

# **Chillan Workshop in Applied Mathematics CWAM 2023**

## **Report of Contributions**

Contribution ID: 1

Type: **not specified**

## Looking for pre-expansivity in two-dimensional cellular automata

*Wednesday, 11 January 2023 10:00 (1 hour)*

In [3] we have introduced the notion of pre-expansivity for cellular automata (CA): it is the property of being positively expansive on asymptotic pairs of configurations (i.e. configurations that differ in only finitely many positions). Pre-expansivity therefore lies between positive expansivity and pre-injectivity, two important notions of CA theory. It is known [5] that expansivity is impossible in two-dimensional CA, but the proof does not apply to the weaker notion of pre-expansivity. Thus we are in the quest for two-dimensional pre-expansive CA. In [3] we showed however that no two-dimensional Abelian CA can be pre-expansive, thus non-linear CAs are our only hope. In this direction, we started by exploring the family of multiplication CA, which is a non-linear CA that resulted to be pre-expansive in dimension one [3]. Then we extended its definition to two-dimensional CAs [2], with surprising experimental results, but very evasive theoretical results. In this talk, we will show our findings, in order to motivate new people in the study of these automata, and receive inspiring ideas.

**Primary author:** GAJARDO, Anahi (Ing-Mat, UdeC)

**Presenter:** GAJARDO, Anahi (Ing-Mat, UdeC)

Contribution ID: 2

Type: **not specified**

## Rompimiento de un Paquete de Ondas de Vorticidad en un Torbellino en Gran Rotación

*Friday, 13 January 2023 09:00 (1 hour)*

This study considers the interaction between a free vorticity wave packet and a rapidly rotating vortex in the slowly-evolving regime, a long time after the initial, unsteady, and strong interaction. The interaction starts when the amplitude modulated neutral mode enters resonance with the vortex on a spiralling critical surface, where the phase angular speed is equal to the rotation frequency. The singularity in the modal equation on this asymmetric surface strongly modifies the flow in its neighbourhood, the three-dimensional helical critical layer, the region where the wave/vortex interaction occurs. This interaction generates a vertically sheared 3D mean flow of higher amplitude than the wave packet. The chosen envelope regime assumes the formation of a mean radial velocity of the same order as the wave packet amplitude, deviating the streamlines in a spiral way with respect to the rotational wind [2]. This spiral motion is frequently observed inside tropical cyclones as spiral rainbands [1]. Through matched asymptotic expansions, we find an analytical solution of the leading-order motion equations inside the 3D critical layer. The system of the coupled evolution equations of the wave amplitude and the low-order critical layer-induced mean flow on the critical radius has been derived in the quasi-steady regime. The main outcome of the first-order mean flow truncated system resolution is that the wave packet/vortex interaction leads to a fast vorticity wave breaking. The vertical wind shear has the highest effect on the wave/mean flow interaction. When the shear is moderate, it enhances intensification but when it is very large, it prohibits it in both the unsteady and slowly evolving stages [3].

**Primary author:** CAILLOL, Philippe (CCE UBB)

**Presenter:** CAILLOL, Philippe (CCE UBB)

Contribution ID: 3

Type: **not specified**

## A posteriori virtual element method for the acoustic vibration problem

*Wednesday, 11 January 2023 18:00 (1 hour)*

In two dimensions, we propose and analyze an a posteriori error estimator for the acoustic spectral problem based on the virtual element method in  $H(\text{div}; \Omega)$ . Introducing an auxiliary unknown, we use the fact that the primal formulation of the acoustic problem is equivalent to a mixed formulation, in order to prove a superconvergence result, necessary to despise high order terms. Under the virtual element approach, we prove that our local indicator is reliable and globally efficient in the  $L^2$ -norm. We provide numerical results to assess the performance of the proposed error estimator.

**Primary author:** RIVERA, Gonzalo (Universidad de Los Lagos)

**Presenter:** RIVERA, Gonzalo (Universidad de Los Lagos)

Contribution ID: 4

Type: **not specified**

## VEM approximation for the Stokes eigenvalue problem: a priori and a posteriori error analysis

*Wednesday, 11 January 2023 17:00 (1 hour)*

We present an inf-sup stable divergence free virtual element method and associated a priori, and a posteriori error analysis to approximate the eigenvalues and eigenfunctions of the Stokes spectral problem in one shot. For the a priori analysis, we take advantage of the compactness of the solution operator to prove convergence of the eigenfunctions and double order convergence of eigenvalues. Additionally we also propose an a posteriori estimator of residual type, which we prove is reliable and efficient, in order to perform adaptive refinements that allow to recover the optimal order of convergence for non smooth eigenfunctions. We report some numerical tests

