Quantum Technologies for Neutrino Mass

Tutorial Session on QTNM and Quantum Electronics

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Q1.1 Why inverted mass ordering is "favourable" for a positive neutrino mass detection? Why m_β cannot be zero in any scenario if $m_{1,2,3} \neq 0$, but $m_{\beta\beta}$ can be?

Q1.2 If Cosmology appears to deliver most sensitive absolute neutrino mass measurement why do we need other methods?

Q1.3 What are pros and cons of the three methods m_β measurement using β-decay?

Q1.4 Why KATRIN sensitivity is limited to 0.2 eV? What limits the sensitivity of the MAC-E method?

Q1.5 How do we know with "pitch angles" β-decay electrons are trapped in a magnetic bottle?

Q1.6 How is the sensitivity of the β-decay method of measuring the neutrino mass is calculated (back-of-the envelope)?

Q1.7 How can the B-field homogeneity be controlled/measured at a sub-ppm level?

Q1.8 Is a fully-fledged fusion centre necessary for a neutrino mass experiment?

Are there other options if CCFE not available?

Q1. Why inverted mass ordering is "favourable" for a positive neutrino mass detection? Why m_β cannot be zero in any scenario if $m_{1,2,3} \neq 0$, but $m_{\beta\beta}$ can be?

There are important differences in expressions for m_β and $m_{\beta\beta}$

$$m_{\beta\beta} = \left| \sum_{i=1}^{3} |U_{ei}^2| e^{i\varphi_i} m_i \right|$$

φ_i → Majorana CP-phases

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2}$$

or more explicitly →

$$\rightarrow m_\beta^2 = m_1^2 + |U_{e2}|^2 \Delta m_{21}^2 + |U_{e3}|^2 \Delta m_{31}^2$$

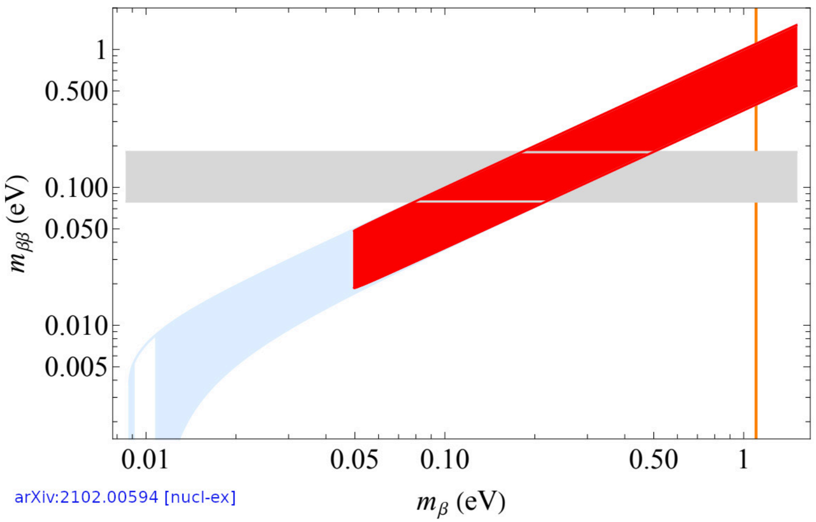
N.B.: $\Delta m_{31}^2 > 0$ for N.O.; $\Delta m_{31}^2 < 0$ for I.O.

No cancellations for m_β !

For $m_{\beta\beta}$:

$$m_{\beta\beta}^{\max} = \sum_{i=1}^{3} |U_{ei}^2| m_i,$$
$$m_{\beta\beta}^{\min} = \max\{2|U_{ei}^2| m_i - m_{\beta\beta}^{\max}, 0\}.$$

cancellation possible due to "tragic" φ_i combinations



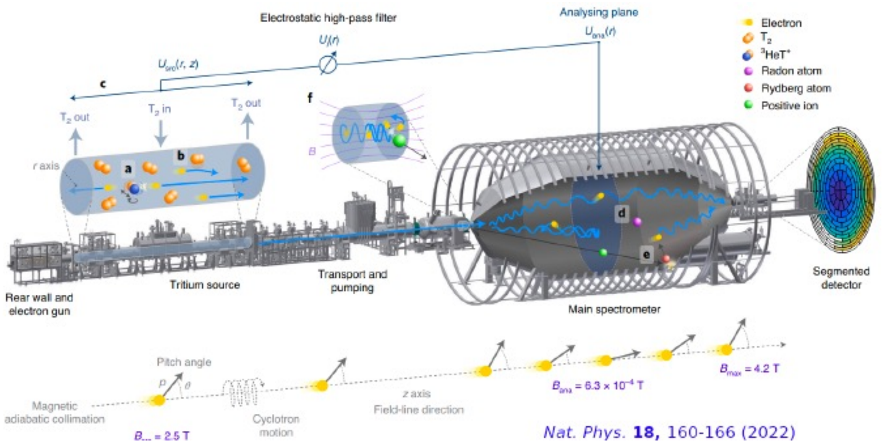
Q1.2 If Cosmology appears to deliver most sensitive absolute neutrino mass measurement why do we need other methods?

