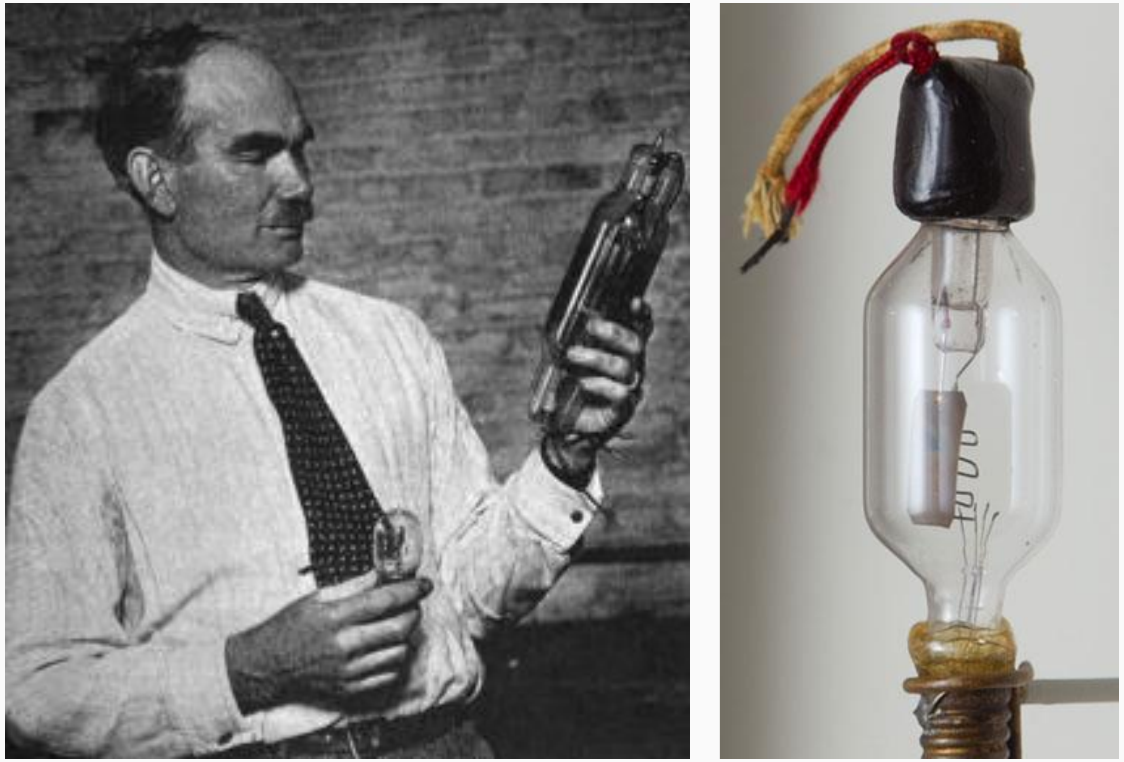
Quantum Technologies for Neutrino Mass

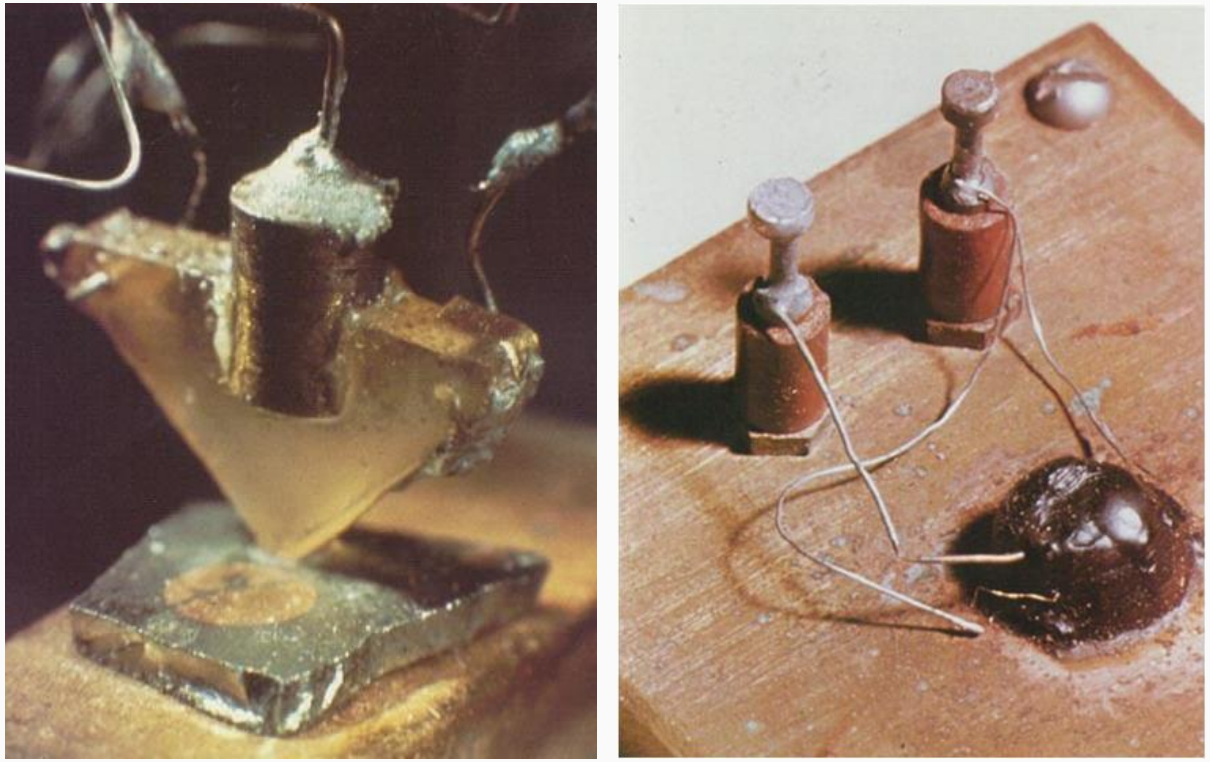
Superconducting Parametric Amplifiers

Songyuan Zhao

History of amplifiers



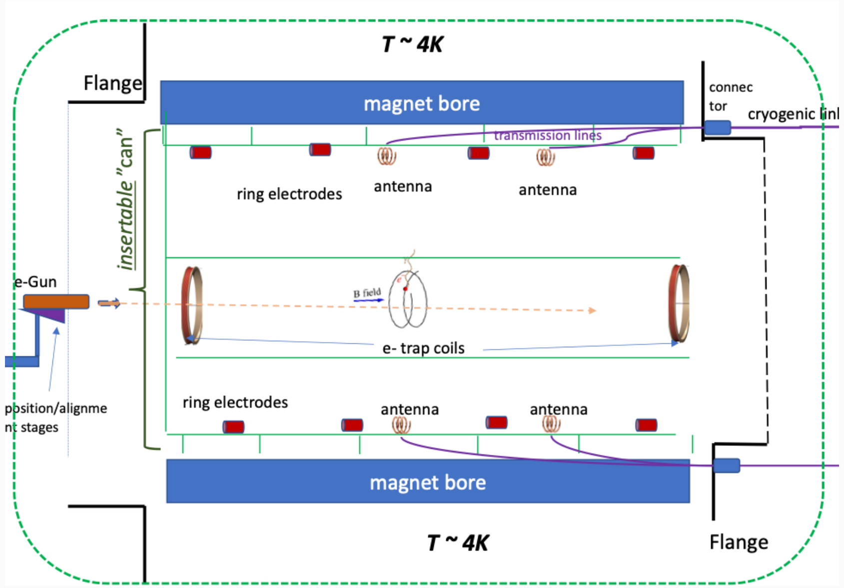
Vacuum tube / valve amplifier
(since 1906)



Transistor / solid state amplifier
(since 1950s)

Amplifiers in science

Experiment