**Primary author:** LEPE, Felipe (DMAT UBB)

**Presenter:** LEPE, Felipe (DMAT UBB)

Contribution ID: 5

Type: **not specified**

## A Weakly Mixing Turing machine which is not Topologically Mixing

*Thursday, 12 January 2023 14:30 (1 hour)*

Over the past few decades, Turing machines have been studied as dynamical systems, focusing on their behavior over their results. Noteworthy results concerning topological and dynamical properties were established, as the existence and undecidability of Topological Transitivity in Turing machines with moving head, and Topological Minimality in Turing machines with moving tape. Both properties are related to reaching finite windows from any or all possible configurations, respectively. Nonetheless, both properties exhibit no restriction over the time a machine takes to reach those finite windows. In this presentation, we focus on the notions over Turing machines with moving tape: Weak mixing and Topological Mixing. These properties are related to a time window or gap where finite configurations must reach one another. In Turing machines, mixing notions are naturally related when presented, as all known examples of weakly mixing Turing machines are also topologically mixing Turing machines, as they are related with coded systems. Nevertheless, the SMART machine, the first known topologically minimal Turing machine, is not related with a coded system. In this presentation, we show that SMART machine is the first example known on Turing machines to be weakly mixing, but it is not topologically mixing.

**Primary author:** TORRES, Rodrigo (DSI FACE UBB)

**Presenter:** TORRES, Rodrigo (DSI FACE UBB)

Contribution ID: 6

Type: **not specified**

## A hybridizable discontinuous Galerkin method for dissimilar meshes

*Thursday, 12 January 2023 10:00 (1 hour)*

We propose and analyze a hybridizable discontinuous Galerkin (HDG) method for dissimilar meshes. This type of meshes are common, for instance, when different parts of the domains are triangulated independently, which may generate gaps or overlaps between these triangulations. The method considers an HDG discretization on separate meshes and tie them together through appropriate transmission conditions.

These transmission conditions are based upon transferring the numerical flux from the first mesh to the second one, and the numerical trace from the second mesh to the first one. Stability and error analysis are shown, where the size of the gap is explicitly written in the estimates. We also present numerical results to validate the theory.

**Primary author:** SOLANO, Manuel (Ing-Mat, UdeC)

**Presenter:** SOLANO, Manuel (Ing-Mat, UdeC)

Contribution ID: 7

Type: **not specified**

## An a priori error analysis for a linear transmission problem using a mixed Hybrid High Order method

*Thursday, 12 January 2023 09:00 (1 hour)*

In this talk we introduce a new Hybrid High-Order (HHO) method for a linear elliptic transmission problem in a bounded domain. In HHO the solution of the problem at hand is approximated by attaching polynomials of degree  $k$  to the mesh cells and to their boundaries. Specific element-local operators are then employed to obtain a high-order reconstruction of the solution. Following this construction, a well-posed nonconforming discrete formulation is obtained. A significant advantage of HHO is that cell-based unknowns can be eliminated locally via a Schur complement, obtaining a global problem posed on the mesh skeleton. This in turn allows to obtain a compact global linear system with a significantly reduced number of unknowns. In our scheme an auxiliary unknown, which plays the role of a Lagrange multiplier, is introduced to deal with the nonhomogeneous transmission conditions. We prove that the proposed method is optimally convergent in the energy norm, as well as in the  $L^2$ -norm for the potential and a weighted  $L^2$ -norm for the Lagrange multiplier, for smooth enough solutions. Finally, we include some numerical experiments that validate our theoretical results, even in situations not covered by the current analysis.

**Primary author:** BUSTINZA, Rommel (Ing- Mat and CI<sup>2</sup>MA, UdeC)

**Presenter:** BUSTINZA, Rommel (Ing- Mat and CI<sup>2</sup>MA, UdeC)

Contribution ID: 8

Type: **not specified**

## Un modelo epidémico con reacción-difusión

*Wednesday, 11 January 2023 11:30 (1 hour)*

En esta charla, presentamos resultados para la dinámica que modela la transmisión de una enfermedad sobre una población, la cual se supone que se subdivide en tres compartimentos: Susceptibles(S), Infectados(I) y Recuperados(R), los cuáles viven sobre una región  $\Omega \subset \mathbb{R}^n$ ,  $i = 1, 2, 3$ . Mediante técnicas de optimización se estudia la existencia de la solución de un problema inverso relacionado con la identificación de las funciones que modelan las tasas de transmisión, recuperación e infección del modelo bajo estudio. Asimismo, se muestra una condición necesaria de optimalidad. Finalmente, probamos la unicidad local para estas funciones.

