Tsung-Dao Lee Institute

Models for the Muon EDM

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with

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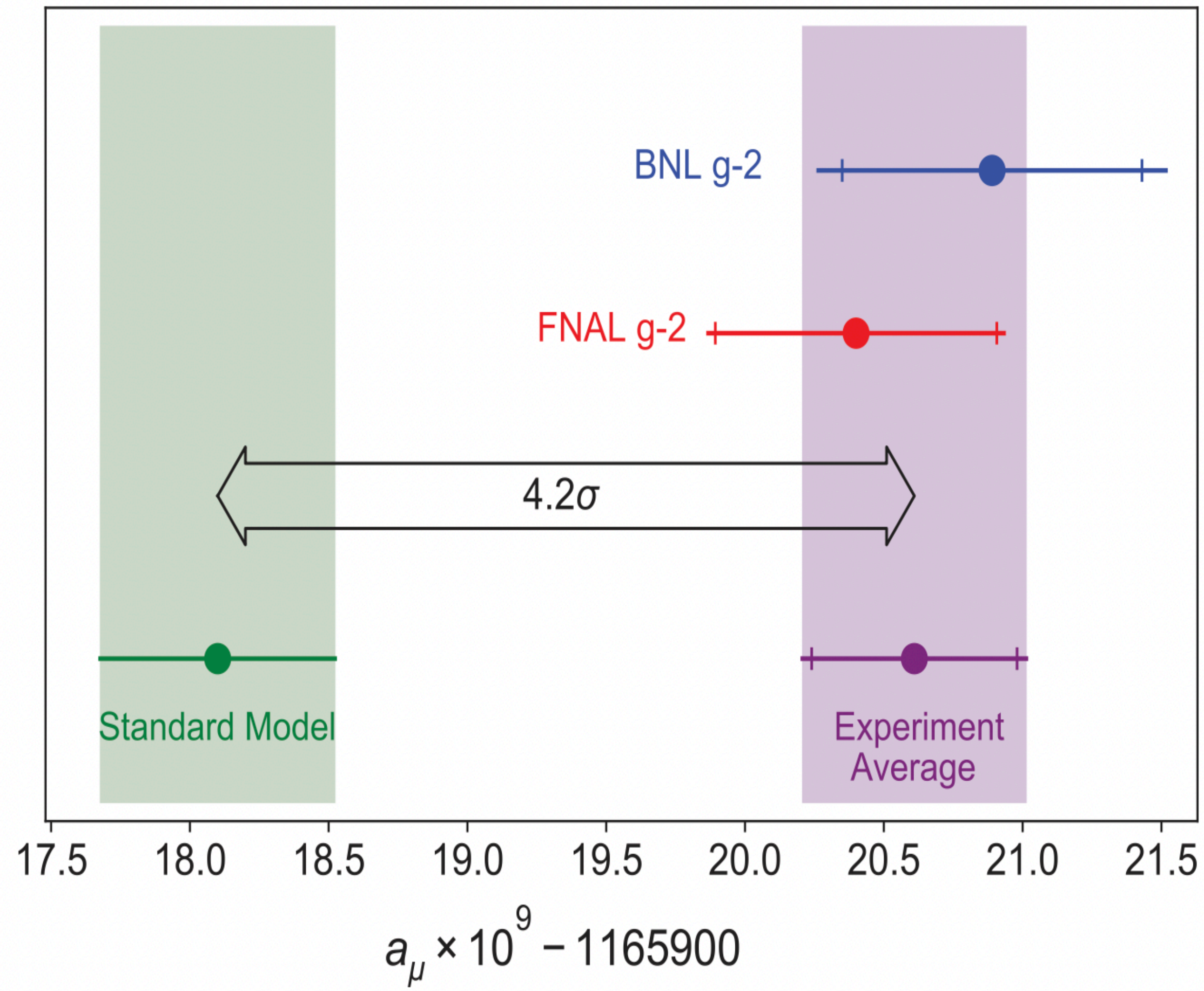
based on PLB831(2022)137194 & 2212.02891.

