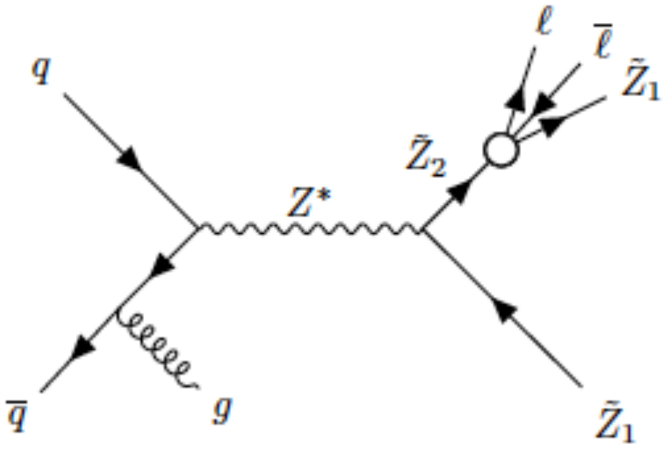
What about $pp \to \tilde{Z}_1\tilde{Z}_2 j$ with $\tilde{Z}_2 \to \tilde{Z}_1 \ell^+\ell^-$?

HB, Barger, Huang, JHEP11 (2011) 031;
Han, Kribs, Martin, Menon, PRD89 (2014) 075007;
HB, Mustafayev, Tata, PRD90 (2014) 115007;

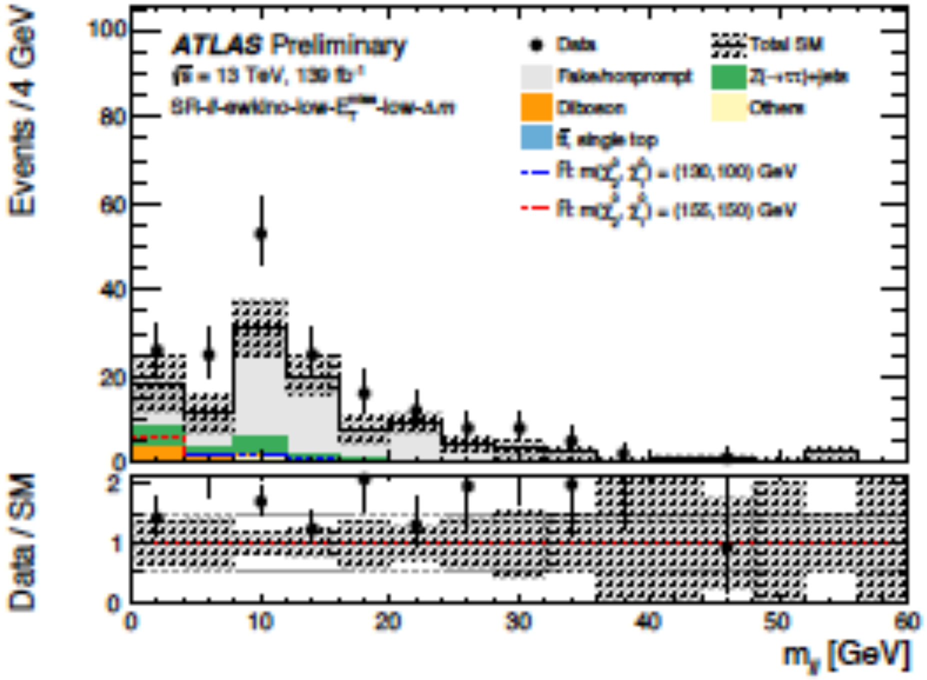


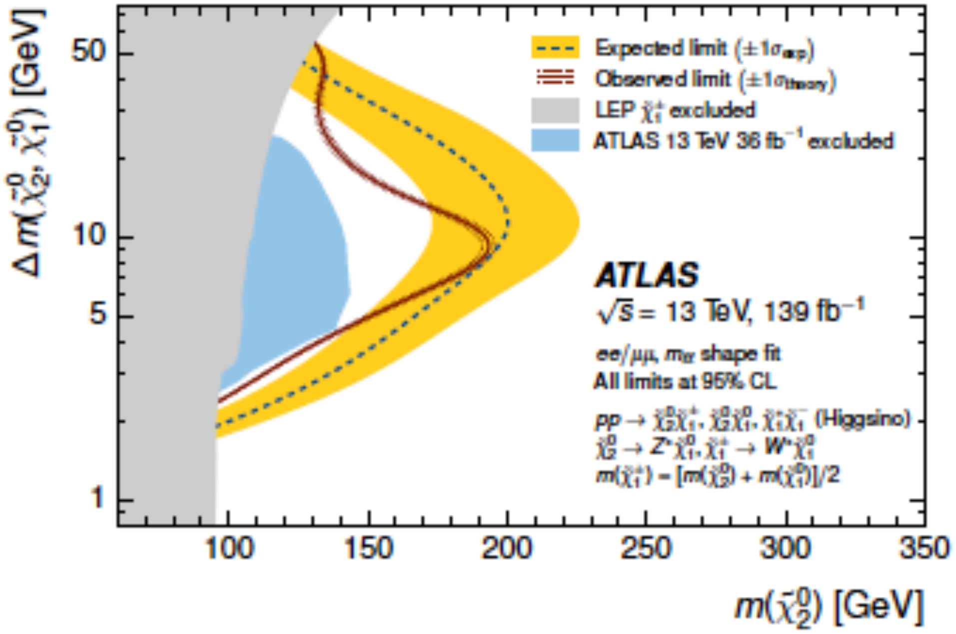
Natural SUSY: only higgsinos need lie close to weak scale

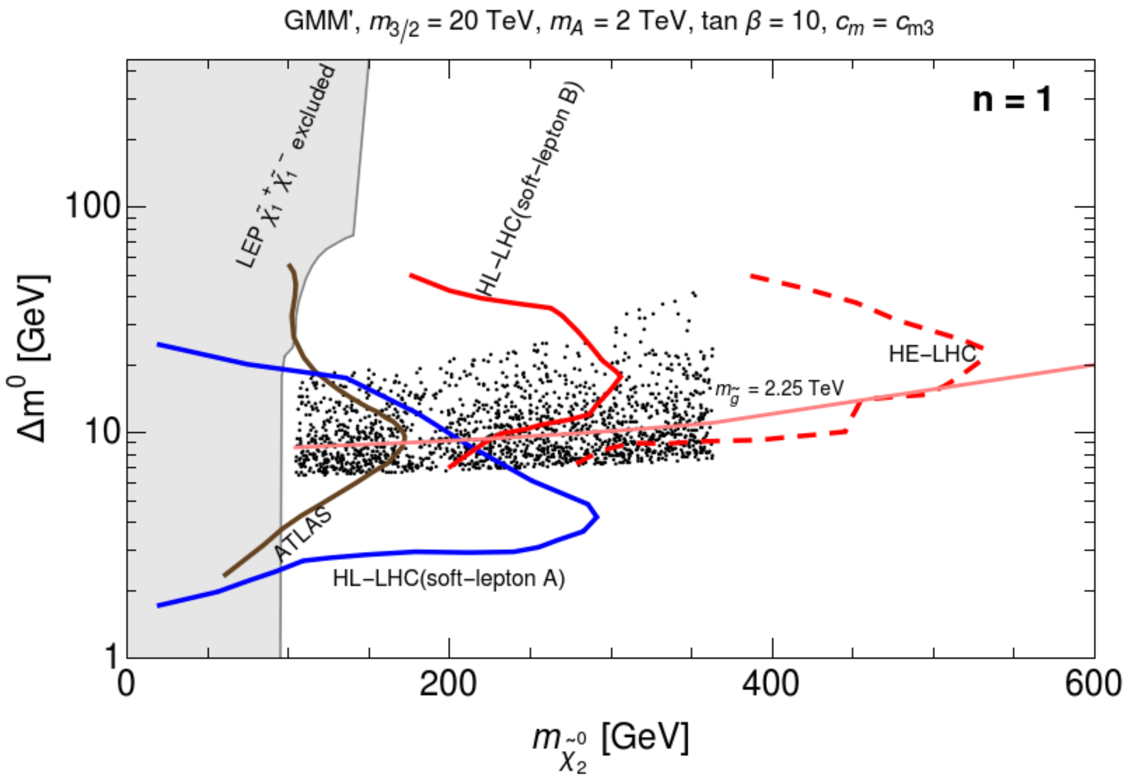
Soft dilepton+jet+MET signature from higgsino pair production



HB, Barger, Huang, 1107.5581;
Z. Han, Kribs, Martin, Menon, 1401.1235;
HB, Mustafayev, Tata; 1409.7058;
C. Han, Kim, Munir, Park, 1502.03734;
HB, Barger, Savoy, Tata, 1604.07438;
HB, Barger, Salam, Sengupta, Tata,2007.09252;
HB, Barger, Sengupta, Tata, 2109.14030

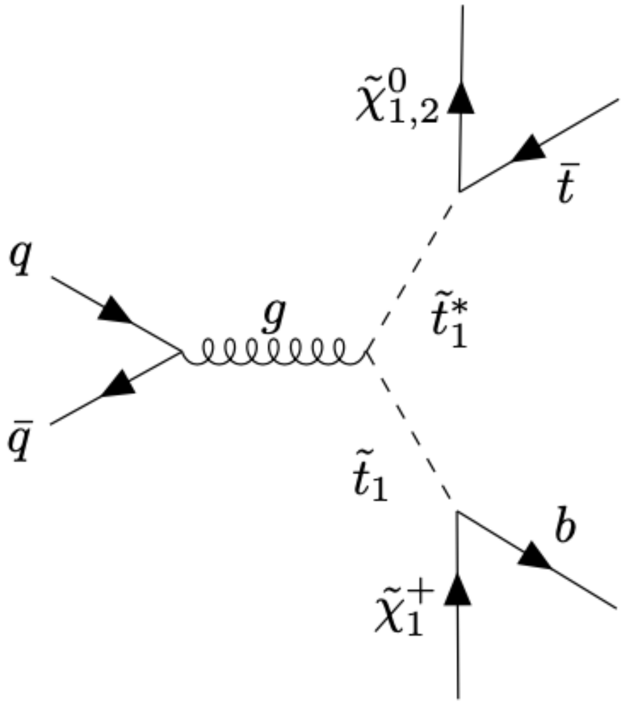


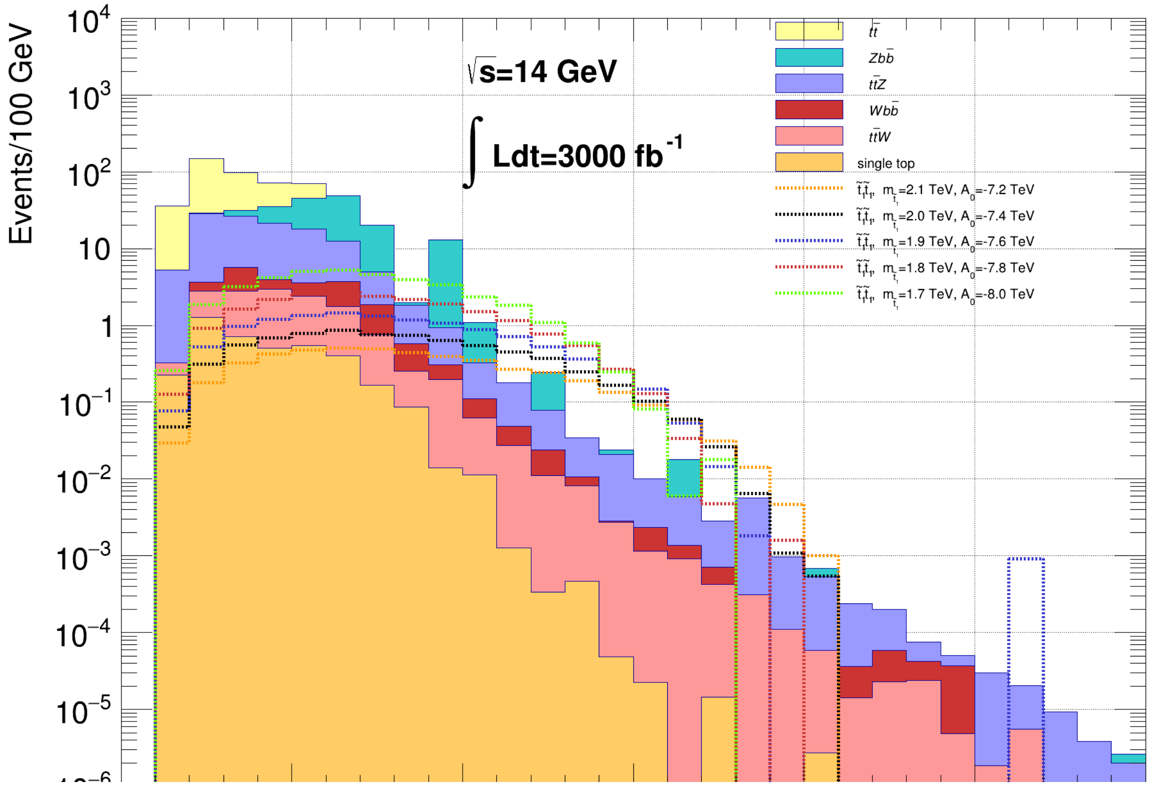




It appears that HL-LHC can see much of natural SUSY p-space;
signal in this channel should emerge slowly as more integrated luminosity accrues