- In the next ~10 years next generation experiments are anticipated to have sensitivity $\Sigma > 0.02$ eV. If this is realised cosmic surveys should detect non-zero Σ at 3σ regardless of neutrino mass ordering.
- However, the extraction of Σ from cosmic surveys is heavily model dependent.
- It assumes Λ-CDM standard cosmological model, and invokes many assumptions of dozens of cosmological parameters to fit Σ. Even with this assumptions the results of different groups and invoking different data sets differ by a factor of few.
- Moreover, it assumes standard neutrinos and "standard" CDM. If e.g. neutrinos are slightly unstable (lifetimes a few billion years) it would relax bound on Σ by $>$ an order of magnitude.
- Cosmological measurement is therefore essential but not a substitute for a laboratory measurement.
- On the contrary, massive neutrinos are a required ingredient of cosmological models but currently have to be treated as fit parameters. They are one of the very few cosmological parameters that are susceptible to laboratory measurements. Their laboratory measurements will therefore reduce the degrees of freedom and allow better determination of those parameters that can only be extracted from cosmology.

Three methods of m_β measurement

Q1.3 What are pros and cons of the three methods m_β measurement using β-decay?

Electrostatic filter (retarding potential)

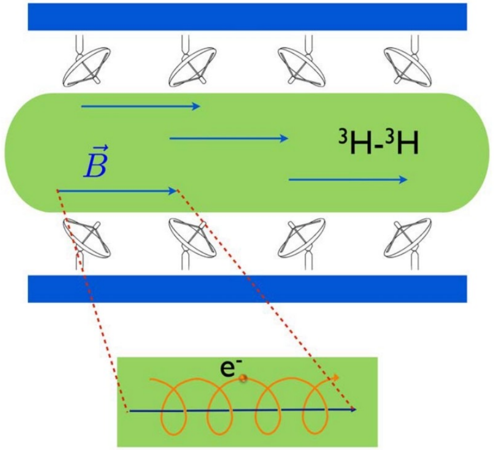


Nat. Phys. **18,** 160-166 (2022)

🙂
- Established technology
- Significant exposures achievable

🙁
- Resolution related to size
- Losses due to e-transport
- Integrated spectrum (background)

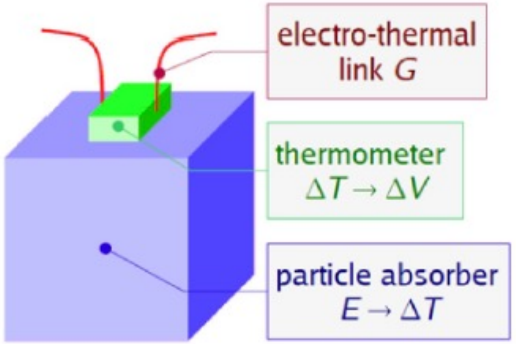
Cyclotron Radiation Emission Spectroscopy (CRES)



- No losses due to e-transport
- Source transparent to radiation -- more compact detectors
- Frequency measurement for superior energy resolution
- Differential spectrum measurement