**Primary author:** HUANCAS, Fernando (DMAT UTEM)

**Presenter:** HUANCAS, Fernando (DMAT UTEM)

Contribution ID: 9

Type: **not specified**

## Strong solutions for the nonhomogeneous MHD equations in thin domains

*Thursday, 12 January 2023 15:30 (1 hour)*

We consider the nonhomogeneous incompressible Magnetohydrodynamic equations in a thin domain  $\Omega := \mathbb{R}^2 \times (0, \varepsilon)$ , with  $\varepsilon \in (0, 1]$ , and show the global existence of strong solutions. In addition, we prove that, as  $\varepsilon \rightarrow 0^+$ , the velocity and magnetic field tends to vanish away from the initial time.

**Primary author:** ROJAS MEDAR, Marko (DMAT UTA)

**Presenter:** ROJAS MEDAR, Marko (DMAT UTA)

Contribution ID: 10

Type: **not specified**

## Machine Learning Control Design for Elastic Composite Materials

*Wednesday, 11 January 2023 14:30 (1 hour)*

A numerical method, based on a machine learning approach, is used to solve an inverse problem involving the Dirichlet eigenfrequencies for the elasticity operator in a bounded domain filled with a composite material. The inhomogeneity of the material under study is characterized by a vector which is designed to control the constituent mixture of homogeneous elastic materials that compose it. Using the finite element method, we create a training set for a forward artificial neural network, solving the forward problem. A forward nonlinear map of the Dirichlet eigenfrequencies as a function of the vector design parameter is then obtained. This forward relationship is inverted and used to obtain a training set for an inverse radial basis neural network, solving the aforementioned inverse problem. A numerical example showing the applicability of this methodology is presented.

**Primary author:** BARRIENTOS, Mauricio (IMA PUCV)

**Presenter:** BARRIENTOS, Mauricio (IMA PUCV)

Contribution ID: 11

Type: **not specified**

## Computation of potential coefficients in quantum mechanics using artificial neural networks

*Wednesday, 11 January 2023 15:30 (1 hour)*

A numerical method based on artificial neural networks is used to solve the inverse Schrödinger equation for a multi-parameter class of potentials. First, the finite element method was used to solve repeatedly the direct problem for different parametrizations of the chosen potential function. Then, using the attainable eigenvalues as a training set of the direct radial basis neural network a map of new eigenvalues was obtained. This relationship was later inverted and refined by training an inverse radial basis neural network, allowing the calculation of the unknown parameters and therefore estimating the potential function. Three numerical examples are presented in order to prove the effectiveness of the method. The results show that the method proposed has the advantage to use less computational resources without a significant accuracy loss.

**Primary author:** OSSANDÓN, Sebastian (IMA PUCV)

**Presenter:** OSSANDÓN, Sebastian (IMA PUCV)

Contribution ID: 12

Type: **not specified**

## Opening

*Wednesday, 11 January 2023 08:50 (10 minutes)*

**Primary author:** FRIZ, Luis (CCE UBB)

**Presenter:** FRIZ, Luis (CCE UBB)

Contribution ID: 13

Type: **not specified**

## Propiedad de continuación única para sistemas discretizados Aplicación a Control y Problemas Inversos

*Wednesday, 11 January 2023 09:00 (1 hour)*

En el estudio de problemas de control o problemas inversos asociados a una EDP, se utilizan propiedades de continuación única (PCU), para las soluciones de las ecuaciones. Este tipo de propiedades están en el centro de los argumentos utilizados para obtener resultados de controlabilidad o estabilidad de sistemas. Sin embargo, no es claro que la solución de alguna discretización del sistema cumpla estas propiedades o cuales son las condiciones que debe cumplir la malla para garantizar alguna convergencia de la PCU.

En esta charla se presentara el concepto de PCU y la importancia que esta tiene en los problemas inversos y de control. Se discutirán las principales dificultades de los problemas discretos y una breve introducción al enfoque utilizado para abordar el estudio de estos problemas. Finalmente se presentaran una serie de ejemplos donde se ha estudiado la PCU en sistemas discretos o semi-discretos.

