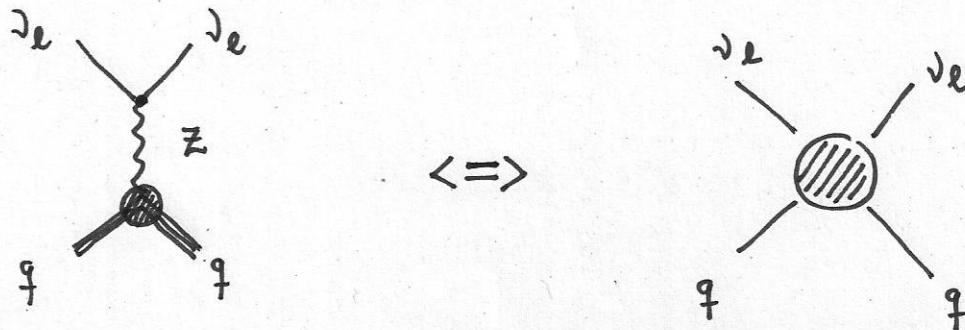


## Lecture 2

- CEvNS cross section
- Form factors and neutron distribution  
rms radius

CEvNS cross section  
calculation



$$E_\nu \approx 10 \text{ MeV} ; \quad M_Z \approx 90 \text{ GeV}$$

$$\mathcal{L}_{NC}^{\text{eff}} = \frac{g_F}{\sqrt{2}} \sum_i [ \bar{\nu} \gamma^\mu (g_V^i - g_A^i \gamma_5) \nu ] [ \bar{q} \gamma_\mu (g_V^q - g_A^q \gamma_5) q ]$$

$$g_V^i = \pm \frac{1}{2} \quad \quad \quad g_A^i = \pm \frac{1}{2}$$

$$g_V^q = T_3^q - 2 Q^q \sin^2 \theta_W = \begin{cases} \frac{1}{2} - \frac{1}{3} \sin^2 \theta_W ; & u, c, t \\ -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W ; & d, s, b \end{cases}$$

$\bar{q} \gamma_\mu \gamma_5 q \rightarrow$  Spin dependent result . It is always suppressed compared with vector currents.

Consider only :  $\bar{q} \gamma_\mu g_V^q q$

Construct the nucleon operator:

$$\langle \eta(p_f) | \bar{q} \gamma^\mu q | \eta(p_i) \rangle = \underline{\underline{N_n^q}} \bar{n} \gamma^\mu n$$

$$\eta = n, p$$

quark number  
operator

Project over the nucleons

$$\sum_{\eta=n,p} \sum_q \langle \eta(p_f) | \bar{q} \gamma^\mu q | g_v^q | \eta(p_i) \rangle$$

$$= \sum_q [ \langle n | \bar{q} \gamma^\mu q | g_v^q | n \rangle + \langle p | \bar{q} \gamma^\mu q | g_v^q | p \rangle ]$$

Recall:

$p \sim uud$
$n \sim udd$

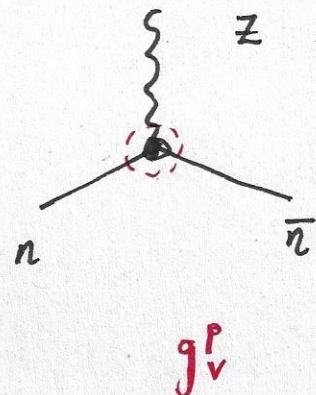
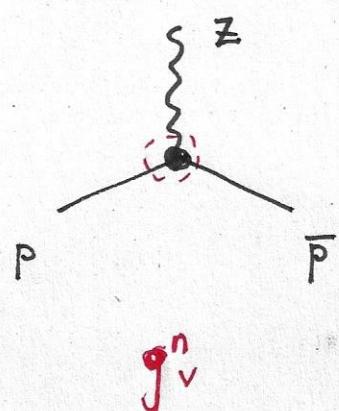
$$\begin{aligned}
 &= g_v^u \langle n | \bar{u} \gamma^\mu u | n \rangle + g_v^d \langle n | \bar{d} \gamma^\mu d | n \rangle \\
 &\quad + g_v^u \langle p | \bar{u} \gamma^\mu u | p \rangle + g_v^d \langle p | \bar{d} \gamma^\mu d | p \rangle \\
 &= g_v^u \textcircled{N_n^u} \bar{n} \gamma^\mu n + g_v^d \textcircled{N_n^d} \bar{n} \gamma^\mu n \\
 &\quad + g_v^u \textcircled{N_p^u} \bar{p} \gamma^\mu p + g_v^d \textcircled{N_p^d} \bar{p} \gamma^\mu p \\
 &= (g_v^u + 2g_v^d) \bar{n} \gamma^\mu n + (2g_v^u + g_v^d) \bar{p} \gamma^\mu p
 \end{aligned}$$

