

- (3) { Everything what you wanted to know about neutrino (particle) theory but were afraid to ask
- (2) { The neutrino and particle theory you need to know to do CEVNS phenomenology

① Neutrinos : Theory
lectures

First lecture:

- History of the neutrino
- Fermi: four-formion interaction
- SM interactions (renormalizable)
- Lagrangian
- Neutrino flavor oscillations:
vacuum and matter
- Neutrino masses : Theory
- Absolute neutrino masses
 $\alpha/\beta/\beta$, kinematic searches,
cosmology
- Neutrino mixing angles
and squared-mass differences

Some historical facts :

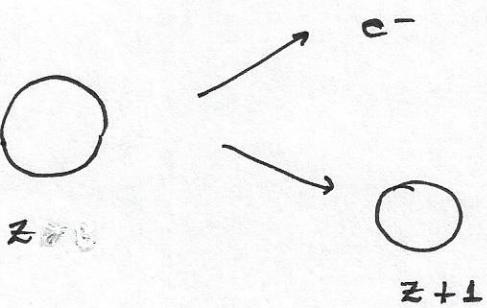
Pauli suggested a new particle as a solution to two seemingly unrelated observations (problems):

1. The problem of continuous /s spectra
2. The problem of spin of certain nuclei

In the late 20's nuclei were believed to be formed of e^- and p^+ , an assumption that could be in perspective understood:

Only e^- and p^+ were known and only EM interactions were known too !

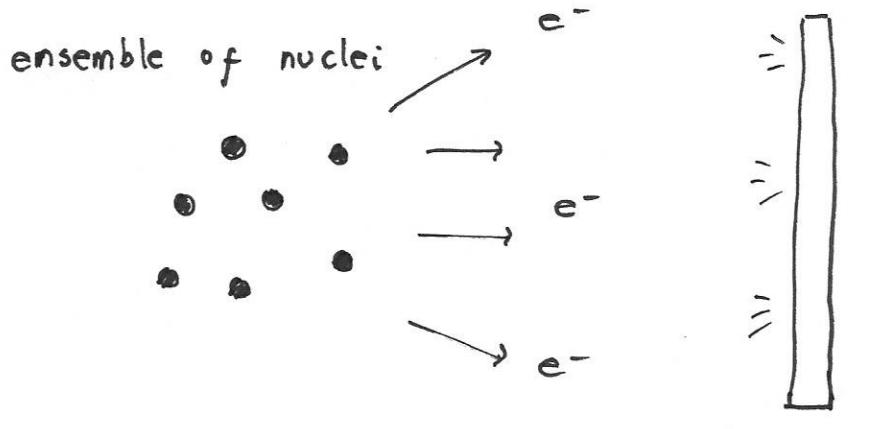
β emission



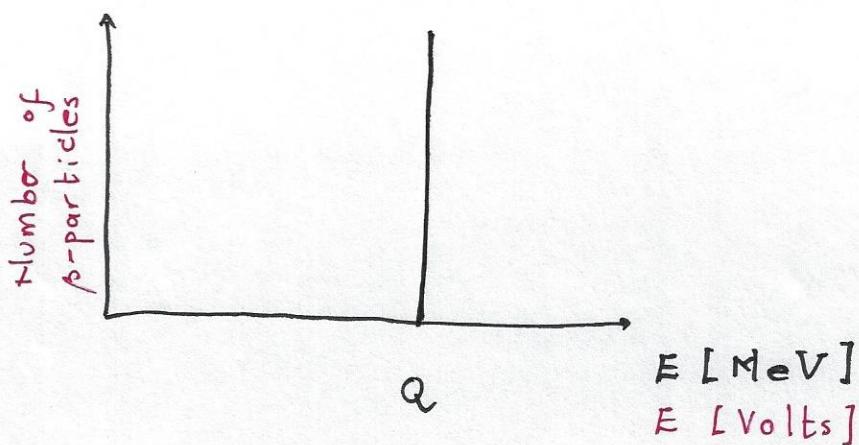
An atomic number z nucleus, being unstable, undergoes decay to an e^- and a $z+1$ nucleus

The electron in that decay is expected to be monochromatic. Its energy amounts to binding energy:

$$Q = m_z - m_{z+1} - m_e$$



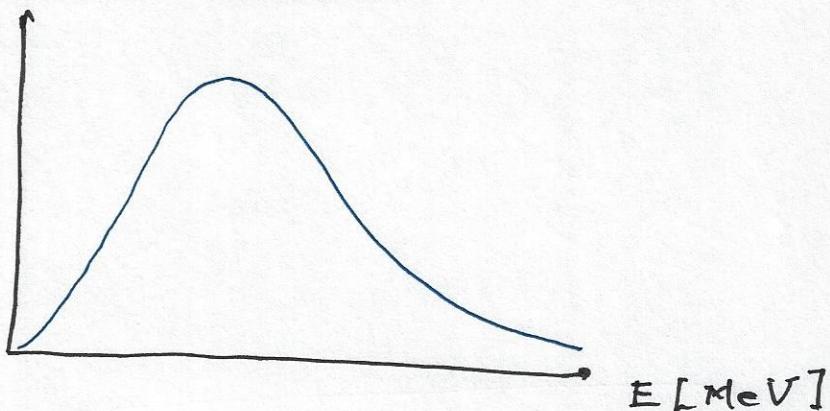
Flux $[\# e/cm^2/s]$



} monochromatic spectrum

Instead...

