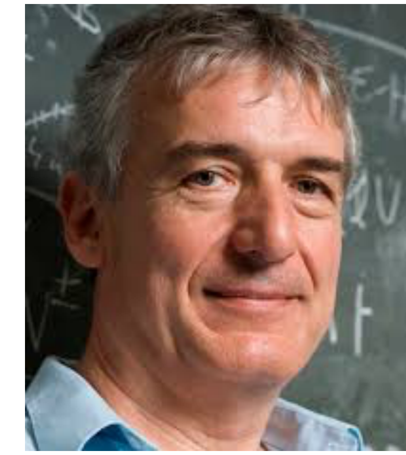
Axion Searches, Feedback Control, Instantons, and Phase Transitions

Some intriguing connections

Ed Daw, QTFP School, Cambridge, 2023

https://www.world-grain.com/articles/13839-grain-storage-and-handling-projects

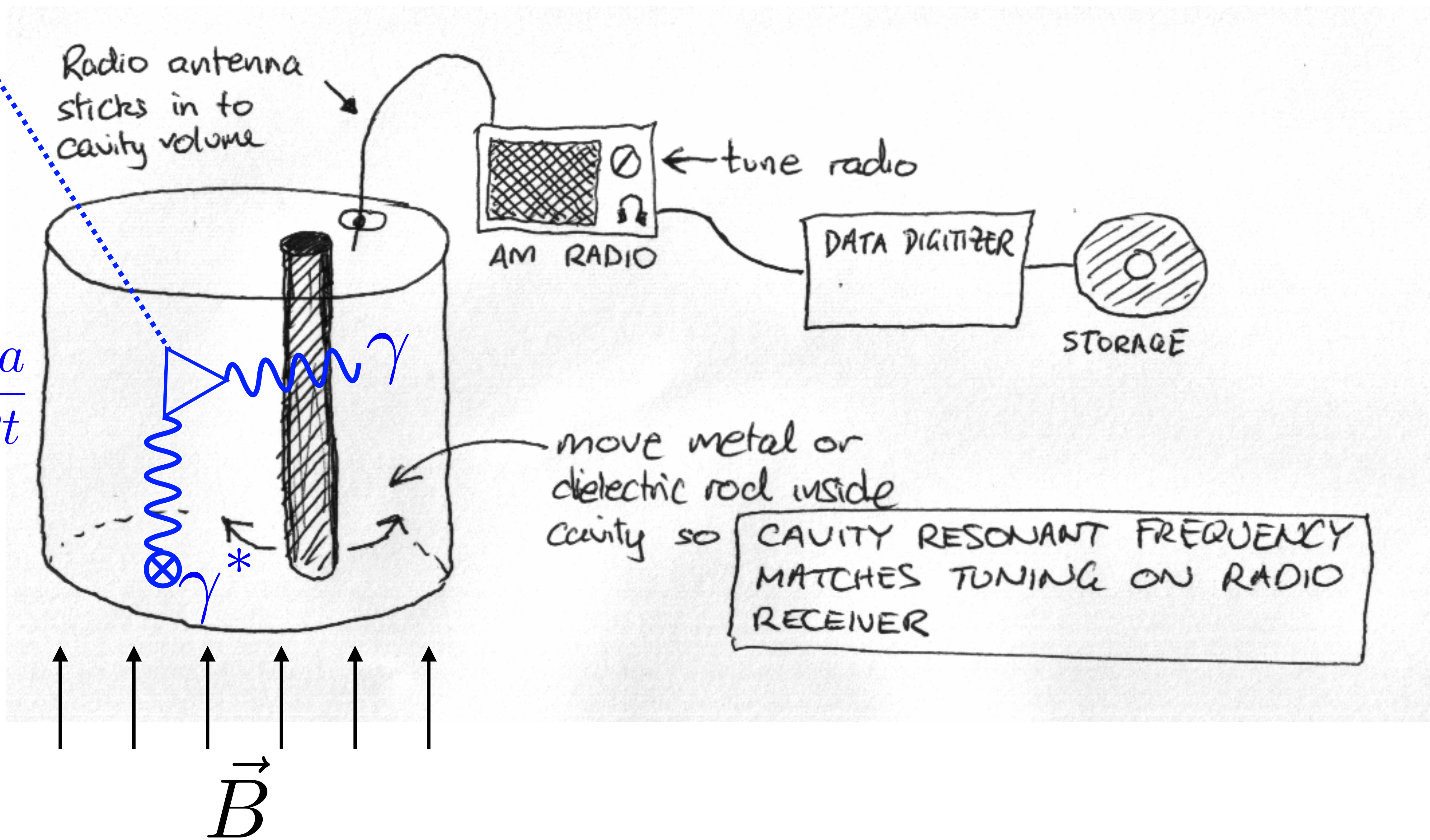
Sikivie-Type Resonant Cavity Axion Search



a

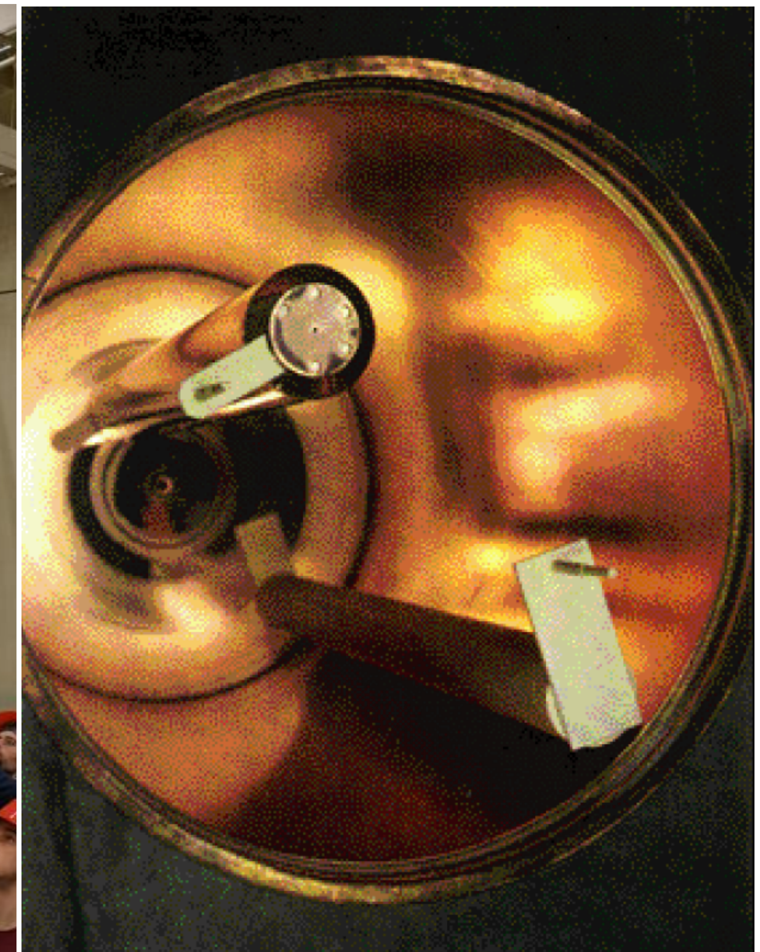
$$h\nu \simeq m_a c^2$$

$$\vec{E}_a = g\vec{B}\frac{\partial a}{\partial t}$$

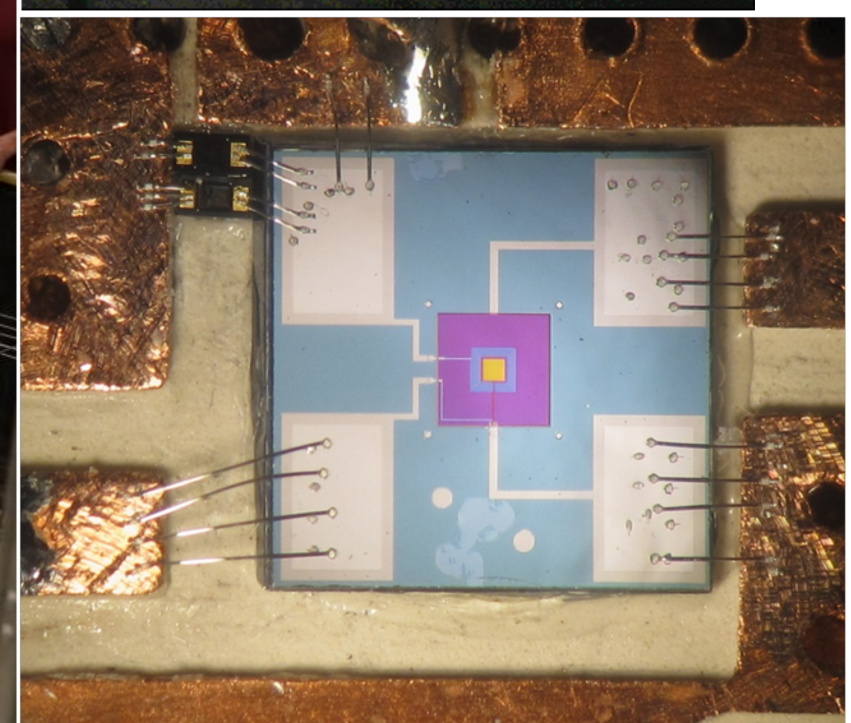


The University Of Sheffield.