$$= g_v^n \bar{n} \gamma^\mu n + g_v^p \bar{p} \gamma^\mu p$$

$$g_v^n = g_v^u + 2g_v^d$$

$$g_v^p = 2g_v^u + g_v^d$$

We are in the effective theory. However one can think of the following couplings:



$g_v^n$  and  $g_v^p$  measure the strength at which nucleons couple to the  $Z^0$ .

We are interested in Nuclei : Construct the Nucleus operator

Project over the nuclei

Nucleon number operator

$$\langle N(k_2) | \bar{\eta} \gamma^\mu \eta | N(k_1) \rangle = \underbrace{N_{\eta} \bar{N}}_{\uparrow} [ \gamma^\mu F_\nu(q^2) + \frac{\sigma^{MN} q^2}{2M_N} F_M(q^2) ] N$$

Two reasons for the second term to be negligible:

- Involves the spin operator (No coherent enhancement)

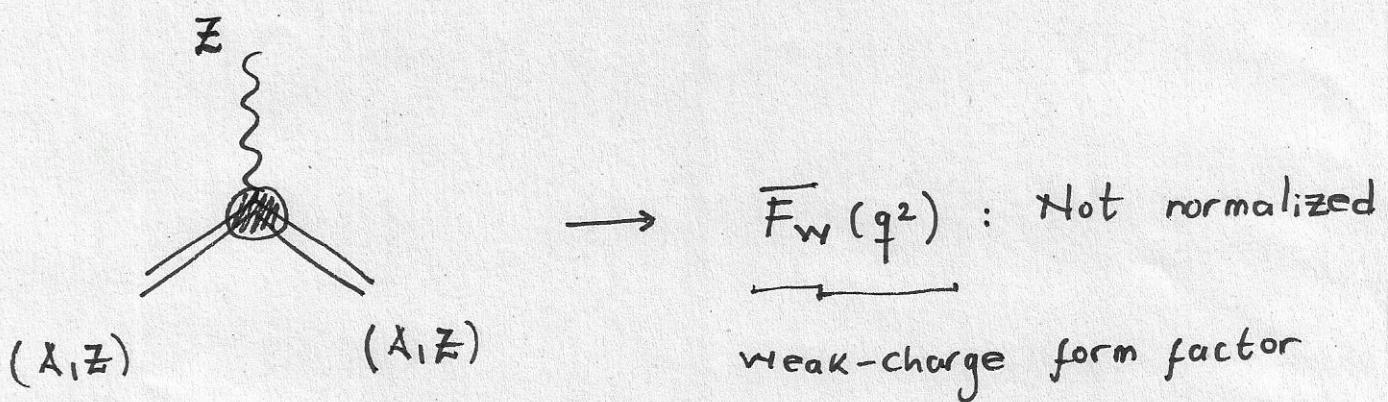
$$\frac{q^2}{2M_N} \sim \frac{10 \text{ MeV}}{10^4 \text{ MeV}} \sim 10^{-3}$$

$$\langle N(k_2) | \bar{\eta} \gamma^\mu \eta | N(k_1) \rangle = N_\eta \bar{N} \gamma^\mu N F_V^n(q^2)$$