Flux $[\# e/cm^2/s]$



1927 : Ellis and Wooster in a calorimetric experiment found a continuous spectrum instead

1934 : Chadwick got the same result.

Ellis and Wooster experiment settled the issue.

To address the conundrum Bohr suggested that for β^- decay energy was not conserved.

Energy is conserved only statistically

1930 : At a conference in Tübingen, Pauli suggested that a third particle, that he named "neutron", was emitted

Properties :

- $L_n \approx L_\gamma$ or $L_n \approx 10 L_\gamma$

Absorption length

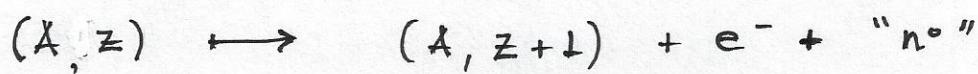
Large absorption length to explain why wasn't observed !

- $m_n \approx m_e$; $m_n \lesssim 10^{-2} m_p$

Has to be light !

- $s(n) = 1/2$. The new state is a fermion.

Implications of a third particle :



■ 2-body decay : Two constraints : Momentum and energy

$$\begin{array}{ccc} E_1, \vec{p}_1 & & \vec{p}_1 = \vec{p}_2 = \vec{p} \\ \diagdown & & \\ E_2, \vec{p}_2 & & E_1 = \sqrt{\vec{p}_1^2 + m_1^2} \end{array}$$

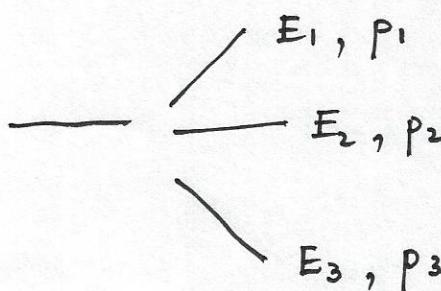
$$E_2 = \sqrt{p_2^2 + M_2^2}$$

$$M_p = E_1 + E_2$$

The final state is fully described by \vec{p}_1, \vec{p}_2 because of the dispersion relations. And \vec{p}_1 and \vec{p}_2 are related by momentum conservation. Final state kinematics is fully determined : $M_p = \sqrt{p_1^2 + M_1^2} + \sqrt{p_2^2 + M_2^2}$

■ 3-body decay

$$M_p = E_1 + E_2 + E_3$$

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$$


$$M_p = \sqrt{|\vec{p}_2 + \vec{p}_3|^2 + M_1^2} + \sqrt{p_2^2 + M_2^2} + \sqrt{p_3^2 + M_3^2}$$

This relation can be satisfied for an infinite set of \vec{p}_2 and \vec{p}_3 values :

$$\begin{aligned}\vec{p}_2 &\rightarrow \vec{p}_2 + \vec{\alpha} \\ \vec{p}_3 &\rightarrow \vec{p}_3 - \vec{\alpha}\end{aligned}$$

With \vec{p}_2, \vec{p}_3 varying so E_1, E_2 and E_3

\Rightarrow Pauli's suggestion addressed as well the problem of the spin of certain nuclei :

$^{14}\text{N}_7$: spin $\frac{1}{2}$. Investigation of molecular

old nuclear model $\left\{ \begin{array}{l} 14 \text{ protons} \\ 7 \text{ electrons} \end{array} \right.$ Nitrogen showed that $^{14}\text{N}_7$ satisfied Bose-Einstein statistics

The new constituents being fermions \Rightarrow

$$S(^{14}\text{N}_7) = \frac{1}{2}$$

1932 : Chadwick discovered the neutron :

$$m_n \approx m_p \quad (\text{Nature})$$

1932 Heisenberg (Z. physics)
1932 Ivanenko (Nature)
1933 Majorana (Z. physics)

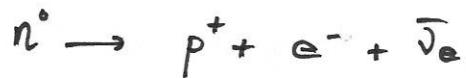
} The nucleus is a neutron-proton bound state

Fermi theory of β -decay

In 1934 Fermi published his theory for β decay.

