QSHS Tutorial - Resonant Microwave Cavities

January 10, 2023

1 Introduction

The constituents of the galactic dark matter halo are unknown, but can be actively searched for using direct dark matter detection experiments, also know as haloscopes. The detection principles depend both on the mass of the particle being searched for, and the nature of its couplings to other particles. Many experiments search for WIMPs which are thought to have masses around the mass of the proton or higher. Another possibility is that the dark matter halo could be made up of much lighter particles. An example of such a particle is the hypothetical pseudoscalar, the axion. Predictions of the allowed mass range for which axions can compose all or the majority of dark matter vary. However, the $1\,\mu\mathrm{eV/c^2}$ to $100\,\mu\mathrm{eV/c^2}$ mass range is generally seen as well-motivated, and is the focus of many experimental searches. Likewise, the exact nature of the couplings of the axion to Standard Model particles are model-dependent, but the existence of an effective coupling between axions and photons is universally predicted (unless removed by fine-tuning). Most axion haloscopes exploit this coupling, and most look for the production of photons from axions passing through strong magnetic fields, via the Primakoff effect. For axions with masses in the mass range identified above, a photon produced via the Primakoff effect in a terrestrial haloscope will have microwave wavelengths. In this tutorial we explore some of the properties of axion haloscopes based on resonant microwave structures coupled to sensitive amplifiers and detector electronics. The questions are intended to promote discussion and draw upon some of the material covered in the first Quantum Sensors for the Hidden Sector lecture.

2 Questions for discussion

1. Calculate the transmission range (for a good signal) from an internet router to a mobile phone. Assume that the power of the router, P_0, is $100\,\mathrm{mW}$, that a typical phone can detect a signal of $-50\,\mathrm{dBm}$. Consider that routers often use frequencies of $f = 2.4\,\mathrm{GHz}$ or $5.0\,\mathrm{GHz}$.

   Hint: You can use dimensional analysis to find an appropriate formula for how the power depends on distance.

2. Consider a haloscope designed to detect microwaves converted from axions via the Primakoff effect. Ignoring any thermal motion of the axions, the power that an experiment could detect from a dark matter halo consisting entirely of axions is given below. The values have been normalized to correspond approximately to those of the ADMX experiment during their recent '1B' run.

$$P_{\text{signal}} \approx 2.2 \times 10^{-23}\,\mathrm{W} \left(\frac{\beta}{1+\beta}\right)\left(\frac{V}{136\,\mathrm{l}}\right)\left(\frac{B}{7.6\,\mathrm{T}}\right)^2\left(\frac{G}{0.4}\right)\left(\frac{g_\gamma}{0.36}\right)^2 \left(\frac{\rho_a}{0.45\,\mathrm{GeVcm^{-3}}}\right)\left(\frac{f}{740\,\mathrm{MHz}}\right)\left(\frac{Q_L}{30,000}\right)$$

where β is the coupling strength of the receiver, V is the volume of the detector cavity, B is the average magnetic field strength over the cavity volume, G is the geometric factor, g_γ is the dimensionless, model-dependent coupling between an axion and two photons, ρ_a is the dark matter density, f is the resonant frequency of the detector cavity, and Q_L is the loaded quality factor of the cavity.

(a) Find the magnetic field strength that would be needed to generate an axion signal of $-50\,\mathrm{dBm}$ in the ADMX cavity, such that a mobile phone could be used as a detector.

(b) How would this equation be modified if the loaded quality factor was close to the axion quality factor?

(c) How would the equation be modfied to take into account a small detuning between the resonant frequency of the cavity and the frequency corresponding to the energy of an axion inside the detector?

3. In addition to detecting a 'signal' of microwaves produced from a halo of dark matter axions, axion haloscopes also detect 'noise' originating from thermal emission in the cavity and from the amplifier / electronics

The ratio of signal power to fluctuations in the noise power is given by

$$\frac{s}{n} = \frac{P_{\text{signal}}}{k_B T_{\text{sys}}}\sqrt{\frac{\Delta t}{\Delta f}}$$

where T_{sys} is the system noise temperature, Δt is the signal integration time and Δf is the frequency bandwidth.

(a) Show that, for the case of ADMX as introduced in question 2, the scan rate is given by

$$\frac{df}{dt} \approx 543\frac{\text{MHz}}{\text{yr}}(\frac{\beta}{1+\beta})^2\left(\frac{g_\gamma}{0.36}\right)^4\left(\frac{f}{740\,\text{MHz}}\right)^2\left(\frac{\rho_a}{0.45\,\text{GeVcm}^{-3}}\right)^2\left(\frac{3.5}{SNR}\right)^2$$
$$\left(\frac{B}{7.6\,\text{T}}\right)^4\left(\frac{V}{136\,\text{l}}\right)^2\left(\frac{Q_L}{30,000}\right)^2\left(\frac{G}{0.4}\right)^2\left(\frac{0.2\,\text{K}}{T_{\text{sys}}}\right)^2$$

(b) What value of β will maximise the scan rate?

4. Finally, for fun, if your mobile was as sensitive as ADMX what would be the effective range of the router in question 1?