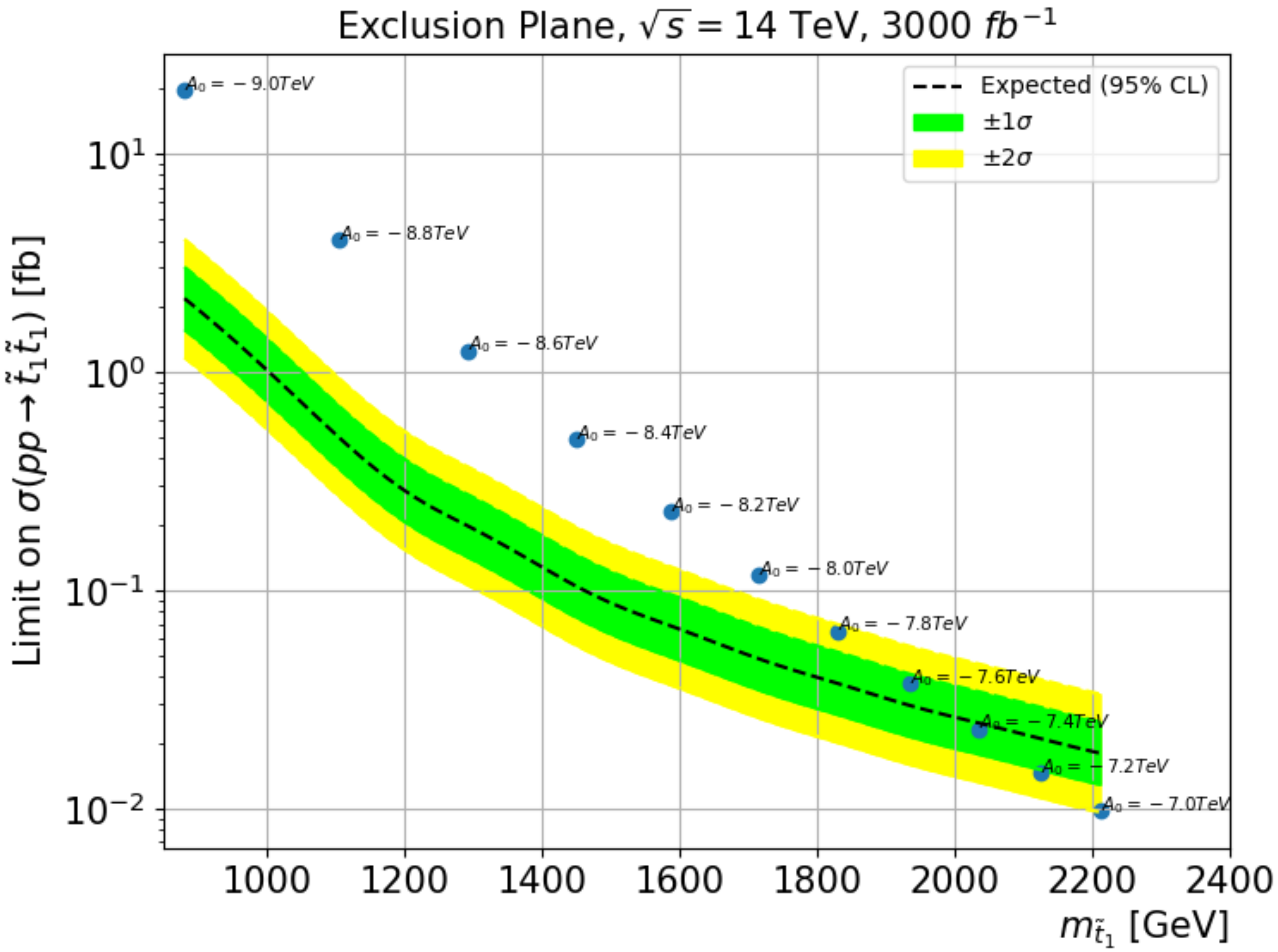
top-squark pair productic



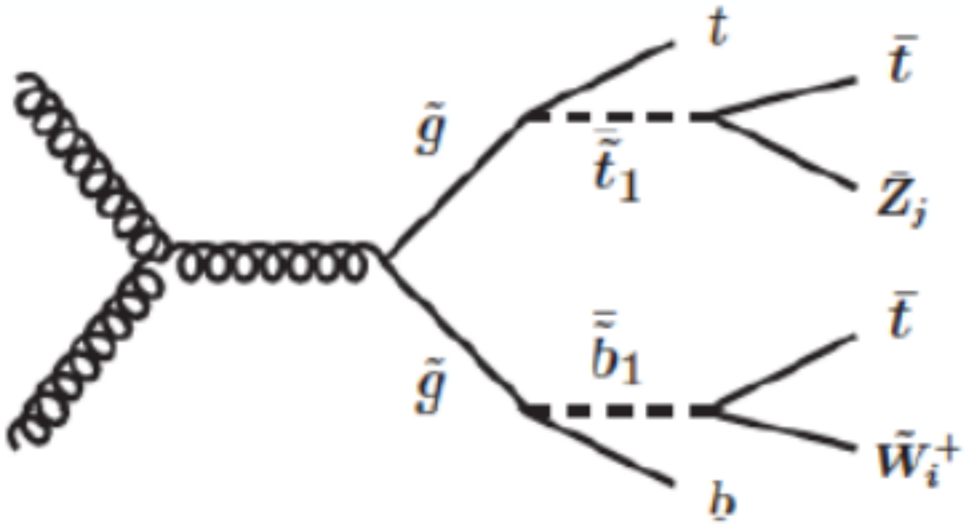


HL-LHC can see
m(t1)~2 TeV
@ 95% CL

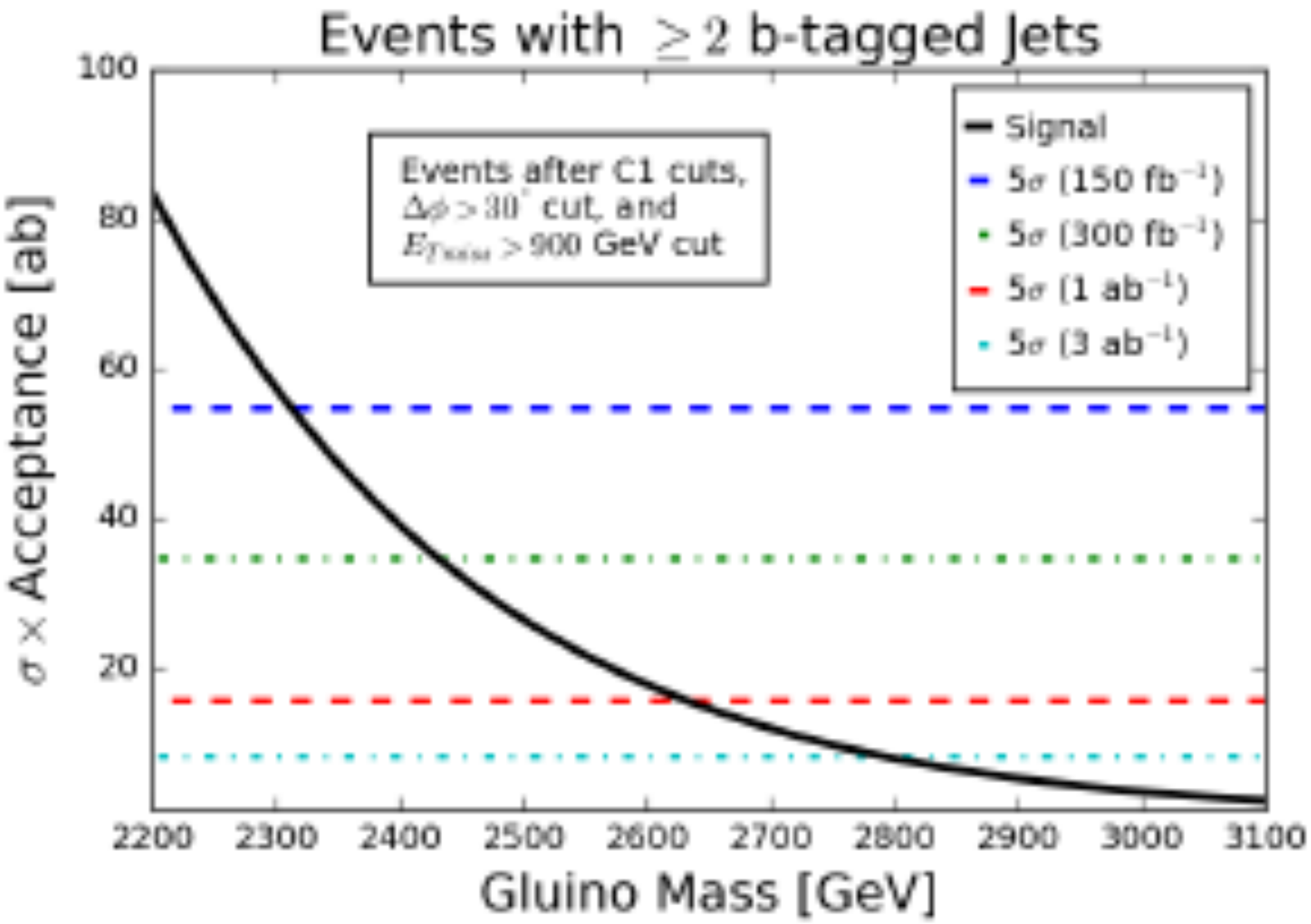
HB, Barger,
Dutta, Sengupta, Zhang



gluino pair cascade decay signatures



LHC14

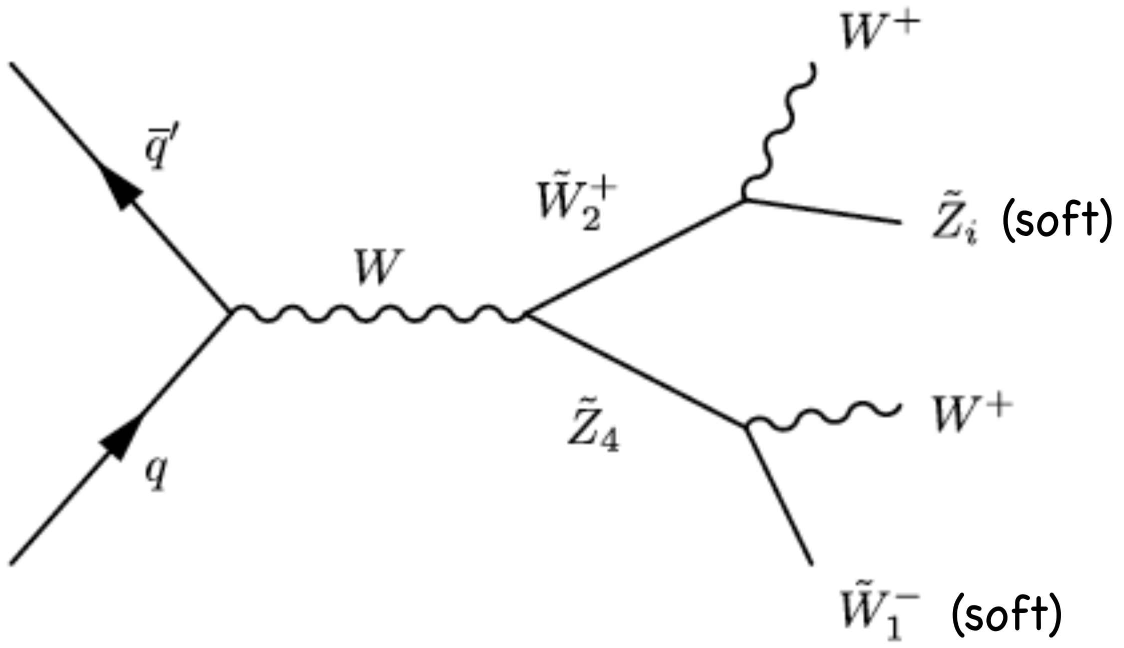


HB, Barger, Gainer, Huang, Savoy, Sengupta,Tata

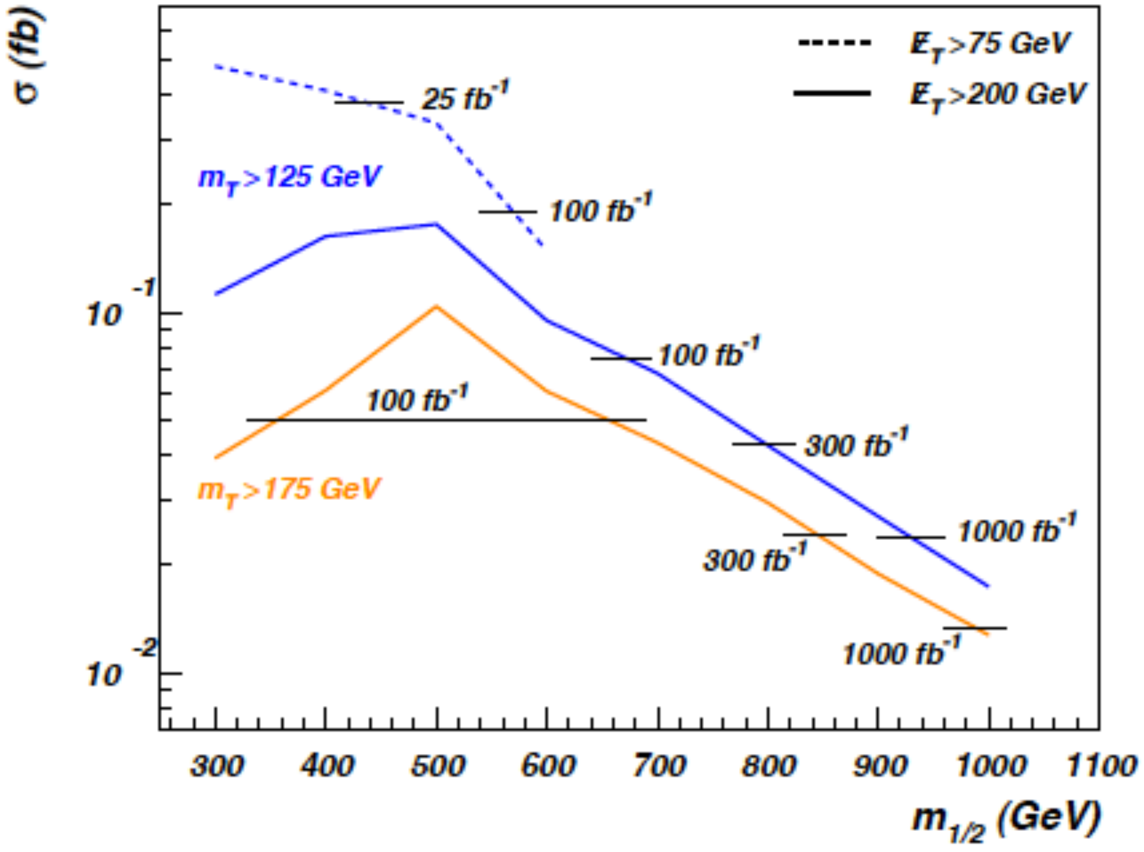
HL-LHC to probe m(gl)~2.8 TeV
HE-LHC to probe m(gl)~5.5-6 TeV

FCC-hh(100) to probe m(gl)~10 TeV

Distinctive new same-sign diboson (SSdB) signature from SUSY models with light higgsinos!



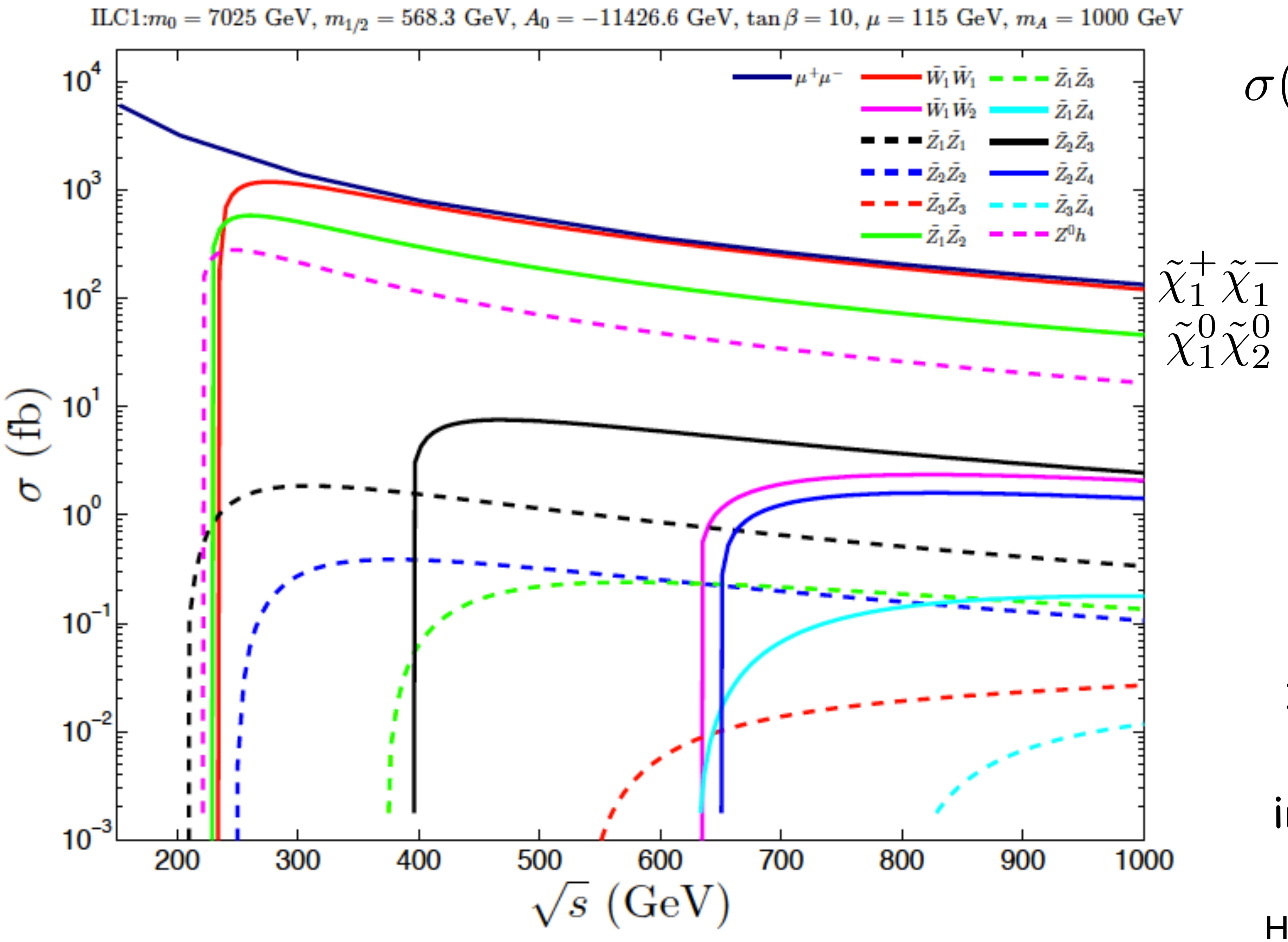
wino pair production



This channel offers added reach of LHC14 for natSUSY; it is also indicative of wino-pair prod'n followed by decay to higgsinos

H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong and X. Tata, *Phys. Rev. Lett.* **110** (2013) 151801.