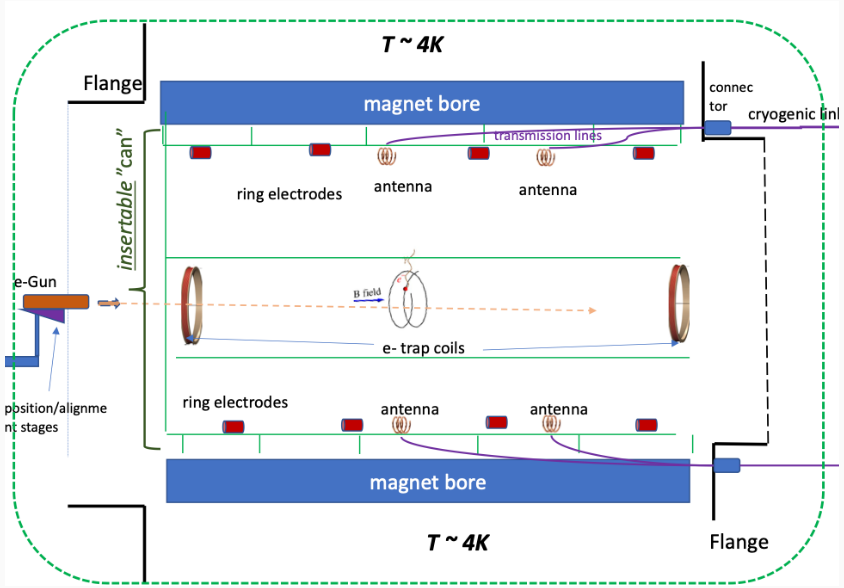
cold environment
small noise
small signal

Data

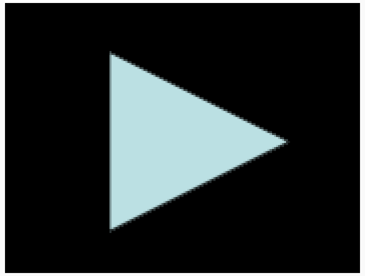
warm environment
big noise
big signal

Amplifiers in science

Experiment



Amplifier



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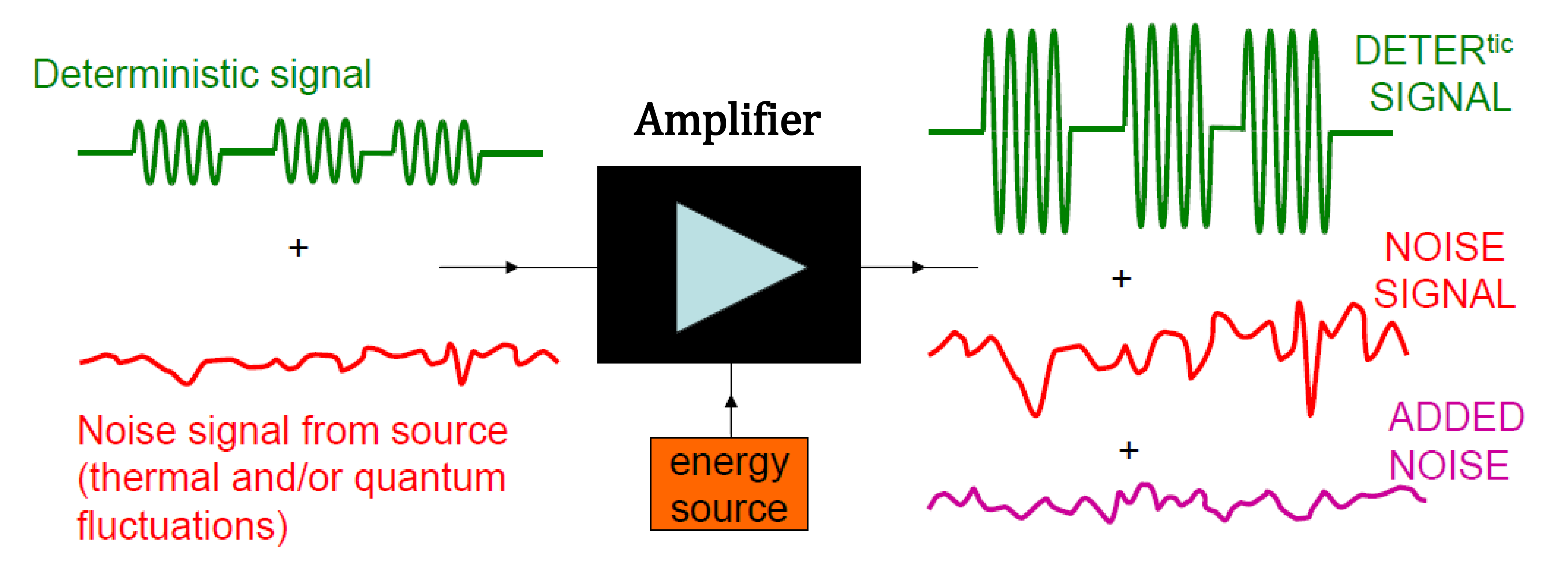
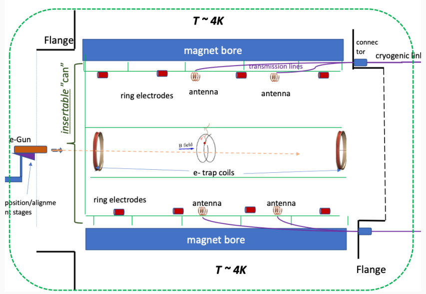
Amplifiers in science