Muon g-2

Anomalous magnetic moment of the muon $a_\mu \equiv (g-2)/2$

shows discrepancy between theory and experiment :

$$\Delta a_\mu^{\rm obs} = a_\mu^{\rm exp} - a_\mu^{\rm theory}$$
$$= (25.1 \pm 5.9) \times 10^{-10}$$



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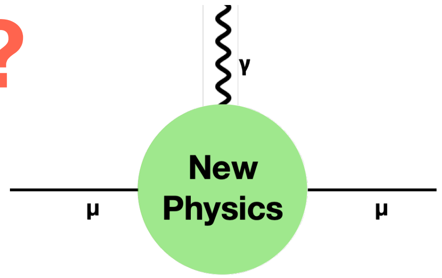
The discrepancy is of the order of the SM electroweak contribution :

$$a_\mu({\rm EW}) = (15.4 \pm 0.1) \times 10^{-10}$$

A hint of TeV scale new physics ?



Muon EDM

The same new physics contribution naturally has the imaginary part.

→ **Muon EDM**

$$\mathcal{H} = -\mu \frac{\mathbf{S}}{|\mathbf{S}|} \cdot \mathbf{B} - d \frac{\mathbf{S}}{|\mathbf{S}|} \cdot \mathbf{E}$$

⇩ Relativistic

$$\text{Time reversal}: \ \mathbf{S} \to -\mathbf{S} \quad \mathbf{B} \to -\mathbf{B} \quad \mathbf{E} \to +\mathbf{E}$$

$$\text{Parity}: \ \mathbf{S} \to +\mathbf{S} \quad \mathbf{B} \to +\mathbf{B} \quad \mathbf{E} \to -\mathbf{E}$$

$$\mathcal{L} \supset -a_\mu \frac{e}{4m_\mu} (\bar{\mu}\sigma^{\alpha\beta}\mu) F_{\alpha\beta} - d_\mu \frac{i}{2} (\bar{\mu}\sigma^{\alpha\beta}\gamma_5\mu) F_{\alpha\beta}$$

The EDM operator requires a chirality flip.

Left- and right-handed fermions carry different charges in the SM.

A Higgs field insertion is needed.

→ The EDM operator is effectively **dimension 6**.

Current Limits

- Current experimental limit (BNL) :

$$d_\mu < 1.8 \times 10^{-19}\, e\,\mathrm{cm}$$

- Indirect limit by measuring EDMs of atoms and molecules :

$$d_\mu < 2 \times 10^{-20}\, e\,\mathrm{cm}$$

Ema, Gao, Pospelov (2022)