Assuming the nucleus to be a $p^+ - n^0$ bound state, the new particle could not be the particle discovered by Chadwick. He coined the word neutrino (small neutron) and

described the electron-neutrino pair as a consequence of the neutron-to-proton transition:



(in analogy with $e^- \rightarrow e^- + \gamma$: Bremsstrahlung)

The simplest Hamiltonian for proton EM interactions has the form

$$H_{EM} = e \bar{p}(x) \gamma^\mu p(x) A_\mu(x)$$

In analogy, Fermi wrote for the β -decay transition

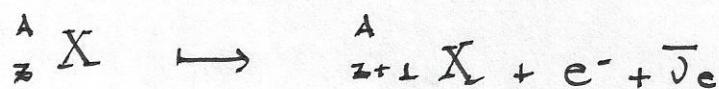
$$H_\beta = G_F \bar{n}(x) \gamma^\mu p(x) \bar{e}(x) \gamma^\nu \nu_e(x) + H.c.$$

↓
Fermi constant

In $n^{\circ} \rightarrow p^+ + e^- + \bar{\nu}_e$ angular momentum (total) should be conserved :

$$\vec{L} = \vec{L}_c + \vec{L}_s$$

$$\vec{J}_i = \vec{J}_f + \vec{L} + \vec{s} \quad \vec{s} = \vec{s}_e + \vec{s}_\nu$$



Let's do

For $e^- \bar{e}^c$: $\vec{s} = \vec{\sigma}$ singlet
 $\vec{s} = \vec{l}$ triplet

Fermi Hamiltonian describes transitions for $\vec{s} = \vec{\sigma}$
 $\vec{s} = \vec{l}$ transitions are described by a more general set of interactions (Gamow - Teller) transitions [Gamow and Teller 1936]

$$H = \sum_i g_i \bar{n}(x) O^i p(x) \bar{e}(x) O_i v(x) + \text{H.c.}$$

$$i = S, V, P, A, T = \{1, \gamma^u, \gamma_5, \gamma^u \gamma_5, \gamma^{uv}\}$$

$$\gamma^{uv} = \frac{i}{2} [\gamma^u, \gamma^v]$$

This Hamiltonian is much more complex than the Fermi Hamiltonian. The Lorentz structure of the interaction responsible for β^- , β^+ , e^- -capture was only clarified through the work of:

- Pontecorvo 1947 and Puppi 1948: Universality of the weak interaction
- Lee and Yang 1956: Parity is violated in weak interactions
- Wu and Lederman (independently) 1957: Experimentally established that parity is not conserved in β^- decay:

To account for Wu and Lederman's result a Hamiltonian of the type

$$H = \sum_{i=S,P,V,A,T} \bar{P}(x) O^i n(x) \bar{\nu}(x) O_i [G_i + G'_i \tau_5] \nu(x) + \text{H.c.}$$

is required. The Hamiltonian implies 10 independent couplings. (constants)

That Hamiltonian was to be further simplified with the advent of the two-component neutrino theory and, more importantly, with the V-A theory of Feynman - Gell-mann and Marshak and Sudarshan. (along with the experimental determination of the neutrino helicity)

Landau, 1957

Lee and Yang, 1957

Salam, 1957

The two-component neutrino theory

$$(i\gamma^\mu - m)\nu(x) = 0$$

$$\tau_5 = \begin{pmatrix} -1_{2 \times 2} & 0 \\ 0 & 1_{2 \times 2} \end{pmatrix}$$

$$P_L = \frac{1 - \tau_5}{2} = \begin{pmatrix} 1_{2 \times 2} & 0 \\ 0 & 0_{2 \times 2} \end{pmatrix}; \quad P_R = \frac{1 + \tau_5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1_{2 \times 2} \end{pmatrix}$$

$$P_L \nu(x) = \begin{pmatrix} \nu_L \\ 0 \end{pmatrix} \equiv \nu_L(x); \quad P_R \nu(x) = \begin{pmatrix} 0 \\ \nu_R \end{pmatrix} \equiv \nu_R(x)$$

$$\sigma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}; \quad \nu(x) = \begin{pmatrix} \nu_L(x) \\ \nu_R(x) \end{pmatrix}$$

$$\begin{pmatrix} -m & i\sigma_\alpha \partial^\alpha \\ -i\sigma_\alpha \partial^\alpha & m \end{pmatrix} \begin{pmatrix} \psi_L(x) \\ \psi_R(x) \end{pmatrix} = 0$$

$$i\sigma_\alpha \partial^\alpha \psi_R - m \psi_L = 0$$

$m = 0 \Rightarrow L$ and R

$$i\sigma_\alpha \partial^\alpha \psi_L + m \psi_R = 0$$

handed components
decouple

$$\boxed{i\sigma_\alpha \partial^\alpha \psi_{L,R} = 0}$$

Eq. of motion for
a massless neutrino
state

Implications of this theory more:

1. $H = \int [\bar{p}(x) 0^i n(x) \bar{\epsilon}(x) 0_i (G_i + G'_i \gamma_5) \psi(x)]$

$$\boxed{G'_i = -G_i}$$

$$\left(\frac{1 - \gamma_5}{2} \right) \psi(x) = \psi_L(x)$$

2. Parity is violated, as required by data

$$P \cdot \psi_L = \psi_R$$

$$\boxed{P = \gamma_0}$$

$$\Rightarrow i \gamma^\alpha \partial_\alpha \psi_L(x) = 0$$

$$i \gamma^\alpha \partial_\alpha P \cdot \psi_L = 0 \Rightarrow i \gamma^\alpha \partial_\alpha \psi_R(x) = 0$$