ADMX experiment



50cm



Microwave Squid Amplifier (MSA)

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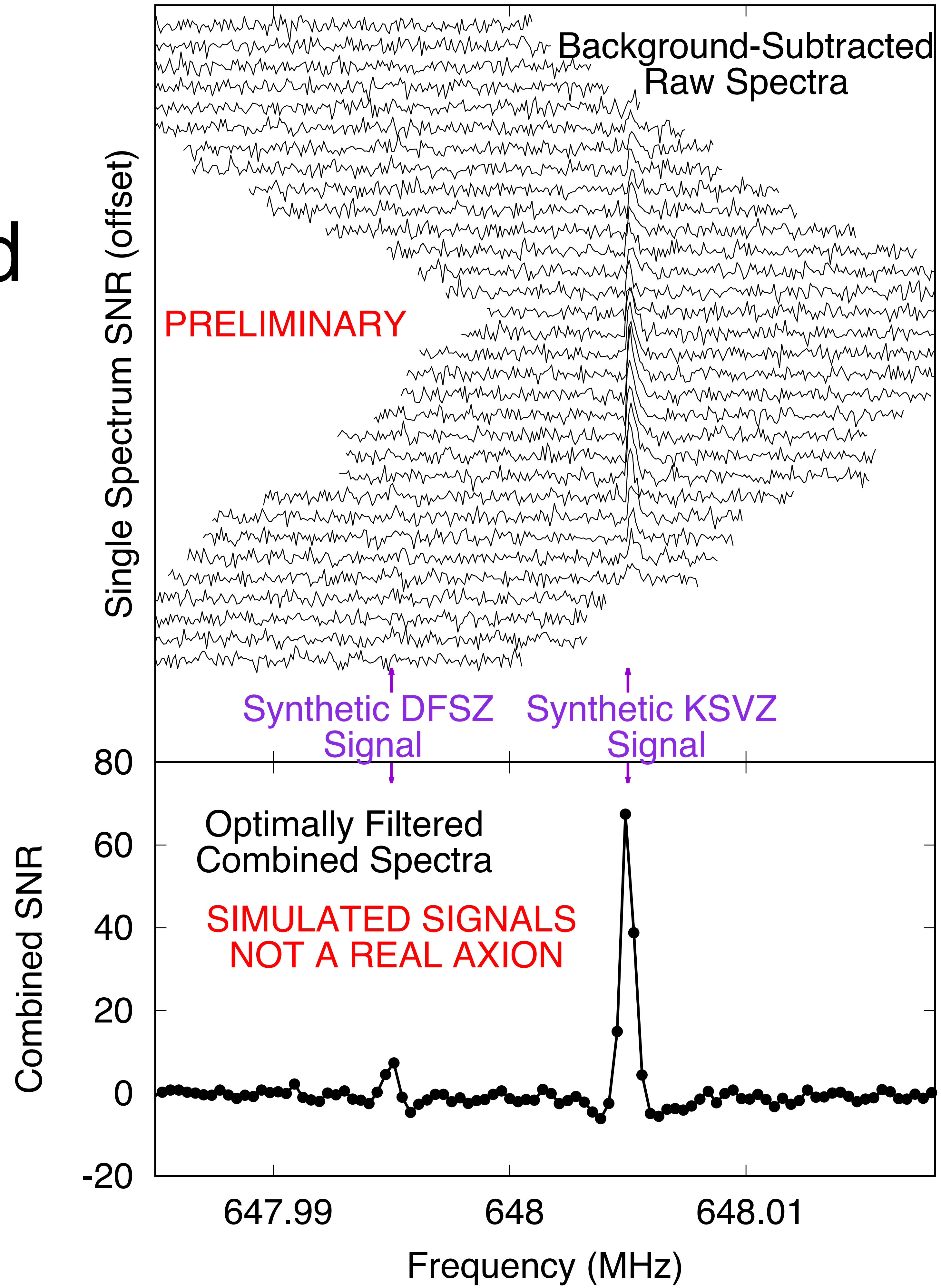
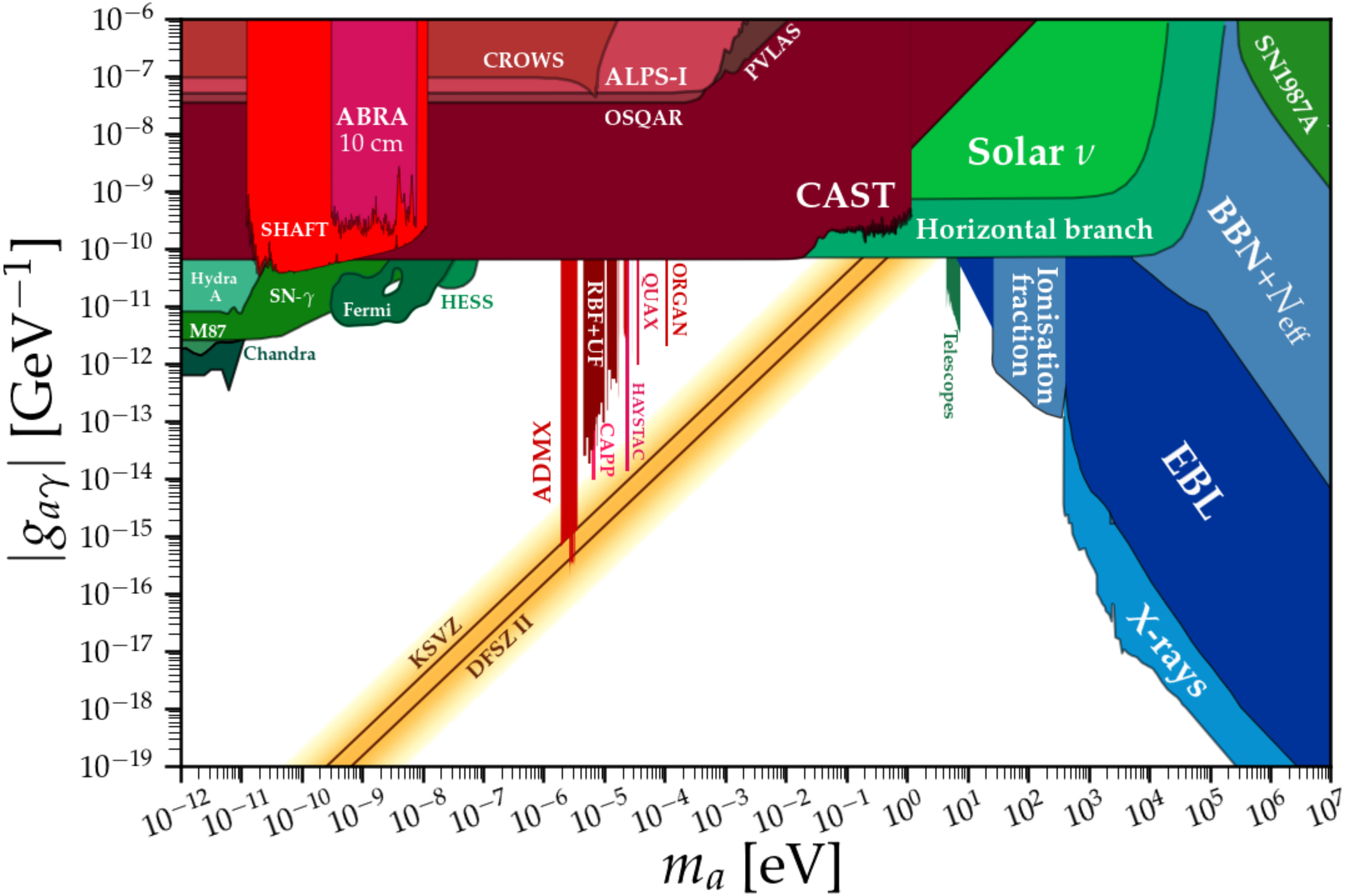
ADMX

Calculated Signal Strengths in ADMX2



Exclusion Limits World-Wide



[4] C. O'Hare, "cajohare/axionlimits: Axionlimits," (2020), URL https://doi.org/10.5281/zenodo.3932430.