projection produces

$$\Rightarrow g_V^n \langle N(k_2) | \bar{n} \gamma^\mu n | N(k_1) \rangle + g_V^p \langle N(k_2) | \bar{p} \gamma^\mu p | N(k_1) \rangle$$

$$= (\underbrace{g_V^n N F_V^n}_{\substack{\text{Accounts} \\ \text{for distribution} \\ \text{of neutrons}}} + \underbrace{g_V^p Z F_V^p}_{\substack{\text{distribution} \\ \text{of protons}}}) \cdot \bar{N} \gamma^\mu N = \bar{F}_W(q^2) \bar{N} \gamma^\mu N$$



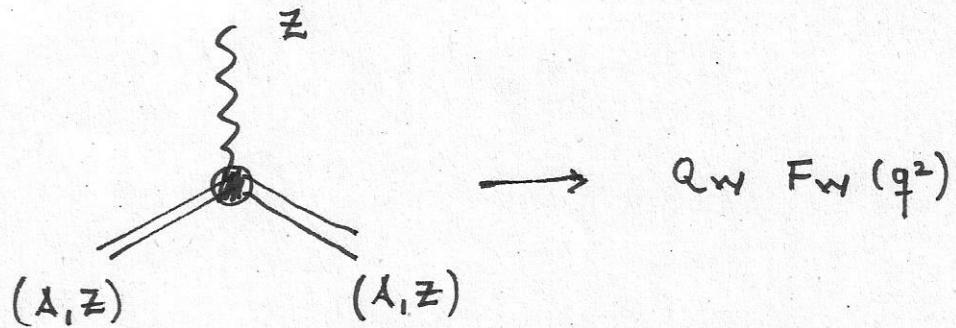
$$F_V^{n,p}(q^2 \rightarrow 0) = 1 \Rightarrow F_W(q^2 \rightarrow 0) = \frac{g_V^n N + g_V^p Z}{Q_W}$$

But we want form factors to be normalized to 1

$$Q_W \equiv g_V^n N + g_V^p Z$$

$$\bar{F}_W(q^2) = Q_W \frac{\bar{F}_W(q^2)}{Q_W} = Q_W F_W(q^2)$$

$$F_W(q^2) = \frac{\bar{F}_W(q^2)}{Q_W} \quad \text{is normalized}$$



Explicit expression for  $Q_W$ :

$$g_V^n = \frac{1}{2} - \frac{4}{3} s_W^2 + \textcircled{2} \left[ -\frac{1}{2} + \frac{2}{3} s_W^2 \right]$$

$$g_V^n = -\frac{1}{2}$$

$$\begin{aligned} g_V^P &= 2 \left( \frac{1}{2} - \frac{4}{3} s_W^2 \right) + \left[ -\frac{1}{2} + \frac{2}{3} s_W^2 \right] \\ &= \frac{1}{2} - \frac{2}{3}(4-1) s_W^2 = \frac{1}{2} - 2 s_W^2 \end{aligned}$$

$$g_V^P = \frac{1}{2} - 2 s_W^2$$

$$Q_W = -\frac{N}{2} + \left( \frac{1}{2} - 2 s_W^2 \right) z$$

### REMARK

~~Weak mixing angle~~

Depends at the scale at which is measured.

Best measurements are done at the LHC  
at ~~100~~ transferred momentum of the order of

$10^2 \text{ GeV}$

$$\sin^2 \theta_W (M_Z) = 0.23 \rightarrow \left(1 - \frac{0.23}{0.25}\right) \times 100\% = 8\%$$

$\frac{1}{2} - 2 \times \frac{1}{4} = 0 \Rightarrow$  the cancellation is exact up to 8%

$$Q_W \approx -\frac{1}{2} \quad \text{up to } 8\% \text{ corrections}$$

Relevant effective Lagrangian :