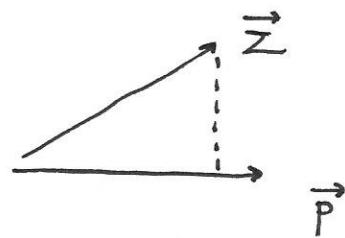
\Rightarrow The equation of motion is not invariant under parity transformations

3. Helicity is "±½" (of the left-handed state) :

$$\vec{\Sigma} = \gamma_5 \gamma_0 \vec{\tau} : \text{Spin operator}$$

$$h = \vec{\Sigma} \cdot \hat{\vec{p}}$$

projection of spin along
the three-momentum



Eigenvalues of the helicity operator :

$$\underbrace{h}_{\vec{\Sigma} \cdot \hat{\vec{p}}} u_L^r = r u_L^r$$

$$\vec{\Sigma} \cdot \hat{\vec{p}} = \gamma_5 \gamma_0 \vec{\tau} \cdot \hat{\vec{p}} \Rightarrow \text{Dirac equation:}$$

$$(\gamma_0 p_0 - \vec{\tau} \cdot \vec{p}) u^r(p) = 0$$

$$(\gamma_0 p_0 - \vec{\tau} \cdot \hat{\vec{p}} |\vec{p}|) u^r(p) = 0$$

$$\left(\gamma_0 - \vec{\tau} \cdot \hat{\vec{p}} \frac{|\vec{p}|}{p_0} \right) u^r(p) = 0$$

$\frac{1}{p_0}$ for
relativistic
particle

$$(\gamma_0 - \vec{\tau} \cdot \hat{\vec{p}}) u^r(p) = 0$$

$$\Rightarrow \vec{\Sigma} \cdot \hat{\vec{p}} = \gamma_5 \Rightarrow \gamma_5 \left(\frac{1 - \gamma_5}{2} \right) u^r = r \left(\frac{1 - \gamma_5}{2} \right) u^r$$

$$\Rightarrow \boxed{r = -1}$$

1958 Measurements by Goldhaber et al. $h = -1$

Current-current V-A theory

Feynman and Gell-Mann
Marshak and Sudarshan 1958

All fields entering the Hamiltonian are LH :

$$H = G_F \sum_i \bar{p}(x) \frac{(1+\gamma_5)}{2} \sigma_i \frac{(1-\gamma_5)}{2} n(x) \\ \bar{e}(x) \frac{(1+\gamma_5)}{2} \sigma_i \frac{(1-\gamma_5)}{2} \nu(x)$$

for $i = \{S, P, T\}$ $H = 0$

only $i = \{V, A\}$ matter

$$H = \frac{G_F}{\sqrt{2}} \bar{p}(x) \gamma^{\mu} (1-\gamma_5) n(x) \bar{e}(x) \gamma_{\mu} (1-\gamma_5) \nu(x) + H.c.$$

For $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_{\mu}$:

$$H = \frac{G_F}{\sqrt{2}} \bar{e}(x) \gamma^{\mu} (1-\gamma_5) \nu_e(x) \bar{\nu}_{\mu}(x) \gamma_{\mu} (1-\gamma_5) \mu(x) + H.c.$$

$$\Gamma(\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_{\mu}) = \frac{G_F^2}{192\pi^3} M_{\mu}^5$$

$$\Rightarrow \bar{L}_{\mu} = \frac{192\pi^3}{G_F^2 M_{\mu}^5}$$

Feynman and Gell-Mann extracted G_F from the β -decay of ^{140}O [$0^+ \rightarrow 0^+$ superallowed transition]

and calculated T_{μ} . in perfect agreement with measurements

To account simultaneously for:

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

$$\mu^- \rightarrow e^- + \bar{\nu}_\mu + \bar{\nu}_e$$

the current

$$j^\kappa = 2 (\bar{p}_L \gamma^\kappa p_L + \bar{e}_L \gamma^\kappa e_L + \bar{\mu}_L \gamma^\kappa \mu_L)$$