Example

Experiment



Noise: 3 K

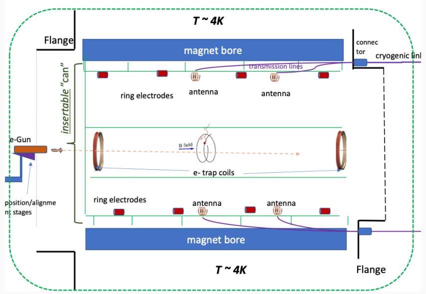
Data

Noise: 300 K

Signal: 10 K
Noise: 3 K
SNR: 3.3

Example

Experiment



Noise: 3 K

Data

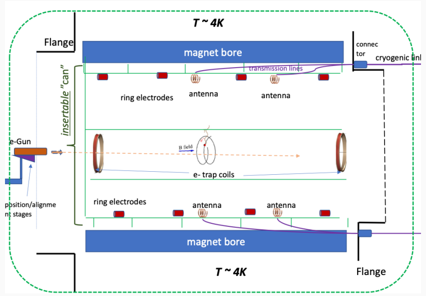
Noise: 300 K

Signal: 10 K
Noise: 3 K
SNR: 3.3

Signal: 10 K
Noise: 303 K
SNR: 0.03

Example

Experiment



Noise: 3 K

HEMT Amplifier

Gain: 100 (or 20 dB)
Noise: 3 K

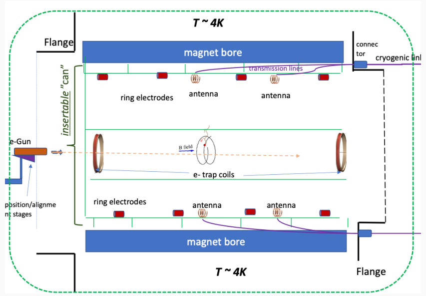
Data

Noise: 300 K

Signal: 10 K
Noise: 3 K
SNR: 3.3

Example

Experiment



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Signal: 10 K
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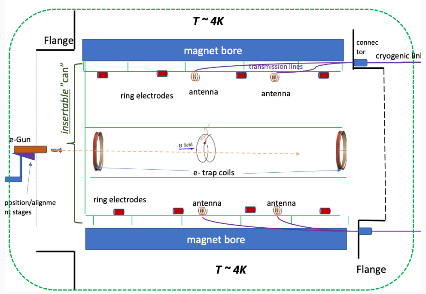
→

Signal: 1000 K
Noise: 600 K
SNR: 1.7

Example

Experiment



Noise: 3 K

HEMT Amplifier

Gain: 100 (or 20 dB)
Noise: 3 K

Data

Noise: 300 K

Signal: 10 K
Noise: 3 K
SNR: 3.3

→

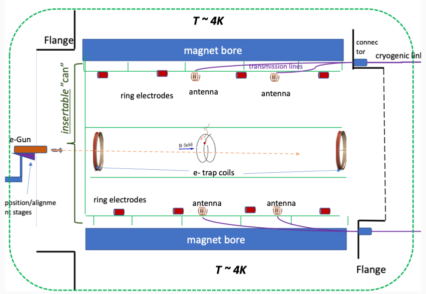
Signal: 1000 K
Noise: 600 K
SNR: 1.7

→

Signal: 1000 K
Noise: 900 K
SNR: 1.1

Example

Experiment



Noise: 3 K

Parametric Amplifier

Gain: 10 (or 10 dB)
Noise: 1 K

HEMT Amplifier

Gain: 100 (or 20 dB)
Noise: 3 K

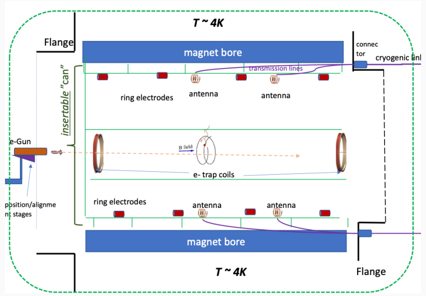
Data

Noise: 300 K

Signal: 10 K
Noise: 3 K
SNR: 3.3

Example

Experiment



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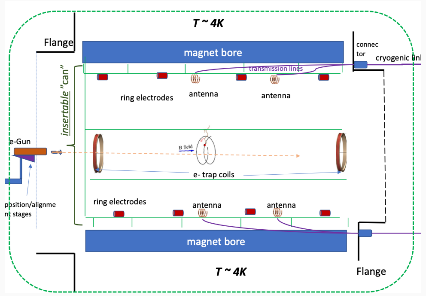
Noise: 3 K
SNR: 3.3

→

Signal: 100 K
Noise: 40 K
SNR: 2.5

Example

Experiment



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Parametric Amplifier

Gain: 10 (or 10 dB)
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HEMT Amplifier

Gain: 100 (or 20 dB)
Noise: 3 K

Data

Noise: 300 K

Signal: 10 K
Noise: 3 K
SNR: 3.3

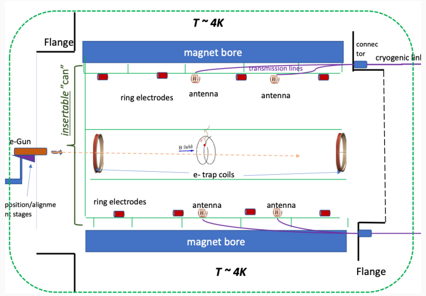
→

Signal: 100 K
Noise: 40 K
SNR: 2.5

→

Signal: 10000 K
Noise: 4300 K
SNR: 2.3

Example

Experiment	Parametric Amplifier	HEMT Amplifier	Data
			
Noise: 3 K	Gain: 10 (or 10 dB) Noise: 1 K	Gain: 100 (or 20 dB) Noise: 3 K	Noise: 300 K

