QSimFP - lecture II

Ultracold atoms as analogue quantum simulators

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Quantum field simulators with ultracold atoms

Analogue gravity

Small perturbations in (quantum) fluids are described by an effective relativistic field theory in curved spacetime

QFT in curved spacetime, black hole physics, ...

Continuous local observers

Study the observer(s) response dependent on its motion / position / etc

Unruh effect, entanglement

Analogue cosmology

Time-dependent effective spacetimes enable the study of analogues for cosmological scenarios

Dynamical casimir effect, Inflation, pre-/re-heating, ...

Quantum many-body systems

Emergent QFTs

Build model QFTs as the coarse grained low-energy description of many-body systems

Bose-Hubbard model, sine-Gordon model, Hydrodynamics, ...

Hamiltonian engineering

Techniques like e.g. Floquet engineering realize inaccessible model systems

FVD, artificial gauge fields, ..

Universality

Study universal properties independent of the microscopic details of the system

Equilibrium universality classes, Non-thermal fixed points, topology, ...

In a nutshell: What's hard in real-time QFT?

All information about non-equilibrium evolution is in the generating functional

$$Z[J,R;\rho_D] = \int \mathscr{D}\varphi\, e^{i\left(S[\varphi]+\int_x J(x)\varphi(x)+\frac{1}{2}\int_{xy}R(x,y)\varphi(x)\varphi(y)+\frac{1}{3!}\int_{xyz}\alpha_3(x,y,z)\varphi(x)\varphi(y)\varphi(z)+\dots\right)}$$

The problem about real time

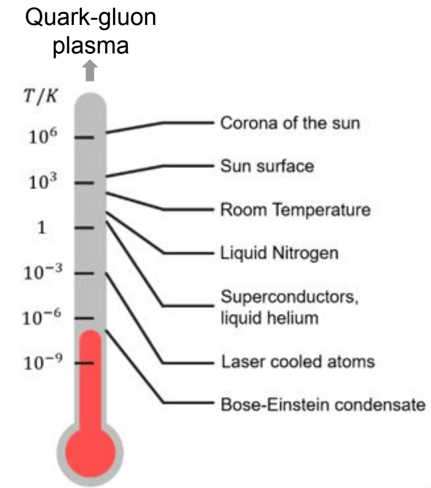
$$\sim e^{iS}$$

non-positive definite probability measure!

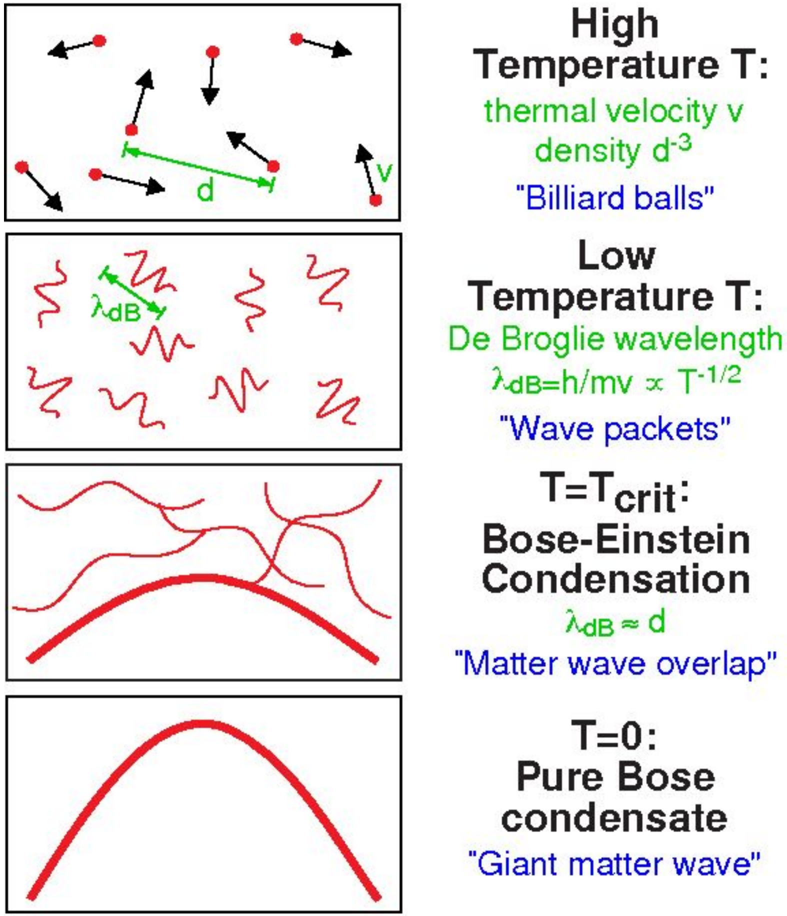
➔ Preempts the use of standard importance sampling techniques ↔ numerical simulations

➔ A quantum simulator using cold atoms would not have this problem ↔ experiments

Ultracold atoms - a brief overview



$$a \ll n^{-1/3} \ll \lambda_{\mathrm{dB}}$$



Quantum degeneracy: phase space density $\lambda_{\mathrm{dB}} \gtrsim n^{-1/3}$

$\lambda_{\mathrm{dB}} = \sqrt{\frac{2\pi\hbar^2}{mk_{\mathrm{B}}T}}$ thermal De Broglie wavelength

$n^{-1/3}$ mean particle distance

Interacting atoms but dilute gas: $na^3 \ll 1$

a scattering length for s-wave interactions

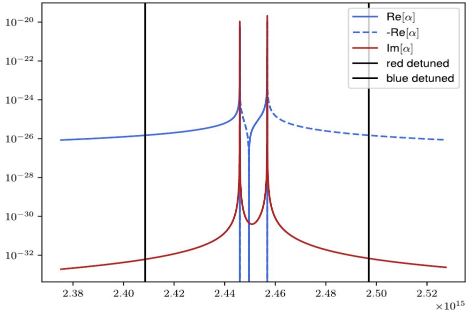
$8\pi a^2$ scattering cross-section (for bosons)

Discovering new phases of matter

Preparation:

- ➔ Catch atoms in magneto-optical trap (MOT)
- ➔ Laser-cooling to □ 50 µK
- ➔ Transfer to conservative trap (non-res. light)
- ➔ Evaporative cooling

$$\alpha(\omega) = -\frac{6\pi c^3}{\omega_r^3}\frac{1}{\delta_0} + \mathrm{i}\frac{6\pi c^3}{\omega_r^3}\frac{1}{\delta_0^2}$$