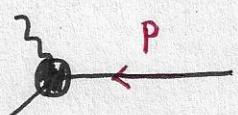
$$\mathcal{L}_{NC}^{\text{Eff}} = \frac{G_F}{\sqrt{2}} \left[ \bar{v} \gamma^\mu \overbrace{\left( \frac{(1-\tau_5)}{2} \right) v}^{P_L} \right] [Q_W F_W \bar{N} \gamma_\mu N]$$

$$\boxed{\mathcal{L}_{NC}^{\text{Eff}} = \frac{G_F}{\sqrt{2}} Q_W F_W L^\mu W_\mu}$$

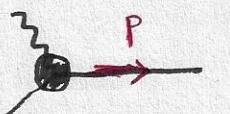
$$\boxed{L^\mu \equiv \bar{v} \gamma^\mu P_L v}$$

$$\boxed{W_\mu \equiv \bar{N} \gamma_\mu N}$$

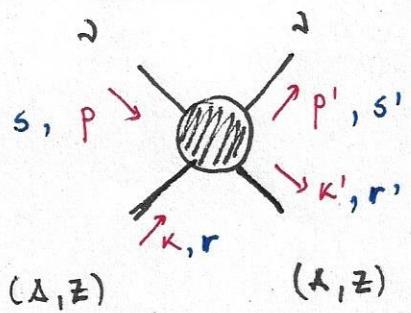
Assume  $N$  is in a fermionic ground state :



$$= u^s(p) \quad (\text{initial state})$$



$$= \bar{u}^s(p) \quad (\text{final state})$$



$$M^{ss'rr'} = \frac{G_F}{\sqrt{2}} Q_W F_W \left[ \bar{u}^{s'}(p') \gamma^\mu P_L u^s(p) \right] \\ \left[ \bar{u}^{r'}(k') \gamma_\mu u^r(k) \right]$$

$$|M|^2 = \sum_{s,s'} \frac{1}{2} \sum_{r,r'} |M^{ss'rr'}|^2$$

The spin  
of the LH  
neutrinos  
is fixed

for the nucleus  
is not, and so  
the  $1/2$  factor

~~Lepton tensor~~

$$|M|^2 = \frac{G_F^2 Q_W^2}{4} L^{\mu\nu} W_{\mu\nu} F_W^2 (q^2)$$

↑                            ↓  
Lepton tensor

$$L^{\mu\nu} = \sum_{s,s'} \left[ \bar{u}^{s'}(p) \gamma^\mu P_L u^s(p) \right] \left[ \bar{u}^s(q) \gamma^\nu P_L u^{s'}(q) \right]$$

$$= \sum_{s'} \underbrace{\bar{u}^s(p) \bar{u}^{s'}(p)}_{s} \gamma^\mu P_L \sum_s \bar{u}^s(p) \bar{u}^s(p) \gamma^\nu P_L$$

$$L^{\mu\nu} = \text{Tr} [ \not{p} \gamma^\mu P_L \not{p} \gamma^\nu P_L ]$$

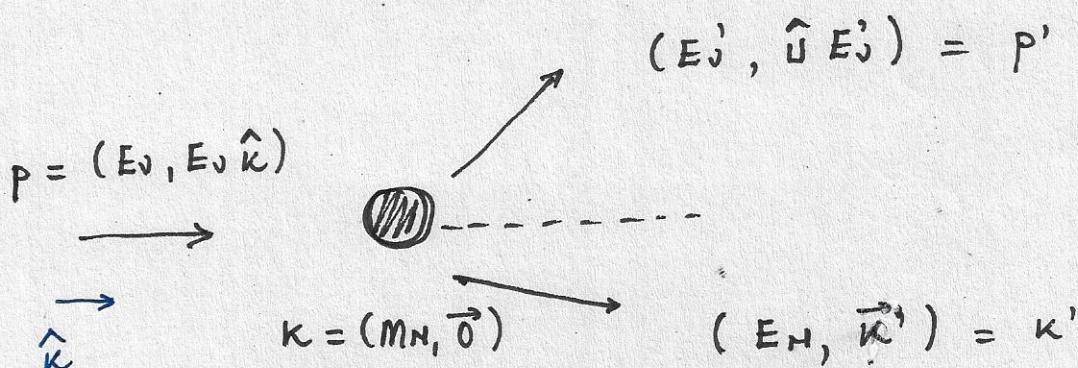
$$W_{\mu\nu} = \sum_{r,r'} [\bar{U}^{r'}(k') \gamma_\mu U^r(k)] [\bar{U}^r(k) \gamma_\nu U^{r'}(k')] \\ = \sum_{r'} U^{r'}(k') \bar{U}^{r'}(k') \gamma_\mu \sum_r U^r(k) \bar{U}^r(k) \gamma_\nu$$