Smoking gun signature: light higgsinos at ILC:
ILC is Higgs/higgsino factory!



$$\sigma(higgsino) \gg \sigma(Zh)$$

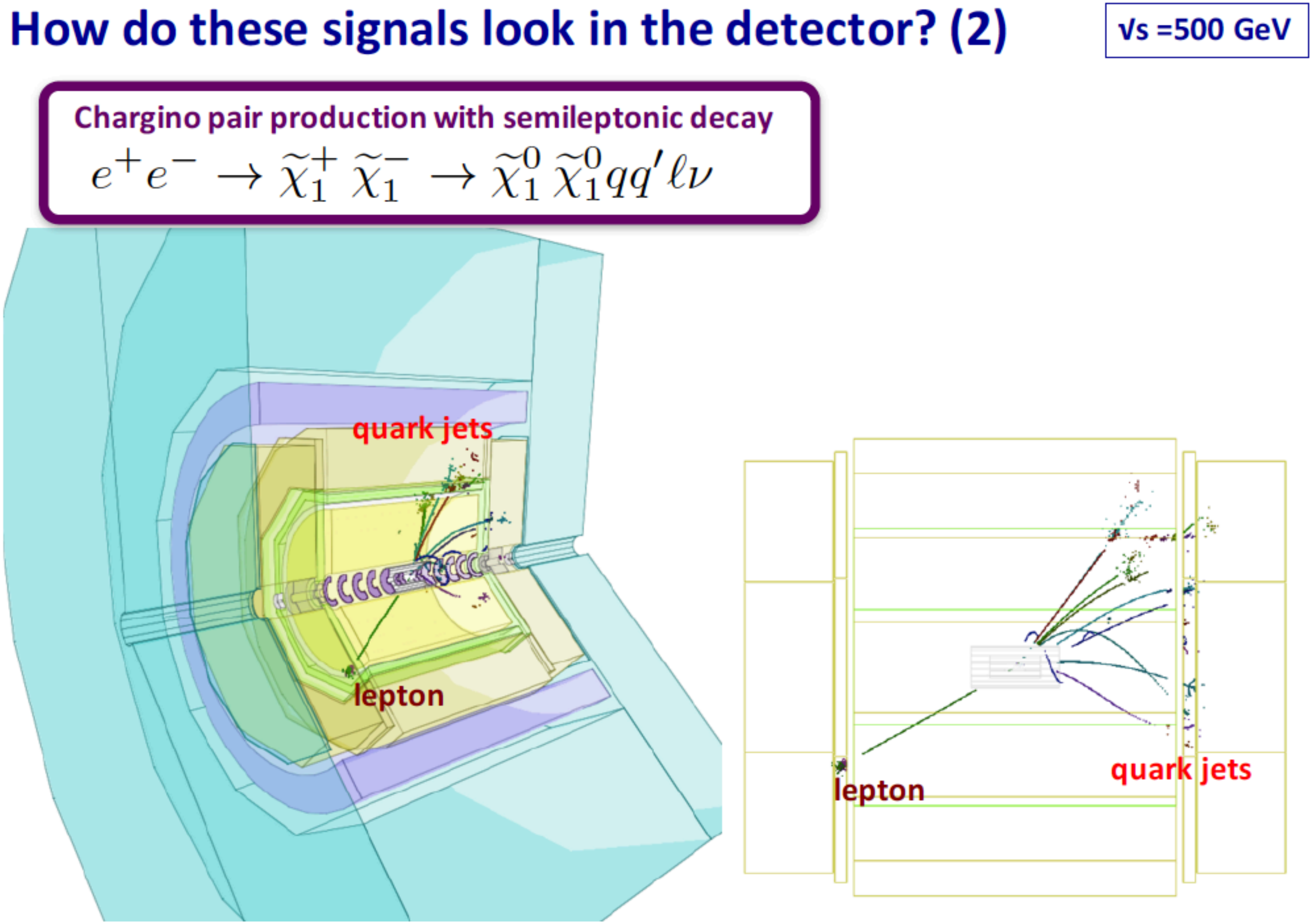
3-15 GeV higgsino mass
gaps no problem
in clean ILC environment

HB, Barger, Mickelson, Mustafayev, Tata
arXiv:1404:7510

$$e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^- \to (\ell\nu_\ell\tilde{\chi}_1^0) + (q\bar{q}'\tilde{\chi}_1^0)$$

measure $m(jj) < m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ and $E(jj)$

soft visible particles since small higgsino mass gaps



How do these signals look in the detector? (2)

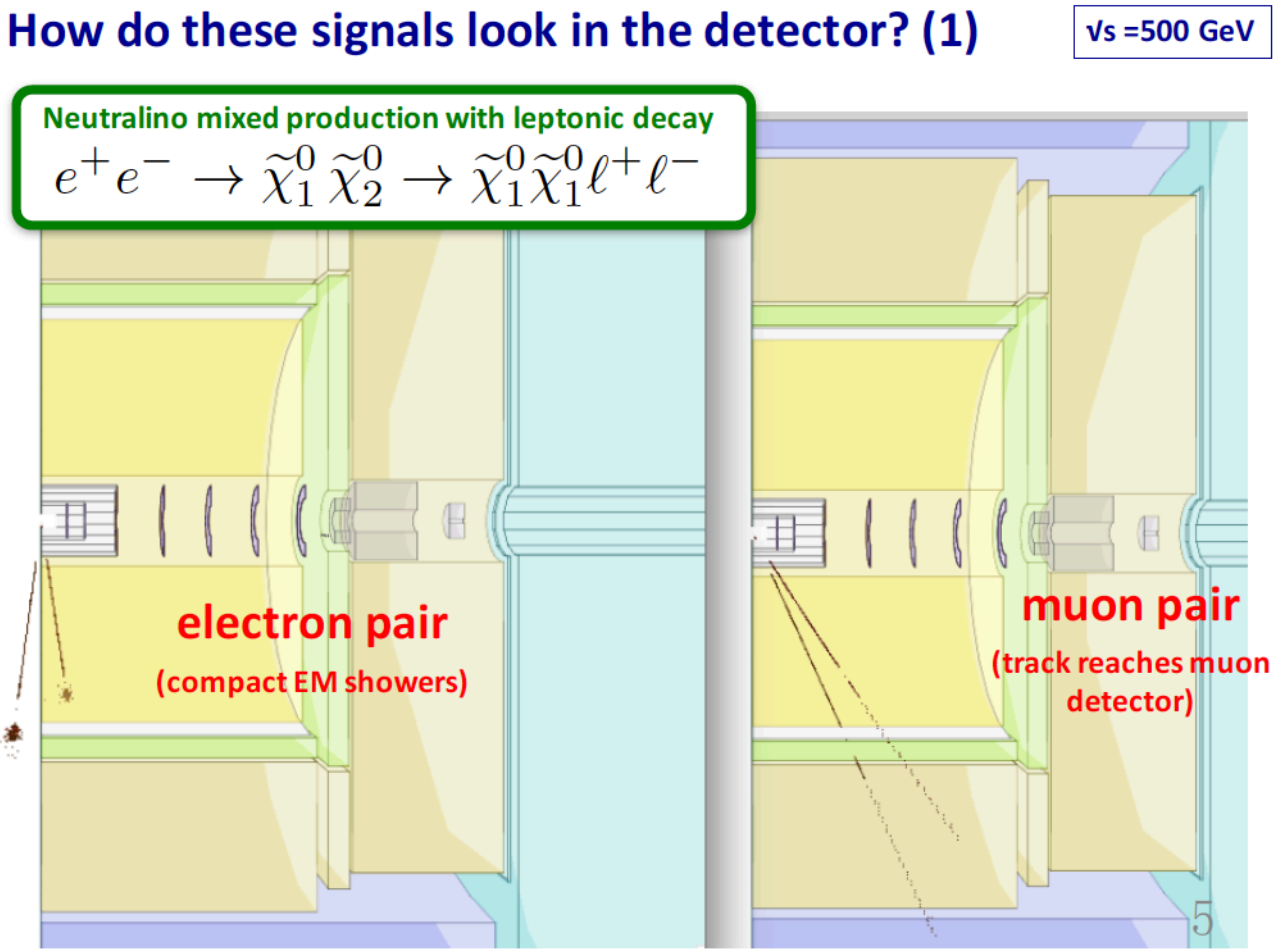
√s =500 GeV

Chargino pair production with semileptonic decay

$$e^+e^- \to \widetilde{\chi}_1^+\,\widetilde{\chi}_1^- \to \widetilde{\chi}_1^0\,\widetilde{\chi}_1^0 qq'\ell\nu$$

$$e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 + (\ell^+\ell^-\tilde{\chi}_1^0)$$

$$\text{measure } m(\ell^+\ell^-) < m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} \text{ and } E(\ell^+\ell^-)$$



How do these signals look in the detector? (1)

√s =500 GeV

Neutralino mixed production with leptonic decay

$$e^+e^- \to \tilde{\chi}_1^0\,\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0\ell^+\ell^-$$

electron pair

(compact EM showers)

muon pair

(track reaches muon detector)

For further reading:

- The string theory landscape, Bousso & Polchinski, Sci. Am. 291 (2004) 60-69
- Midi-review: Status of weak scale supersymmetry after LHC Run 2 and ton-scale noble liquid WIMP searches, HB, Barger, Salam, Sengupta, Sinha, arXiv: 2002.03013

Dark matter from SUSY with radiatively-driven naturalness

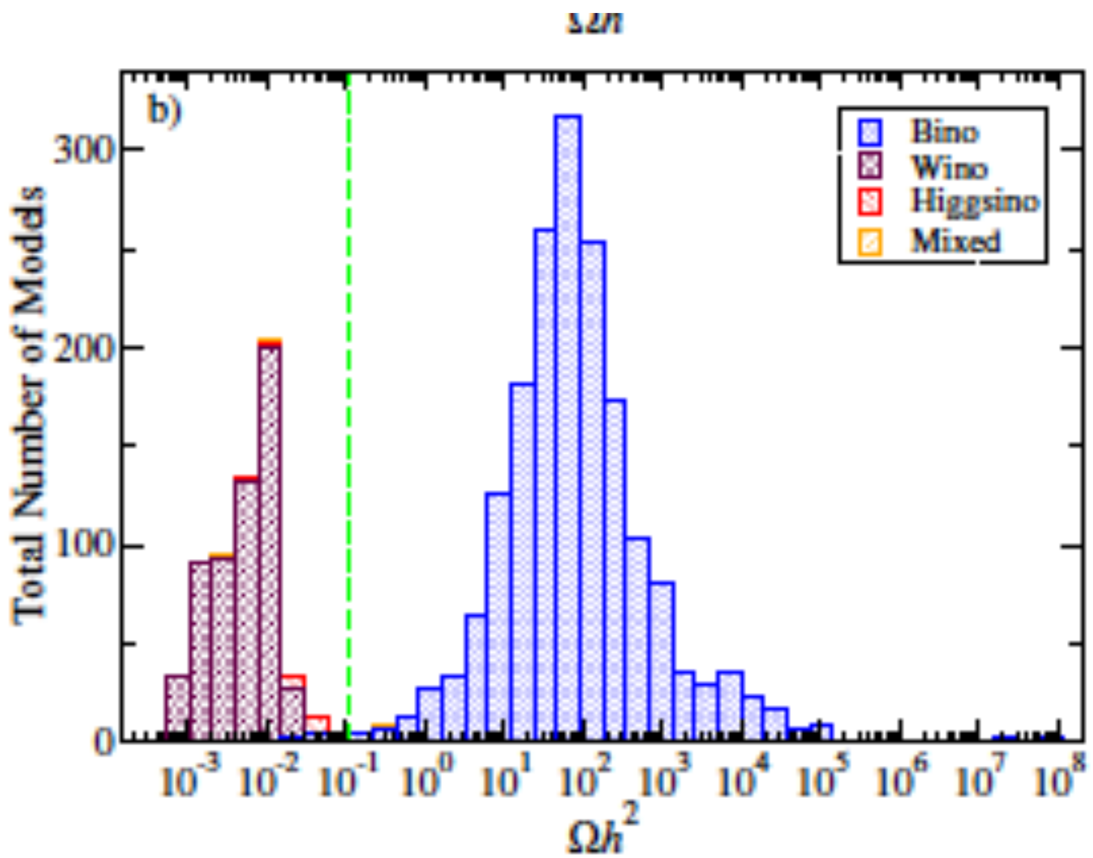
SUSY dark matter

- ⋆ R-parity conservation⇒ conserved B and L ⇒ proton stability
  - $R(particle) = 1$; $R(sparticle) = -1$
- ⋆ Naturally occurs in $SO(10)$ SUSY GUT theories
- ⋆ Some consequences:
  - Sparticles are produced in pairs
  - Sparticles decay to other sparticles
  - Lightest SUSY particle (LSP) is absolutely stable (good candidate for dark matter)
- ⋆ LSP must be charge, color neutral (bound on cosmological relics)
- ⋆ Sneutrino would have been detected in direct detection experiments
- ⋆ lightest neutralino $\tilde{Z}_1$ is LSP in wide range of models
- ⋆ $\tilde{Z}_1$ is weakly interacting, massive particle (WIMP)

Calculating the relic density of neutralinos

- ⋆ At very high T, neutralinos in thermal equilibrium with cosmic soup
- ⋆ As universe expands and cools, expansion rate exceeds interaction rate (freeze-out)
- ⋆ number density is governed by Boltzmann eq. for FRW universe
  - $dn/dt = -3Hn - \langle\sigma v_{rel}\rangle(n^2 - n_0^2)$
  - $\Omega_{\widetilde{Z}_1} h^2 = \frac{s_0}{\rho_c/h^2}\left(\frac{45}{\pi g_*}\right)^{1/2}\frac{x_f}{m_{Pl}}\frac{1}{\langle\sigma v\rangle}$
  - $\Omega_{CDM} h^2 \sim 0.1 \Rightarrow \langle\sigma v\rangle \sim 0.9$ pb!
  - $\langle\sigma v\rangle = \pi\alpha^2/8m^2 \Rightarrow m \sim 100$ GeV
  - "The WIMP miracle!": cosmic motivation for new physics at weak scale
- ⋆ SUSY: 1722 annihilation/co-annihilation reactions; 7618 Feynman diagrams
- ⋆ IsaReD program (HB, A. Belyaev , C. Balazs)

The WIMP miracle was always overhyped



HB, Box, Summy, 2010

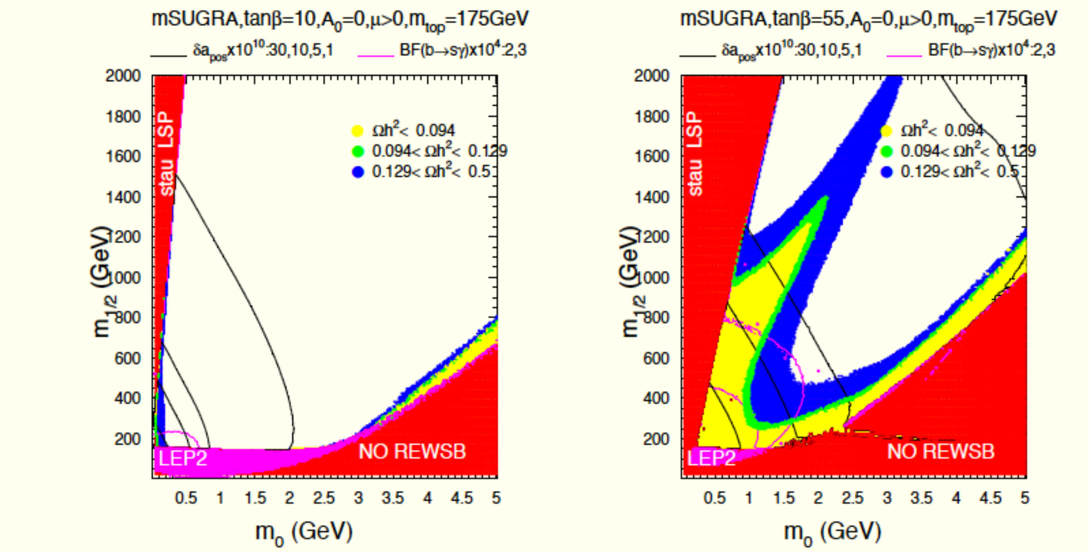
scan over sugra19 model

bino => too much DM

wino, higgsino => to little DM

need special conditions to gain Oh2~0.1:

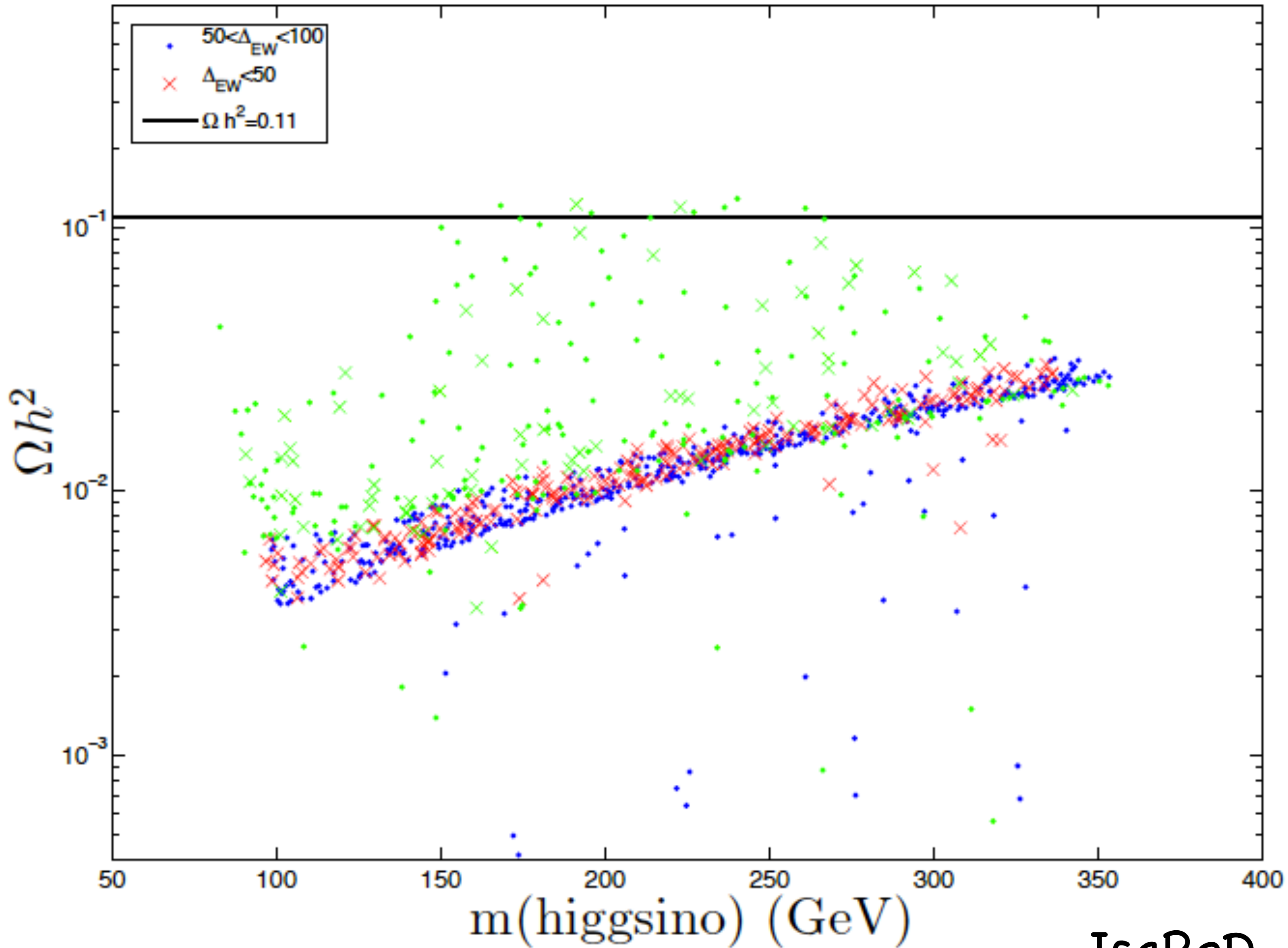
1. well-tempered
2. coannihilation
3. resonance annihilation



HB, A. Belyaev, T. Krupovnickas and A. Mustafayev

natural SUSY WIMP= lightest higgsino

Mainly higgsino-like WIMPs thermally underproduce DM



IsaReD

green: excluded;
red/blue:allowed

HB, Barger, Mickelson

Factor of 10-15 too low

But so far we have addressed only Part 1
of fine-tuning problem:

In QCD sector, the term $\frac{\bar{\theta}}{32\pi^2} F_{A\mu\nu}\tilde{F}_A^{\mu\nu}$ must occur

But neutron EDM says it is not there: strong CP problem

(frequently ignored by SUSY types)

Best solution after 35 years:
PQWW/KSVZ/DFSZ invisible axion

In SUSY, axion accompanied by axino and saxion

Changes DM calculus:
expect mixed WIMP/axion DM (2 particles)

PQ axions need SUSY

- PQ: need new scale f_a~10^11 GeV; but don't want m(h)-> newly introduced high scale
- global PQ inconsistent with quantum gravity: no global symmetries! But PQ can emerge as accidental, approximate global symmetry from more fundamental discrete R-symmetries (intrinsically SUSY) which arise from string compactifications: similar to B and L conservation arising accidentally from SM gauge symmetries
- why f_a~10^11 GeV? link to SUSY breaking scale sqrt{F_x}~10^11 GeV
- axion quality problem: higher dim op's can destroy thetabar<10^-10: but e.g. discrete R-symmetries can sufficiently suppress these terms
- axion quality: stringy instantons can destroy but not for MSSM as LE-EFT (McAllister et al., PQ axiverse)

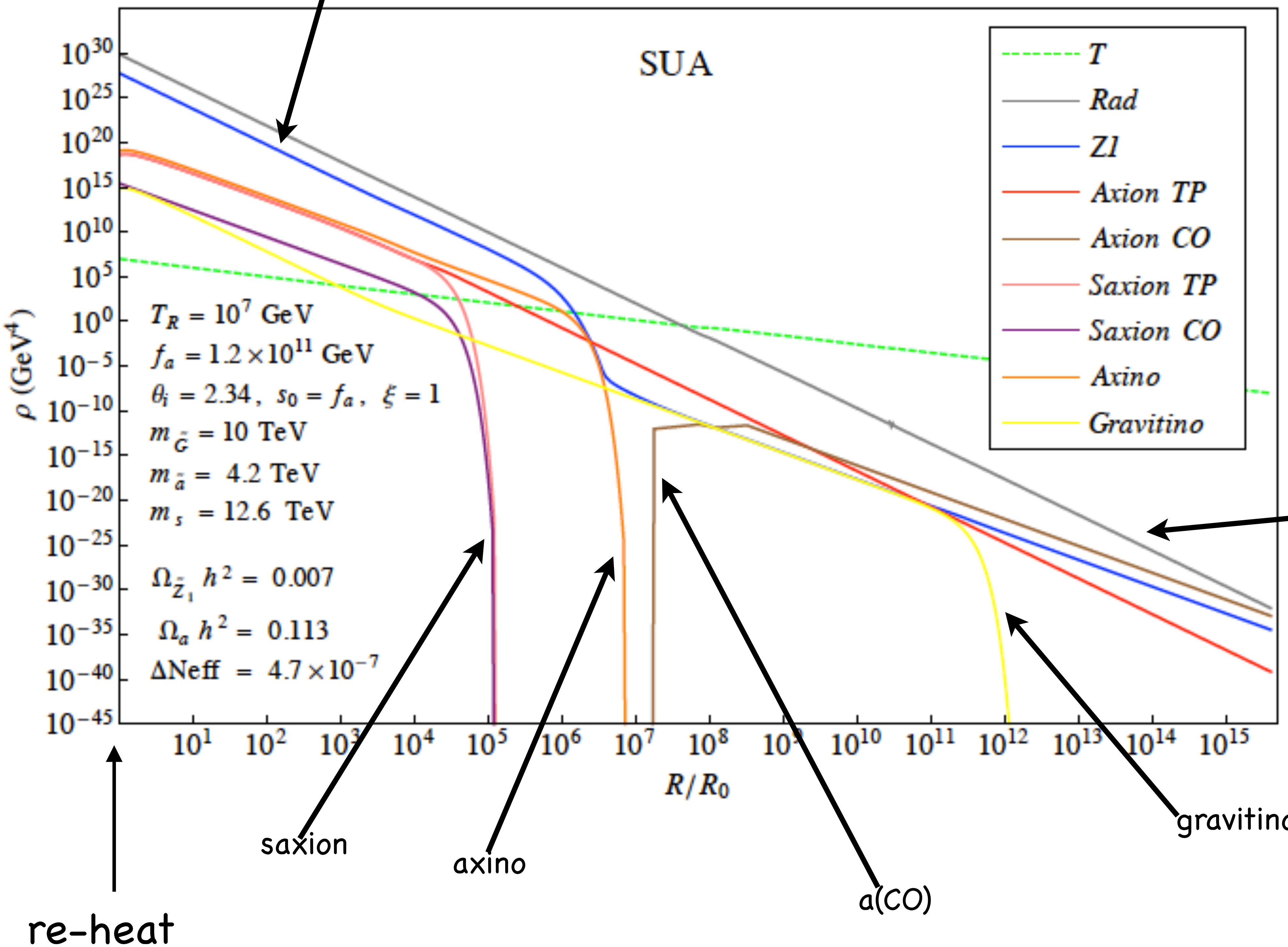
and SUSY needs axion

- SUSY mu problem: superpotential mu term is SUSY conserving, not SUSY breaking: then expect mu~m(Planck) unless forbidden by e.g. PQ symmetry (Kim-Nilles solution to SUSY mu problem in SUSY DFSZ axion model [DFSZ fits well with MSSM as both require two Higgs doublets])
- naturalness => SUSY LSP is light higgsino: thermally underproduced by typically factor of 10
- marriage of SUSY with PQ axion => multicomponent DM: DFSZ axion plus higgsino-like WIMP admixture
- R-parity, B/L conservation, PQ can all emerge from discrete R-symmetry
- related work: see Harigaya, Yanagida et al.

mixed axion-neutralino production in early universe

- neutralinos: thermally produced (TP) or NTP via $\tilde{a}$, s or $\tilde{G}$ decays
  - re-annihilation at $T_D^{s,\tilde{a}}$
- axions: TP, NTP via $s \to aa$, bose coherent motion (BCM)
- saxions: TP or via BCM
  - $s \to gg$: entropy dilution
  - $s \to SUSY$: augment neutralinos
  - $s \to aa$: dark radiation ($\Delta N_{eff} < 1.6$)
- axinos: TP
  - $\tilde{a} \to SUSY$ augments neutralinos
- gravitinos: TP, decay to SUSY

DM production in SUSY DFSZ: solve eight coupled Boltzmann equations



wimp

radiation

saxion

axino

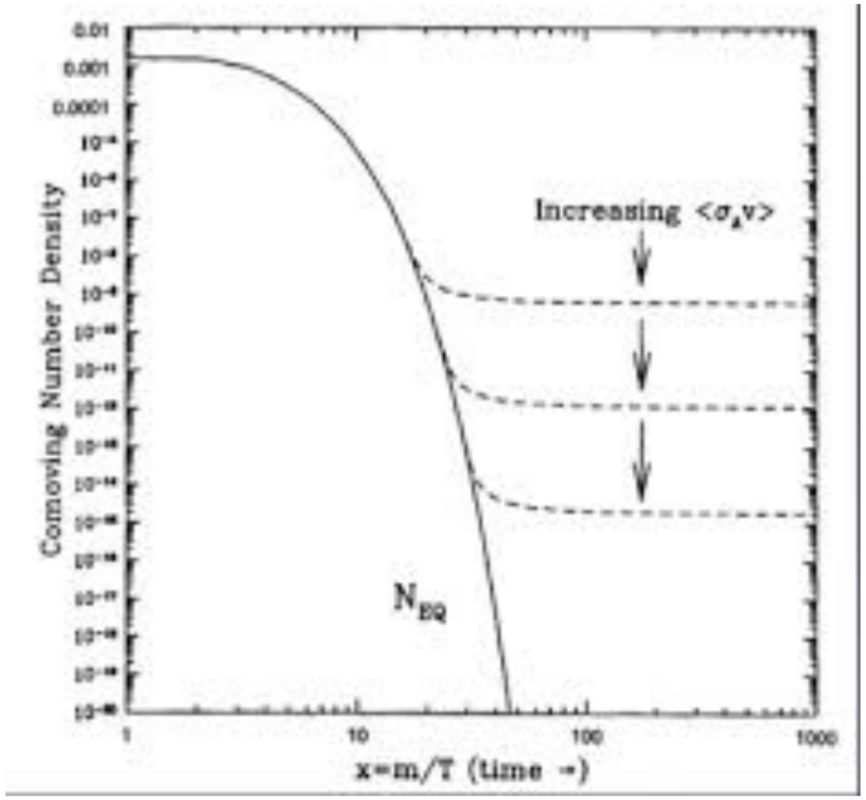
a(CO)