was introduced. And the Hamiltonian

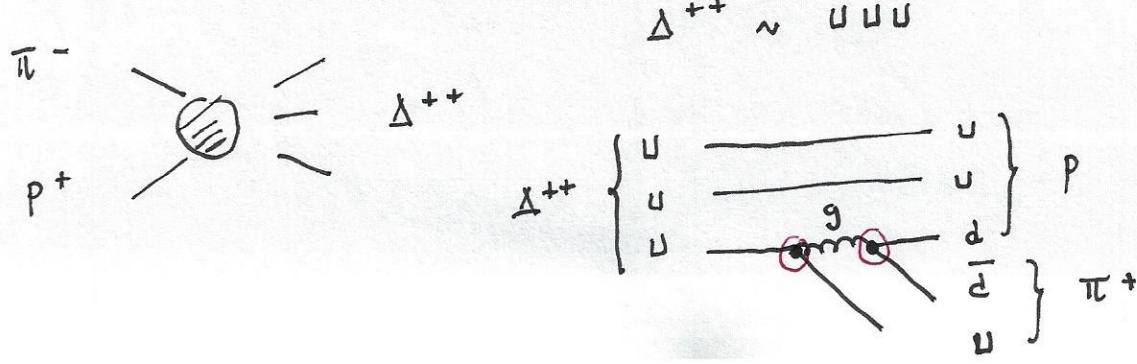
$$H = \frac{g_F}{\sqrt{2}} j^\kappa j^\kappa$$

was assumed to be responsible for weak interactions.

Klein studied the vector intermediate vector hypothesis,

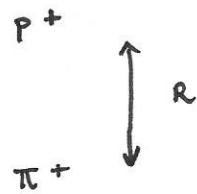
$$\mathcal{L} = -\frac{g}{2\sqrt{2}} j^\kappa W^\kappa + \text{H.c}$$

Strength of the interaction
and mass of the mediator



$$\tau_{\Delta^{++}} \sim 10^{-23} \text{ s}$$

$$\tau_{\Delta^{++}} = \frac{R}{c}$$



$\tau_{\Delta^{++}}$: Time that take the $p^+ - \pi^+$ pair

to be separated

1 fm

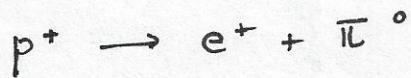
Distance @

which the strong
interaction matters !

$$\tau_{\Delta^{++}} \sim \frac{10^{-15} \text{ m}}{10^8 \text{ m/s}} \sim 10^{-23} \text{ sec} = \tau_{\Delta^{++}}$$

\Rightarrow Baryons have lifetimes in that ballpark

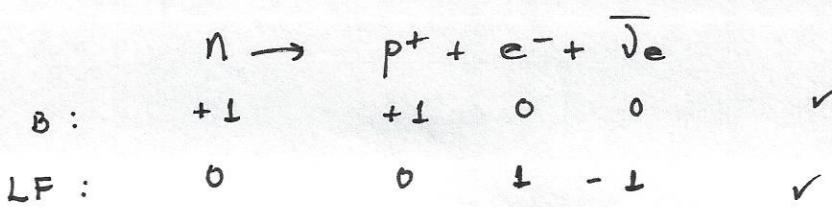
What about the proton ?



$$B(p^+) = 1 \quad B(\bar{\nu}_e) = 0$$

Baryon number behind
stability !

What about neutron decay ?



$$m_n > m_p : \Delta m_{np} = 1.293 \text{ MeV}$$

$$\boxed{\Delta m = m_n - m_p = 1.293 \text{ MeV}}$$

$$\boxed{\tau_{n^0} \sim 13 \text{ min} = 780 \text{ sec}}$$

Could it be that n^0 decays via EM interactions?

$$\pi^0 \rightarrow \gamma \gamma : \tau_{\pi^0} \sim 10^{-16} \text{ sec}$$

$$\tau \sim \Gamma(\pi^0 \rightarrow \gamma \gamma) ; \boxed{\Gamma \propto \alpha_{EM}^2}$$

$$\Rightarrow \tau_{EM} \propto \bar{\alpha}_{EM}^2$$

$$\tau_{\Delta^{++}} \sim \alpha_s^{-2} \quad \left. \begin{array}{l} \text{The larger the coupling, the} \\ \text{shorter the lifetime} \end{array} \right\}$$

$$\kappa_{EM} < \alpha_s \Rightarrow \frac{\tau_{\pi^0}}{\tau_{\Delta^{++}}} \sim 10^7$$

$$\frac{\tau_{EM}}{\tau_{strong}} \sim 10^4 - 10^6 \approx \left(\frac{\kappa_{strong}}{\alpha_{EM}} \right)^2$$

EM interactions cannot account for this!

\Rightarrow Not only the n^0 but other particles have lifetimes that does not fit strong or EM expectations

$$\tau(\Sigma^+ \rightarrow n^0 + \pi^+) \sim 10^{-20} \text{ sec}$$

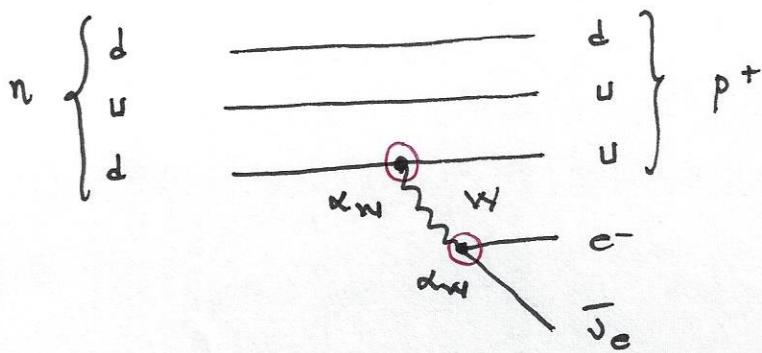
$$\tau(\Delta^+ \rightarrow n^0 + \pi^+) \sim 10^{-24} \text{ sec}$$