Dynamics:

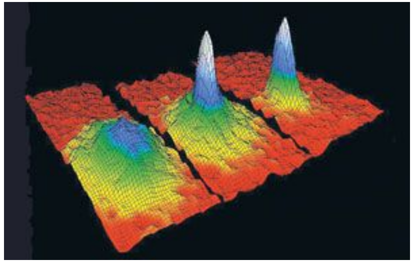
- ➔ E.g. quench of Hamiltonian to initiate non-equilibrium dynamics

Ultracold atoms as model systems for quantum many body physics

- ➔ Dilute but *interacting* gases
- ➔ Tunability
- ➔ Microscopic properties well characterized (atomic physics)
- ➔ Energies & timescales in experimentally accessible range
- ➔ Well isolated from the environment

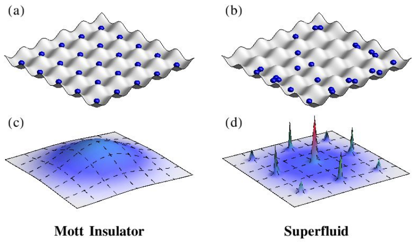
Superfluid gas

JILA, MIT, Rice (1995)



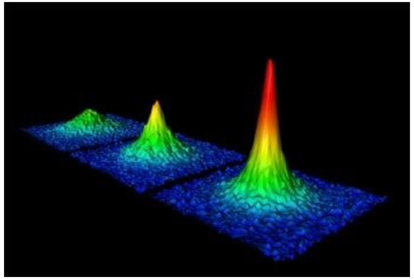
superfluid Mott insulator

Munich (2002)



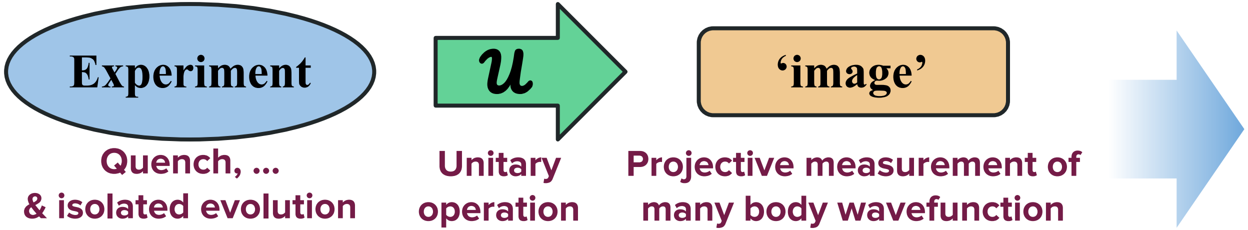
BEC-BCS crossover

JILA, MIT, ENS (2003/4)



Measuring quantum many-body systems (QMBS)

Commonly: **Destructive measurements** ➡ The best we can **measure** is **every constituent** (and internal states)



Evaluation

Extracted observables, full counting statistics, (higher-order) correlations, …

Measuring quantum many-body systems (QMBS)

Commonly: **Destructive measurements** ➡ The best we can **measure** is **every constituent** (and internal states)

Experiment → $\mathcal{U}$ → **'image'** → **Evaluation**

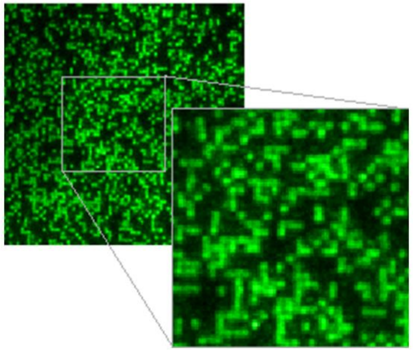
Quench, … & isolated evolution

Unitary operation

Projective measurement of many body wavefunction

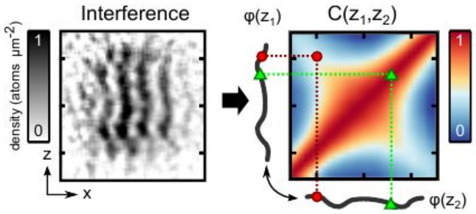
Extracted observables, full counting statistics, (higher-order) correlations, …

in situ ➔ position



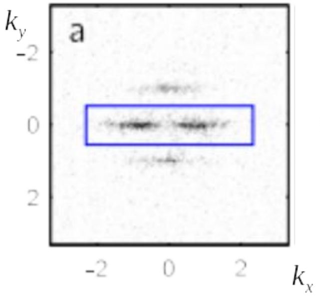
W.S. Bakr, et al.,Nature 462, 74 (2009).
J.F. Sherson, et al. Nature 467, 68 (2010).

interference ➔ phase



Langen et al., Nat. Phys. 9, 460 (2013)
Gring et al., Science 337, 1318 (2012)
Langen et al., Science 348, 207 (2015)
Schweigler et.al, Nature 545, 323, (2017)
Rauer et al., Science 360, 307 (2018)
Zache et.al, PRX 10, 011020 (2020)
…

time of flight ➔ momentum



R. Bücker et al. NJP 11, 103039 (2009)
R. Bücker et al. Nature Physics 7, 608 (2011)

$$n(k,t) \equiv \alpha\rho\left(\alpha^{-1} z_{\mathrm{tof}}\right)$$

$$= \int \mathrm{d}z\, f\left(z, k - \alpha^{-1}(z + v t_{\mathrm{tof}}), t\right)$$

Additional bulk velocity v

➔ **condensate focusing**

Enables measuring momenta in IR for finite time of flight

The single component Bose gas - from atoms to fields

$$\hat{H} = \int d\mathbf{r}\, \hat{\Psi}^\dagger(\mathbf{r},t)\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) - \mu\right)\hat{\Psi}(\mathbf{r},t)$$
$$+ \iint d\mathbf{r}\, d\mathbf{r}'\ \hat{\Psi}^\dagger(\mathbf{r},t)\hat{\Psi}^\dagger(\mathbf{r}',t)U_{\text{eff}}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}',t)\hat{\Psi}(\mathbf{r},t)$$