- B-field uniformity
- Detection efficiency
- Ultra-low power signals

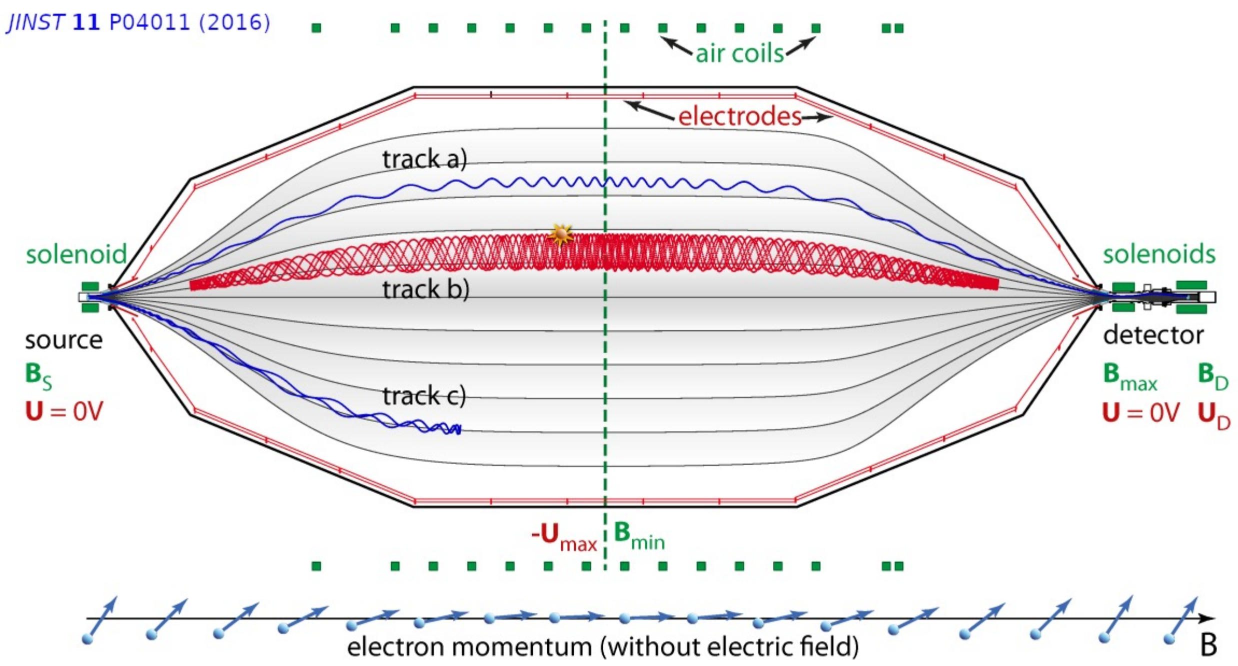
Calorimetry



- Source=Detector – no invisible energy losses
- Modularity
- Some isotope flexibility

- Scalability to large exposures
- Instrumental energy resolution
- Pile-up

Q1.4 Why KATRIN sensitivity is limited to 0.2 eV? What limits the sensitivity of the MAC-E method?

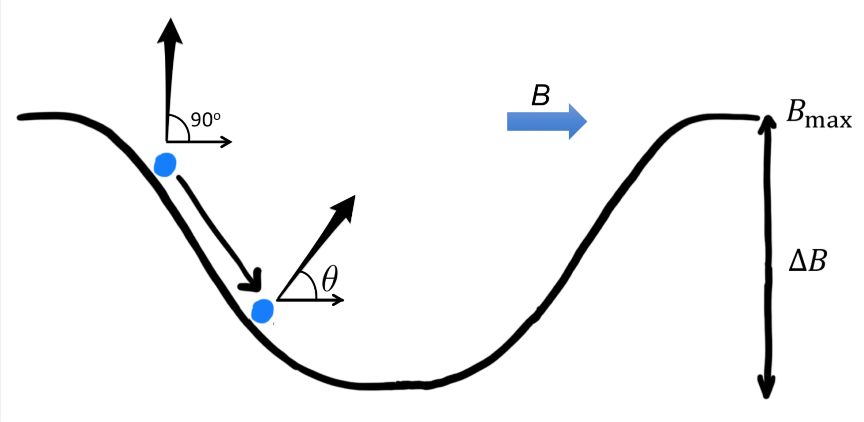


- Electrons must be "transported" to detector → requires very "thin" source $\sigma n < 1$ to avoid collisional losses → large volume with low density to have sufficient intensities

- $\frac{\Delta E}{E} \sim \frac{B_{ana}}{B_{src}} = \left(\frac{R_{src}}{R_{ana}}\right)^2$ → Huge detectors needed

Sensitivity scales with spectrometer size. Already 10m in diameter and 24m in length for **KATRIN**. MAC-E cannot be scaled up beyond **KATRIN**

Q 1.5 How do we know which pitch angles of β-decay electrons are trapped in a magnetic bottle?