$$\frac{\tau_{\Delta^+}}{\tau_{\Sigma^+}} = \left(\frac{\kappa_w}{\kappa_s} \right)^2 = \frac{10^{-24}}{10^{-10}} = 10^{-14}$$

$$\kappa_s \approx 1; \quad \kappa_{EW} \approx 10^{-2} \Rightarrow$$

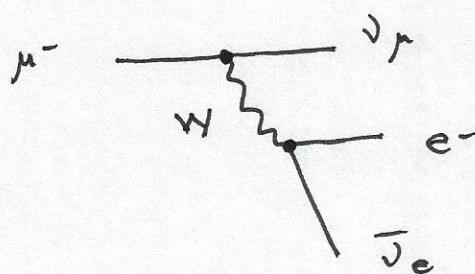
$$\boxed{\kappa_w \sim 10^{-7}}$$

New interaction



Intermediate new gauge boson:

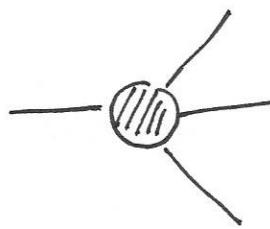
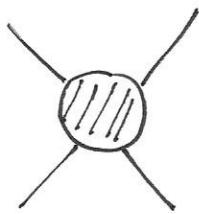
$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \quad (\text{Take this case})$$



Back to V-A:

$$H = g_F \bar{e} \gamma^\mu (1 - \gamma_5) v_e \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu$$

=> Diagrammatically :



Four fermion operator : Fermion fields are dimension (in mass) $3/2$:

$$\mathcal{L} = \bar{\psi} (\not{p} - m) \psi \Rightarrow S = \int dt \mathcal{L}$$

$$= \int d^4x \mathcal{L}$$

$$[S] = [h] = 1 \Rightarrow [x] = [E^{-2}]$$

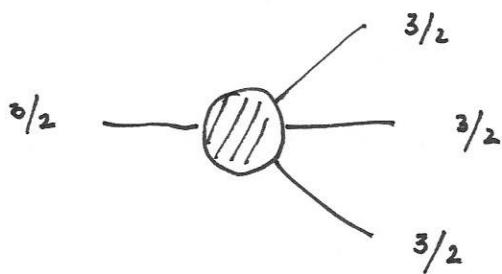
$$d^4x \mathcal{L} = [E^{-4}] [E^4]$$

The Lagrangian (density) is dim +

$$\psi m \psi = \psi^2 m = [E^1]$$

$$\psi^2 = [E^5]$$

$$[\psi] = [E^{3/2}]$$



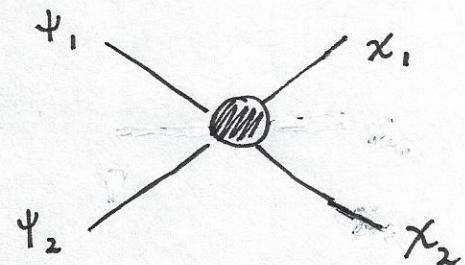
$$\dim = 1 \times \frac{3}{2} = 6$$

■ Why effective?

\Rightarrow Renormalizable couplings are $\dim = 4$

\Rightarrow Any other $\dim > 4$ is not renormalizable (or effective)

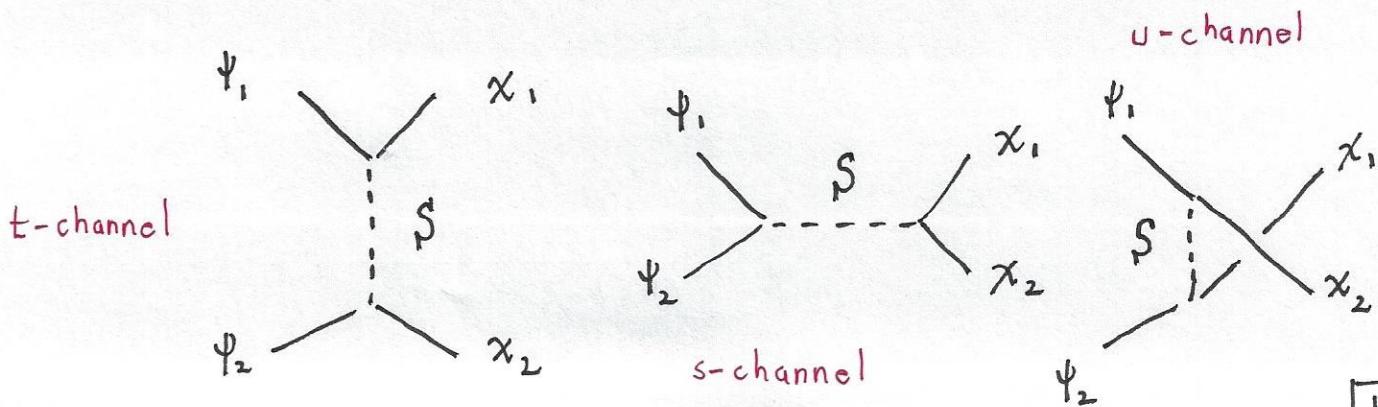
$$\dim = 6 : \mathcal{L} \sim \frac{C}{\Lambda^2} (\bar{\psi}_1 \psi_2) (\bar{x}_1 x_2)$$