One order of magnitude smaller than the direct bound

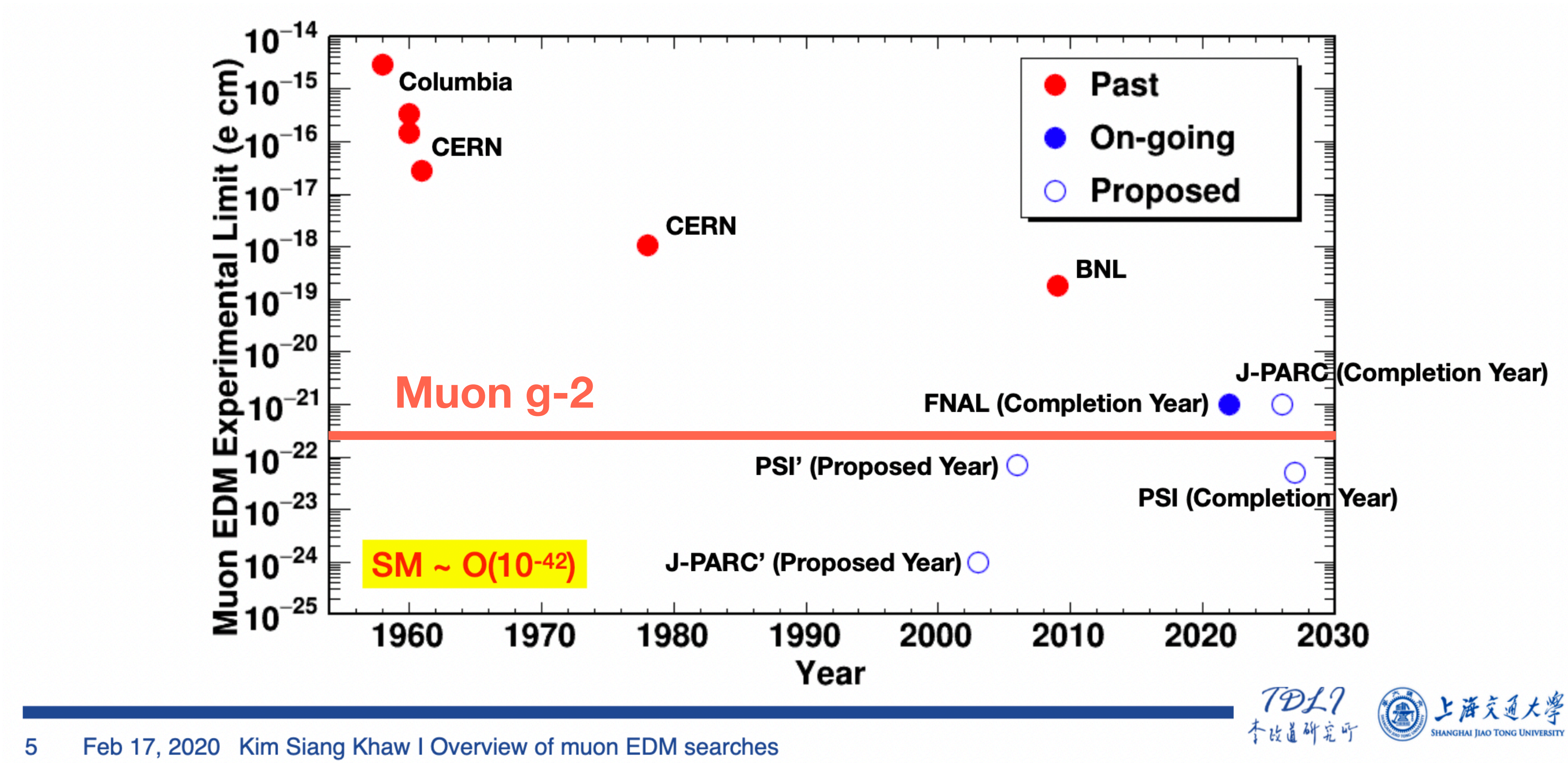
cf. Contribution to the muon g-2 implies …

$$|d_\mu| \sim \frac{e}{2m_\mu}\Delta a_\mu \sim 2.34 \times 10^{-22}\, e\,\mathrm{cm}$$

Still orders of magnitude smaller than the current limits

Future Prospects

Sensitivity to the muon EDM will be improved in the near future.



Explore to what extent new physics to explain the muon g – 2 anomaly is probed by searches for the muon EDM.

Models

Four classes of models to generate the muon g-2 and EDM

- **Spurion approach**

  The chirality flip is provided by the muon Yukawa or some coupling proportional to it.

$$d_\mu \sim \delta_{\rm CPV} \left(\frac{\lambda^2}{16\pi^2}\right)^k \frac{m_\mu}{M^2}$$

$\delta_{\rm CPV}$: CPV phase

M : New physics mass scale

λ : Coupling in loop

- **Flavor changing approach**

  If the muon is converted to the tau by a lepton flavor violating (LFV) interaction, the chirality flip can be provided by the tau Yukawa.

$$d_\mu \sim \delta_{\rm CPV} \frac{y_{\mu\tau}^2}{\lambda^2} \left(\frac{\lambda^2}{16\pi^2}\right)^k \frac{m_\tau}{M^2}$$