Signal: 10 K
Noise: 3 K
SNR: 3.3

→

Signal: 100 K
Noise: 40 K
SNR: 2.5

→

Signal: 10000 K
Noise: 4300 K
SNR: 2.3

→

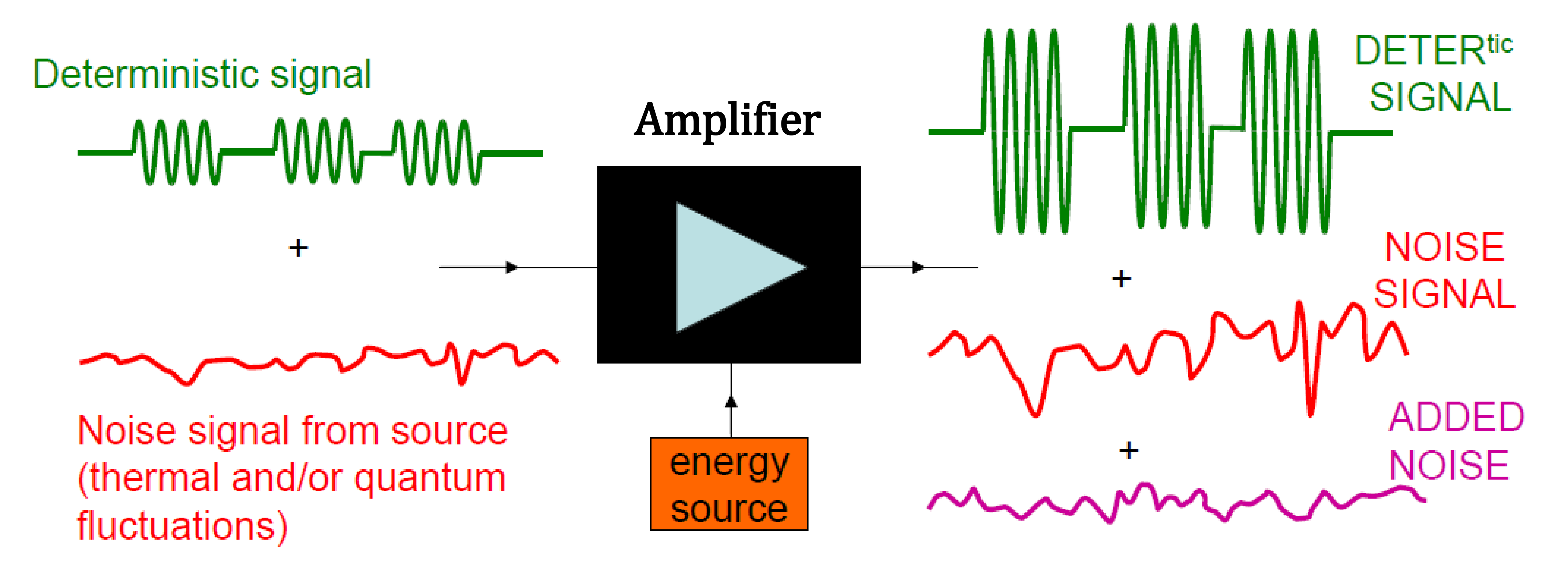
Signal: 10000 K
Noise: 4600 K
SNR: 2.2

Standard Quantum Limit

$$\hat{a}_{\text{out}} = g\,\hat{a}_{\text{in}} + \hat{N}$$

$$\frac{\langle|\Delta\hat{a}_{\text{out}}|^2\rangle}{g^2} \geq \langle|\Delta\hat{a}_{\text{in}}|^2\rangle + \frac{1}{2}$$

Standard Quantum Limit



Nonlinearity

The Superposition Principle:

*For all **linear** systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus **individually**.*

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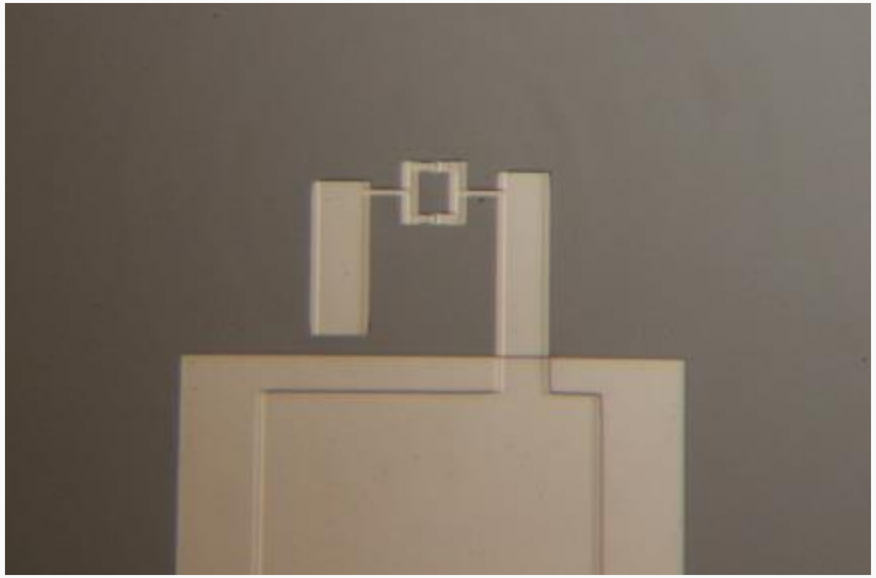
Types of nonlinearities