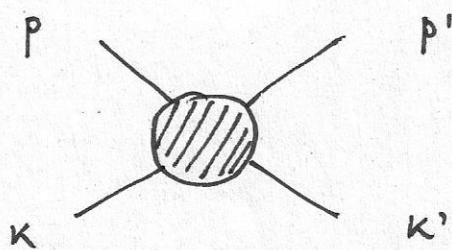
$$W_{\mu\nu} = \text{Tr} [(x^\mu + m_N) \gamma_\mu (x^\nu + m_N) \gamma_\nu]$$

- Write the differential cross section in terms of Lorentz invariant quantities. The resulting expression is valid regardless of reference frame

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} \frac{1}{(s - m_N^2)} |M|^2$$

Write this expression in terms of the quantities of interest. Those that can be measured  $E_V, E_R$  (Folks use  $T \equiv E_R$ )





$$s = (p + k)^2 = (p' + k')^2$$

$$t = (p - p')^2 = (k' - k)^2$$

$$2E_J M_N$$

$$s = \cancel{p^2} + \underbrace{2p \cdot k}_{m_N^2} + \underbrace{k^2}_{m_N^2} = m_N^2 + 2E_J M_N$$

$$s = m_N^2 + 2E_J M_N$$

$$t = \underbrace{k'^2}_{m_N^2} - \underbrace{2k' \cdot k}_{2M_N E_N} + \underbrace{k^2}_{m_N^2}$$

Rewrite  $E_N$ :

Energy conservation:  $M_N + E_J = E'_J + E_N$

$$\Rightarrow E_N = M_N + \frac{E_J - E'_J}{E_r}$$

$$E_N = M_N + E_r$$

$$t = 2m_N^2 - 2M_N(M_N + E_r)$$

$$t = -2M_N E_r$$

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dE_r} \left| \frac{dE_r}{dt} \right| = \frac{d\sigma}{dE_r} \frac{1}{2m_N}$$

$$\Rightarrow \boxed{\frac{d\sigma}{dE_r} = 2m_N \frac{d\sigma}{dt}}$$

$$\frac{d\sigma}{dE_r} = \frac{2m_N}{16\pi} \frac{1}{4E_J^2 m_N^2} |M|^2$$

$$= \frac{1}{32\pi} \frac{1}{E_J^2 m_N} |M|^2$$

$$\boxed{\frac{d\sigma}{dE_r} = \frac{G_F^2}{128\pi} \frac{Q_W^2}{E_J^2 m_N} L^{\mu\nu} W_{\mu\nu} F_W^2(E_r)}$$

Form factors and neutron  
distribution rms radius

$$F(q^2) = \frac{1}{Q} \int e^{-iq \cdot r} \rho(r) d^3r$$

$Q$ : Charge of the entire distribution

$$Q = \int \rho(r) d^3r$$

For spherical symmetric distributions

$$F(q^2) = \frac{2\pi}{Q} \int e^{-iqr \cos\theta} \rho(r) r^2 dr d\cos\theta$$

$$y = iq r \cos\theta \Rightarrow dy = iq r d\cos\theta$$

$$F(q^2) = \frac{2\pi}{iQq} \int e^{-y} \rho(r) r dr \underbrace{\frac{iqr d\cos\theta}{dy}}$$

$$= - \frac{2\pi}{iQq} \int \rho(r) r dr \left[ e^{-y} \right]_{iqr}^{-iqr}$$

$$= - \frac{2\pi}{iQq} \int \rho(r) r dr [e^{-iqr} - e^{iqr}]$$

$$= \frac{4\pi}{Qq} \int \rho(r) r dr \sin(qr)$$

$$F(q^2) = \frac{4\pi}{Qq} \int r \rho(r) \sin(qr) dr$$

### Parametrizations

Hard sphere    surface thickness

$$- F_H(q^2) = \underbrace{\frac{3}{j_1(qR_0)}}_{\substack{qR_0 \\ \text{Hard sphere}}} e^{-q^2 s^2 / 2}$$