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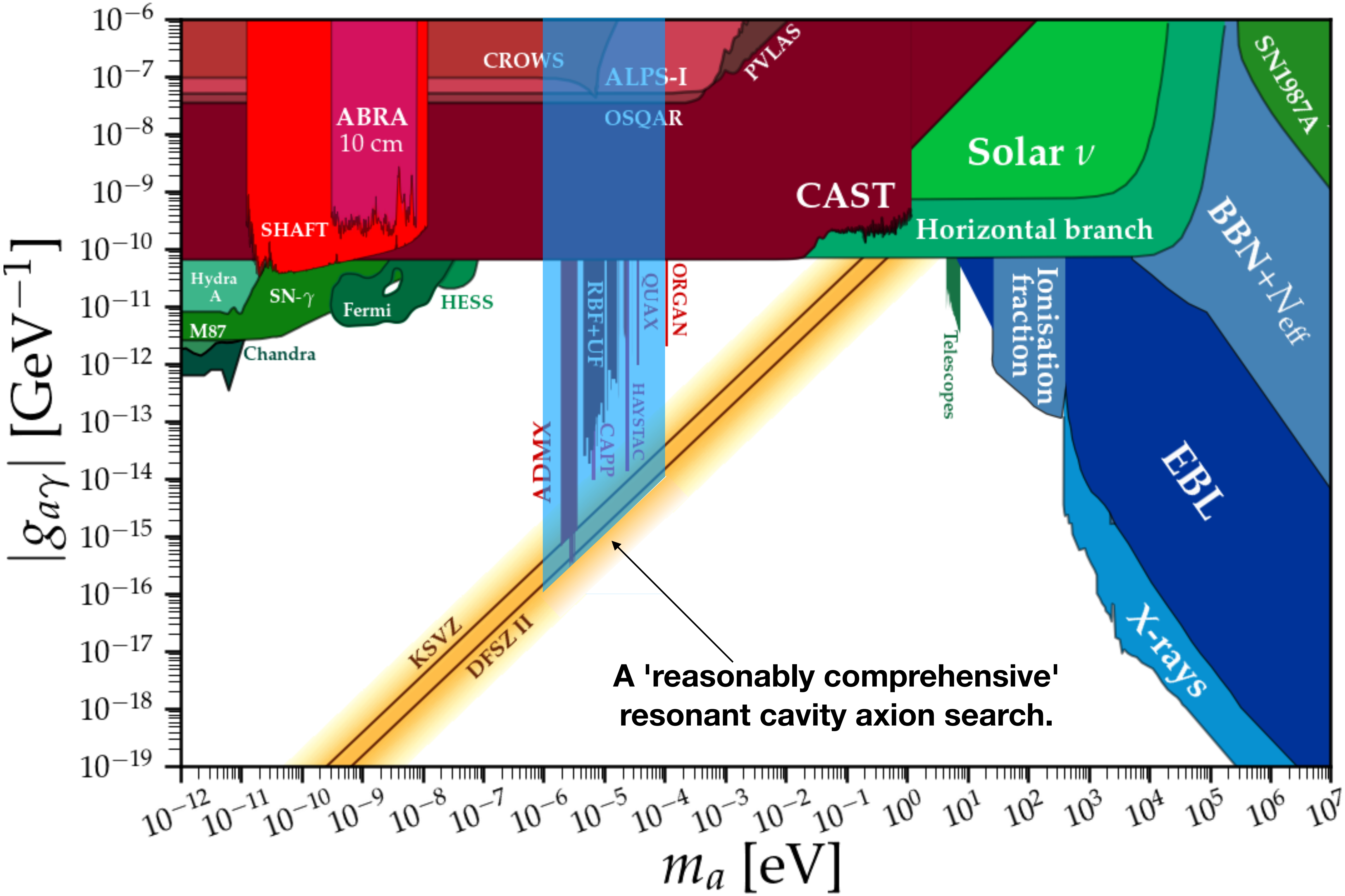
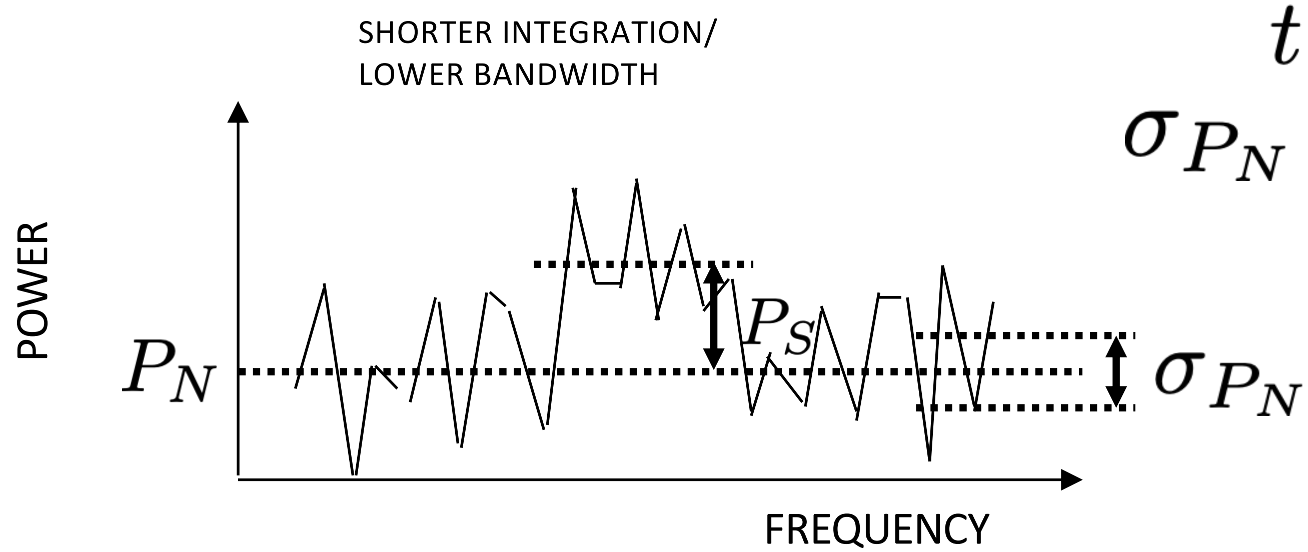


[4] C. O'Hare, "cajohare/axionlimits: Axionlimits," (2020), URL https://doi.org/10.5281/zenodo.3932430.

How Long Does A Reasonably Comprehensive Resonant Cavity Axion Search Campaign Take?

$$\mathrm{SNR} = \frac{P_S}{\sigma_{P_N}} = \frac{P_S}{P_N}\sqrt{Bt}$$

- B : bandwidth
- P_S : signal power
- P_N : noise power
- t : integration time
- σ_{P_N} : r.m.s. of bin-to-bin fluctuations in noise



A really long time!

$$\mathrm{SNR} = 4$$
$$\mathrm{P_S} = 2 \times 10^{-23}\,\mathrm{W}$$
$$\mathrm{P_N} = \mathrm{k_B} \times (0.2\,\mathrm{K}) \times (500\,\mathrm{Hz})$$
$$= 1.4 \times 10^{-21}\,\mathrm{W}$$

Ps and Pn calculated at 700MHz but both scale linearly with frequency. fmin is 240MHz (1 micro eV axion) fmax is 24GHz (100 micro eV axion)

$$\frac{P_S}{P_N} = \frac{1}{70} \qquad 4 = \frac{1}{70}\sqrt{Bt} \qquad T = \int_{\mathrm{fmin}}^{\mathrm{fmax}} \frac{D\,t\,df}{f/Q_L}$$

$$T = DQ_L(\mathrm{SNR})^2 Q_a \left(\frac{P_N}{P_S}\right)^2 \int_{\mathrm{fmin}}^{\mathrm{fmax}} \frac{df}{f^2}$$

$$D = 20$$
$$Q_L = 5 \times 10^4$$
$$\mathrm{SNR} = 4$$
$$Q_a = 10^7$$

$$T = 100\,\mathrm{years}$$

This isn't great news. Is there any way to make this go faster?

The Elephant in the Room

We are basically stuck tuning a single cavity resonance. Only get +/- 30% tunability per cavity geometry. Takes forever to cover a significant mass range.

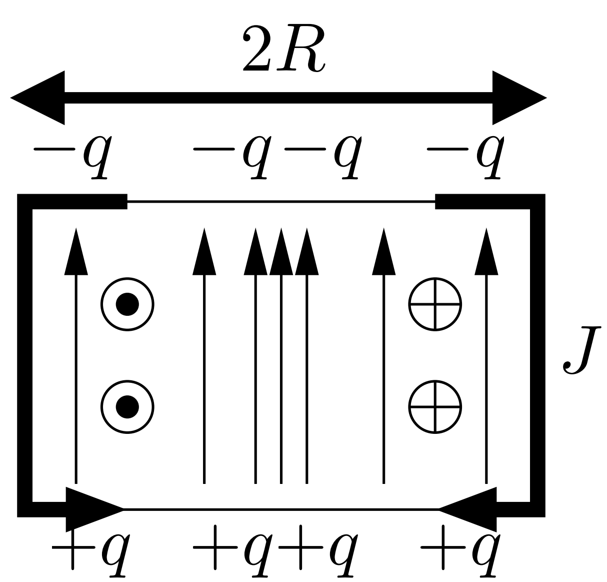
The University Of Sheffield.

A practical resonant feedback circuit

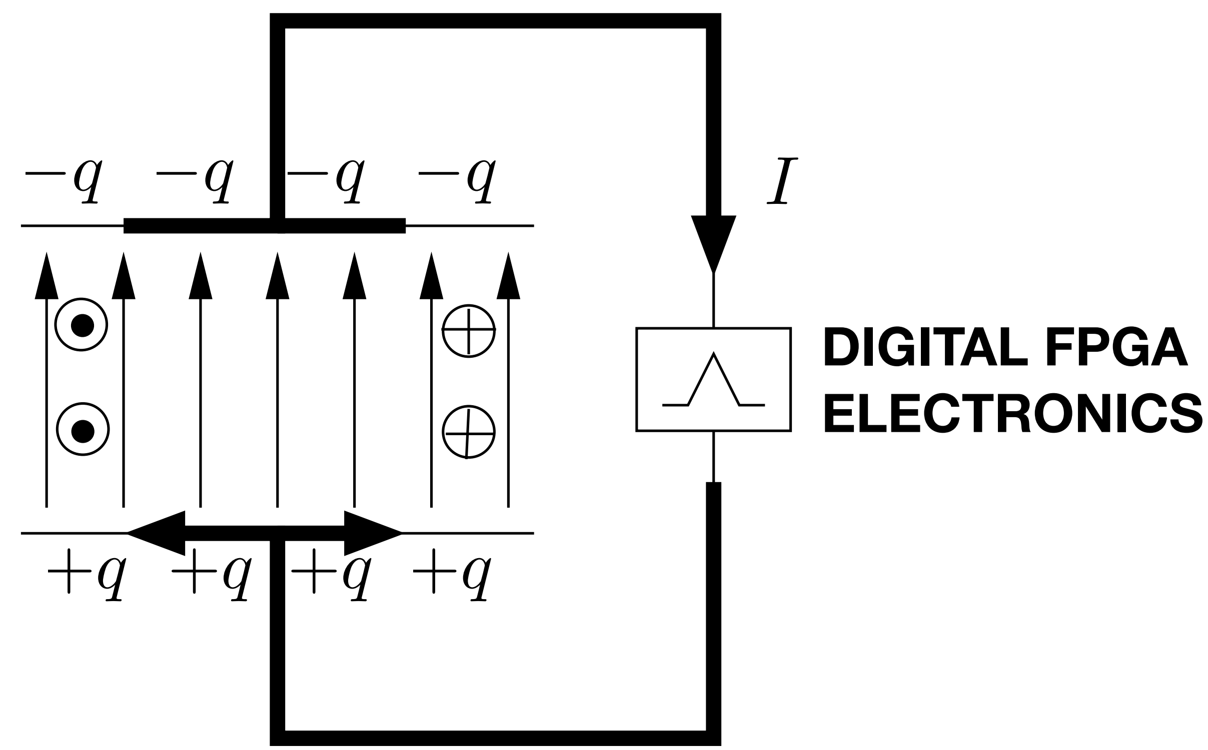
Courtesy of Holger Notzel, www.kometamps.com

Resonant feedback concept

Cavity



Resonant feedback



IN COLLABORATION WITH CHELSEA BARTRAM, PANOWSKY FELLOW (STANFORD)