Ashtari Esfahani *et al.*, Phys. Rev. C 99, 055501 (2019)

Instantaneous electron KE can be divided into components parallel and perpendicular to B

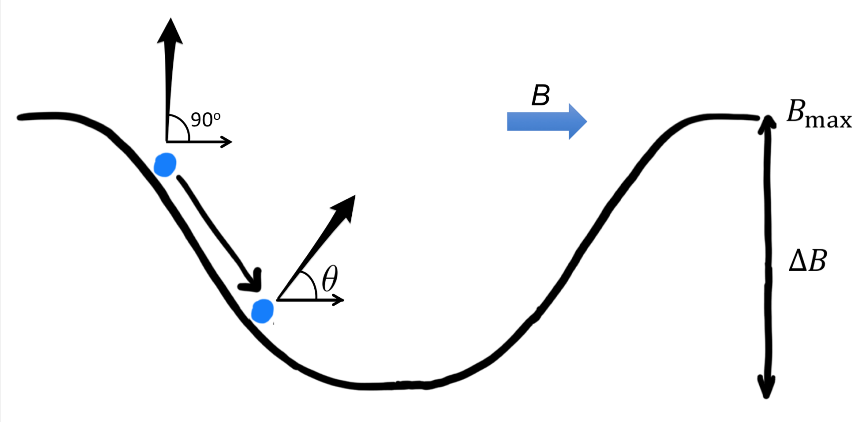
$$E_k = E_{k\|} + E_{k\perp}$$
$$= \frac{1}{2}\frac{p_0^2}{m_e}\cos^2\theta(t) + \mu(t)B(t)$$
$$\mu(t) = \frac{1}{2}\frac{p_0^2}{m_e}\frac{\sin^2\theta(t)}{B(t)}$$

Adiabatic approximation – slowly changing B field means that μ is constant with time

At the bottom of the trap: $\theta = \theta_{\text{bot}}, \quad B = B_{\text{max}} - \Delta B$

For electrons that are *just* trapped: $\theta = \pi/2, \quad B = B_{\text{max}}$

Q 1.5 How do we know which pitch angles of β-decay electrons are trapped in a magnetic bottle?



Ashtari Esfahani *et al.*, Phys. Rev. C 99, 055501 (2019)

Equating the expressions for μ...

$$\frac{1}{2}\frac{p_0^2}{m_e}\frac{\sin^2\theta_{\text{bot}}}{B_{\text{max}}-\Delta B}=\frac{1}{2}\frac{p_0^2}{m_e}\frac{1}{B_{\text{max}}}$$

Rearranging this, find that pitch angle at the bottom of the trap for a *just* trapped electron given by:

$$\theta_{\text{bot}}=\arcsin\left(\sqrt{1-\frac{\Delta B}{B_{\text{max}}}}\right)$$

Therefore, trapping condition given by:

$$\theta_{\text{bot}}\geq\arcsin\left(\sqrt{1-\frac{\Delta B}{B_{\text{max}}}}\right)$$

Q1.6 Back-of-the-envelope neutrino mass sensitivity calculation

$$\frac{dN}{dE_e} = 3rt\,(E_0 - E)\left[(E_0 - E)^2 - m_\beta^2\right]^{1/2}$$

Rate in last eV of spectrum with no mass

Running time

Endpoint energy

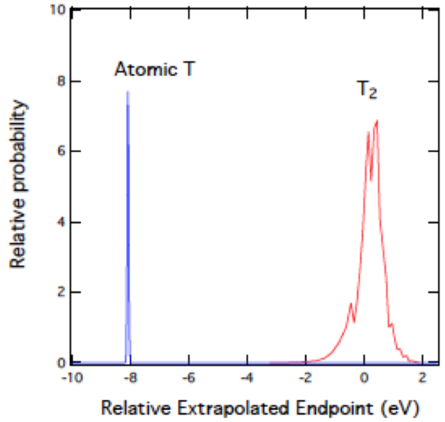
Effective neutrino mass can be determined from single measurement of N events in energy interval, $\Delta E = E_0 - E_1$

Total number of signal events obtained by integrating over energy interval

$$N_{\text{tot}} = rt\,(\Delta E)^3\left[1 - \frac{3}{2}\frac{m_\beta^2}{(\Delta E)^2}\right] + bt\Delta E$$

Background assumed to be constant across energy window proportional to ΔE

Q1.6 Back-of-the-envelope neutrino mass sensitivity calculation



We can define the statistical uncertainty on the effective neutrino mass, $\sigma_{m_\beta^2}$

$$N_{\text{tot}} = rt\,(\Delta E)^3 \left[1 - \frac{3}{2}\frac{m_\beta^2}{(\Delta E)^2}\right] + bt\Delta E$$