Josephson Junction Inductance

$$L = L_0 (\cos\phi)^{-1}$$
$$I = I_c \sin\phi$$

Kinetic Inductance of Superconductors

$$L \sim L_0 \left(1 + \frac{I^2}{I_*^2}\right)$$

Nonlinear kinetic inductance

$$L \sim L_0\left(1+\frac{I^2}{I_*^2}\right)$$

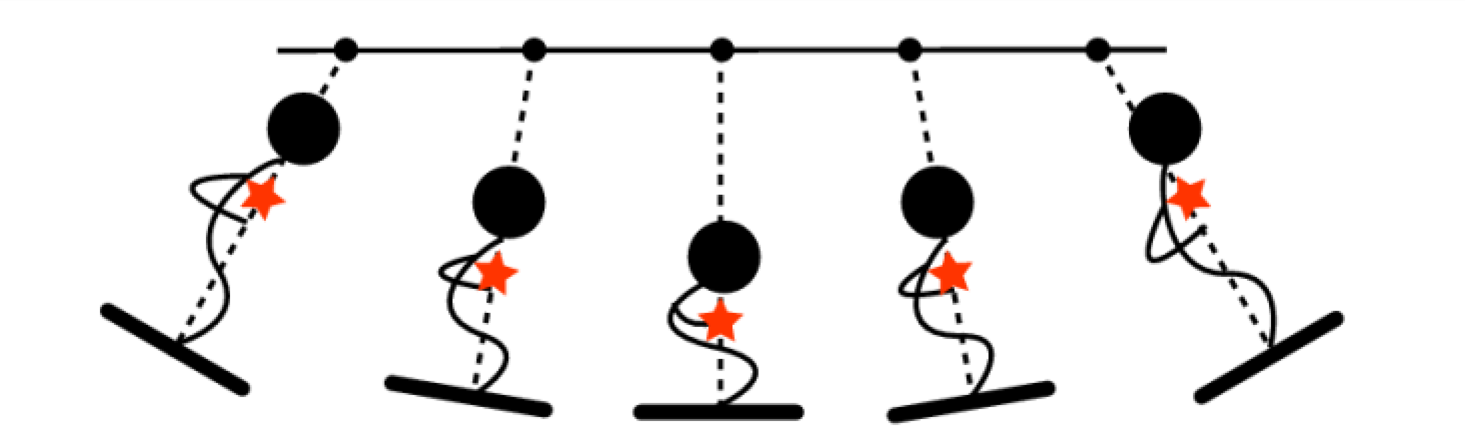
- Current results in a change in Cooper pair density
  - Phonon-mediated pair breaking
  - Superfluid velocity suppresses equilibrium pair density

$$F = F_0 + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}|(-i\hbar\nabla - q\mathbf{A})\psi|^2 + \frac{|B|^2}{2\mu_0}$$

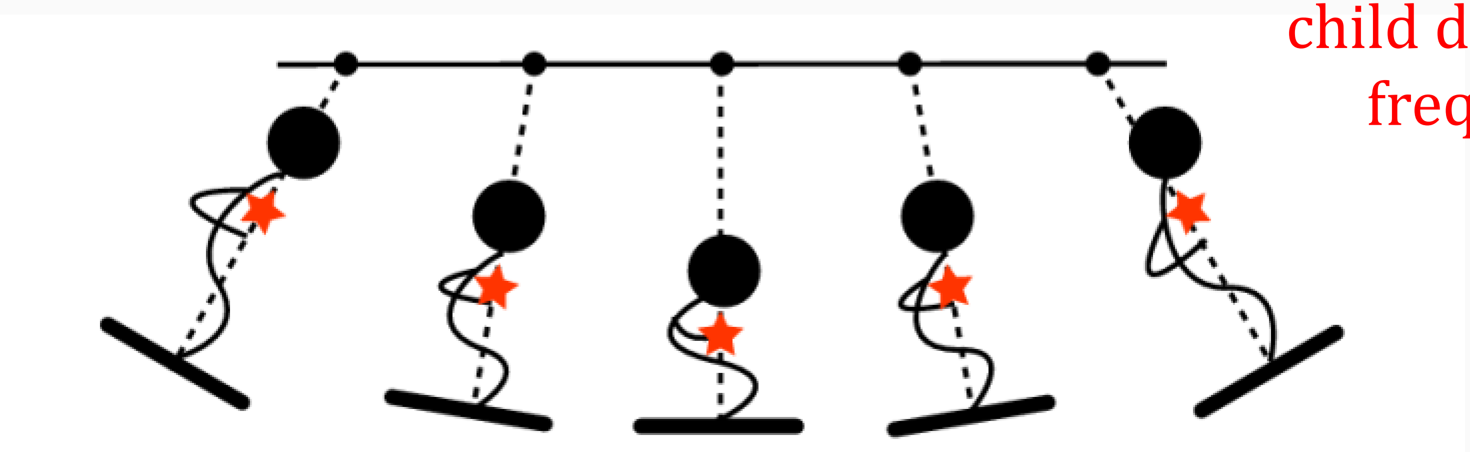
$$|\psi|^2 = \frac{1}{\beta}\left[\alpha_0(T_c - T) - \frac{1}{2}m\mathbf{v}_s^2\right]$$