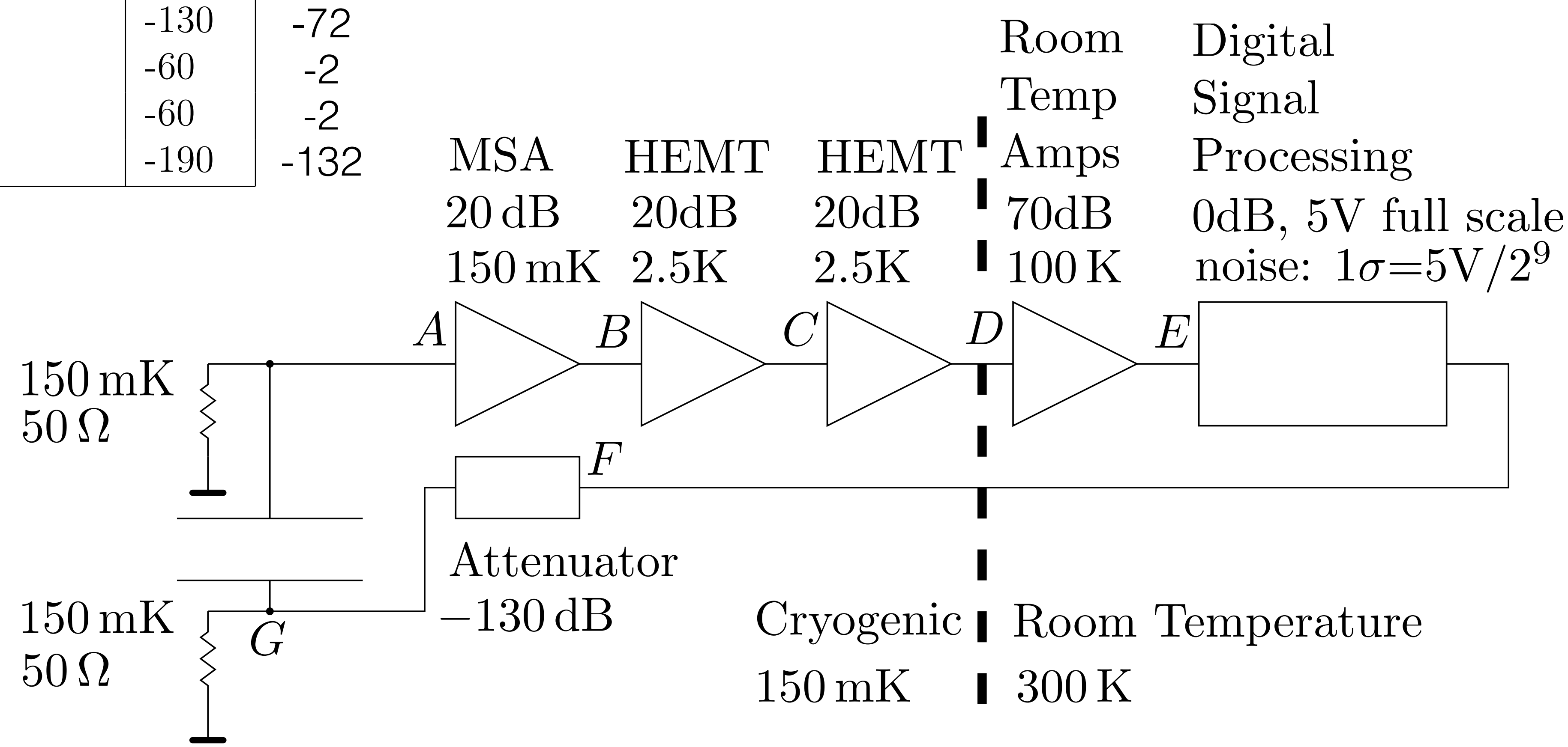
Nuclear Inst. and Methods in Physics Research, A, Volume 921, p. 50-56.
https://arxiv.org/abs/1805.11523

Advantages of Resonant Feedback

- You can make more than one parallel resonator.
- The form factor for 'capacitor like' fields is 1.
- No tuning rods! No moving parts in the fridge.
- The mode frequencies are not controlled by the dimension of the apparatus.
- Because you are injecting energy through feedback, you can achieve very high mode Q, and hence very high sensitivity.

Realistic electronics

Location	Total summed noise into 750Hz bandwidth [dBm]	Noise from local component into 750Hz bandwidth [dBm]	Signal power [dBm]	**Noise in 15MHz bandwidth [dBm]**
A	-175	-178	-190	-132
B	-155	-166	-170	-112
C	-135	-166	-150	-92
D	-115	-150	-130	-72
E	-45	-76	-60	-2
F	-45	-178	-60	-2
G	-175	-178	-190	-132



Nuclear Inst. and Methods in Physics Research, A 921 (2019) 50-56
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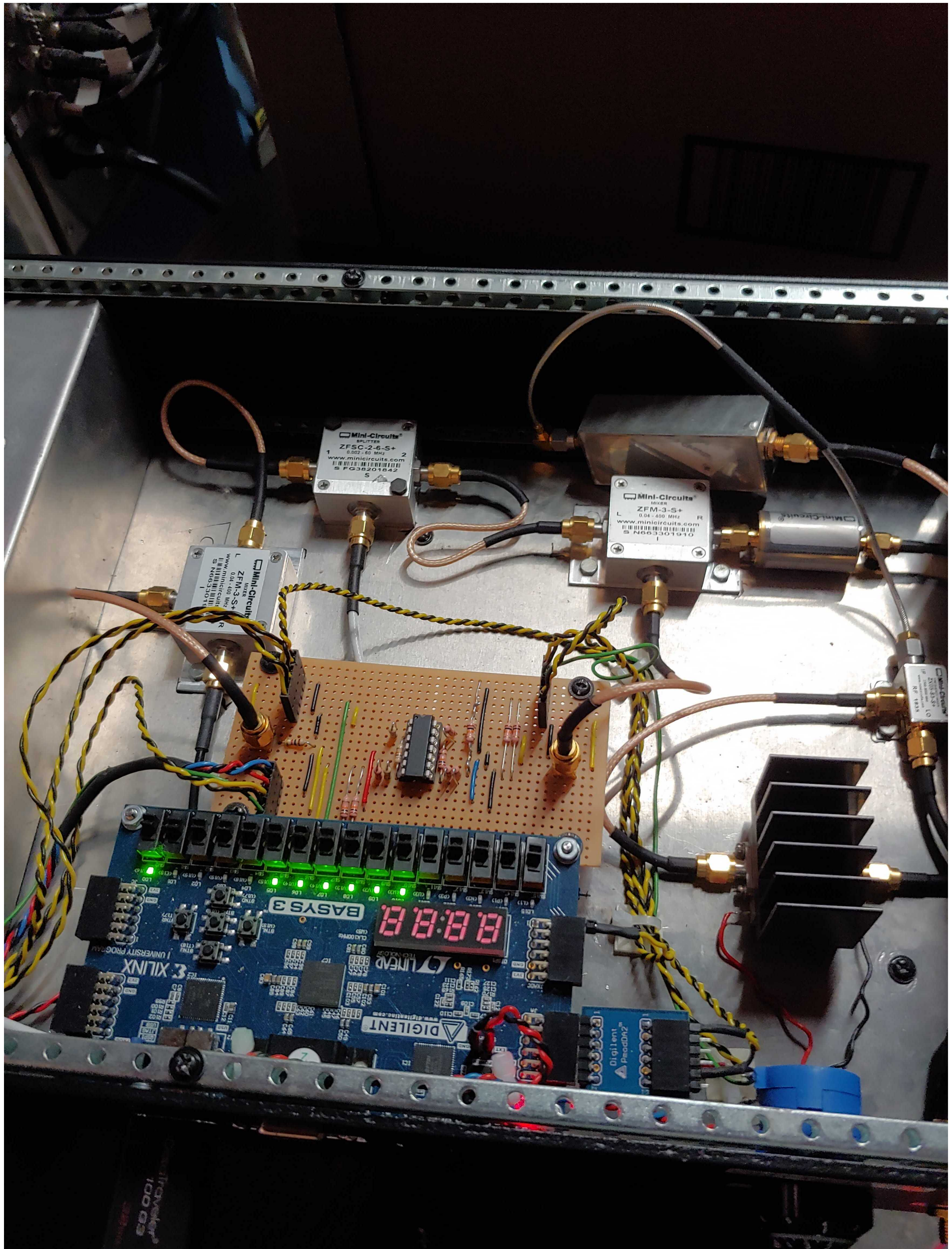
FPGA digital filter implementation