This is related to the variance in the total number of events by:

$$\sigma_N^2 = \left(\frac{\partial N_{\text{tot}}}{\partial m_\beta^2}\right)^2 \sigma_{m_\beta^2}$$

$$\frac{\partial N_{\text{tot}}}{\partial m_\beta^2} = -\frac{3rt\Delta E}{2}$$

$$\sigma_{m_\beta^2} = \frac{2}{3rt\Delta E}\sqrt{N_{\text{tot}}}$$

$$= \frac{2}{3rt}\sqrt{rt\Delta E + \frac{bt}{\Delta E}}$$

There is an optimum choice of ΔE that minimises $\sigma_{m_\beta^2}$

$$\Delta E_{\text{opt}} = \sqrt{\frac{b}{r}}$$

Full calculation includes contributions from FSD, instrumental res., etc...

$$\Delta E = \sqrt{\frac{b}{r} + C^2\left(\sigma_{\text{FSD}}^2 + \sigma_{\text{instr}}^2 + \ldots\right)}$$

$$C = \sqrt{8\ln 2}$$

Q1.6 Back-of-the-envelope neutrino mass sensitivity calculation

Detected decay rate can be expressed as

$$r = \Delta\Omega \frac{nV}{\tau_m} \eta$$

Number density (n)

Volume (V)

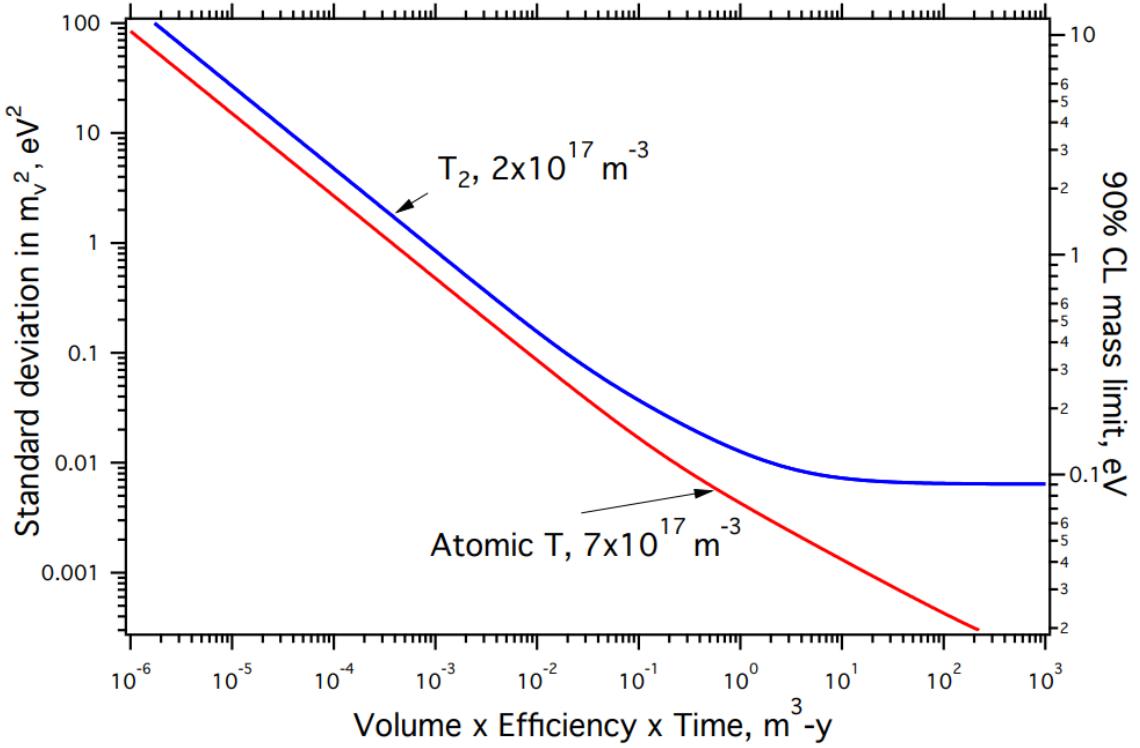
Branching ratio in last eV ~ 2 x 10^{-13} (η)

Trapping and detection eff. ($\Delta\Omega$)

Mean lifetime ~ 12 yr / ln2 (τ_m)

$$\text{90\% CL mass limit} = \sqrt{1.28\sigma_{m_\beta^2}}$$

Derivation in arXiv:1309:7093 (2013)