Bosonic commutation relations

$$[\hat{\Psi}(\mathbf{r}), \hat{\Psi}^\dagger(\mathbf{r})] = \delta(\mathbf{r}-\mathbf{r}')$$

Pseudopotential approximation (s-wave): $U_{\text{eff}}(\mathbf{r}-\mathbf{r}') \simeq g_{3\text{D}}\delta(\mathbf{r}-\mathbf{r}')$ $\quad g_{3\text{D}} = \frac{4\pi\hbar^2}{m}a_s$

Madelung representation (quantum hydrodynamics): $\hat{\psi}^\dagger(z,t) = \sqrt{\hat{\rho}(z,t)}\,\mathrm{e}^{-\mathrm{i}\hat{\theta}(z,t)}$ $\quad \left[\hat{\rho}(z)\,,\,\hat{\theta}(z')\right] = \mathrm{i}\delta(z-z')$

Expand to second order in *small density perturbations* $\hat{\rho}(z) = \rho_0(z) + \delta\hat{\rho}(z)$ and *phase gradients*

$$\hbar\partial_t\delta\rho = -\frac{\hbar^2}{m}\rho_0\nabla^2\theta$$

$$\hbar\partial_t\theta = \frac{\hbar^2}{4m\rho_0}\nabla^2\delta\rho - g\delta\rho$$

for general back-ground flow ***v***

$$\partial_a\left(\sqrt{-g}\,g^{ab}\partial_b\phi\right) = 0$$

$$g_{ab} \propto \begin{bmatrix} -(c^2 - v^2) & -v_j \\ -v_i & \delta_{ij} \end{bmatrix}$$

Bogoliubov transformation

Introducing the operators $B = \delta\hat{\rho}/2\sqrt{\rho_0} + \mathrm{i}\sqrt{\rho_0}\hat{\theta}$ mapps the problem to the usual Bogoliubov equations

$$\mathrm{i}\hbar\partial_t \begin{pmatrix} B \\ B^\dagger \end{pmatrix} = \begin{pmatrix} -\frac{\hbar^2}{2m}\nabla^2 + \mu & \mu \\ -\mu & \frac{\hbar^2}{2m}\nabla^2 - \mu \end{pmatrix} \begin{pmatrix} B \\ B^\dagger \end{pmatrix}$$

$$\begin{pmatrix} B \\ B^\dagger \end{pmatrix} = \sum_m \left[\begin{pmatrix} u_m \\ v_m \end{pmatrix} \mathrm{e}^{-\mathrm{i}\omega_m t} b_m + \begin{pmatrix} \bar{v}_m \\ \bar{u}_m \end{pmatrix} \mathrm{e}^{\mathrm{i}\omega_m t} b_m^\dagger \right]$$

$$\int \mathrm{d}z \, \left[|u_m|^2 - |v_m|^2 \right] = 1$$

➡ $f_m^\pm = u_m \pm v_m \qquad \frac{1}{2}\int \mathrm{d}z \, \left[\bar{f}_m^+ f_m^- + f_m^+ \bar{f}_m^- \right] = 1$

Diagonalizing the quadratic Hamiltonian results in the modal expansion of fluctuations in the quasiparticle basis

$$\hat{\theta}(t, \boldsymbol{r}) = \frac{1}{2\sqrt{V\rho_0}} \sum_k \sqrt{\frac{\epsilon_k}{E_k}} \left(\hat{b}_k \, \mathrm{e}^{-\mathrm{i}(\epsilon_k t/\hbar - \boldsymbol{k}\boldsymbol{r})} + \mathrm{H.c.} \right)$$

$$\delta\hat{\rho}(t, \boldsymbol{r}) = \frac{\rho_0}{V} \sum_k \sqrt{\frac{E_k}{\epsilon_k}} \left(\hat{b}_k \, \mathrm{e}^{-\mathrm{i}(\epsilon_k t/\hbar - \boldsymbol{k}\boldsymbol{r})} + \mathrm{H.c.} \right)$$

- ➜ phononic excitations in the long wavelength limit
- ➜ effective relativistic scalar field with "speed of light" c_s

Chemical potential:	$\mu = \rho_0 g$
Speed of sound:	$c_s = \sqrt{\mu/m}$
Healing length:	$\xi = \hbar/\sqrt{2\,\mu m}$
Dispersion relation:	$\epsilon_k = \sqrt{E_k(E_k + 2\mu)}$
with:	$E_k = \hbar^2 k^2/2m$

Designing QFT simulators

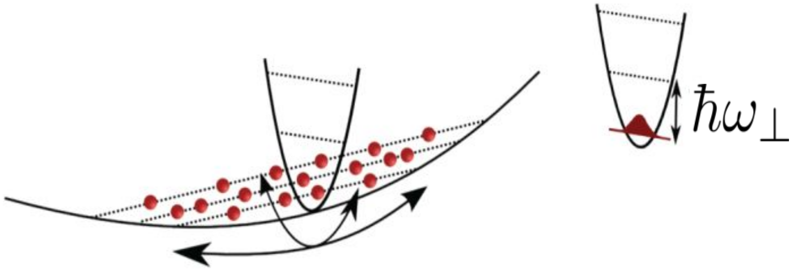
Basic cold atom primitives enable to **tune the effective parameters of simulated QFTs**

- ❑ **Dimensionality**
- ❑ **Local density**
- ❑ **Microscopic interaction properties**
- ❑ **Dispersion relation, boundary conditions**, …
- ❑ **Internal states**, **multi-component systems**

New possibilities arise when considering **time-dependent systems**

- ❑ **Floquet engineering, Artificial gauge fields, …**

Dimensional reduction (the physicist's way)



$$\Psi(\mathbf{r},t) = \psi(z)\,\varphi(x,y,\sigma(z))$$

$$\varphi(x,y,\sigma(z)) = \frac{1}{\sqrt{\pi}\sigma(z,t)} \exp\left(-\frac{x^2+y^2}{2\sigma(z,t)^2}\right)$$

Integrating out the radial directions leads to effective lower-dimensional systems.

