QTFP School QUEST-DMC Tutorial Problems

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1 Question 1

The local dark matter density at Earth can be measured using vertical kinematics of stars near the Sun or extrapolation from galactic rotation curves. Use the resulting value of $\rho_0 = 0.3$ GeV/c^2 cm^{-3} to estimate the number of dark matter particles in a typical mug, for dark matter masses of 100 GeV/c^2 and 0.5 GeV/c^2.

2 Question 2

Galaxy dynamics show that typical dark matter particle velocity in the Milky Ways dark matter halo at our radius from the galactic centre would be 220 km/s (the circular velocity of the galaxy).

a) What kinetic energy would this give for dark matter masses of 100 GeV/c^2 and 0.5 GeV/c^2?
Use this to explain why we can assume that interactions with a nucleus will result in non-relativistic elastic scattering (typical binding energy per nucleon is 6 – 8 MeV).

b) Using conservation of energy and momentum show that the maximum recoil energy in dark matter nucleus scattering is:

$$E_R^{max} = \frac{2\mu^2 v^2}{m_N},$$

where $\mu = m_N m_\chi/(m_N + m_\chi)$ is the reduced mass of the system.

c) Calculate the maximum recoil energy for dark matter masses of 100 GeV/c^2 and 0.5 GeV/c^2 in Xe and ^{3}He targets. Note that the maximum possible velocity of dark matter in the galaxy is the sum of the circular velocity and escape velocity (544 km/s).

3 Question 3

The superfluid gap $\Delta = 1.76\ k_B T_c$ is the pairing energy of Cooper pairs (per helium atom) in superfluid ^{3}He. Here the superfluid transition temperature is T_c=1mK.

a) How many quasiparticles (unpaired helium atoms) are produced if 10 eV of heat is released into the superfluid? Compare this to the number corresponding to the maximum energies calculated in 2. c).

b) Each quasiparticle carries the Fermi momentum $p_F = v_F m*$. What is the momentum distribution of the produced quasiparticles? Here v_F=59 m/s is the Fermi velocity and $m*$=2.8 m_{3He} is the mass of a quasiparticle, where m_{3He} is the mass of a bare ^{3}He atom.