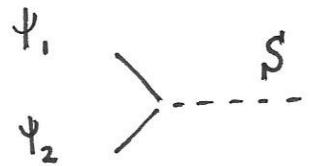


C: Dimensionless (Wilson coefficient)

$$[\Lambda] = [M]$$

$$E_{CM} = E\psi_1 + E\psi_2 \Rightarrow E_{CM} \rightarrow \Lambda :$$





: S can be produced
on shell

$E_{cm} < E_{\Psi_1} + E_{\Psi_2}$: S is a virtual
state and the
effective formulation
is valid

In β decay, is the effective formulation valid?

$E_\mu = m_\mu$ in the μ rest frame

$m_\nu = ?$ Heuristic approach

From the effective Hamiltonian:

$$\Gamma(\mu \rightarrow e \bar{\nu}_e \nu_\mu) = \frac{G_F^2}{192\pi^3} M_\mu^5$$

$$\tau = \frac{192\pi^3}{G_F^2 M_\mu^5}$$

[τ] = [T] = [E^{-2}] : In natural units

$[G_F] = [E^{-2}]$; G_F is the scale of the effective theory



$$\frac{g^2}{M_W^2}$$

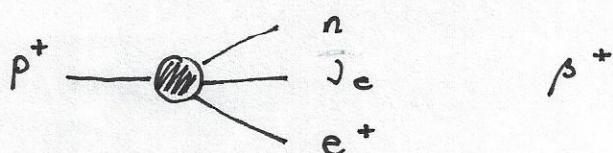
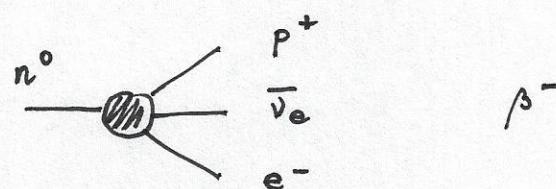
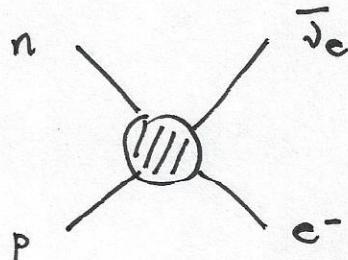
 G_F

$$G_F \sim \frac{g^2}{M_W^2}$$

$$G_F = 1.166 \times 10^{-5} \frac{1}{\text{GeV}^2}$$

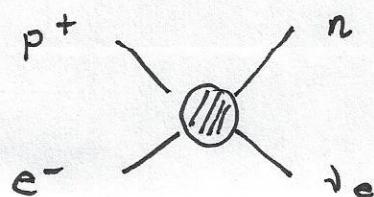
$$M_W^2 \sim \frac{g^2}{G_F} \sim 100 \text{ GeV}$$

For muon decay the effective theory formulation is valid. It is valid as well in β decay related processes :



$$E \sim 1 \text{ GeV}$$

$$M_W \sim 10^2 \text{ GeV}$$



e^- - capture
(K or L shells)

At the fundamental level:

$$H = \frac{g_F}{\sqrt{2}} \left[\overline{e} \gamma^\mu (1-\gamma_5) \nu e W_\mu^- + \overline{n} \gamma^\mu (1-\gamma_5) \bar{\nu} W_\mu^- \right]$$

Renormalizable
Renormalizable

$[W] = [M]$: Vector bosons are dimension 1 as any other boson

dof : $2J+1$: $\xrightarrow[J=1]{\text{spin}}$ \Rightarrow 3

For muon decay

$$H = \frac{g_F}{\sqrt{2}} \left[\overline{\mu} \gamma^\sigma (1-\gamma_5) \nu_\mu \times \vec{v} \right. \\ \left. + \overline{e} \gamma^\sigma (1-\gamma_5) \nu e \times \vec{v} \right]$$

