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An a priori error analysis for a linear transmission problem using a mixed Hybrid High Order method

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Abstract

In this talk we introduce a new Hybrid High-Order (HHO) method for a linear elliptic transmission problem in a bounded domain. In HHO the solution of the problem at hand is approximated by attaching polynomials of degree k to the mesh cells and to their boundaries. Specific element-local operators are then employed to obtain a high-order reconstruction of the solution. Following this construction, a well-posed nonconforming discrete formulation is obtained. A significant advantage of HHO is that cell-based unknowns can be eliminated locally via a Schur complement, obtaining a global problem posed on the mesh skeleton. This in turn allows to obtain a compact global linear system with a significantly reduced number of unknowns.

In our scheme an auxiliary unknown, which plays the role of a Lagrange multiplier, is introduced to deal with the nonhomogeneous transmission conditions. We prove that the proposed method is optimally convergent in the energy norm, as well as in the L^2-norm for the potential and a weighted L^2-norm for the Lagrange multiplier, for smooth enough solutions. Finally, we include some numerical experiments that validate our theoretical results, even in situations not covered by the current analysis.

Joint work with:
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