  - Other unknown mechanisms…

Amplification

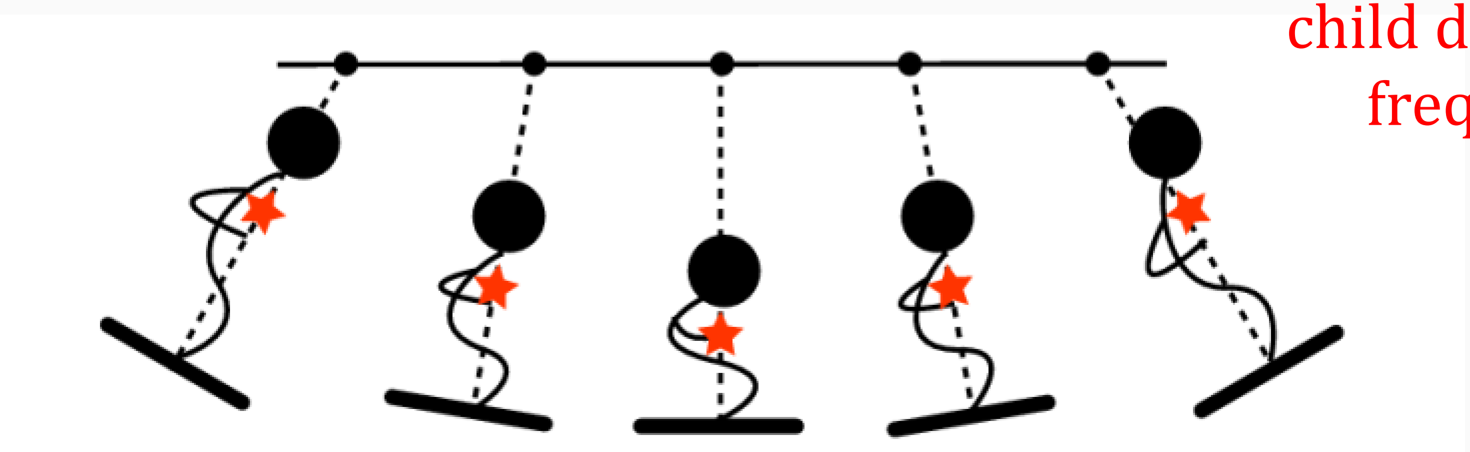


Amplification



child drives swing at frequency $2\omega_0$

Amplification



child drives swing at frequency $2\omega_0$

swing amplified at frequency ω_0

Amplification

- Four wave mixing $$L \sim L_0\left(1+\frac{I^2}{I_*^2}\right)$$

- Important frequencies:
  - Strong pump at ω_p
  - Signal of interest at ω_s
  - By-product idler at $\omega_i = 2\omega_p - \omega_s$

Amplification

$$L \sim L_0\left(1 + \frac{I^2}{I_*^2}\right)$$

- Heisenberg picture

$$i\hbar\frac{d\hat{A}}{dt} = [\hat{A}, \hat{H}]$$

- Transmission line Hamiltonian

$$\hat{H} \propto \frac{1}{2}C\hat{V}^2 + \frac{1}{2}L\hat{I}^2$$

- Expansion into Fourier modes

$$\hat{I} \propto \sum_m (\hat{a}_m + \hat{a}_m^\dagger)$$

$$[\hat{a}_m, \hat{a}_n^\dagger] = \delta_{m,n}$$

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contains term $\hat{a}_p\hat{a}_p\hat{a}_s^\dagger\hat{a}_i^\dagger$ and Hermitian conjugate

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Amplification

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$$\frac{d\hat{a}_s}{dt} \propto \hat{a}_p\hat{a}_p\hat{a}_i^\dagger$$

$$\frac{d\hat{a}_i}{dt} \propto \hat{a}_p\hat{a}_p\hat{a}_s^\dagger$$

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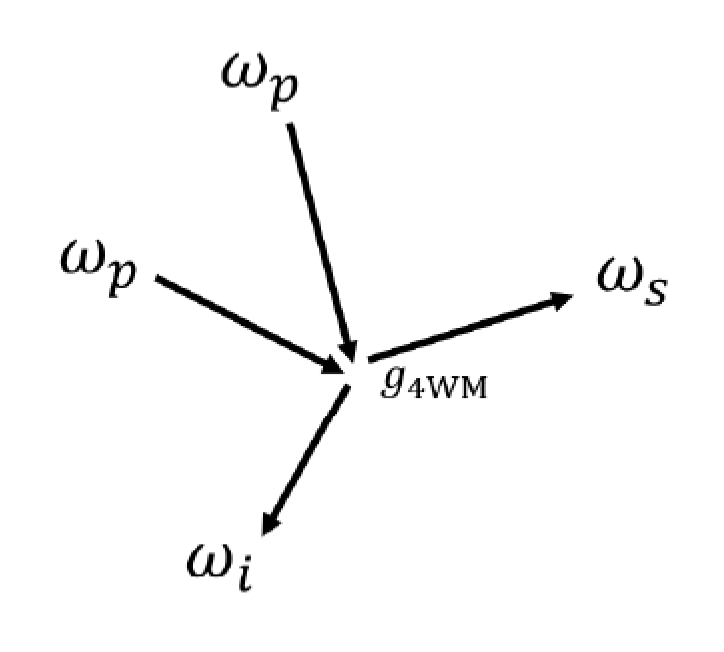
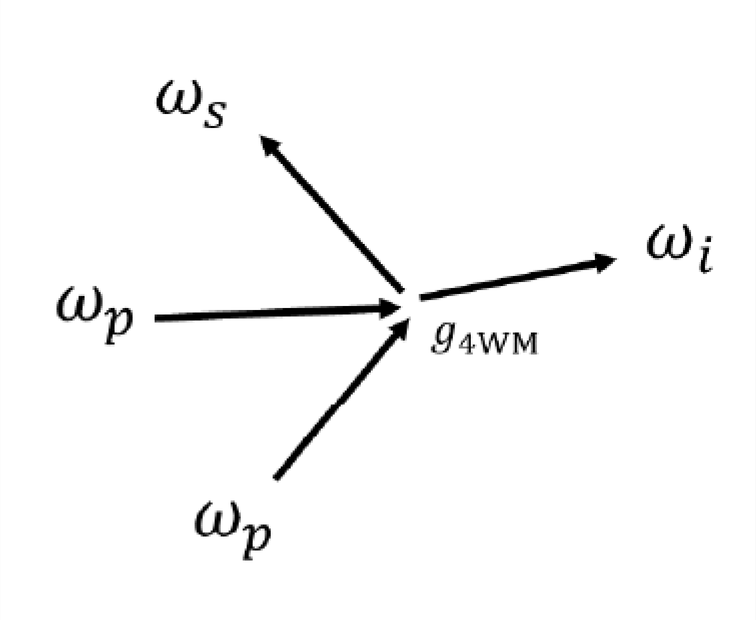


$$\frac{d\hat{a}_i}{dt} \propto \hat{a}_p\hat{a}_p\hat{a}_s^\dagger$$



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- Strong pump $\hat{a}_p$ can be approximated by $a_{p,0}$

$$\hat{a}_s = \hat{a}_{s,0} \cosh(kt) - i\hat{a}_{i,0}^\dagger \sinh(kt)$$

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$$\hat{a}_s = \hat{a}_{s,0}\cosh(kt) - i\hat{a}_{i,0}^\dagger \sinh(kt)$$

- Set $\langle \hat{a}_i \rangle = 0$ as the starting condition

$$\langle \hat{a}_s \rangle = \langle \hat{a}_{s,0} \rangle \cosh(kt)$$

Squeezing amplification

- Introduce DC to current $I \to I_{\mathrm{DC}} + I_{\mathrm{RF}}$

$$L_0\left(1+\frac{I^2}{I_*^2}\right) \quad \rightarrow \quad L_0\left(1+\frac{{I_{\mathrm{DC}}}^2}{I_*^2}+2\,\frac{I_{\mathrm{DC}}I_{\mathrm{RF}}}{I_*^2}+\frac{{I_{\mathrm{RF}}}^2}{I_*^2}\right)$$