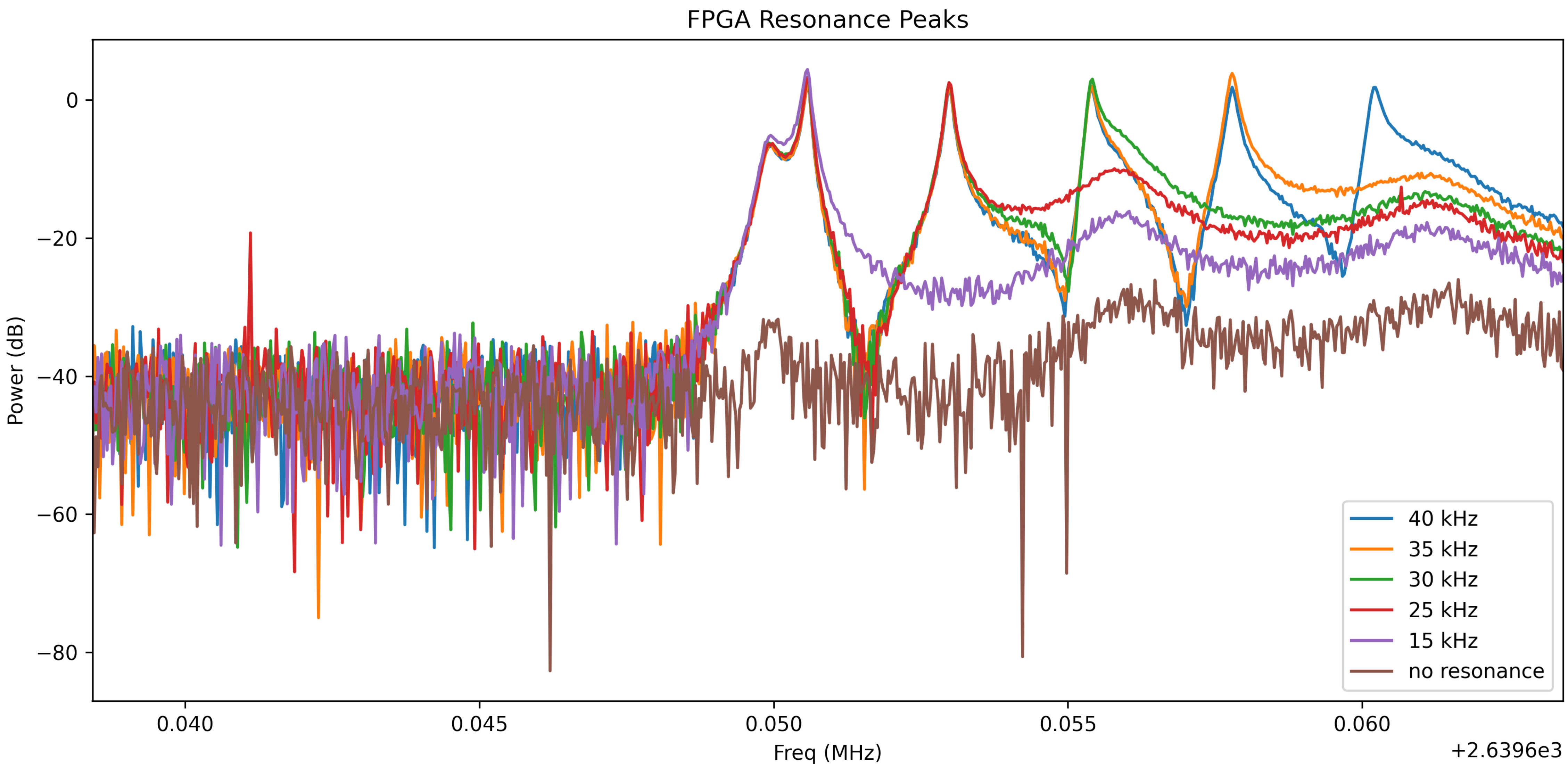


Digital filters implemented on an FPGA (Xilinx Artix 7) housed on a low budget development board.

ADC/DAC have 493kHz bandwidth - just borrowed the on-board ADC used for measuring the chip temperature.

Several parallel resonances can be created even on this cheap board. True real time deterministic - no operating system to get in the way. Same number of clock cycles every calculation.

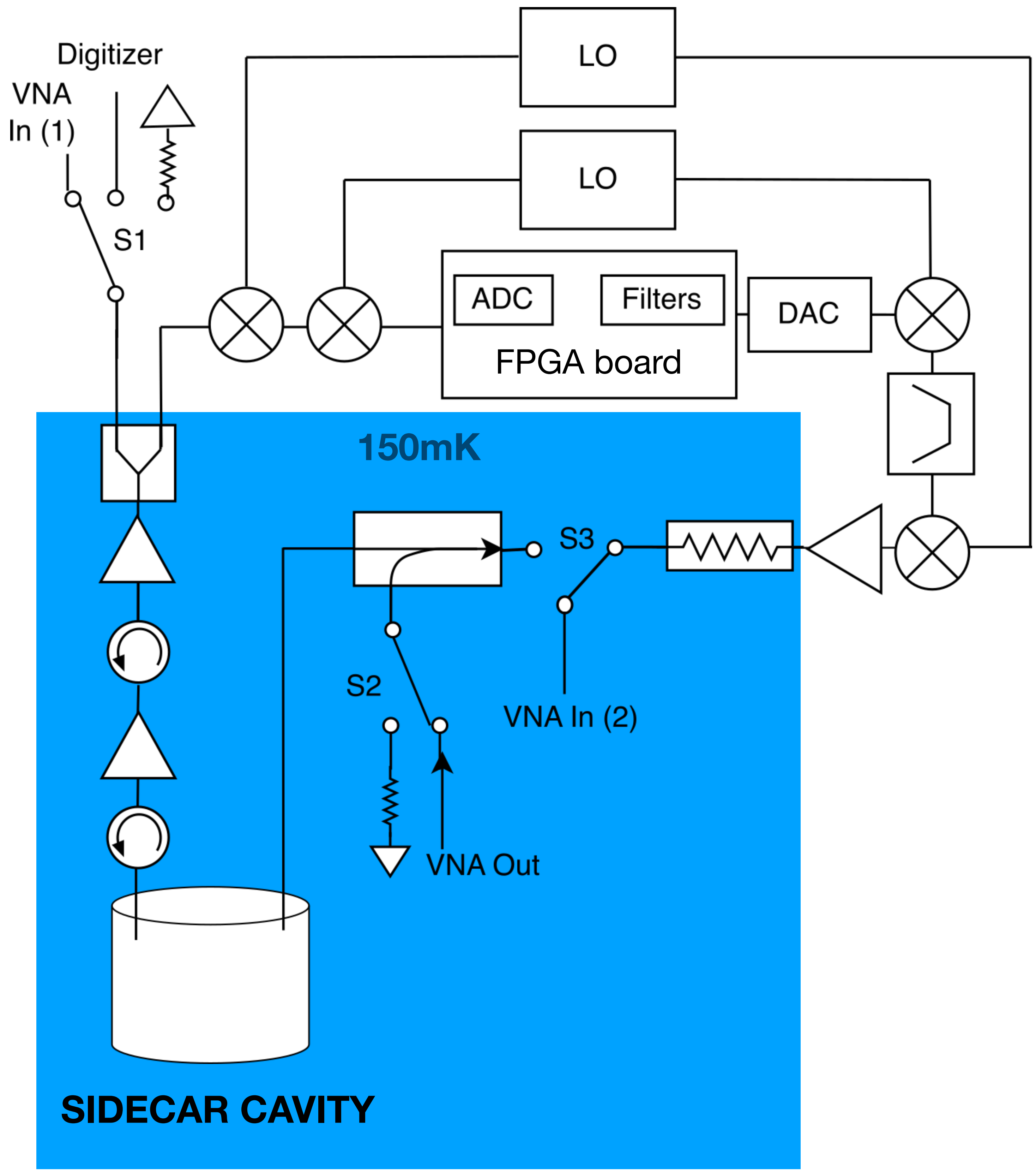
Room temperature tests at ADMX



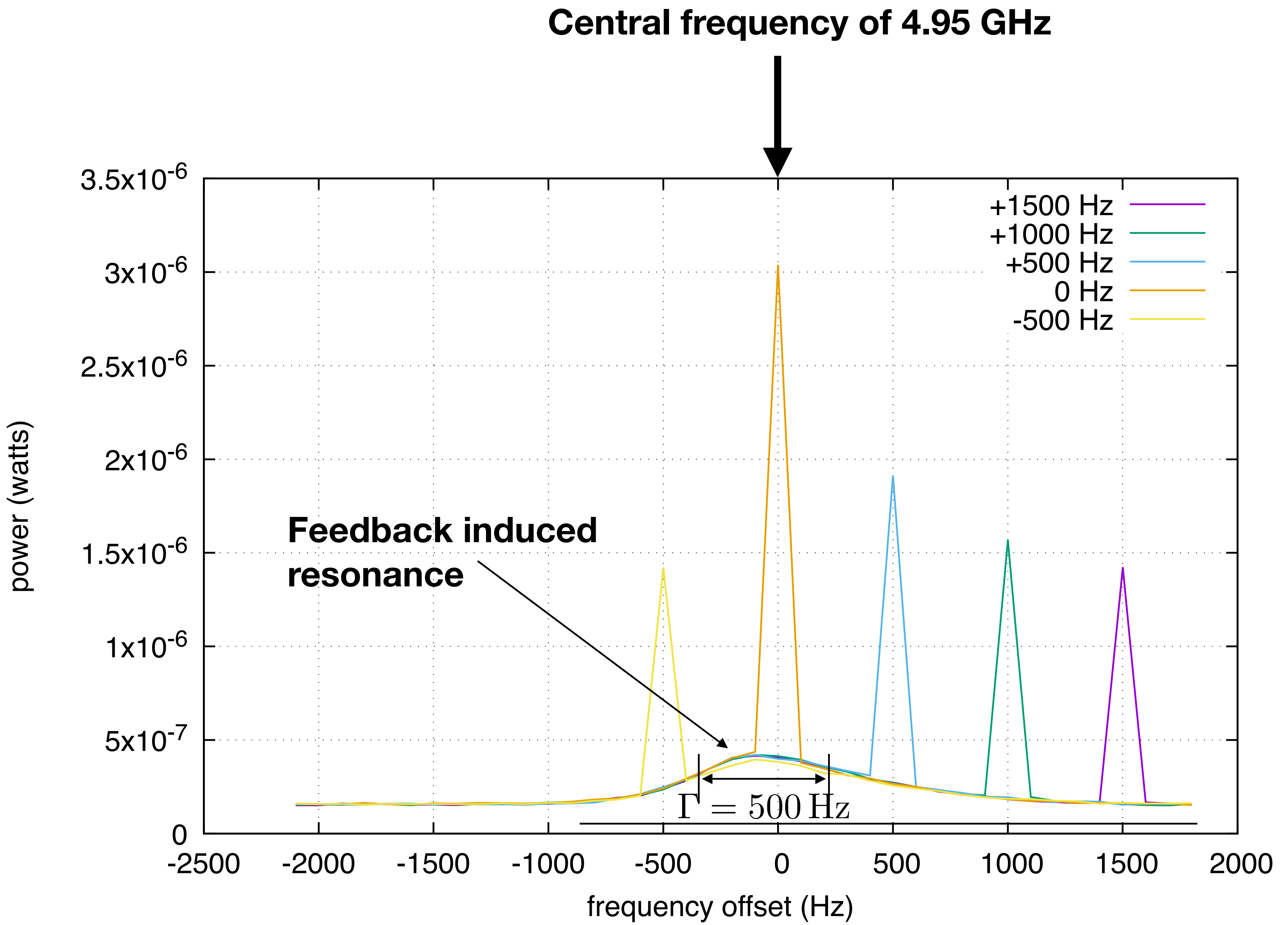
Frequency offset in MHz from 2.6396 GHz

Dr. Chelsea Bartram, ADMX, Spring 2021.