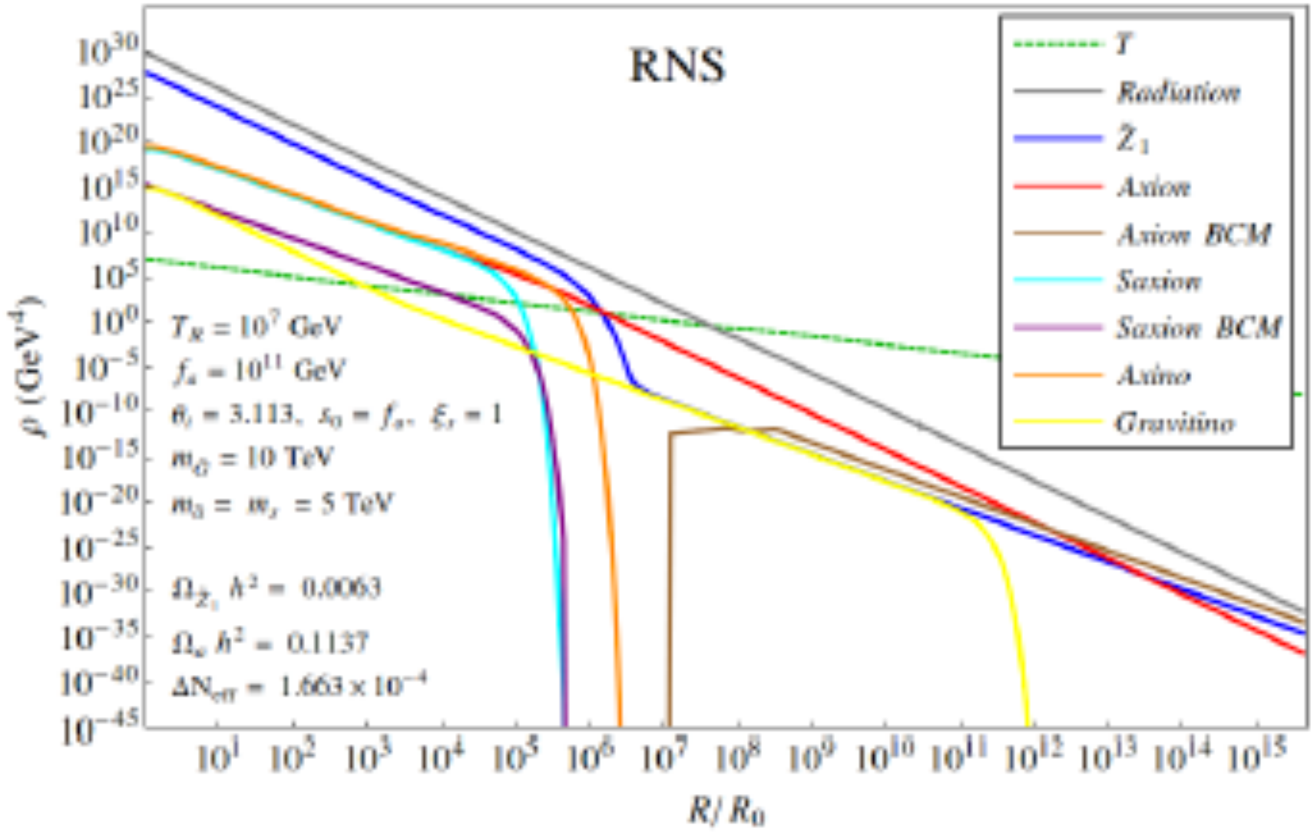
gravitino

re-heat

Bae, HB, Chun;
Bae, HB, Lessa, Serce

usual picture => mixed axion/WIMP

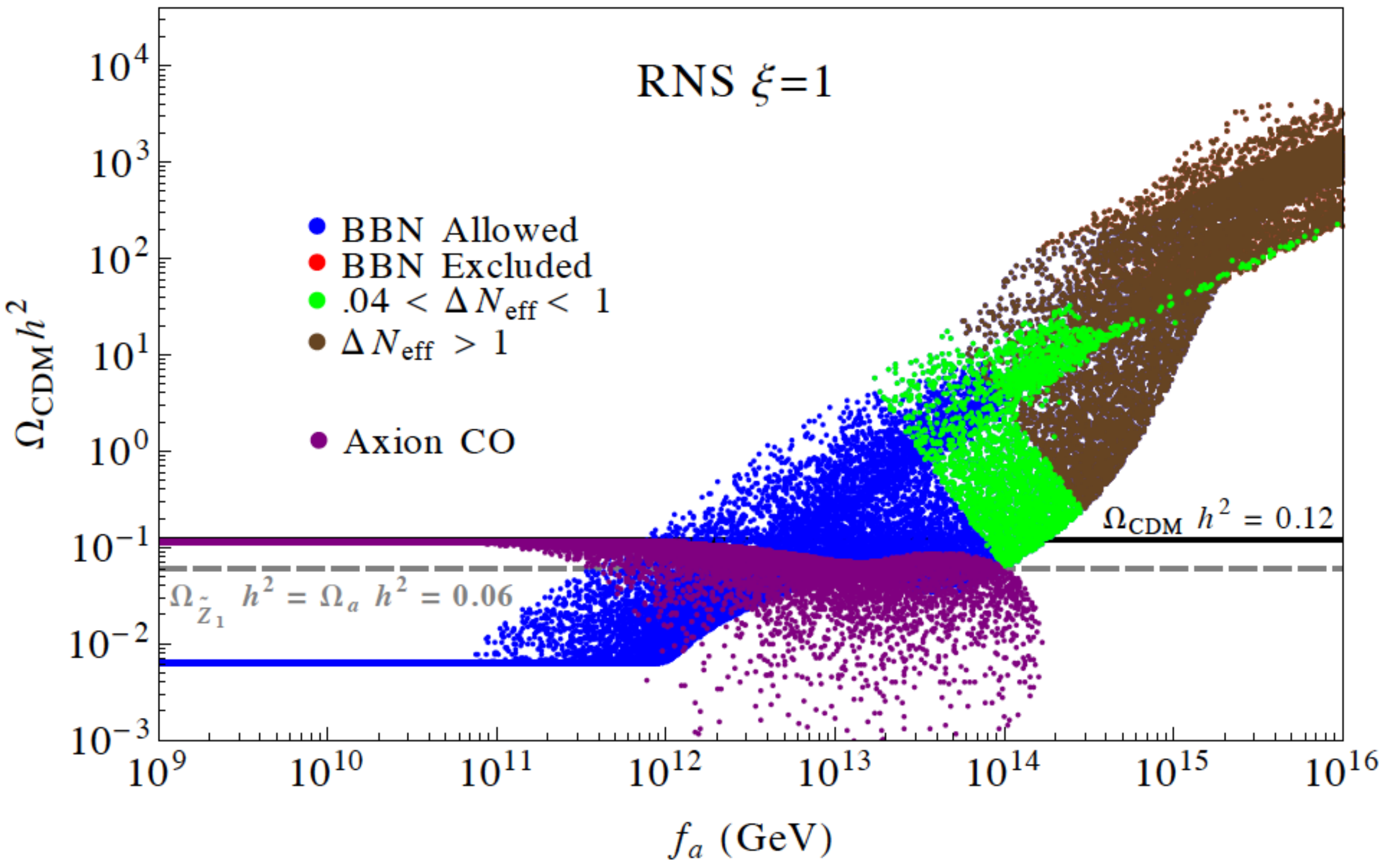




KJ Bae, HB, Lessa, Serce

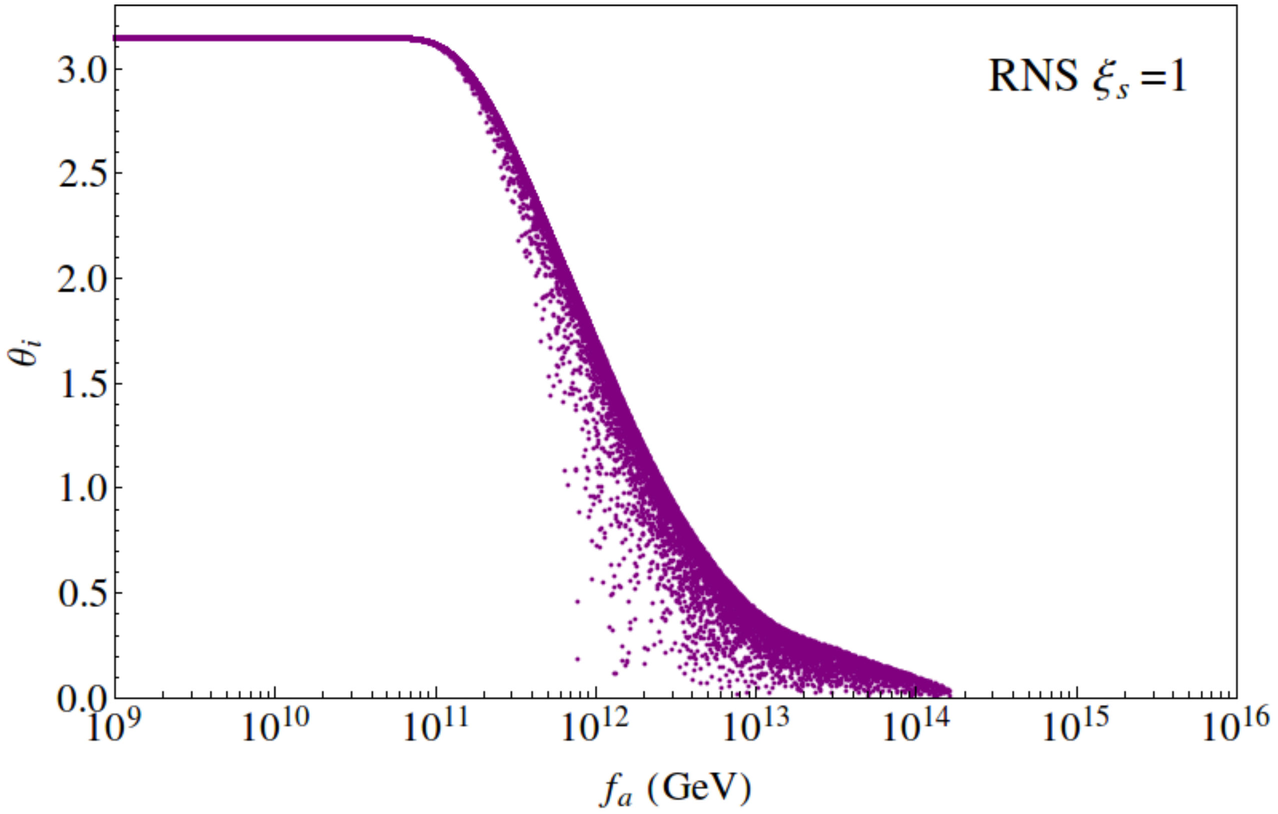
much of parameter space is axion-dominated
with 10-15% WIMPs

=>



higgsino abundance

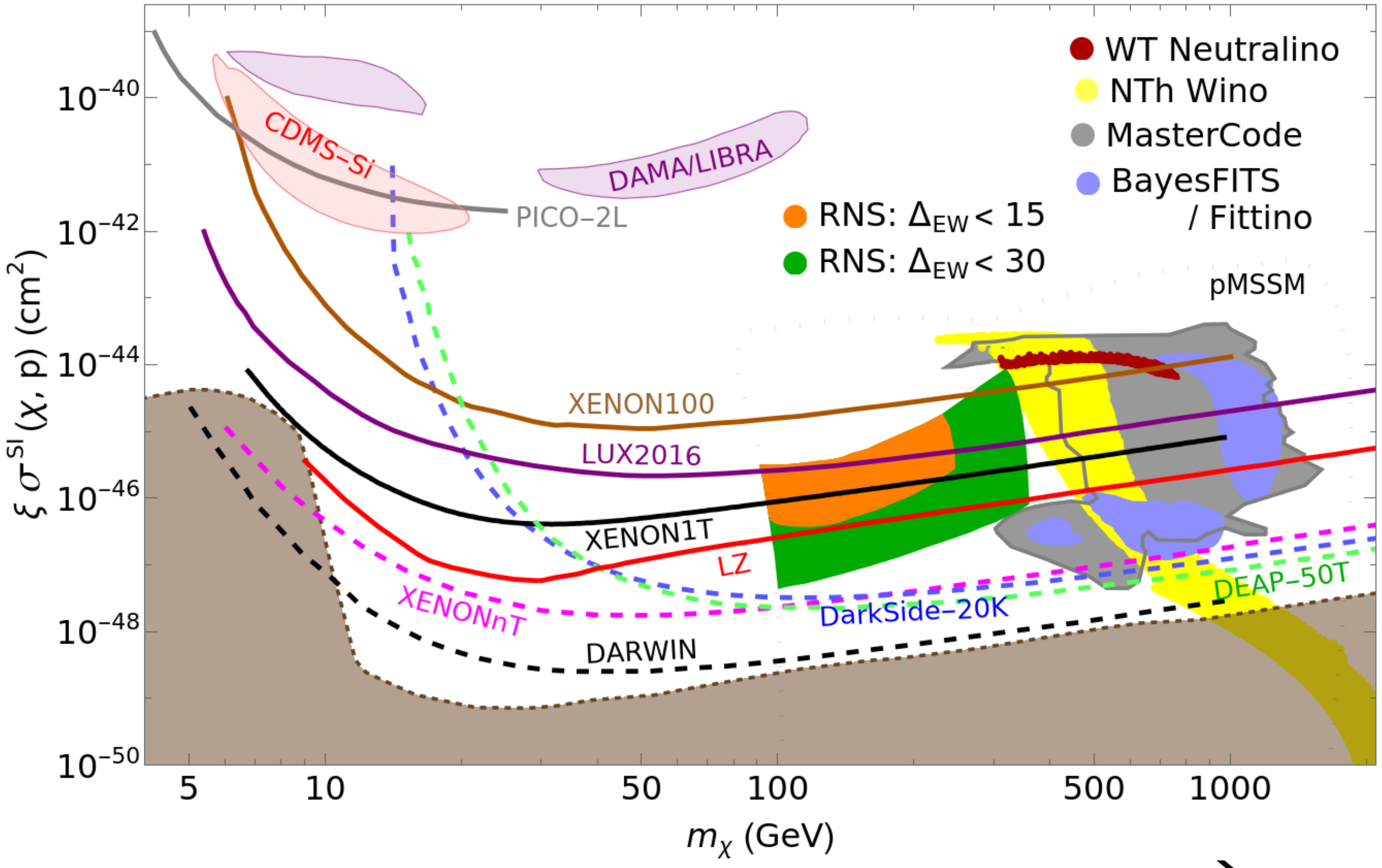
axion abundance



mainly axion CDM
for fa<~10^12 GeV;
for higher fa, then get increasing wimp abundance

Bae, HB,Lessa,Serce

Direct higgsino detection rescaled for minimal local abundance $\xi \equiv \Omega_\chi^{TP} h^2/0.12$



Bae, HB, Barger,Savoy,Serce

$$\mathcal{L} \ni -X_{11}^h \overline{\widetilde{Z}}_1 \widetilde{Z}_1 h$$

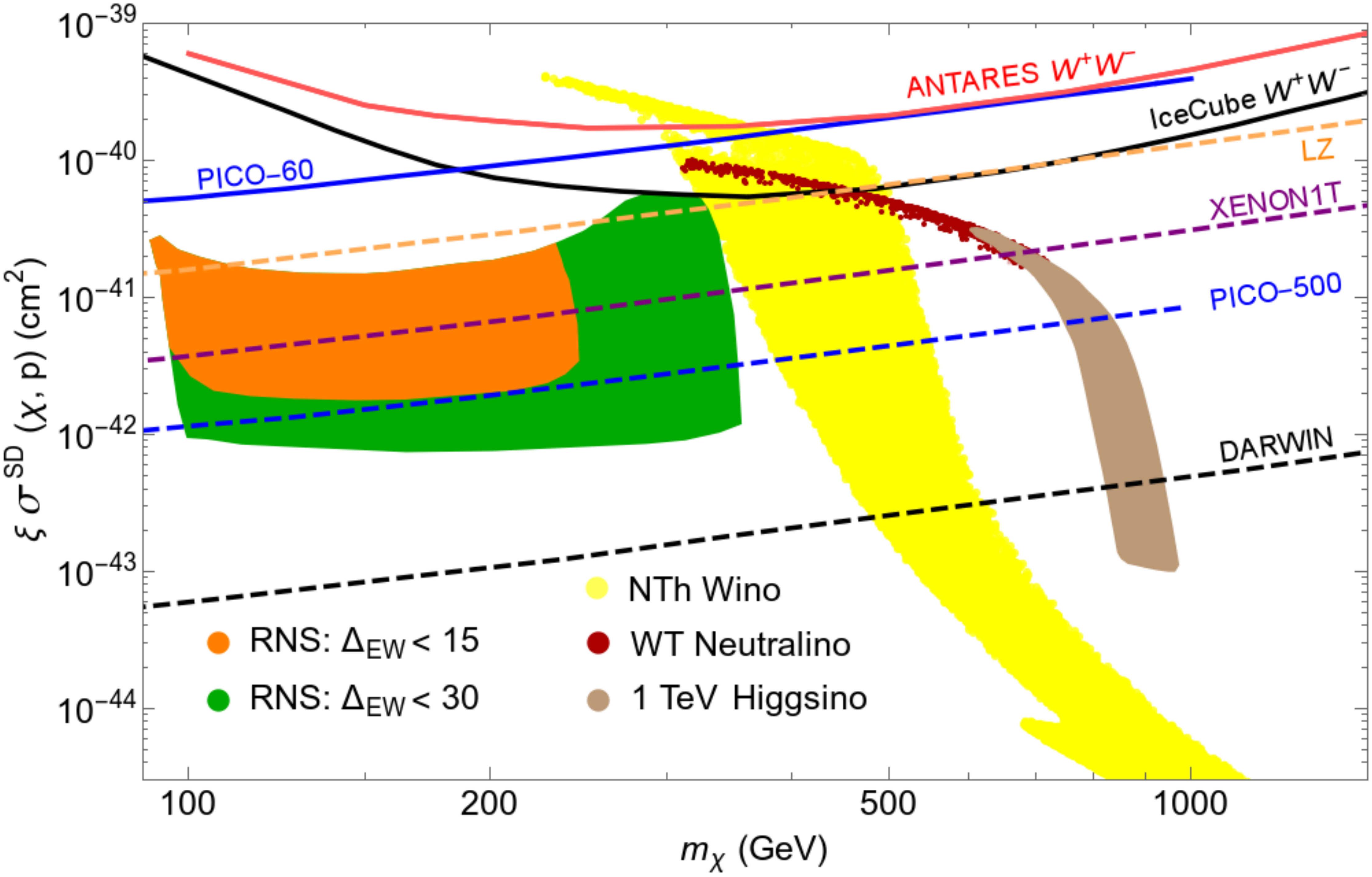
$$X_{11}^h = -\frac{1}{2}\left(v_2^{(1)}\sin\alpha - v_1^{(1)}\cos\alpha\right)\left(g v_3^{(1)} - g' v_4^{(1)}\right)$$

includes latest LZ2022 results!

