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# A Weakly Mixing Turing machine which is not Topologically Mixing

R. Torres-Avilés \*

Departamento de Sistemas de Información  
Universidad del Bío-Bío, Concepción, Chile

### Abstract

Over the past few decades, Turing machines have been studied as dynamical systems, focusing on their behavior over their results. Noteworthy results concerning topological and dynamical properties were established, as the existence and undecidability of Topological Transitivity in Turing machines with moving head, and Topological Minimality in Turing machines with moving tape. Both properties are related to reaching finite windows from any or all possible configurations, respectively. Nonetheless, both properties exhibit no restriction over the time a machine takes to reach those finite windows. In this presentation, we focus on the *Mixing* notions over Turing machines with moving tape: *Weak Mixing* and *Topological Mixing*. These properties are related to a time window or gap where finite configurations must reach one another. In Turing machines, mixing notions are naturally related when presented, as all known examples of weakly mixing Turing machines are also topologically mixing Turing machines, as they are related with coded systems. Nevertheless, the SMART machine, the first known topologically minimal Turing machine, is not related with a coded system. In this presentation, we show that SMART machine is the first example known on Turing machines to be weakly mixing, but it is not topologically mixing.

Joint work with:

A. Gajardo<sup>1</sup>, Universidad de Concepción, Chile.

## References

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<sup>1</sup>e-mail: angajardo@udec.cl

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