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$$L_0\left(2\,\frac{I_{\mathrm{DC}} I_{\mathrm{RF}}}{I_*^2}\right)$$

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Three wave mixing

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$$L_0\left(2\frac{I_{\mathrm{DC}}I_{\mathrm{RF}}}{I_*^2}\right)$$

Three wave mixing

- For squeezing

$$\omega_s = \omega_i = \omega_p/2$$

Squeezing amplification

- Hamiltonian contains $\hat{a}_p \hat{a}_s^\dagger \hat{a}_s^\dagger$ and $\hat{a}_p^\dagger \hat{a}_s \hat{a}_s$

Squeezing amplification

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- Two quadrature of signal

$$\hat{X}_s = \hat{a}_s + \hat{a}_s^\dagger = \exp(kt)\,\hat{X}_{s,0}$$
$$\hat{Y}_s = \hat{a}_s - \hat{a}_s^\dagger = \exp(-kt)\,\hat{Y}_{s,0}$$

Squeezing amplification

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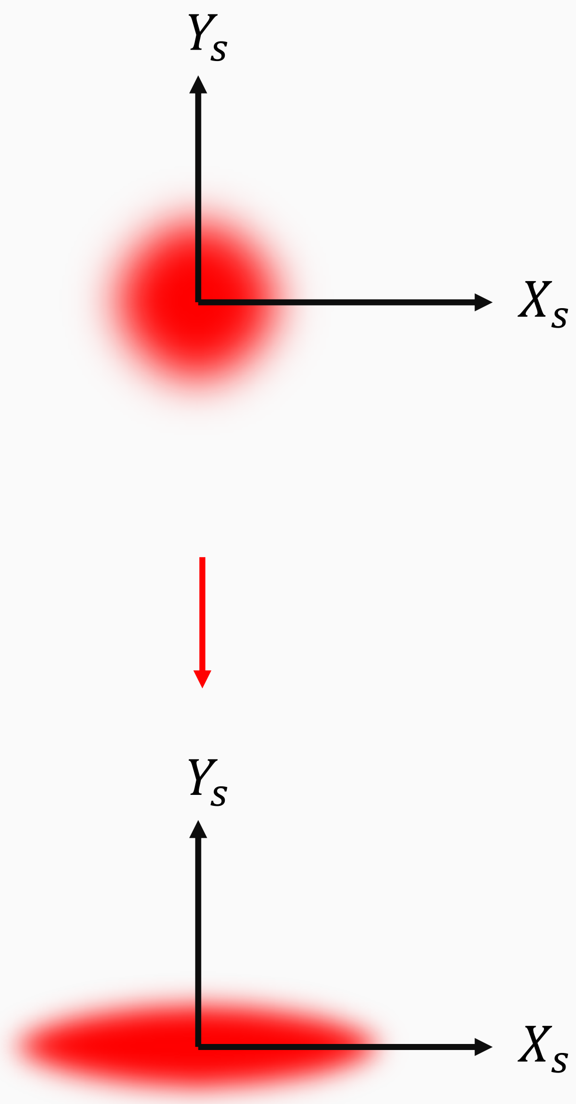
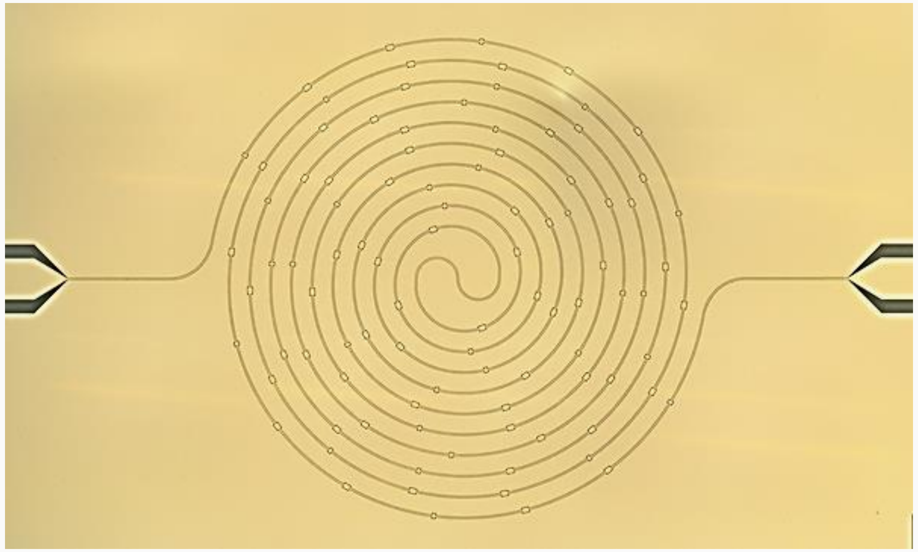


Types of SPAs geometries



Travelling-Wave Parametric Amplifier (TWPA)

Resonator Parametric Amplifier

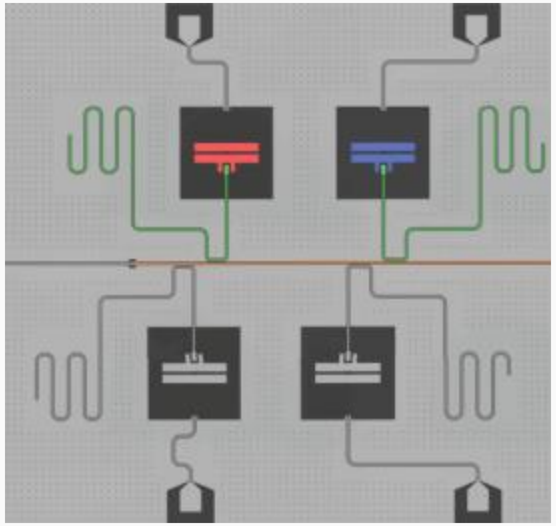
Research

- **Material** investigation – minimising unwanted nonlinearities, lower pump power requirements
- **Performance** optimisation – noise, operation frequency, bandwidths, operation temperature
- **Squeezing** amplification – generate squeezed states, amplification without adding noise