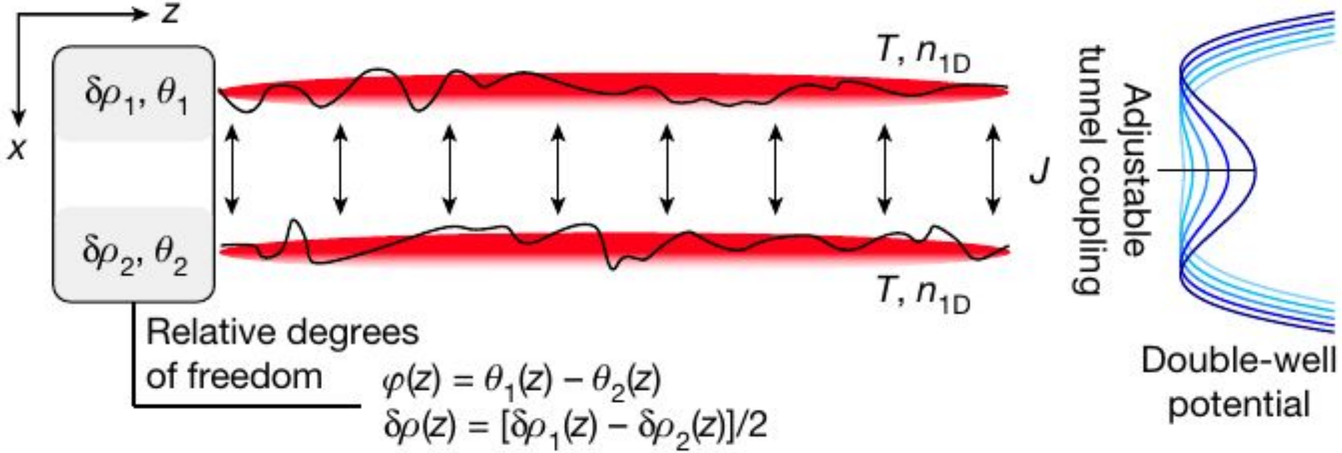
Simplest approximation (single-particle ground state) leads to the Lieb-Liniger model

$$\hat{H}_{\mathrm{LL}} = \int \mathrm{d}z\, \hat{\psi}^\dagger(z,t)\left(-\frac{\hbar^2}{2m}\partial_z^2 + V(z) - \mu + \frac{g_{1D}}{2}\hat{\psi}^\dagger(z,t)\hat{\psi}(z,t)\right)\hat{\psi}(z,t)$$

with an effective interaction constant $g_{1D} = 2\hbar a_s \omega_\perp \left(1 - C\frac{a_s}{l_\perp}\right)^{-1} \overset{a_s \ll l_\perp}{\simeq} 2\hbar a_s \omega_\perp$

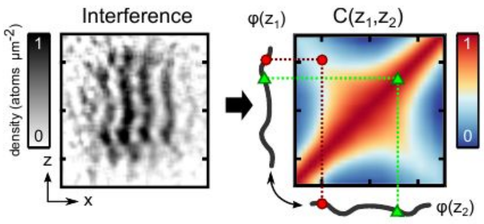
Adiabatic approximation with radial width as variational parameter leads to Non-polynomial Schrödinger Equation

Spinor Bose gases and double well system



$$H = \sum_{j=1}^{2} \int dz \left[\frac{\hbar^2}{2m} \frac{\partial \psi_j^\dagger}{\partial z} \frac{\partial \psi_j}{\partial z} + \frac{g_{1D}}{2} \psi_j^\dagger \psi_j^\dagger \psi_j \psi_j + U(z) \psi_j^\dagger \psi_j - \mu \psi_j^\dagger \psi_j \right] - \hbar J \int dz \left[\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1 \right]$$

Measurement of the relative phase
through matterwave interference



Measures single shot realization of the simulated quantum field

Perturbation theory and the sine-Gordon model

Low-energy effective field theory through **bosonization**

Coleman, Mandelstam, Mattis, Luther, Tomonaga, Gogolin, Giamarchi, ...

$$\hat{\psi}^\dagger(z,t) = \sqrt{\hat{\rho}(z,t)}\, e^{-i\hat{\theta}(z,t)} \qquad \hat{\rho}(z) = \rho_0(z) + \delta\hat{\rho}(z)$$

$$H = \int dz \left[\frac{\hbar^2}{4mn_{1D}} \left(\frac{\partial \delta\rho}{\partial z} \right)^2 + g\delta\rho^2 + \frac{\hbar^2 n_{1D}}{4m} \left(\frac{\partial \varphi}{\partial z} \right)^2 - 2\hbar J n_{1D} \cos\varphi \right]$$

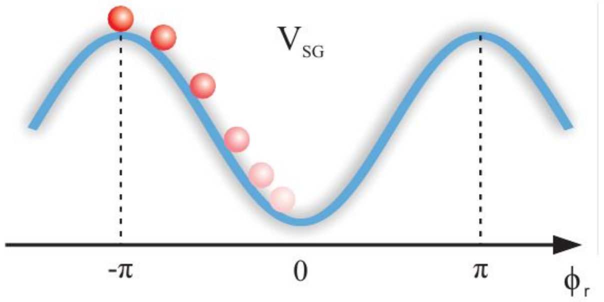
quantum pressure

relativistic free theory
Tomonaga-Luttinger model
Haldane (1980 & 1981)

Interactions
gapped spectrum,
topological excitations

sine-Gordon model

Gritsev, Polkovnikov, Demler Phys. Rev. B 75, 174511 (2007)



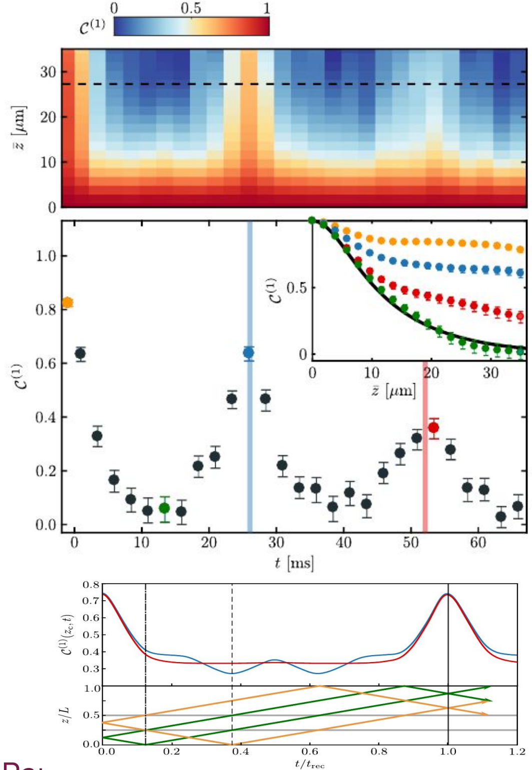
Quadratic approximation expected to be valid for large J:

$$2\hbar J n_{1D} \cos\varphi \;\longrightarrow\; \hbar J n_{1D} \varphi^2$$

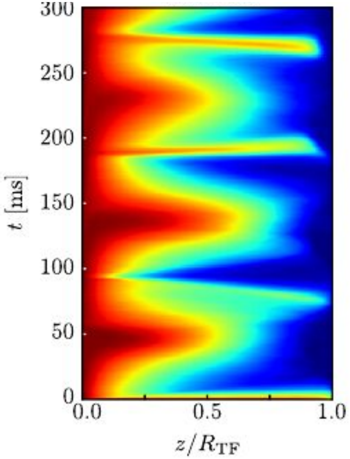
Experiments in the linear regime

Quantum recurrences

Macroscopic recurrences via dispersion-relation engineering



In contrast for inhomogeneous metric



Rauer et.al Science 360, 6386 pp. 307-310

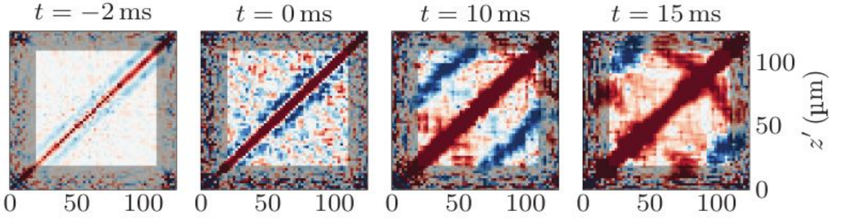
Curved light-cone propagation

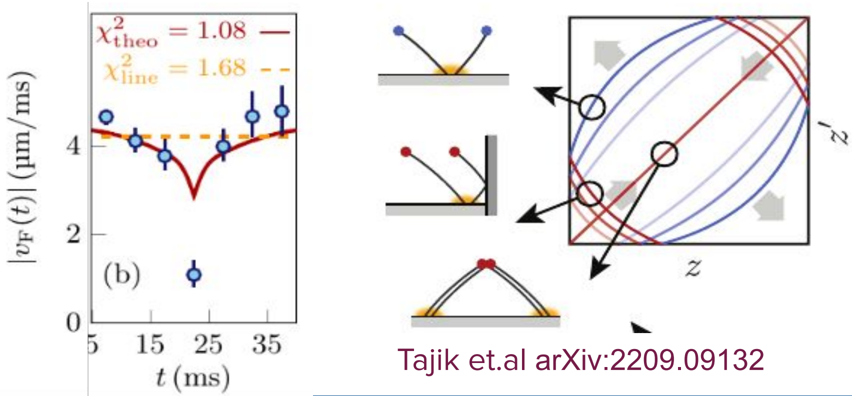
Curved light-cone propagation in inhomogeneous background

$$\mathcal{S}[\phi] \sim \int \mathrm{d}z\mathrm{d}t \sqrt{-g} K(z) \left[g^{\mu\nu}(\partial_\mu \phi)(\partial_\nu \phi) + \tfrac{1}{2} M^2 \phi^2 \right]$$

$$\hat{u}(z) = (\hbar/m)\partial_z \hat{\phi}(z) \qquad \langle \hat{u}(z) \rangle = 0$$

$$\Rightarrow \quad C_u(z,z') = \langle \hat{u}(z)\hat{u}(z') \rangle$$





Tajik et.al arXiv:2209.09132

Analogue cosmology - Inflation

Engineer a **time-dependent** analogue metric

$$c_s = \sqrt{\mu/m}$$

$$\mu = \rho_0 g$$

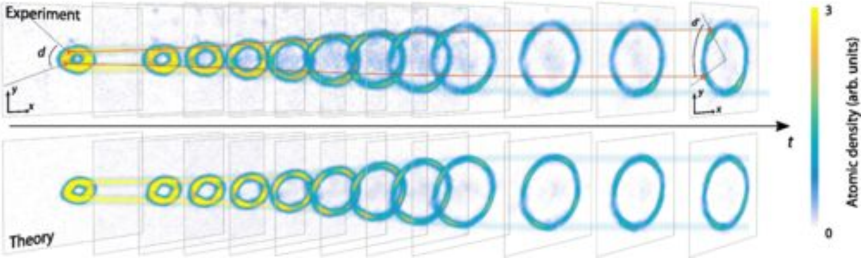
$$g_{ab} \propto \begin{bmatrix} -(c^2 - v^2) & -v_j \\ -v_i & \delta_{ij} \end{bmatrix}$$

$$\epsilon_k = \sqrt{E_k(E_k + 2\mu)}$$

Expanding ring system

Time-dependent speed of sound through changing background density

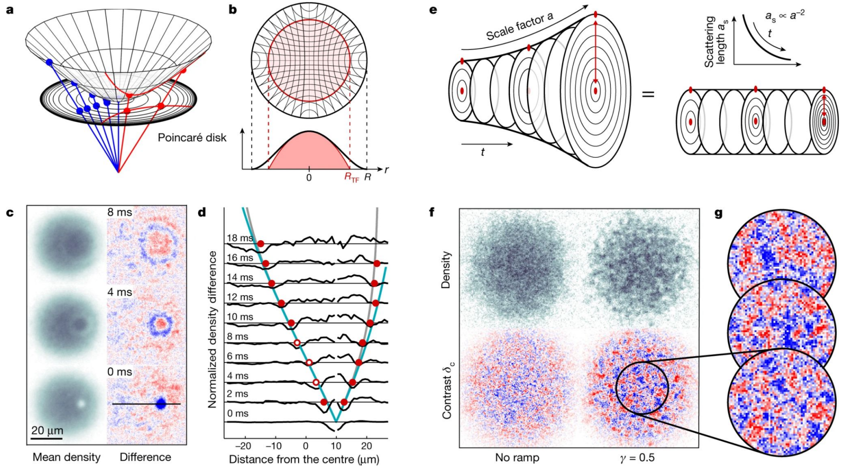
Eckel et.al Phys. Rev. X 8, 021021



Effective expansion

Time-dependent speed of sound through Feshbach resonance

Vierman et.al Nature volume 611, p. 260-264

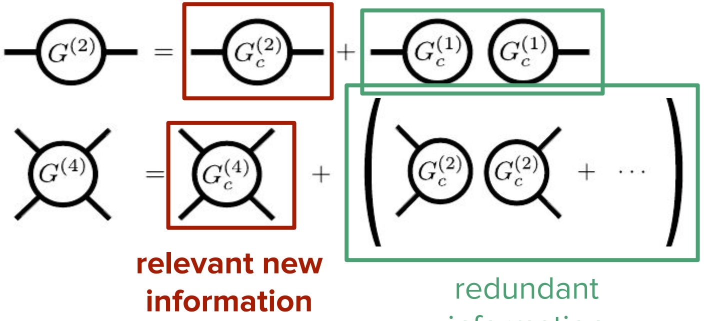


From correlations to generating functionals

I) Correlation functions:

$$Z[J] = \int \mathcal{D}\varphi\, e^{i(S[\varphi]+\varphi\cdot J)}$$

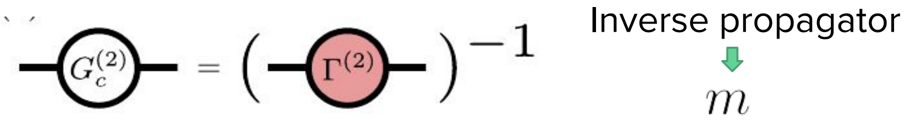
II) Connected correlation functions:

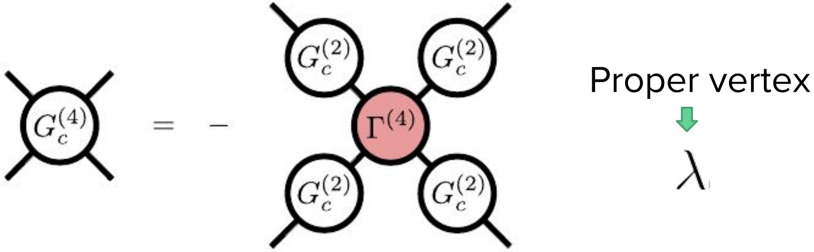


$$W[J] = -i \log Z[J]$$

III) 1-PI correlation functions:

Are the fundamental building blocks of all correlations!





$$\Gamma[\phi] = -i \log Z[J(\phi)] - J(\phi)\cdot\phi$$

Renormalization group and emergent QFTs

$$\Gamma\,[\Phi] = \sum_{n=2}^{\infty} \frac{1}{n!} \Gamma^{(n)}_{\mathbf{x}_1,\ldots,\mathbf{x}_n} \prod_{j=1}^{n} \left(\Phi_{\mathbf{x}_j} - \bar{\Phi}_{\mathbf{x}_j}\right)$$

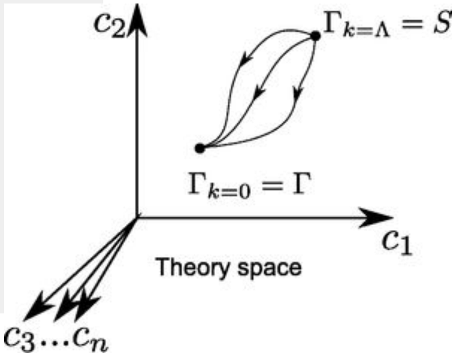
- Expansion coefficients are the proper vertices of **n-body interactions**
- Note that all **higher-order couplings are dynamically created** (even if bare action only has two-body interactions)
- They are the **momentum dependent, running couplings**

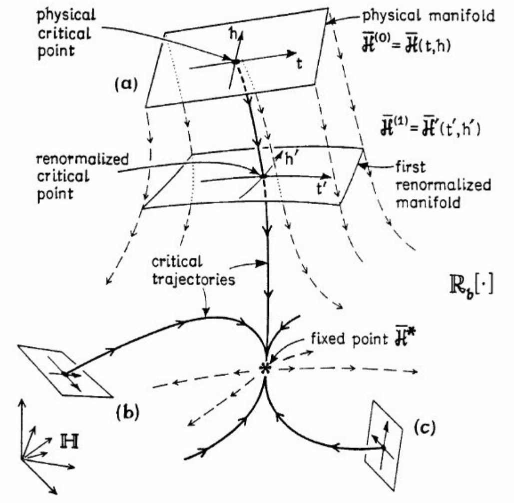
However: So far only shifted the problem, since $\Gamma[\phi]$ in general not solvable / calculable

Renormalization Group enables to calculate momentum dependence of coupling beyond perturbation theory
Kadanoff, Wilson, Fisher, Polchinski, Morris, Wetterich, ...

E.g.: Exact flow equation for the effective action (Functional Renormalization Group)

$$k\,\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr}\, k\,\partial_k R_k \left(\Gamma_k^{(1,1)} + R_k\right)^{-1}$$