Gaussian profile

$$- F_{KH}(q^2) = \frac{3}{\frac{j_1(qR_A)}{qR_A}} \frac{1}{1 + q^2 a_K}$$

potential

Helmholtz : Hard sphere smoothed with  
a Gaussian profile

Radius of the hard sphere  $R_0$   
(diffraction radius)

Klein-Nystrand : Hard sphere with  
a Yukawa potential

$R_A$  : Radius of the hard sphere

$a_K$  : Range of the potential

## Hard sphere

$$\rho = \frac{Q}{4\pi a^3/3}$$

a: radius of the hard sphere

$$F(q^2) = \frac{3}{q a^3} \int_0^a r \sin(qr) dr$$

$$= \frac{3}{q a^3} \left[ \frac{\sin(qr)}{q^2} - r \frac{\cos(qr)}{q} \right]_0^a$$

$$= \frac{3}{q a^3} \left[ \frac{\sin(qa)}{q^2} - a \frac{\cos(qa)}{q} \right]$$

$$= \frac{3}{(qa)^3} [\sin(qa) - qa \cos(qa)]$$

$$= \frac{3}{qa} \left[ \frac{\sin(qa) - qa \cos(qa)}{(qa)^2} \right]$$

$$F(q^2) = \frac{3}{qa} j_\perp(qa)$$

Take the Helm parametrization

$$\langle r^2 \rangle = -6 \frac{dF_H}{dq^2} \Big|_{q^2=0}$$

$\Rightarrow$

$$\boxed{\langle r^2 \rangle = \frac{3}{5} R_0^2 + 3 \sigma^2}$$

$F_H^P$  : Fix  $R_0$  with the aid of data

$F_H^n$  : Assume something (in an actual calculation) or let  $\langle r_n^2 \rangle$  vary and compare with data

## Radial moments

$$r_n = \sqrt[n]{\langle r^n \rangle} = \left[ \frac{\int d^3r r^n \rho(r)}{\int d^3r \rho(r)} \right]^{1/n}$$

The rms radius of the distribution

$$r_2 = \frac{\int d^3r r^2 \rho(r)}{\int d^3r \rho(r)}$$

They can be calculated as follows:

$$\begin{aligned} F(q^2) &= \frac{4\pi}{Qq} \int r \rho(r) \left[ qr - \frac{(qr)^3}{3!} + \frac{(qr)^5}{5!} - \dots \right] dr \\ &= \frac{4\pi}{Q} \int r^2 \rho(r) \left[ 1 - \frac{q^2 r^2}{3!} + \frac{q^4 r^4}{5!} - \dots \right] dr \end{aligned}$$

Term by term:

$$F(q^2) = 1 - \frac{q^2}{3!Q} \int \rho(r) r^2 d^3r + \frac{q^4}{5!Q} \int \rho(r) r^4 d^3r - \dots$$

$$F(q^2) = \left[ 1 - \frac{q^2}{3!} \langle r^2 \rangle + \frac{q^4}{5!} \langle r^4 \rangle - \dots \right]$$

$$F(q^2) = 1 + \left. \frac{dF}{dq^2} \right|_{q^2=0} q^2 + \left. \frac{d^2F}{dq^4} \right|_{q^2=0} q^4 + \dots$$

$$-\frac{\langle r^2 \rangle}{3!} = \left. \frac{dF}{dq^2} \right|_{q^2=0}$$

$$\boxed{\langle r^2 \rangle = -6 \left. \frac{dF}{dq^2} \right|_{q^2=0}}$$