When E of the process is $E \ll m_W$ then:

$$H \rightarrow H_{\text{eff}}$$

Standard model

Based on $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$

Leptons

$$\mathcal{L}_{cc}^L = -\frac{g}{2\sqrt{2}} [\bar{\nu}_l \gamma^\mu (1-\gamma_5) l W_\mu^+ + \bar{l} \gamma^\mu (1-\gamma_5) \nu W_\mu^-]$$

$$\begin{aligned} \mathcal{L}_{NC}^L &= -\frac{g}{2c_W} \sum_{i=1,2} \bar{\Psi}_i \gamma^\mu (g_V^{ii} - g_A^{ii} \gamma_5) \Psi_i Z_\mu \\ &\quad - g_{SW} \bar{l} \gamma^\mu l \end{aligned}$$

Quarks

$$\mathcal{L}_{cc}^q = \frac{g}{2\sqrt{2}} \bar{q}_L \gamma^\mu (1-\gamma_5) q_L W_\mu^+ + H.c.$$

$$\mathcal{L}_{NC}^q = -\frac{g}{2c_W} \sum_i \bar{\Psi}_i \gamma_\mu (g_V^{ii} - g_A^{ii} \gamma_5) \Psi_i Z^\mu$$

$$g_V^{ii} = T_3^i - 2 Q_i s_W^2$$

$$g_A^{ii} = -T_3^i$$

\rightarrow Dictated by
the gauge group and
the gauge principle

T_3

$$\Psi_i = L_i = \begin{pmatrix} \nu_e \\ l \end{pmatrix}$$

$$\begin{matrix} 1/2 \\ -1/2 \end{matrix}$$

$$\Psi_i = Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$$

$$g_V^e = -\frac{1}{2} + 2 s_w^2$$

$$g_A^e = -\frac{1}{2}$$

$$g_V^j = \frac{1}{2}$$

$$g_A^j = \frac{1}{2}$$

$$q=u,c,t : g_V^q = \frac{1}{2} - \frac{4}{3} s_w^2$$

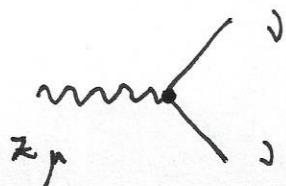
$$g_A^q = -\frac{1}{2}$$

$$q=d,s,b : g_V^q = -\frac{1}{2} + \frac{2}{3} s_w^2$$

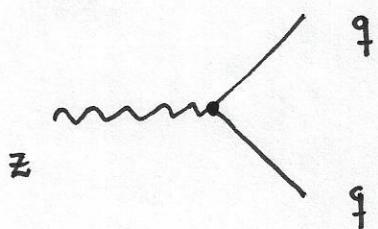
$$g_A^q = -\frac{1}{2}$$

Neutral current

Feynman rules



$$\frac{g}{2c_w} \gamma_\mu P_L$$



$$\frac{g}{2c_w} \gamma_\mu (g_V^q - g_A^q \gamma_5) : q = \begin{cases} u, c, t \\ d, s, b \end{cases}$$

\Rightarrow A comment on $\gamma_\mu \gamma_5$ couplings (see page 24)

Exercise : Prove that $\bar{\Psi} \gamma^\mu \gamma_5 \Psi$ involves the Nuclear spin operator for $\Psi = Q$

Coupling to nucleons:

Step I: $O_q \rightarrow O_n$ $\eta = n^0, p^+$

Project vector current in nucleon states

$$\langle n(p_f) | \bar{q} \gamma^\mu q | n(p_i) \rangle = \underbrace{N_{q\bar{q}}^n}_{\downarrow} \bar{n} \gamma^\mu n$$

quark number operator

$$\mathcal{L}_{NC}^q = -\frac{g}{2c_W} (\bar{u} \gamma_\mu u g_v^u + \bar{d} \gamma_\mu d g_v^d) z^\mu$$

1st term: $g_v^u \langle p | \bar{u} \gamma_\mu u | p \rangle + g_v^u \langle n | \bar{u} \gamma_\mu u | n \rangle$

$$g_v^u \underbrace{N_u^p}_2 \bar{p} \gamma_\mu p + g_v^u \underbrace{N_u^n}_1 \bar{n} \gamma_\mu n$$

$p \sim uud$ $n \sim udd$	\Rightarrow	$2g_v^u \bar{p} \gamma_\mu p + g_v^u \bar{n} \gamma_\mu n$
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2nd term:

$$g_v^d \underbrace{N_d^p}_1 \bar{p} \gamma_\mu p + g_v^d \underbrace{N_d^n}_2 \bar{n} \gamma_\mu n$$

$$\Rightarrow g_v^d \bar{p} \gamma_\mu p + 2g_v^d \bar{n} \gamma_\mu n$$

$$(2g_v^u + g_v^d) \bar{p} \gamma_\mu p + (g_v^u + 2g_v^d) \bar{n} \gamma_\mu n$$

$$g_v^p \bar{p} r_{\mu p} + g_v^n \bar{n} r_{\mu n}$$

Coupling to nuclei:

Step II:

$$O_n \rightarrow O_N$$