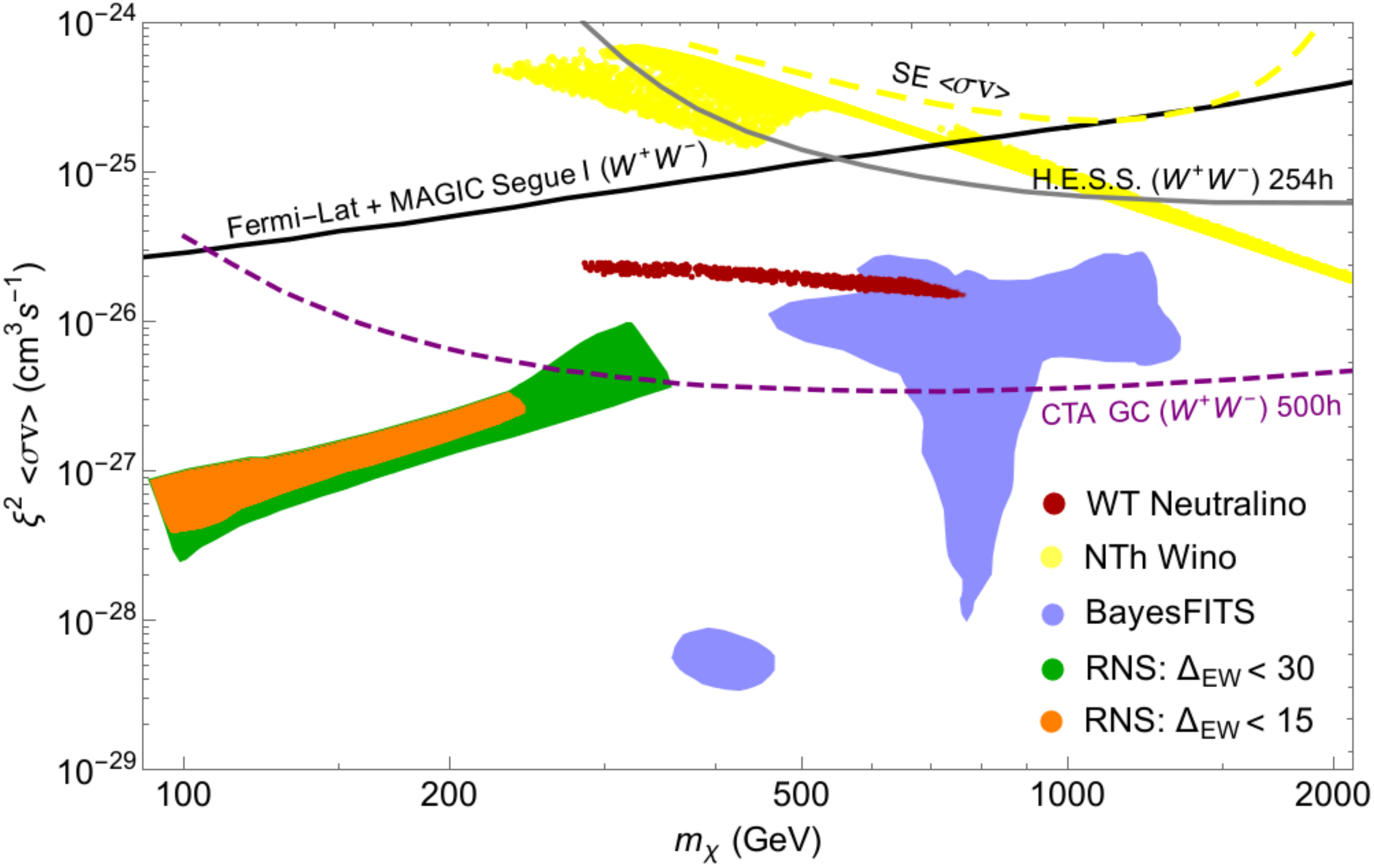
natural SUSY

Can test completely with ton scale detector or equivalent (subject to minor caveats)

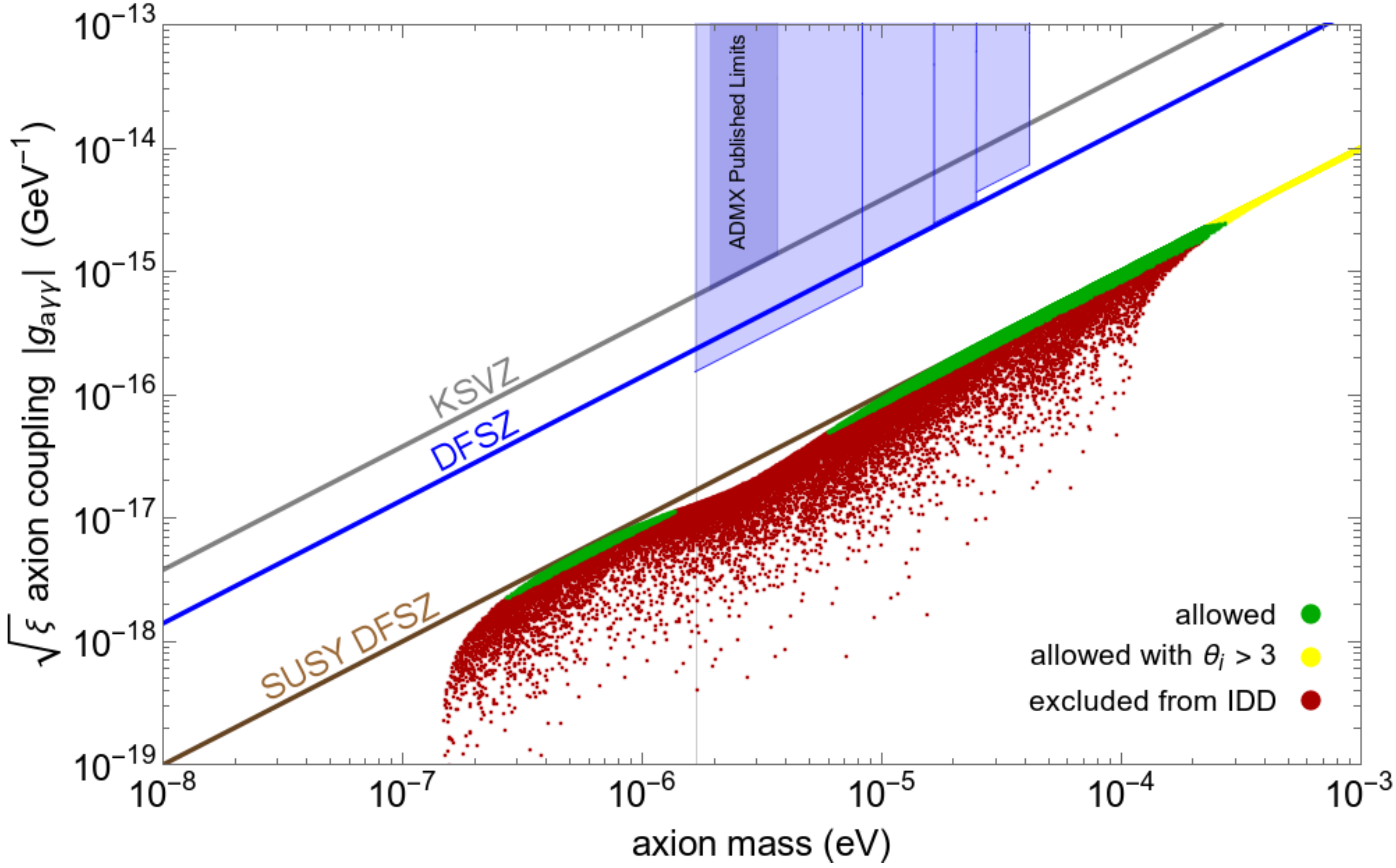
Prospects for SD WIMP searches:



Prospects for IDD WIMP searches:

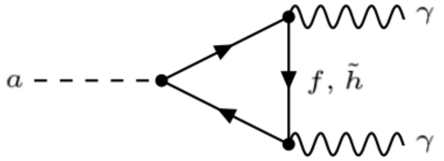


suppressed by square of diminished WIMP abundance



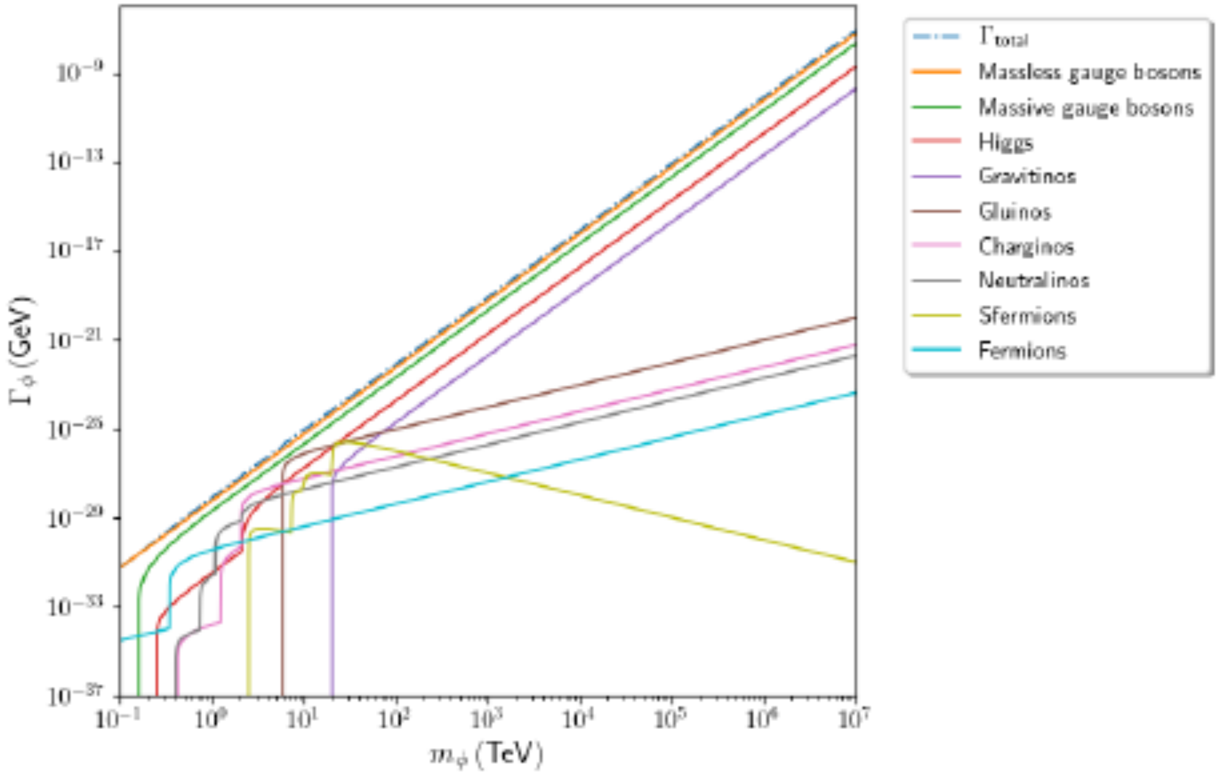
SUSY DFSZ axion: large range in m(a) but coupling reduced

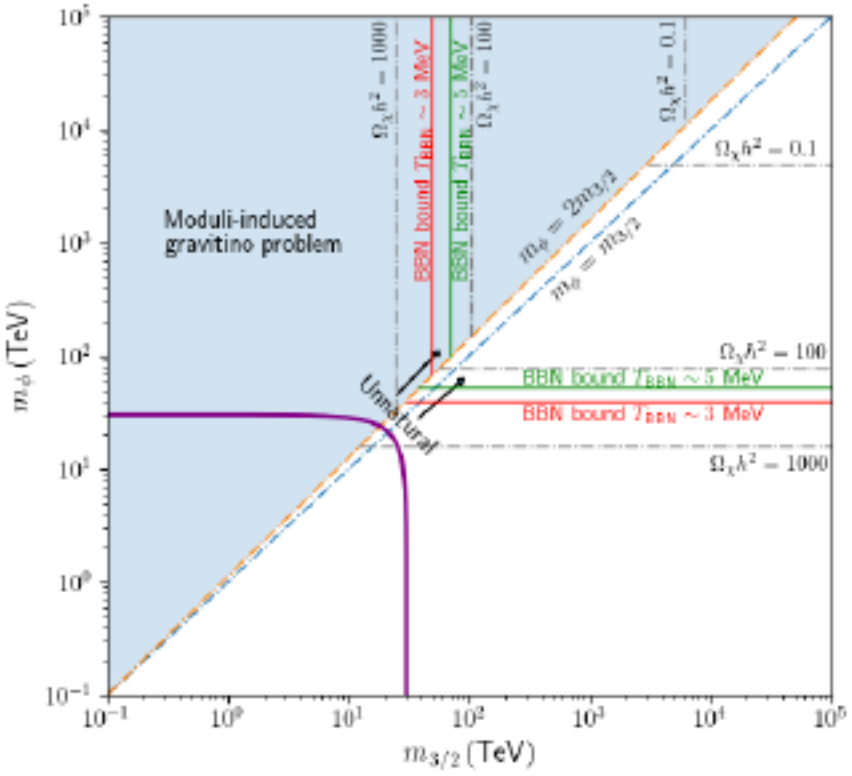
may need to probe broader and deeper!



Also: string remnant moduli fields in early universe: CMP?