Cryogenic Test at ADMX



Closed loop power spectra with an injected sine wave

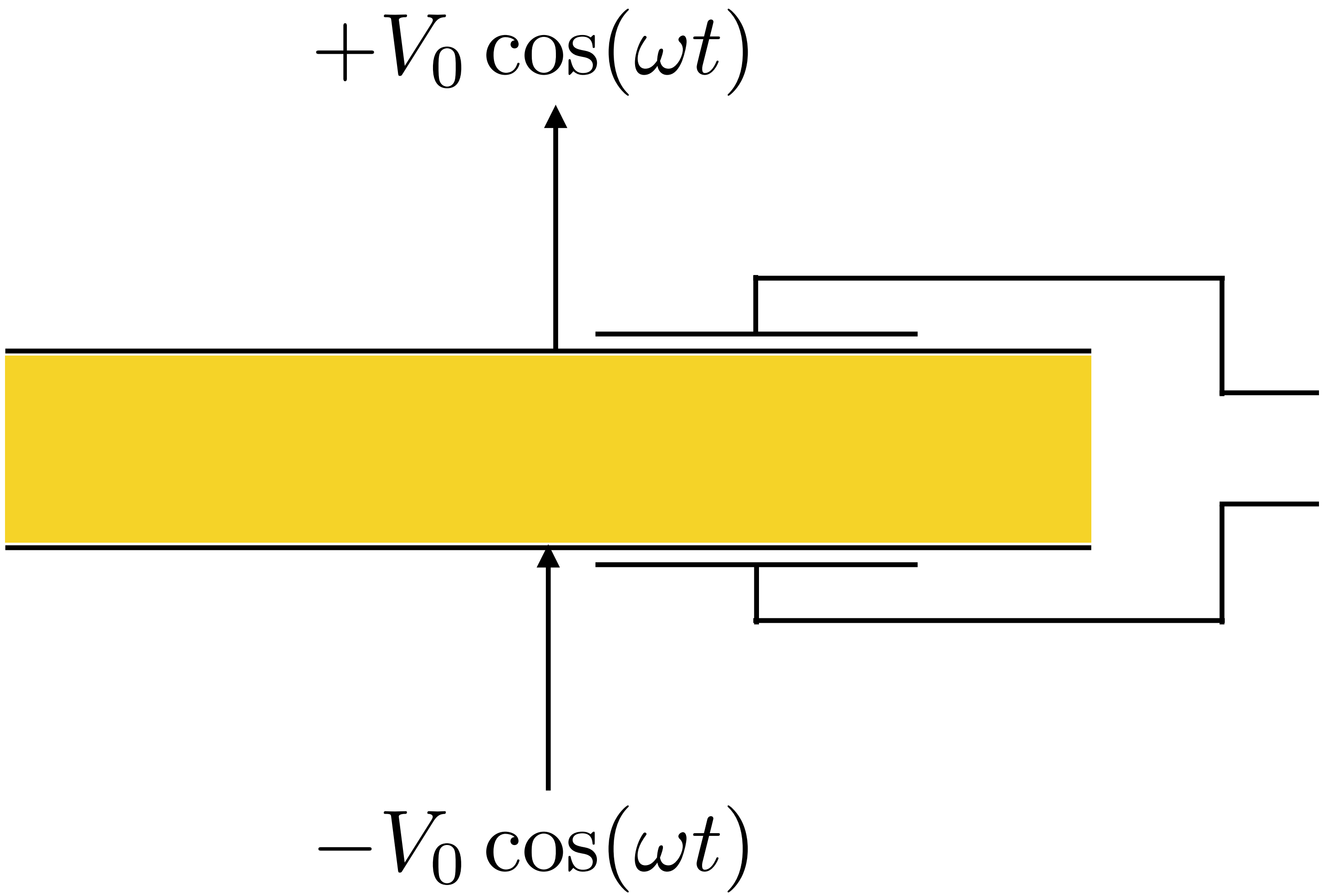


Peaks at 5 injection frequencies. All signals were injected at the same power level.

$$Q = \frac{4.949\,\mathrm{GHz}}{500\,\mathrm{Hz}} \sim 10^7$$

Note that the power level induced by the injected signal is enhanced in the vicinity of the induced resonance, exactly as we see for natural cavity resonances

Feedback control of loop gain



Differential signal feeds back to control resonant feedback gain

It is also important to control the PHASE of the output signal so that the signals on the two plates are exactly in antiphase. This requires some **frequency** control, as well as **amplitude** control.

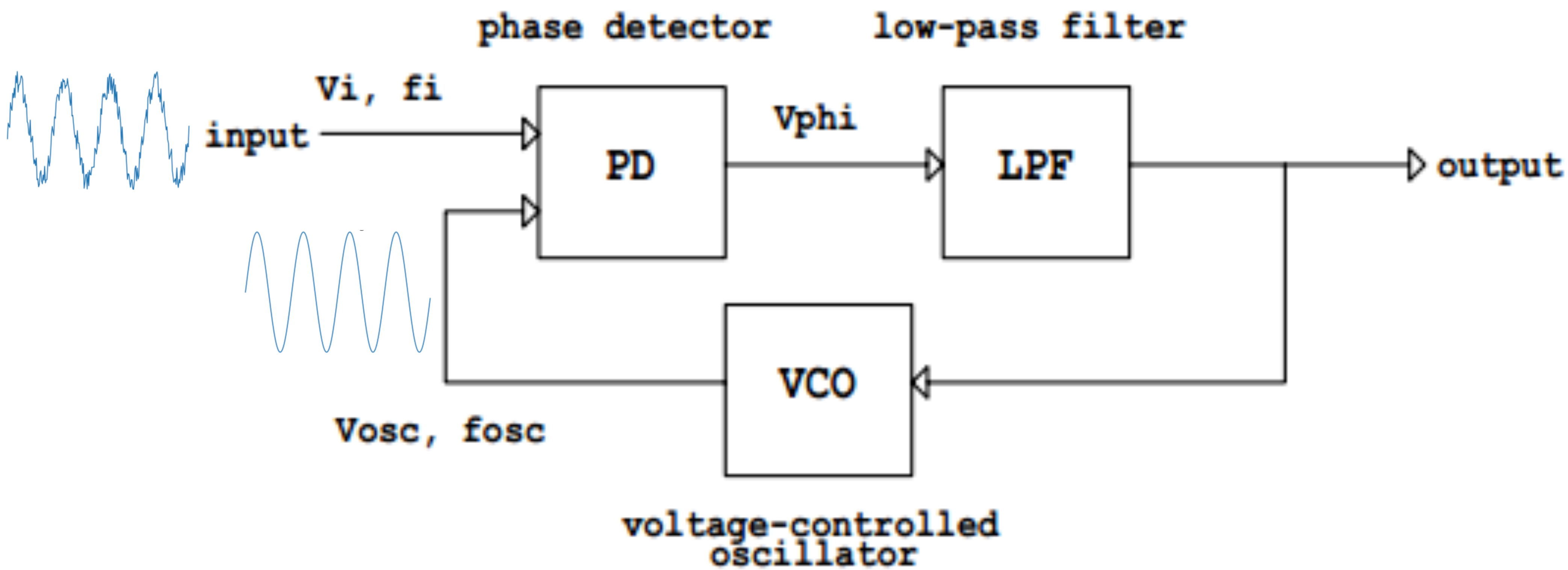
Some Further Thoughts on Feedback Control

Feedback controlled oscillators used in a variety of physics experiments also allow the frequency of the wave in the loop to wander and track signals. Can this be applied here? Possibly.

An undergraduate student project (Adam Carter, 2021-22) showed how the sensitivity could be enhanced by using a PHASE LOCKED LOOP circuit so that the feedback also allowed the frequency of the wave in the feedback circuit to follow the drive frequency from the axion as it varies naturally due to the axion linewidth.

What is a phase locked loop, anyway?

Principle of Phase Locked Loops

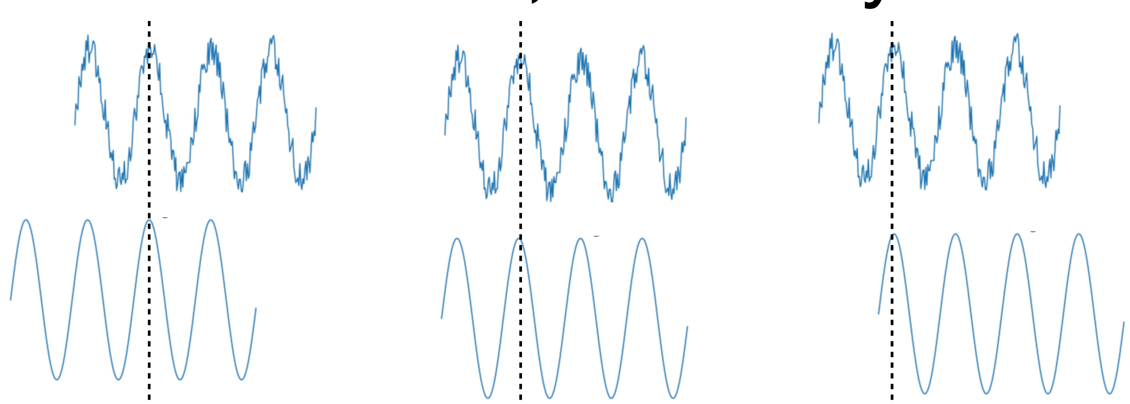


The ERROR SIGNAL is the output of the phase detector. It is LINEAR in departures of the phase, or frequency, of the input signal from that of the Voltage Controlled Oscillator (VCO) output signal. If the PLL is perfectly locked on the signal, the ERROR SIGNAL is zero.

BUT if this were true, you would just disconnect the loop, therefore the error signal is NEVER exactly zero, rather it fluctuates about zero, with (relatively) small fluctuations.