Magnetic Field Mapping in CRES Region

Q1.7 How can the B-field homogeneity be controlled/measured at a sub-ppm level?

$$f = \frac{1}{2\pi} \frac{eB}{m_e + E_{\mathrm{kin}}/c^2}$$

$$\Downarrow$$

$$\frac{\Delta f}{f} \sim 10^{-6}$$

$$\Downarrow$$

$$\frac{\Delta B}{B} \sim 10^{-6}$$

- Measuring electron energy with a $<$ **1ppm** resolution requires **B-field** known to $\leq$ level
- **Rydberg Magnetometry** can be used to achieved that
- Using D/T atoms as **quantum sensors** for B-field mapping with a precision of **0.1ppm** or better
- **Spatial** resolution of **0.1 mm** achievable

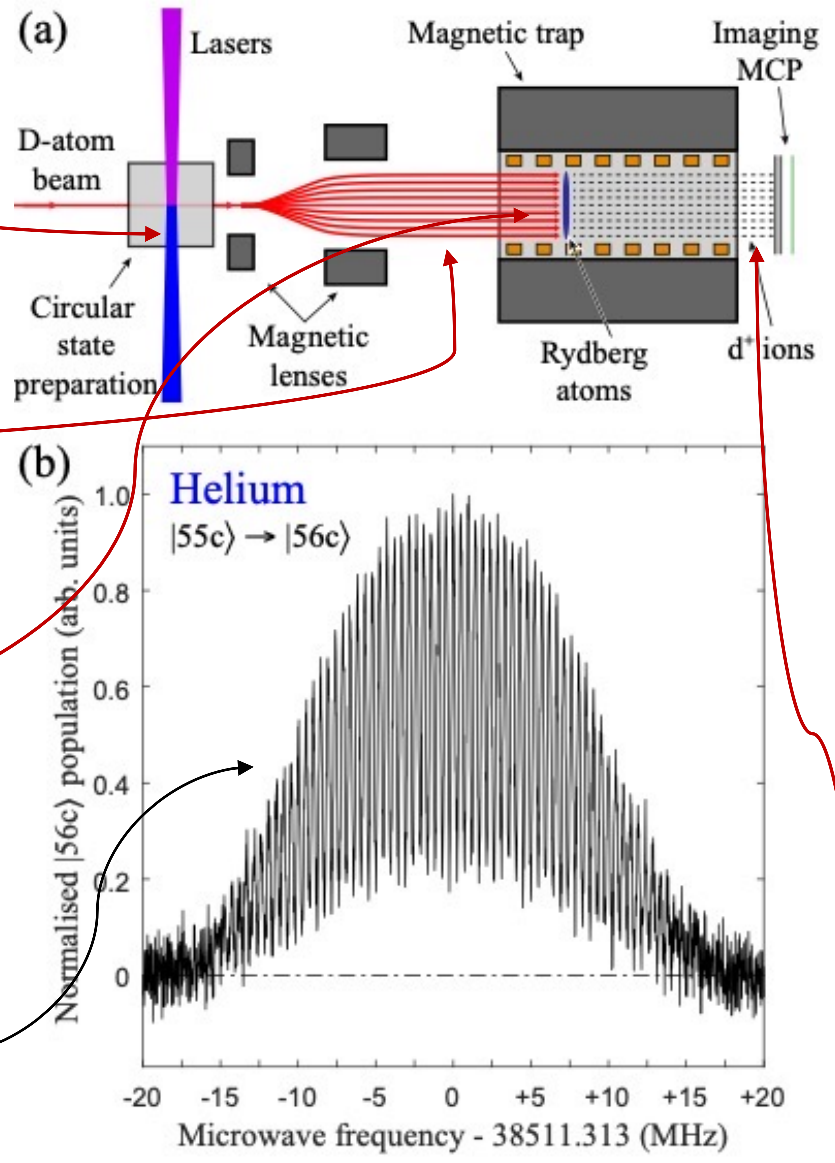
Palmer and Hogan, Mol. Phys. 117, 3108 (2019)

Precise B-field mapping using D/T atoms as *quantum sensors* in the QTNM experiment

D/T atoms are prepared in circular Rydberg states

Beam is expanded to fill the CRES region

At selected time pulses of MW-radiation applied within CRES volume drive Rydberg-Rydberg transition. These transitions are sensitive to B-field variations at
1 part in 10^7 for B=1T (<0.1ppm)



Ramsey spectrum of MW-transition between circular Rydberg states (Helium example)

Transitions are detected by state-selective ionisation