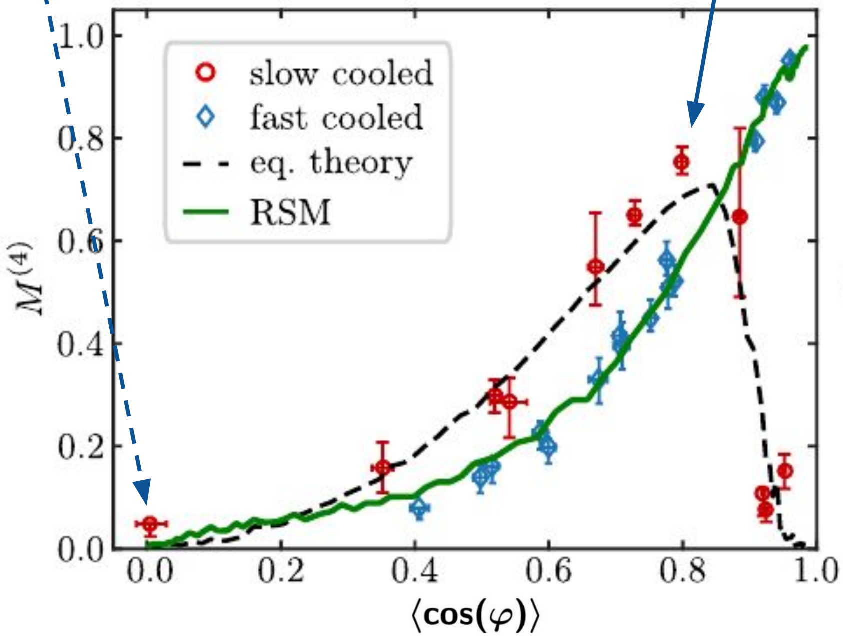


Testing nonlinear field theories I

$$M^{(N)} = \frac{\sum_z |G^{(N)}_{\text{con}}(z,0)|}{\sum_z |G^{(N)}(z,0)|}$$

free (gaussian) theory

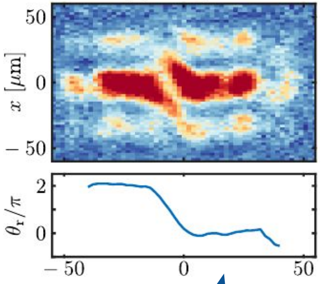
strongly correlated theory

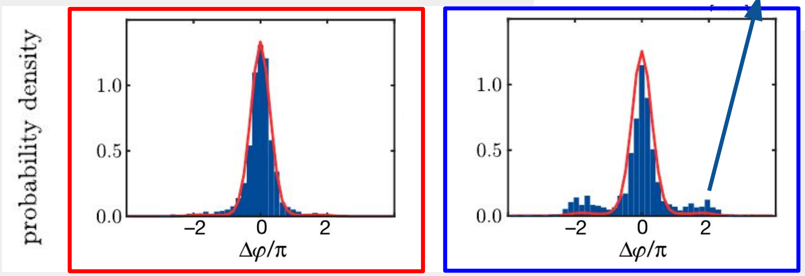


Schweigler et.al Nature 545 p. 232-236

sine-Gordon solitons

Nonlinear stable solution





$$\mathcal{G}^{(N)}(\mathbf{z},\mathbf{z}') = \langle \varphi(z_1, z_1') \ldots \varphi(z_N, z_N') \rangle$$

Factorization in **connected** and **disconnected** parts:

$$\mathcal{G}^{(N)}(\mathbf{z},\mathbf{z}') = \mathcal{G}^{(N)}_{\text{con}}(\mathbf{z},\mathbf{z}') + \mathcal{G}^{(N)}_{\text{dis}}(\mathbf{z},\mathbf{z}')$$

Genuine new information
⟹ particle interactions

Fully determined by
lower order correlations

Testing nonlinear field theories I

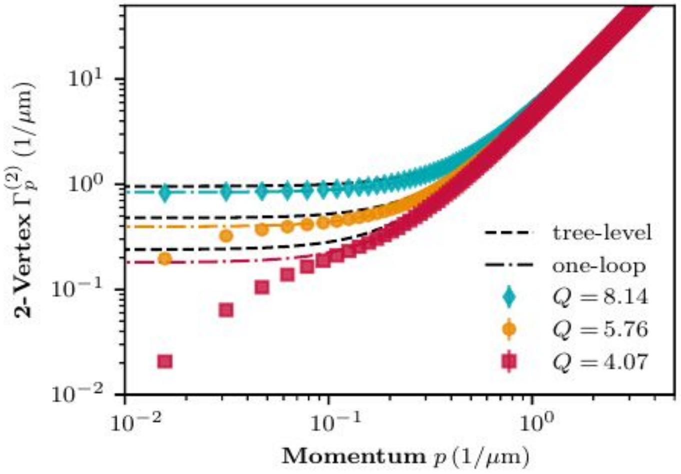
Analysis can be extended to **1PI correlations**

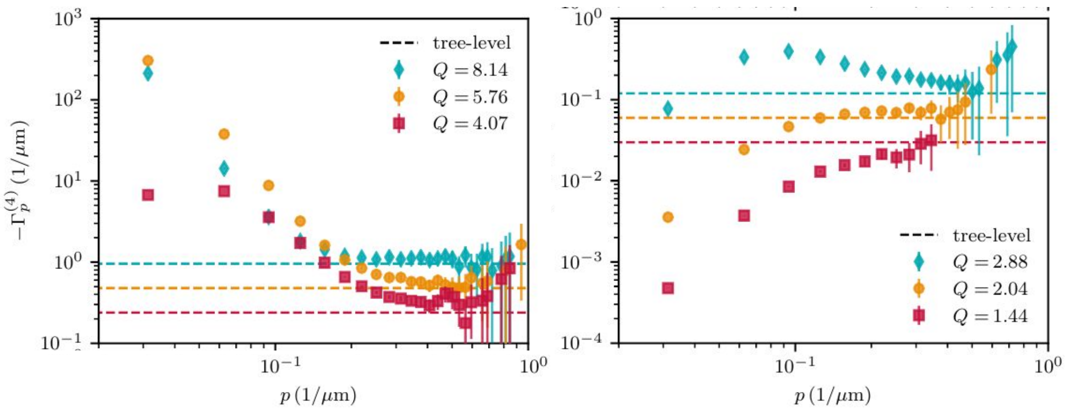
➜ **direct measurement of the momentum dependent effective field theory parameters!**

Rescaling to **canonical form** leads to **two expansion parameters**

$\epsilon_{\mathrm{q}} = \sqrt{4\gamma}$	$\epsilon_{\mathrm{th}} = \frac{4\ell_J}{\lambda_T} = \frac{4}{Q}$
$\epsilon_{\mathrm{q}} \ll \min[1, \epsilon_{\mathrm{th}}]$	$\epsilon_q \ll \epsilon_{\mathrm{th}} \ll 1$
classical approximation valid	tree-level approximation valid