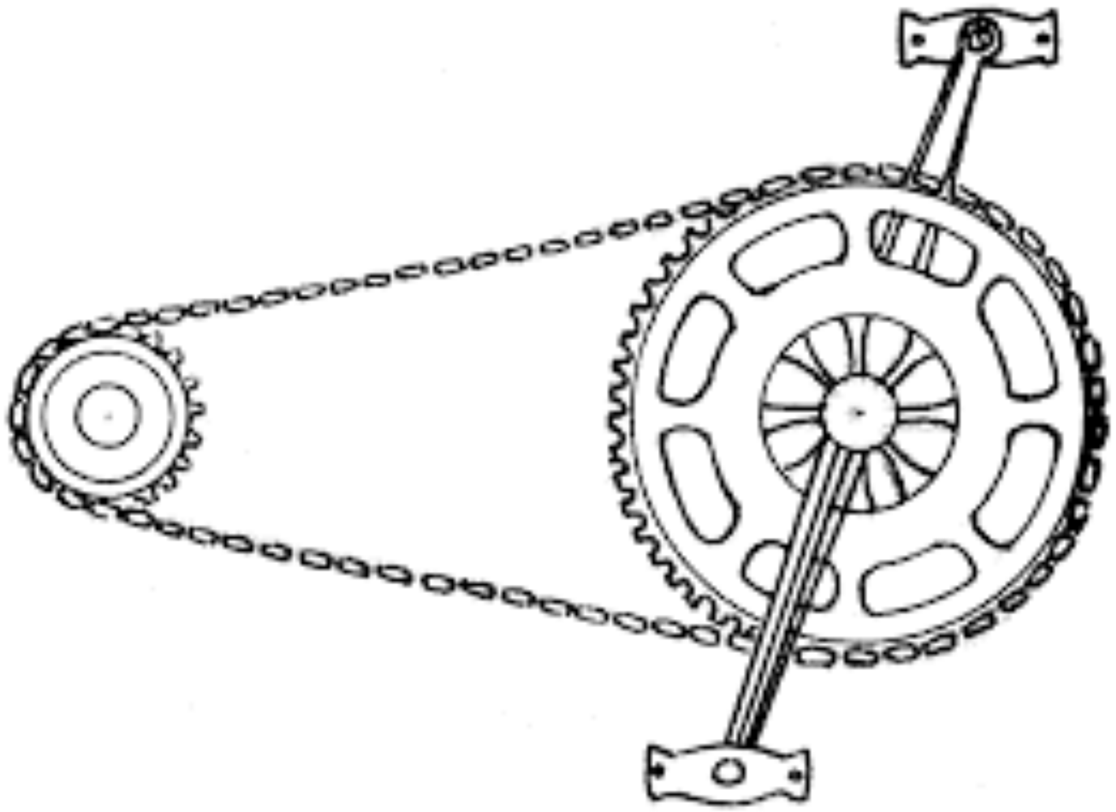
The PLL solves a Non-Convex optimisation problem

Whenever the input wave is in phase with the oscillator reference wave, the error signal is zero. The behaviour of the PLL error signal has TRANSLATIONAL INVARIANCE with respect to displacement of the Voltage Controlled Oscillator by ANY INTEGER MULTIPLE of the wave period. Any one of these minima of the error signal is an equally good result. There are MULTIPLE MINIMA of the error signal, or the COST FUNCTION, and they are all equivalent.



Classically, you are always in ONE of these minima, whilst the PLL is 'locked'.

Phase locked loop failure
- a bicycle analogy.



Phase transitions of bicycle chains

Worn bicycle chain drives have two distinct phases

Unmeshed, or (rather terrifying, especially on a steep slope) **unlocked** phase, where the chain slips abruptly over multiple teeth, jarring and uncomfortable, and loss of motive power. Danger of falling off the bike !

Meshed, or **locked** phase, where the chain meshes with the teeth of the drive wheel, power is restored, progress resumes, a PARTICULAR set of chain links meshes with a PARTICULAR set of teeth on the drive wheel, at least in the classical limit.

Our thought experiment suggests the conclusion that the 'locking' of feedback control loops is an example of a phase transition, from a disordered to a more ordered phase. Furthermore, in the ordered phase there is a SYMMETRY (translational by multiples of the period) that is not evident in the higher energy disordered phase....

Operating points of transistors

The purpose of a control system is to stabilise some plant so that its quiescent state is stable, and that any future evolution of the system can be treated as fluctuations about that quiescent 'operating point'

Example - a transistor used as an amplifier. In a common emitter configuration, a bipolar transistor has the DC voltages at the base lead set to some operating point. A small fluctuation in the base voltage then leads to a fluctuating collector current, resulting in an oscillating response in the collector voltage. Transistor bias is (especially in high tech transistors) maintained by a control system, for example a 'constant voltage' source.

As a 'device theorist', you can forget about the bias voltages and just use a small signal model of the transistor - what small signal occurs at the output in response to a small(er) signal at the input.

Axions and the Strong CP problem

Standard model symmetry group is $\underbrace{SU(3)} \times \underbrace{SU(2)} \times \underbrace{U(1)}$

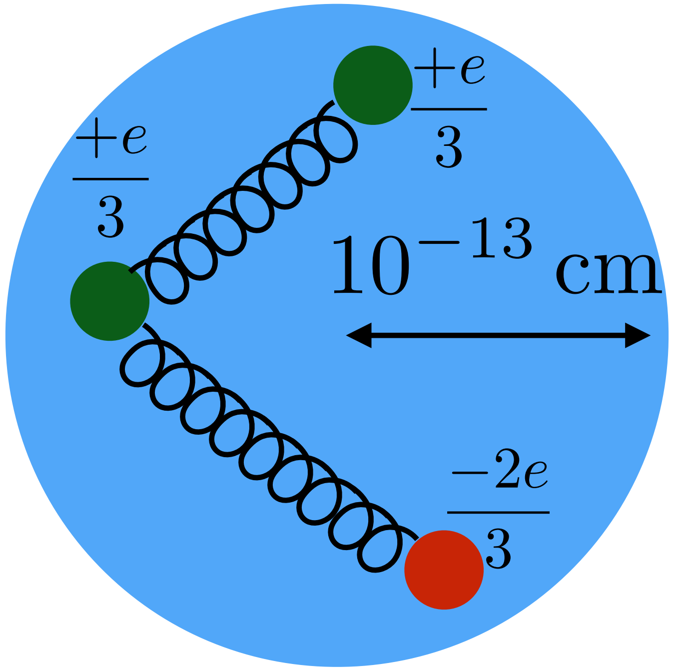
$SU(3)$	$SU(2)$	$U(1)$
NON-ABELIAN	**NON-ABELIAN**	**ABELIAN**
CP CONSERVING!	**CP VIOLATING**	**CP CONSERVING**

$$\mathcal{L}_{\rm CPV} = \frac{(\Theta + \arg\det M)}{32\pi^2}\vec{E}_{\rm QCD}\cdot\vec{B}_{\rm QCD}$$

Evidence for CP conservation in the SU(3) strong interactions from multiple measurements of neutron and nuclear electric dipole moments. For example, neutron EDM < 10^{-26} e-cm.

Even simple dimensional arguments show that this is unexpected. Why do the SU(3) QCD interactions conserve CP when SU(2) QED interactions do not? This is the strong CP problem.

NEUTRON



Operating points in field theory.

In field theory you work with operating points all the time, it's just that for simple systems it's so obvious what the operating point is that you don't ever notice it. For example, a mass on a spring or a pendulum.

In a more sophisticated example, the operating point in classical electrodynamics and classical gravity is a vacuum - the electromagnetic classical vacuum consists of zero electric and magnetic fields, the gravitational classical vacuum consists of zero spacetime curvature. Fluctuations about this operating point yield electromagnetic and gravitational waves, respectively.

In quantum field theory, vacuum fluctuations ensure that spacetime is never completely empty, but instead is populated with virtual fields, particle-antiparticle pairs if you like. Therefore even in the vacuum state there is a lot going on!

However, even in quantum field theory, there is a single 'vacuum' state, a single configuration of vacuum fluctuations, isn't there? The vacuum is complicated, certainly, but surely it is at least unique?

Physical Motivation for Positing CP violation in Quantum Chromodynamics

It's one thing to identify interaction terms in a theory that correspond to CP violating effects, but quite another to find a physical basis for them.

Instantons provide one physical basis. Start with the QCD Lagrangian for just the fermion fields representing the quarks masses m^a.

$$\mathcal{L}_{\mathrm{QCD}} = \overline{\psi^a} \left(i\gamma^\mu \partial_\mu - m^a \right) \psi^a$$

QCD is a local gauge theory. This means that given some quark fields, $\psi^a(x)$, all the terms in the Lagrangian for the theory are invariant under a class of gauge transformations that mix these quark fields into each other:

$$\psi^a(x) \to e^{-iT^b\theta^b(x)/2}\psi^a(x) = G\psi^a(x)$$

Here T^b are matrices called the generators of the group of transformations under which QCD is invariant. In QCD there are 8 of these generators, and the index b sums over them. Each generator has a coefficient $\theta^b(x)$ that parameterises the transformation. The gauge invariance is local because these coefficients are functions of position. The exponential of a square matrix is also a matrix, by series expansion of the exponent.

Gauge Covariant Derivative

The 8 SU(3) group generators obey commutation relations defining the Lie Algebra of the group SU(3).

$$[T^a, T^b] = if^{abc}T^c$$

The f^{abc} are numbers called the structure constants of the group. There is an implied sum over all 8 values of c on the right. You've seen this before in electrodynamics, where the Pauli matrices are generators of the SU(2) group, and $[\sigma^j, \sigma^k] = 2i\varepsilon_{ijk}\sigma^k$. You can find this in the wikipedia article on the Pauli matrices, for example. SU(3) is more complicated because there are more structure constants, more of them are nonzero and they differ in magnitude.