- Finally: all modulus decay rates to MSSM and PQMSSM fields
- moduli must:
- 1. decay before BBN
- 2. not overproduce WIMPs
- 3. not over produce gravitinos
- 4. not overproduce dark radiation
- need m(phi)>~10^4 TeV!
- can do in e.g. KKLT: msoft<<m32<<m(phi)

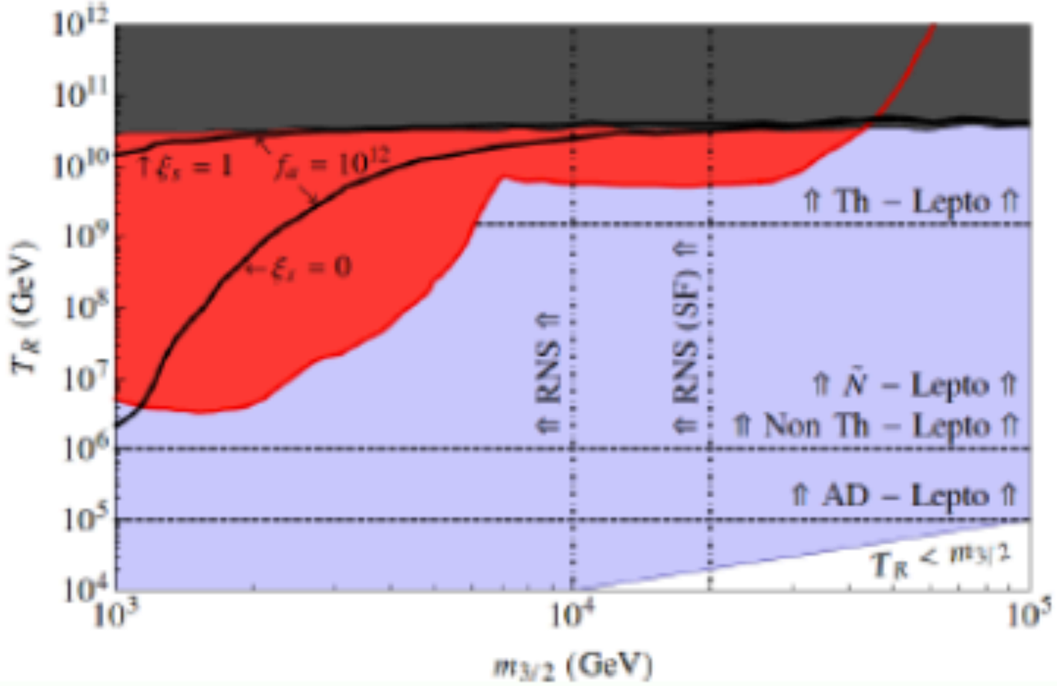




HB, Barger, Wiley Deal: 2201.06633 and 2301.12546

Baryogenesis scenarios for radiative natural SUSY

- thermal leptogenesis
- non-thermal (inflaton decay)
- oscillating sneutrino
- Affleck-Dine (AD)



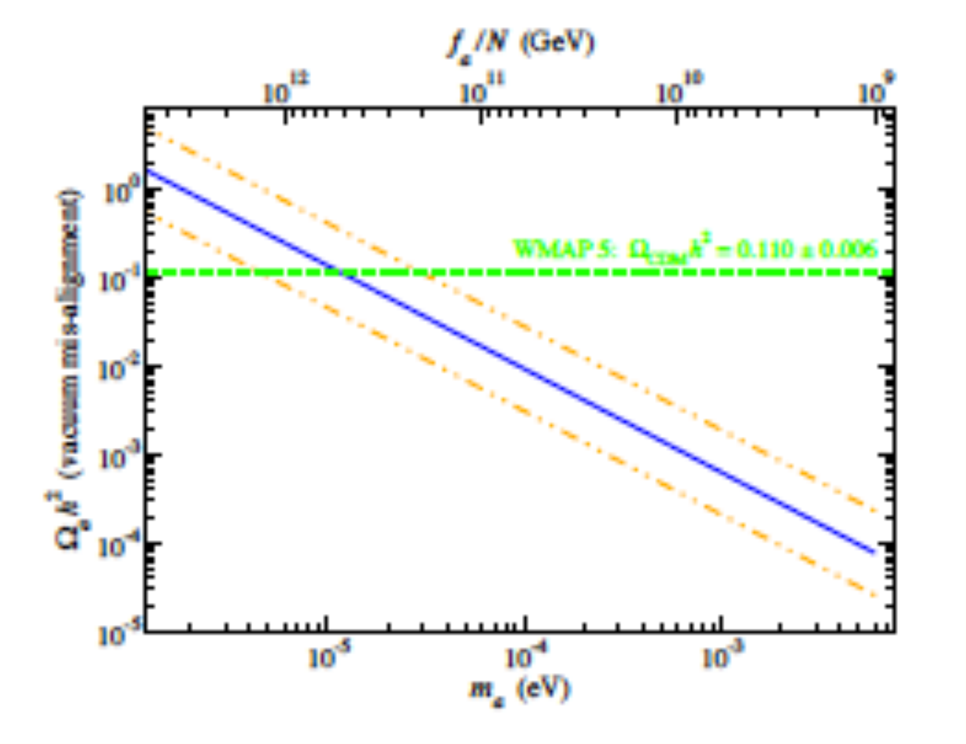
gravitino problem plus
axino/saxion problem:
still plenty room

$$f_a = 10^{11},\ 10^{12}\ \text{GeV}$$

Bae,HB, Serce, Zhang, arXiv:1510.00724

Axion cosmology

- Axion field eq'n of motion: $\theta = a(x)/f_a$
  - $\ddot{\theta} + 3H(T)\dot{\theta} + \frac{1}{f_a^2}\frac{\partial V(\theta)}{\partial \theta} = 0$
  - $V(\theta) = m_a^2(T) f_a^2 (1 - \cos\theta)$
  - Solution for T large, $m_a(T) \sim 0$:
    $\theta = const.$
  - $m_a(T)$ turn-on ~ 1 GeV
- a(x) oscillates,
  creates axions with $\vec{p} \sim 0$:
  production via vacuum mis-alignment
- $\Omega_a h^2 \sim \frac{1}{2}\left[\frac{6\times10^{-6}eV}{m_a}\right]^{7/6}\theta_i^2 h^2$
- astro bound: stellar cooling $\Rightarrow f_a \gtrsim 10^9 GeV$



Why might mu<<m(soft)?

SUSY mu problem: mu term is SUSY, not SUSY breaking: expect mu~M(Pl) but phenomenology requires mu~m(Z)

- NMSSM: mu~m(soft); but beware singlets!
- Giudice-Masiero: mu forbidden by some symmetry: generate via Higgs coupling to hidden sector: mu~m(soft)
- Kim-Nilles: invoke SUSY version of DFSZ axion solution to strong CP:

KN: PQ symmetry forbids mu term, but then it is generated via PQ breaking

$$\mu \sim \lambda_\mu f_a^2/m_P$$

$$m(soft) \sim m_{3/2} \sim m_{hidden}^2/m_P$$

Little Hierarchy due to mismatch between PQ breaking and SUSY breaking scales?

$$f_a < m_{hidden} \Rightarrow$$
$$\mu \ll m(soft)$$

Higgs mass m(h)~mu tells us where to look for axion!

$$m_a \sim 6.2\mu\text{eV}\left(\frac{10^{12}\ \text{GeV}}{f_a}\right)$$

Gravity safe, electroweak natural axionic solution to strong CP and SUSY μ problems

HB, Barger, Sengupta, arXiv:1810.03713

1. Global symmetries fundamentally incompatible with gravity completion
2. Expect global symmetry to emerge as accidental (approximate) symmetry from some more fundamental gravity-safe (e.g. gauge or R-) symmetry.
3. Discrete R-symmetries:

intrinically supersymmetric and expected to emerge from string compactification

A model which works: Z(24) R symmetry (see also Lee et al.), arXiv:1102.3595

$$W \ni f_u Q H_u U^c + f_d Q H_d D^c + f_\ell L H_d E^c + f_\nu L H_u N^c + \\ M_N N^c N^c/2 + \lambda_\mu X^2 H_u H_d/m_P + f X^3 Y/m_P + \lambda_3 X^p Y^q/m_P^{p+q-3}$$

- Lowest dimension PQ breaking operator contributing to scalar PQ potential $\sim 1/m_P^8$: enough suppression so that PQ is gravity-safe
- Also forbids/suppresses RPV/p-decay operators
- $\mu \sim \lambda_\mu f_a^2/m_P$

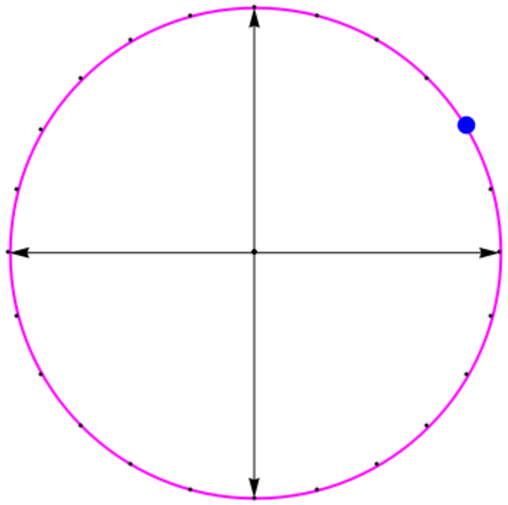
This two-extra -field model based on Z(24)^R symmetry forbids mu term, RPV terms and dim 6 p-decay operators,
while maintaining MSSM Yukawa and Majorana nu mass term and to-be mu parameter

$$\begin{aligned} W_{hyCCK} &\ni f_u Q H_u U^c + f_d Q H_d D^c + f_\ell L H_d E^c + f_\nu L H_u N^c + M_N N^c N^c/2 \\ &+ f X^3 Y/m_P + \lambda_\mu X^2 H_u H_d/m_P . \end{aligned}$$

Also W contains an X^8Y^2/mP^7 superpotential; scalar pot'l suppressed by 1/mP^8, gravity safe!

multiplet	H_u	H_d	Q_i	L_i	U_i^c	D_i^c	E_i^c	N_i^c	X	Y
$\mathbb{Z}_{24}^R$ charge	16	12	5	9	5	9	5	1	-1	5
PQ charge	-1	-1	1	1	0	0	0	0	1	-3

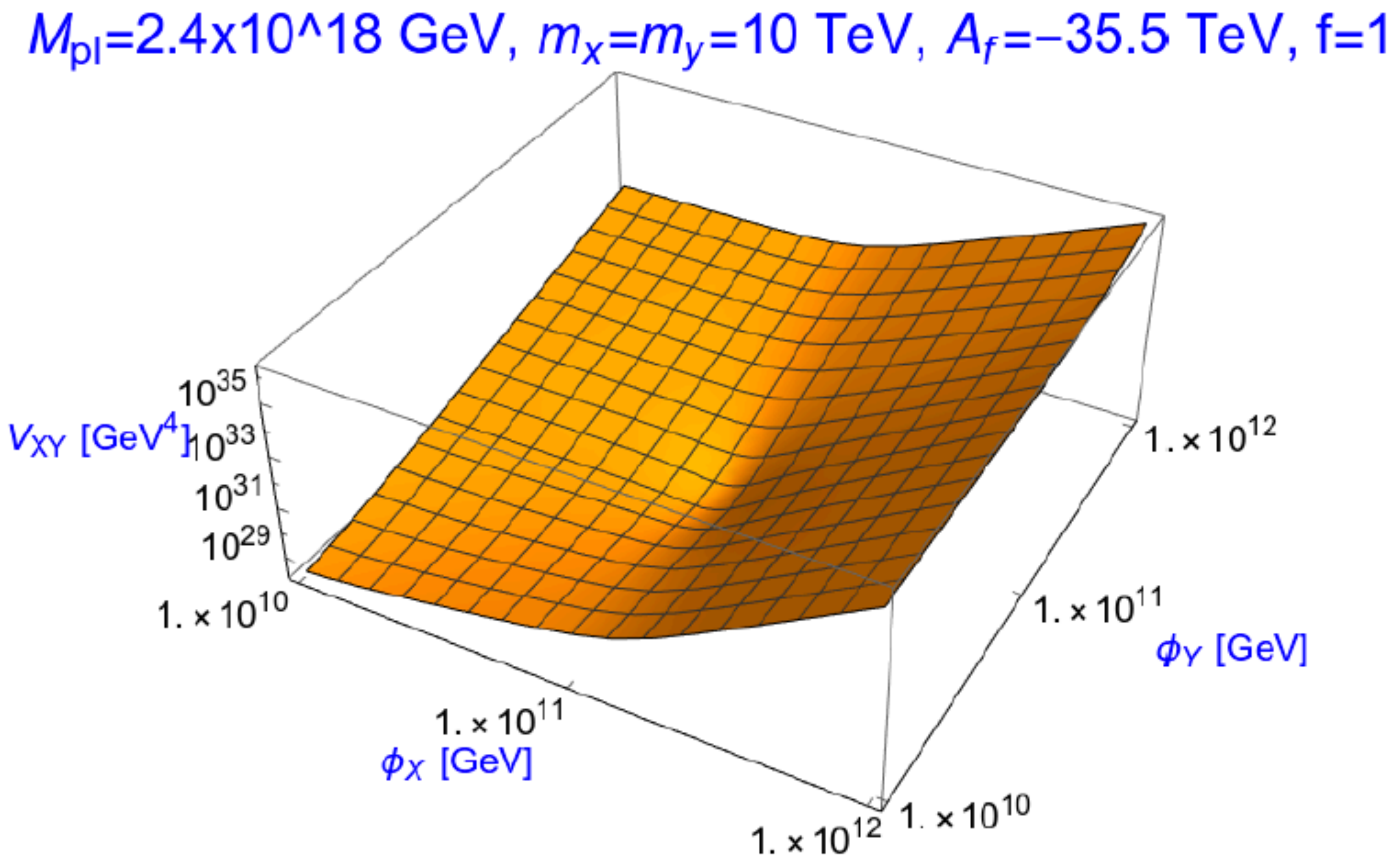
Z(24)^R and PQ charge assignments



HB, Barger, Sengupta, arXiv:1810.03713

See also SPMartin and Bhattiprolu, arXiv:2106.14964

For large A_f soft terms, Z(24)^R and U(1)_PQ spontaneously broken due to SUSY breaking with vevs~10^11 GeV => f_a~10^11 GeV!



M_{pl}=2.4x10^18 GeV, m_x=m_y=10 TeV, A_f=−35.5 TeV, f=1

Figure 1: Scalar potential V_{GSPQ} versus ϕ_X and ϕ_Y for $m_X = m_Y \equiv m_{3/2} = 10$ TeV, $f = 1$ and $A_f = -35.5$ TeV.