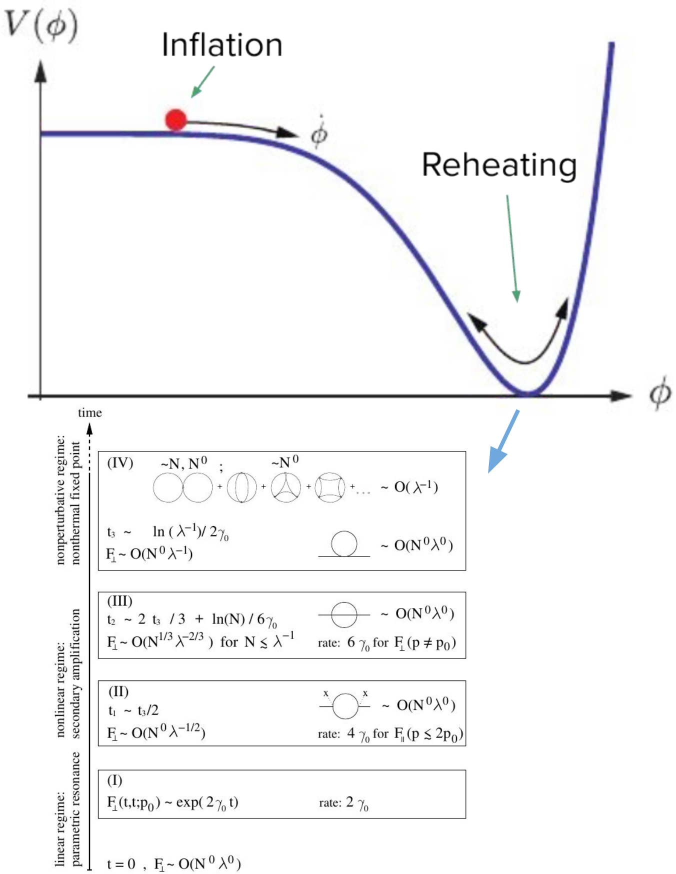
1PI correlations = Hamiltonian parameter!



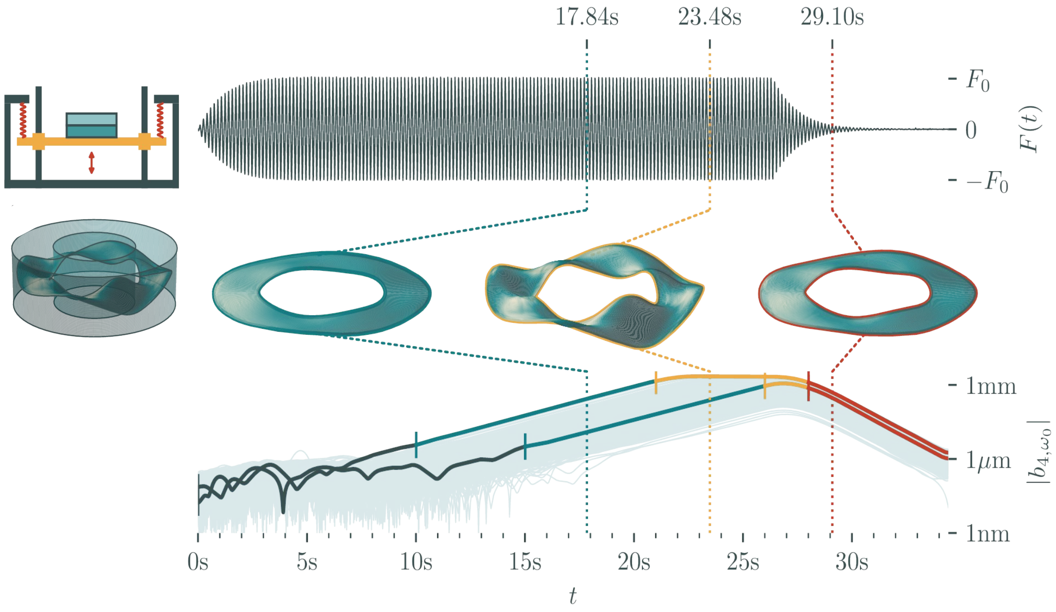


Zache et.al PRX 10, 011020 (2020)

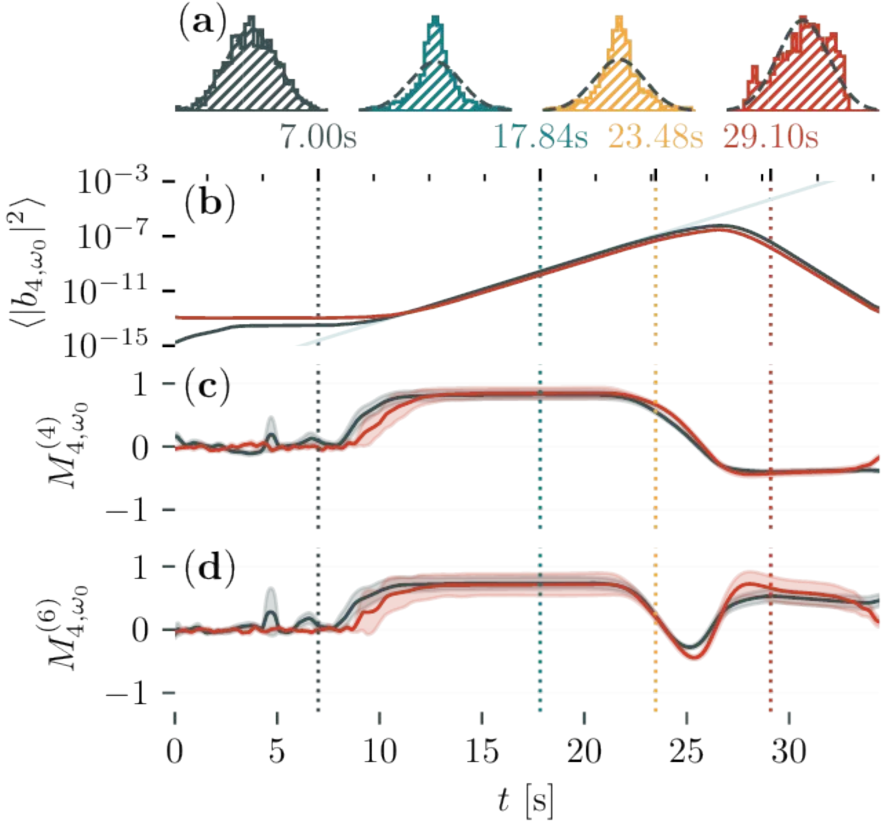
Testing nonlinear field theories II



Analogy to parametrically driven two-fluid interfaces (Silke's lecture)



Testing nonlinear field theories II



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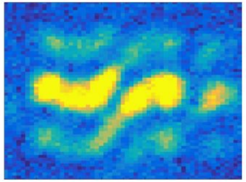
Thank you for your attention!

Realizing and probing quantum fields with ultra-cold Atoms

QuFT-Lab



PhD positions available!