Research

Quantum information systems:

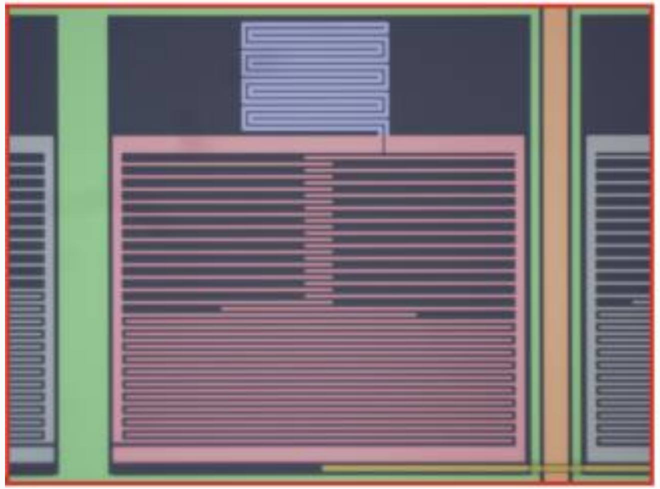
Qubit readout



(Ranzani, 2018)

Observational astronomy:

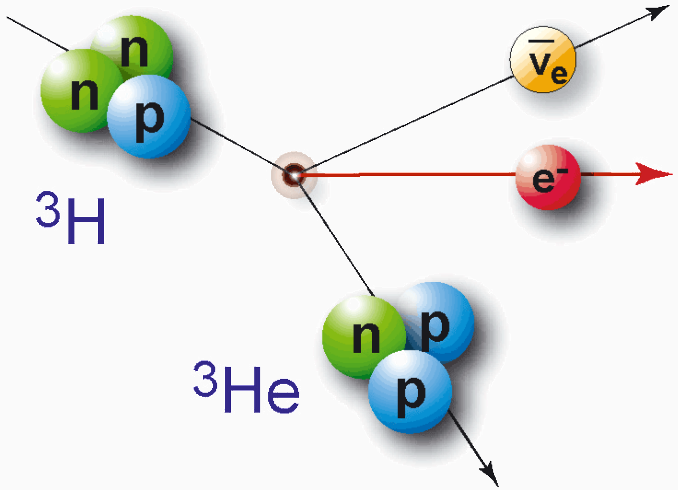
Detector readout



(Zobrist, 2022)

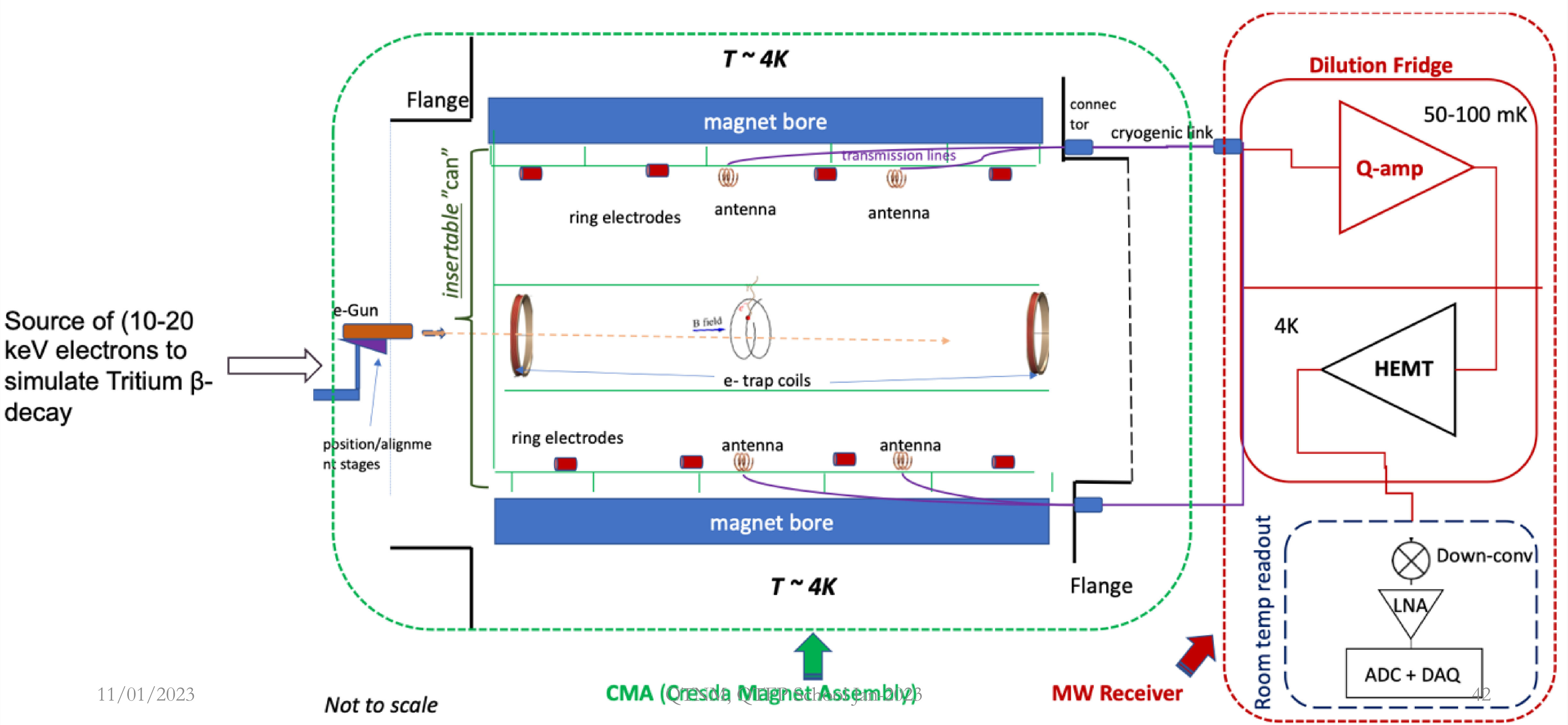
Fundamental physics:

Neutrino mass measurement



(QTNM, 2022)

QTNM: Quantum Technologies for Neutrino Mass



QTNM: Quantum Technologies for Neutrino Mass

- Quantum Technologies for Neutrino Mass

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NPL National Physical Laboratory

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