**Primary author:** LECAROS, Rodrigo (DMAT UTFSM)

**Presenter:** LECAROS, Rodrigo (DMAT UTFSM)

Contribution ID: 14

Type: **not specified**

## **Modelos de dinámica de daños en mecánica de rocas y aplicaciones a minería subterránea**

*Thursday, 12 January 2023 11:30 (1 hour)*

El objetivo central de esta charla es el estudio de la dinámica de fracturas y del daño en la mecánica de rocas. Este es un problema de gran interés en la minería y en particular en la minería subterránea. El objetivo central es estudiar el efecto de la sismicidad en la expansión del daño en macizos rocosos, y de esta forma buscar un modelo matemático que permita explicar el llamado “block caving”, que es la forma más usada para la extracción del mineral en las minas subterráneas. En esta charla presentaremos un modelo matemático y su implementación numérica.

**Primary author:** ORTEGA, Jaime (CMM DIM UCHILE)

**Presenter:** ORTEGA, Jaime (CMM DIM UCHILE)

Contribution ID: 15

Type: **not specified**

## Regularity criteria for 3D MHD flows in terms of spectral components

*Thursday, 12 January 2023 17:00 (1 hour)*

We extend the spectral regularity criteria of the Prodi-Serrin kind for the Navier-Stokes equations in a torus to the MHD equations. More precisely, the following is established: for any  $N > 0$ , let  $x_N$  and  $y_N$  be the sum of all spectral components of the velocity and magnetic field whose wave numbers possess absolute value greater than  $N$ ; then, it is possible to show that for any  $N$  the finiteness of the Prodi-Serrin norm of  $x_N$  implies the regularity of the weak solution  $(u, h)$ ; thus, no restriction on the magnetic field is needed.

**Primary author:** NOTTE CUELLO, Eduardo (DMAT ULS)

**Presenter:** NOTTE CUELLO, Eduardo (DMAT ULS)

Contribution ID: 16

Type: **not specified**

## Levels of Chaoticity on Action groups and its applications of Turing Machines

*Thursday, 12 January 2023 18:00 (1 hour)*

Iâll introduce this context to some levels of chaoticity in group actions, at the local level, seeking to verify that some classic properties are fulfilled for a given group. Afterward, it is proposed to study the relationship between notions of Chaos (in a distributional and Li-Yorke sense) with the property of undecidability. Finally, some examples of this will be given.

**Primary author:** DIAZ RABY, Mauricio (DMAT UBB)

**Presenter:** DIAZ RABY, Mauricio (DMAT UBB)

Contribution ID: 17

Type: **not specified**

## Analytical solution to integro-differential equations via complex maps in domains of contour integrals

*Friday, 13 January 2023 10:00 (1 hour)*

A simple model for QCD dynamics in which the DGLAP integro-differential equation may be solved analytically has been considered in our previous papers arXiv:1611.08787 [hep-ph] and arXiv:1906.07924 [hep-ph]. When such a model contains only one term in the splitting function of the dominant parton distribution, then Bessel function appears to be the solution to this simplified DGLAP equation. To our knowledge, this model with only one term in the splitting function for the first time has been proposed by Blümlein in arXiv:hep-ph/9506403. In arXiv:1906.07924 [hep-ph] we have shown that a dual integro-differential equation obtained from the DGLAP equation by a complex map in the plane of the Mellin moment in this model may be considered as the BFKL equation. Then, in arXiv:1906.07924 we have applied a complex diffeomorphism to obtain a standard integral from Gradshteyn and Ryzhik tables starting from the contour integral for parton distribution functions that is usually taken by calculus of residues. This standard integral from these tables appears to be the Laplace transformation of Jacobian for this complex diffeomorphism. We find out certain important points for further development of this strategy. We verify that the inverse Laplace transformation of the Laplace image of the Bessel function may be represented in a form of Barnes contour integral. We consider other integro-differential equations which may be solved analytically by this method.

**Primary author:** KONDRASHUK, Igor (CCE UBB)

**Presenter:** KONDRASHUK, Igor (CCE UBB)

Contribution ID: **18**

Type: **not specified**

## Charla divulgativa

*Thursday, 12 January 2023 20:00 (1 hour)*

Conversatorio Programa Copernicus. La charla se realizara en el Sport Burger Bar, Avenida Argentina 295, Chillan

**Primary author:** ORTEGA, Jaime (CMM DIM UCHILE)

**Presenter:** ORTEGA, Jaime (CMM DIM UCHILE)