All this leads to a problem with differentiating. The derivative of ψ^a is obtained by taking the difference between the ψ^as at neighbouring points, but under a gauge transformation, the fields at neighbouring points transform in different ways, which muddies the water in terms of the conventional derivatives. The solution is to define a covariant derivative $D_\mu = \partial_\mu + ig_s A^b_\mu T^b/2$

where the A^b_μ are defined such that terms like $\overline{\psi^a}\gamma^\mu D_\mu \psi^a$ are invariant under gauge transformations. So the locally gauge invariant QCD Lagrangian is

$$\mathcal{L}_{\text{QCD}} = \overline{\psi^a}\left(i\gamma^\mu D_\mu - m^a\right)\psi^a$$

Gauge fields in the Lagrangian

The gauge fields $A_\mu(x)$ themselves don't appear in the Lagrangian, but as in electrodynamics they can be used to define field strength tensors that can be formed into gauge invariant terms that do appear. However they are more complex than the electromagnetic field strengths.

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - i g_s f^{abc} A^b_\mu A^c_\nu$$

These fields appear in the Lagrangian through the term $-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a}$

These represent the energy stored in the QCD analogies to the electric and magnetic fields, which are the gluon fields. Back in SI (MKS) units,

$$-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} = \frac{\varepsilon_0}{2} \left| \vec{E}^a \right|^2 + \frac{1}{2\mu_0} \left| \vec{B}^a \right|^2$$

Including these gauge field terms, the locally gauge invariant QCD lagrangian is now

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \overline{\psi^a} \left(i\gamma^\mu D_\mu - m^a \right) \psi^a$$

where again a sum over quark flavours a is implied. This lagrangian is invariant under local gauge transformations and the discrete CP transformation.

Definition of gauge field operators

We define the operators $A_\mu = A_\mu^a \frac{T^a}{2}$

leading to $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i g_s [A_\mu, A_\nu]$

You can show, after some not particularly pleasant algebra, that in order for $\overline{\psi^a} \gamma^\mu D_\mu \psi^a$ to be invariant under local gauge transformations, you require that the corresponding transformation on these fields are

$$A_\mu \to G A_\mu G^{-1} - \frac{i}{g_s} (\partial_\mu G) G^{-1}.$$

The $A_\mu(x)$ act as both the connection between the gauge transformations at neighbouring points, and as the gauge fields analogous to the vector 4-potential in electromagnetism.

Vacuum of QCD - or is it vacua?

Because we are interested in the vacuum state of QCD, then in that state we might require $A_\mu^b = 0$. Gauge transformations starting with this vacuum state are therefore of the form

$$\frac{A_\mu^b T^b}{2} \to -\frac{i}{g_s}(\partial_\mu G)G^{-1}.$$

Are there multiple distinct vacuum states? That is, are there G such that the resultant A are also A=0, but separated from the original A only by gauge field configurations that do not correspond to vacuum states? Are there separated minima in A? In other words, are there multiple, classically distinct, QCD vacua?

It turns out that there are, as was shown by Polyakov and Tyupin in the 1970s. It is sufficient to show that such vacua exist in an SU(2) subspace of SU(3), so we're back to the ordinary Pauli matrix generators, and in imaginary time, so that we have 'Wick Rotated' the time axis, $x_4 = ix_0 = ict$
Consider the gauge transformation

$$G = \frac{x_4 + i\vec{x}\cdot\vec{\sigma}}{x_4^2 + |\vec{x}|^2}$$

Polyakov and Tyupin's Gauge Transformation

$$G = \frac{x_4 + i\vec{x}\cdot\vec{\sigma}}{x_4^2 + |\vec{x}|^2}$$

Starting at one gauge vacuum, A=0, applying this gauge transformation leads to

$$A_\mu \to -\frac{i}{g_s}(\partial_\mu G)G^{-1}.$$

so that

$$A_4 = \frac{-i}{g_s}\frac{\vec{x}\cdot\vec{\sigma}}{x_4^2 + |\vec{x}|^2}$$

$$A_i = \frac{\sigma_i x_4 + \varepsilon_{ikl}\sigma_l x_k}{g_s(x_4^2 + |\vec{x}|^2)}$$

It turns out that, after some extremely tedious and lengthy algebra,

$$F^a{}_{\mu\nu}\tilde{F}^{a\,\mu\nu} = F^a{}_{\mu\nu}\frac{\varepsilon_{\mu\nu\sigma\omega}}{2}F^a{}_{\sigma\omega}$$
$$= \partial_\mu K_\mu$$

where

$$K_\mu = \frac{2x_\mu}{g_s^2(x_4^2 + |\vec{x}|^2)}$$

but, why do we care?

Instantons

$$\int d^4x F^a{}_{\mu\nu} \tilde{F}^{a\,\mu\nu} = 4 \int d^4x K_\mu$$

$$= 4 \oint_S \vec{K} \cdot d\vec{S} = \frac{16\pi^2}{g_s^2}$$

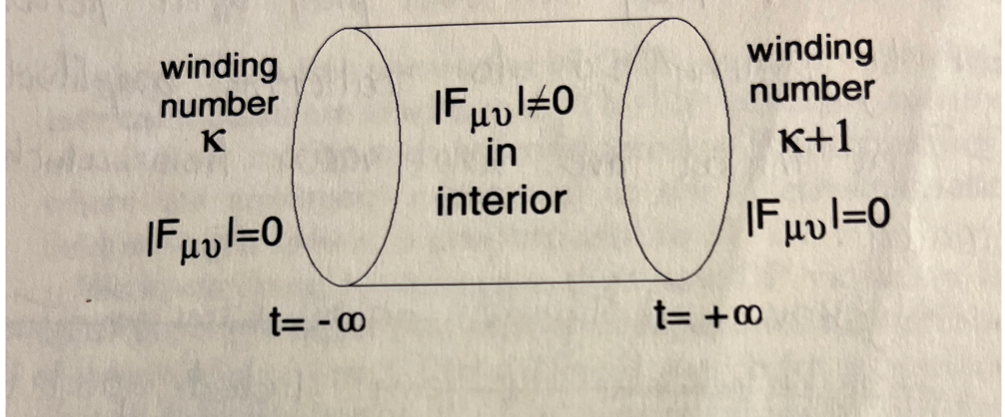


FIG. 1. A pictorial representation of instanton tunnelling.

The physical interpretation is that this gauge transformation connects two distinct vacua having different gauge field configurations

Now, starting at A=0, apply the same gauge transformation ν times, and you get

$$\int d^4x \frac{g_s^2\theta}{16\pi^2} F^a{}_{\mu\nu} \tilde{F}^{a\,\mu\nu} = \nu\theta$$

There are an infinite number of distinct vacua labelled by a winding number ν. There is therefore NOT a single unique QCD vacuum. Polyakov and Tyupin's discovery implies there are an infinite number of them.

Effective Lagrangian for QCD incorporating Polyakov's vacuum states.

Classically, you can have many vacua and only physically be in one of them (think of the bicycle chain again). Quantum mechanically, tunneling connects the vacua, so the physical vacuum is a superposition of them.

$$|\theta\rangle = \sum_n e^{in\theta}\,|n\rangle$$

The 'partition function', or 'generating functional' for the theory, used to generate all the Feynmann rules for processes (fluctuations) about the vacuum state is then

$$\begin{aligned} Z_J &= \left\langle \theta' \,\middle|\, e^{-iHt} \,\middle|\, \theta \right\rangle_J \\ &= \sum_{m,n} e^{-im\theta'} e^{in\theta} \left\langle m \,\middle|\, e^{-iHt} \,\middle|\, n \right\rangle_J \\ &= \delta(\theta - \theta') \sum_{\nu} e^{-i\nu\theta} \left\langle m \,\middle|\, e^{-iHt} \,\middle|\, n \right\rangle \end{aligned}$$

Effective Lagrangian in the presence of Instanton Tunnelling between vacuum states.

Re-write the time evolution of the hamiltonian as an integral over all possible configurations of the gauge fields

$$Z_J = \sum_\nu e^{-i\nu\theta} \int [dA]_\nu \exp\left[-i \int d^4x(\mathcal{L} + JA)\right]$$

Bring the exponential inside the integral over the fields

$$Z_J = \sum_\nu \int [dA]_\nu \exp -i\left[\nu\theta + \int d^4x(\mathcal{L} + JA)\right]$$

but

$$\nu\theta = \int d^4x \frac{g_s^2\theta}{16\pi^2} F^a{}_{\mu\nu}\tilde{F}^{a\,\mu\nu}$$

so

$$Z_J = \sum_\nu \int [dA]_\nu \exp -i\left[\int d^4x(\mathcal{L} + \frac{\theta g_s^2}{16\pi^2} F\tilde{F} + JA)\right]$$

What have we done here?

$$Z_J = \sum_\nu \int [dA]_\nu \exp -i \left[\int d^4x (\mathcal{L} + \frac{\theta g_s^2}{16\pi^2} F\tilde{F} + JA) \right]$$

We've shown that Polyakov and Tyupin's discovery of a rich basis of classically distinct QCD vacuum states can be re-cast into a new term in the QCD Lagrangian. Once you have this term, it gives rise to new Feynmann graphs that express fluctuations about the vacuum state.

Consider, we also showed that in a feedback circuit, there may exist multiple distinct locked states, which are just like classical vacuum states. Classically, you are only in one of them. Imagine that you now have a feedback control circuit operating in the quantum limit. QSHS may soon have such a circuit used as a resonant axion detector.

This mathematics strongly implies that the behaviour of such a circuit may also be described by an effective Lagrangian analysis.

Conclusions

1. Sensitive axion searches are rendered slow by the faintness of the axion signal. You must wait and average away noise.
2. Feedback circuit induced resonances can sidestep this problem by allowing multiple simultaneous resonant frequencies.
3. A feedback circuit with multiple locked states at very low temperatures is an example of quantum feedback.
4. In a quantum feedback circuit, multiple distinct phase-locked resonant configurations should be superposed.
5. The physics of instantons shows us how to analyse the consequences of this behaviour using field theory.
6. We can also think of the transition from an 'unlocked' configuration of a feedback circuit to a 'locked' configuration as a phase transition from a high energy vacuum state without multiple distinct vacua to a low energy vacuum state having distinct vacua. This may allow the analysis of feedback circuits using the machinery developed to understand phase transitions.
7. Thanks for listening. And, if you are theoretically inclined, there should be a quantum systems theory postdoc advertised in a few weeks. We'd really like to get a good applicant! It's fascinating stuff.