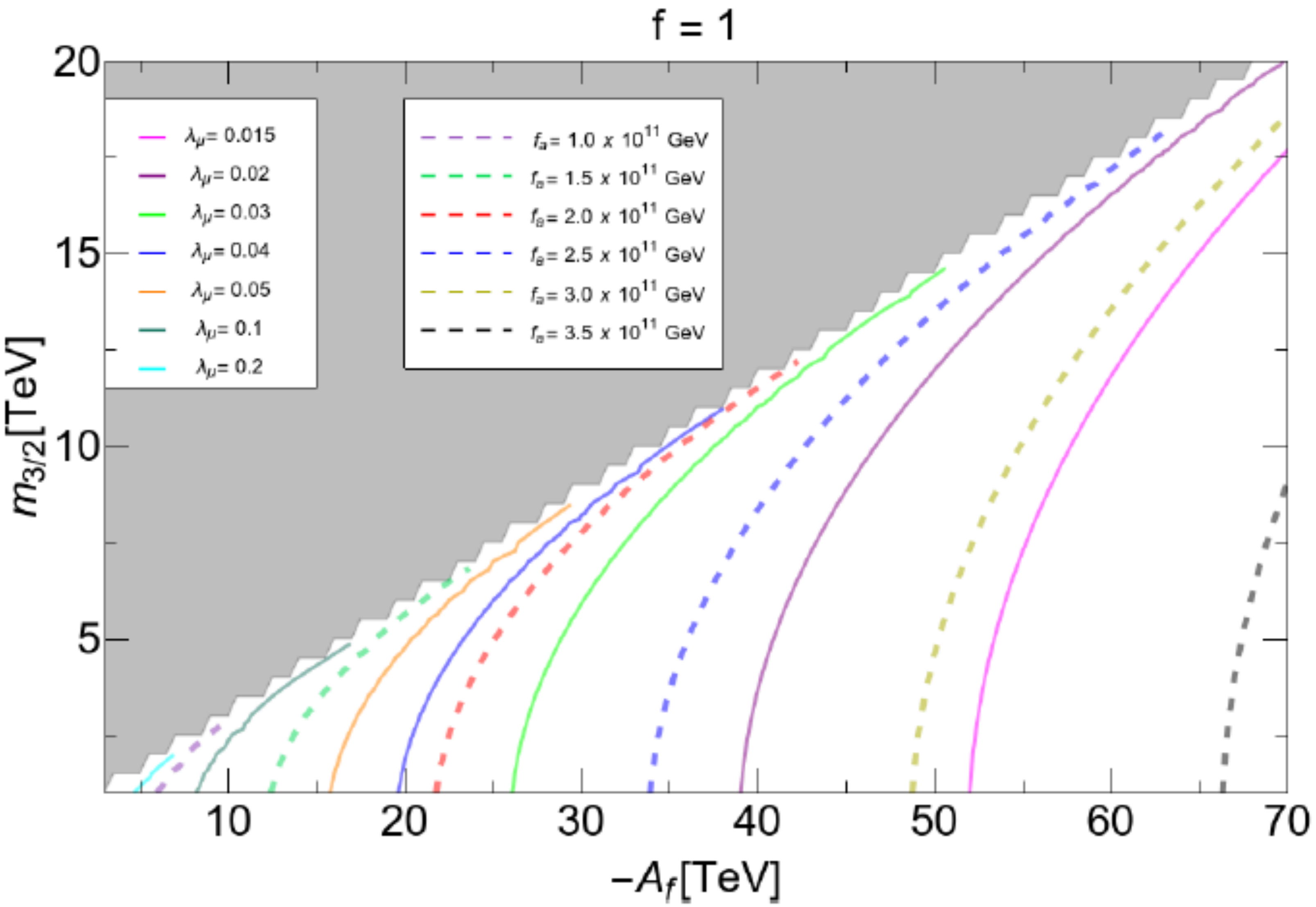


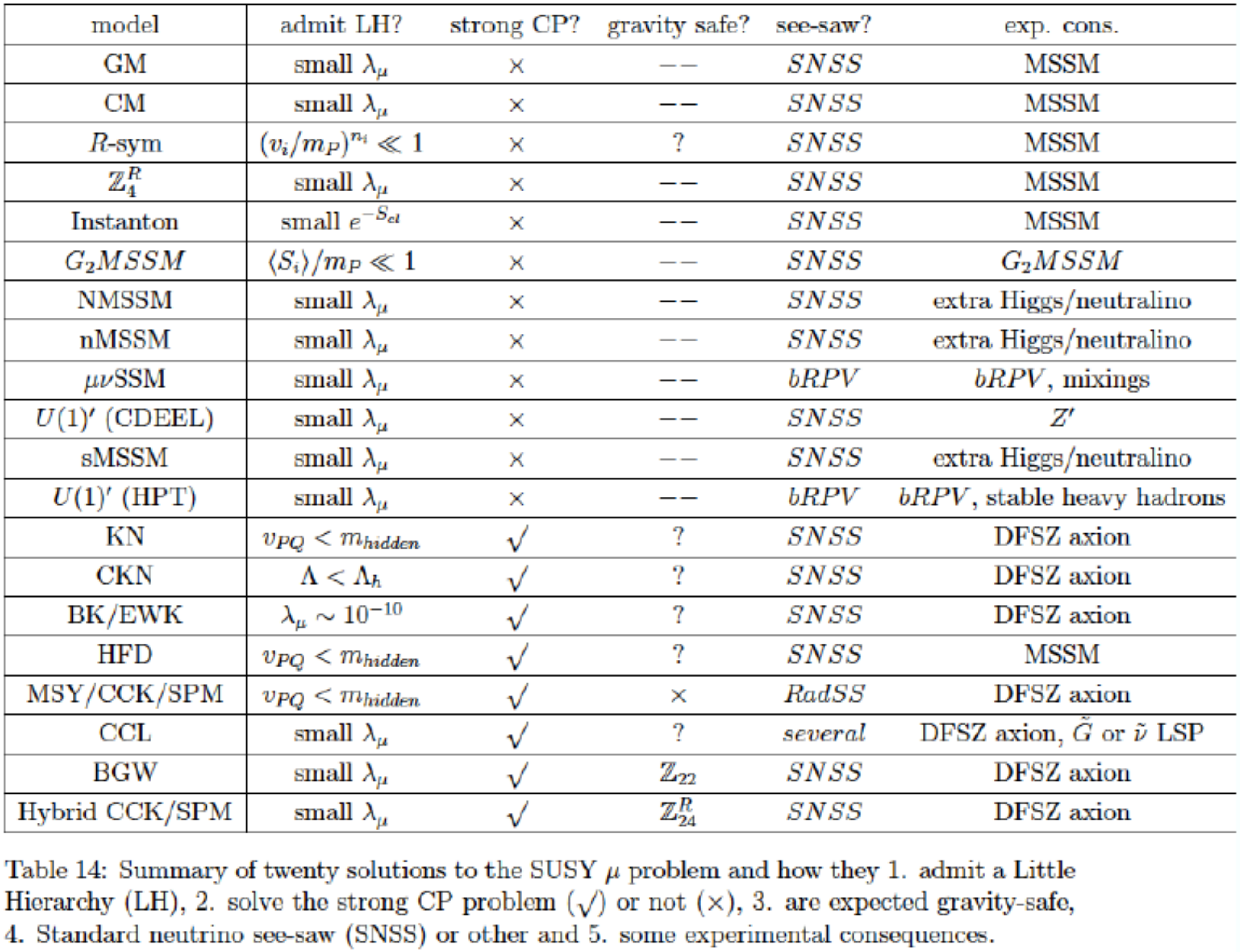
Figure 2: Representative values of λ_μ required for $\mu = 150$ GeV in the $m_{3/2}$ vs. $-A_f$ plane of the GSPQ model for $f = 1$. We also show several contours of f_a.

Z(24)^R model can easily accommodate mu~100-300 GeV consistent with EW naturalness

axion quality problem/SUSY mu problem/f_a problem: all solved!

20 solutions to mu problem:

Bae, HB, Barger, Sengupta, 1902.10748

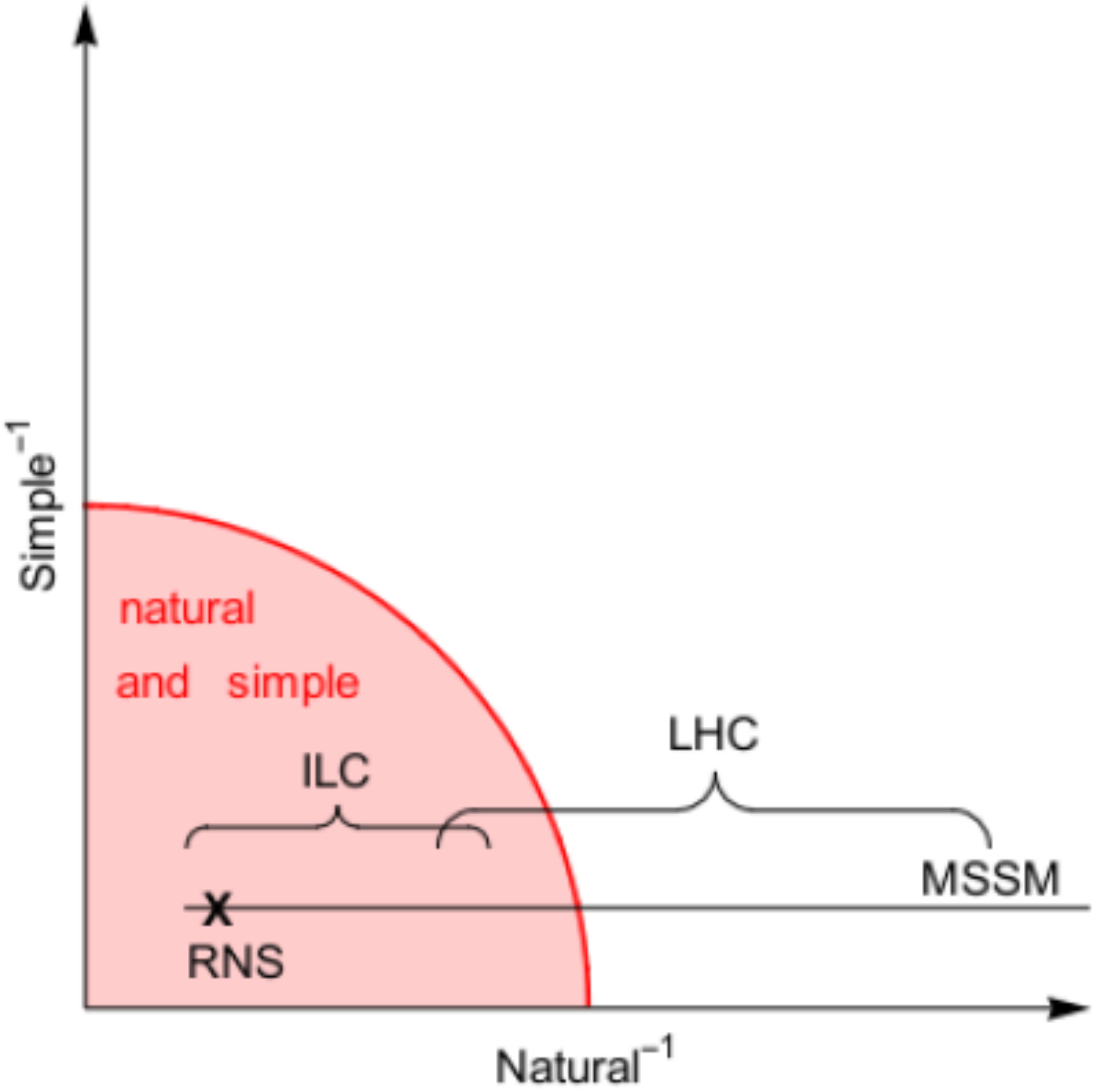


model	admit LH?	strong CP?	gravity safe?	see-saw?	exp. cons.
GM	small λ_μ	$\times$	$--$	$SNSS$	MSSM
CM	small λ_μ	$\times$	$--$	$SNSS$	MSSM
R-sym	$(v_i/m_P)^{n_i} \ll 1$	$\times$	?	$SNSS$	MSSM
$\mathbb{Z}_4^R$	small λ_μ	$\times$	$--$	$SNSS$	MSSM
Instanton	small $e^{-S_{cl}}$	$\times$	$--$	$SNSS$	MSSM
G_2MSSM	$\langle S_i \rangle / m_P \ll 1$	$\times$	$--$	$SNSS$	G_2MSSM
NMSSM	small λ_μ	$\times$	$--$	$SNSS$	extra Higgs/neutralino
nMSSM	small λ_μ	$\times$	$--$	$SNSS$	extra Higgs/neutralino
$\mu\nu$SSM	small λ_μ	$\times$	$--$	$bRPV$	$bRPV$, mixings
$U(1)'$ (CDEEL)	small λ_μ	$\times$	$--$	$SNSS$	Z'
sMSSM	small λ_μ	$\times$	$--$	$SNSS$	extra Higgs/neutralino
$U(1)'$ (HPT)	small λ_μ	$\times$	$--$	$bRPV$	$bRPV$, stable heavy hadrons
KN	$v_{PQ} < m_{hidden}$	$\surd$	?	$SNSS$	DFSZ axion
CKN	$\Lambda < \Lambda_h$	$\surd$	?	$SNSS$	DFSZ axion
BK/EWK	$\lambda_\mu \sim 10^{-10}$	$\surd$	?	$SNSS$	DFSZ axion
HFD	$v_{PQ} < m_{hidden}$	$\surd$	?	$SNSS$	MSSM
MSY/CCK/SPM	$v_{PQ} < m_{hidden}$	$\surd$	$\times$	$RadSS$	DFSZ axion
CCL	small λ_μ	$\surd$	?	$several$	DFSZ axion, $\tilde{G}$ or $\tilde{\nu}$ LSP
BGW	small λ_μ	$\surd$	$\mathbb{Z}_{22}$	$SNSS$	DFSZ axion
Hybrid CCK/SPM	small λ_μ	$\surd$	$\mathbb{Z}_{24}^R$	$SNSS$	DFSZ axion

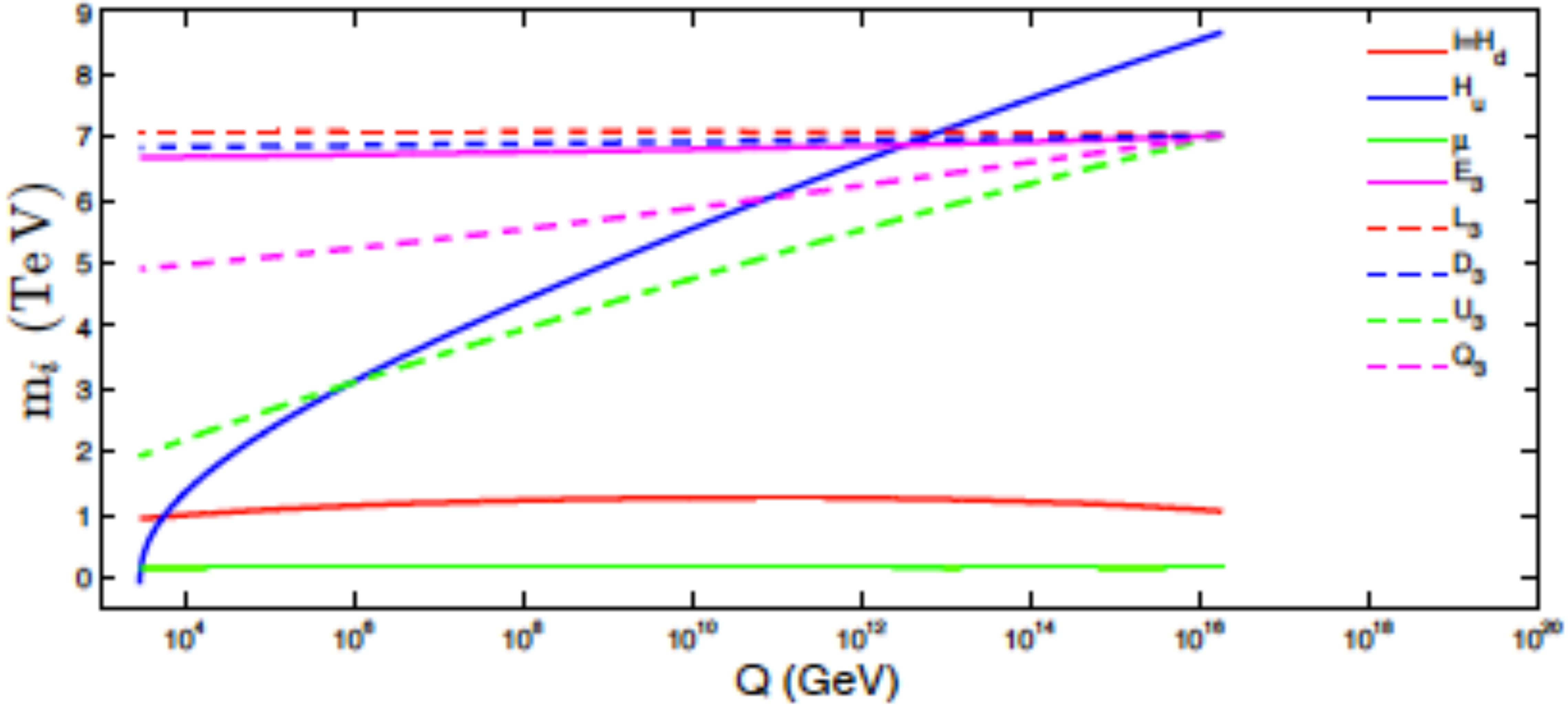
Table 14: Summary of twenty solutions to the SUSY μ problem and how they 1. admit a Little Hierarchy (LH), 2. solve the strong CP problem ($\surd$) or not ($\times$), 3. are expected gravity-safe, 4. Standard neutrino see-saw (SNSS) or other and 5. some experimental consequences.

Conclusions:

- Time to set aside old notions of naturalness:
- Plenty of natural parameter space under model independent measure DEW
- mu~100-350 GeV: light higgsinos!
- other sparticle contributions to m(weak) are loop suppressed- masses can be TeV->multi-TeV
- stringy naturalness: what the string landscape prefers
- draw to large soft terms provided m(weak)~(2-5)*100 GeV
- predicts LHC sees mh~125 GeV but as yet no sign of sparticles
- under stringy naturalness, a 3 TeV gluino more natural than 300 GeV gluino
- landscape-> non-universal 1st/2nd gen. scalars at 20-40 TeV: natural but gives quasi-degeneracy/decoupling sol'n to SUSY flavor, CP and cosmological moduli problems
- dark matter: a mix of axions+higgsino-like WIMPs (typically mainly axions)


Simple^{-1}
natural
and simple
ILC
LHC
MSSM
RNS
Natural^{-1}

Applied properly, all three measures agree:
naturalness is unambiguous and highly predictive!



Radiatively-driven natural SUSY, or RNS:

(typically need mHu~25-50% higher than m0)

H. Baer, V. Barger, P. Huang, A. Mustafayev and X. Tata, *Phys. Rev. Lett.* **109** (2012) 161802.

H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev and X. Tata, *Phys. Rev.* **D 87** (2013) 115028 [`arXiv:1212.2655` [hep-ph]].