$y_{\mu\tau}$: LFV coupling

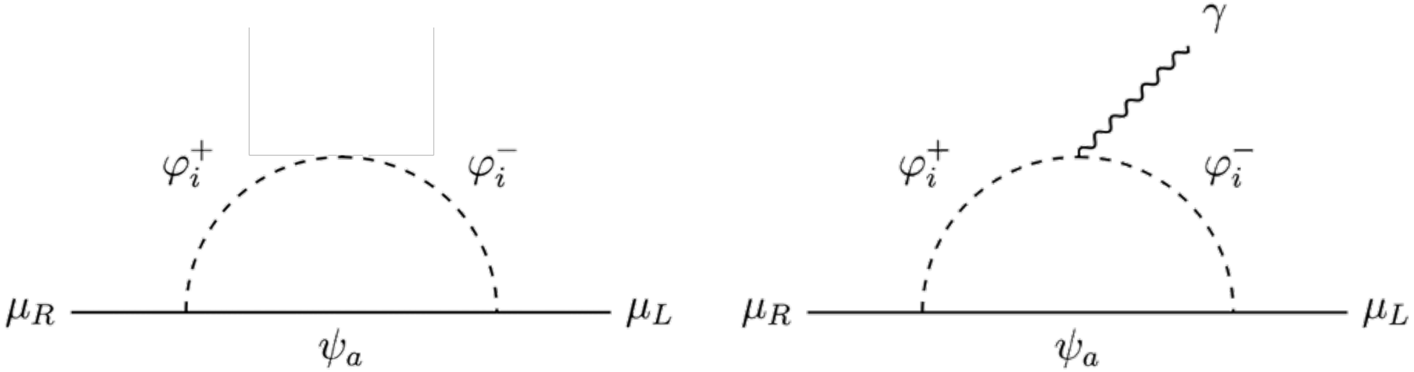
Models

- **Radiative stability approach**

  New physics that produces the muon EDM also generates the muon mass by removing the attached photon.

  Such a contribution to the muon does not exceed the correct value :

$$d_\mu \sim \delta_{\mathrm{CPV}} \frac{m_\mu}{M^2}$$



- **Tuning approach**

  Logical possibility of fine-tuning to cancel a large contribution to the muon mass :

$$d_\mu \sim \delta_{\mathrm{CPV}} \lambda \left(\frac{\lambda^2}{16\pi^2} \right)^k \frac{v_H}{M^2} \lesssim \delta_{\mathrm{CPV}} \frac{4\pi v_H}{M^2}$$

Models

Mass scales of new physics that produce the muon EDM probed by PSI (Fermilab / J-PARC) :

use $\lambda \approx 0.65,\ y_{\mu\tau} \approx 0.3$	1-loop	2-loop
Spurion	300 GeV (75 GeV)	16 GeV (4 GeV)
Flavor changing	580 GeV (140 GeV)	30 GeV (7 GeV)
Radiative stability	5900 GeV (1400 GeV)	
Tuning	1.0×10^6 GeV (2.5×10^5 GeV)	

- ✓ Aside from the tuning approach, the radiative stability approach generates the largest muon EDM.
- ✓ Its near-future measurements can probe mass scales larger than the TeV scale.

Our Focus

- **CP-violating muon specific 2HDM** (spurion approach)

YN, Sato, Shigekami (2022)

CP violation ⬅ $V_\Phi \supset -m_{12}^2 \Phi_1^\dagger \Phi_2 + \frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_2)^2 + \mathrm{h.c.}$

$|d_\mu| \sim \mathcal{O}(10^{-23})\, e\, \mathrm{cm}$

Relative phase is physical

- **Radiative muon mass model** (radiative stability approach)

Khaw, YN, Sato, Shigekami, Zhang (2022)

CP violation ⬅ $\mathcal{L} \supset -m_D \bar{\psi}_L \psi_R - \frac{m_{LL}}{2} \bar{\psi}_L \psi_L^c - \frac{m_{RR}}{2} \overline{\psi_R^c} \psi_R + \mathrm{h.c.}$

$|d_\mu| \sim \mathcal{O}(10^{-22})\, e\, \mathrm{cm}$

Both models can be probed by near-future experiments !

Cμ2HDM

CP-violating muon specific 2HDM (Cμ2HDM)

Two Higgs doublet model that the muon exclusively couples to one scalar doublet

	q_L^a	u_R^a	d_R^a	ℓ_L^e	ℓ_L^τ	ℓ_L^μ	e_R	τ_R	μ_R	Φ_1	Φ_2
$SU(3)_C$	**3**	**3**	**3**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$SU(2)_L$	**2**	**1**	**1**	**2**	**2**	**2**	**1**	**1**	**1**	**2**	**2**
$U(1)_Y$	$1/6$	$2/3$	$-1/3$	$-1/2$	$-1/2$	$-1/2$	-1	-1	-1	$1/2$	$1/2$
Z_4	1	1	1	1	1	i	1	1	i	-1	1

Z_4 symmetry makes only the muon couple to Φ_1

2 scalar doublets

$$\mathcal{L}_Y = -\bar{q}_L\widetilde{\Phi}_2 Y_u u_R - \bar{q}_L \Phi_2 Y_d d_R - \sum_{E=e,\tau} y_E \bar{\ell}_L^E \Phi_2 E_R - y_\mu \bar{\ell}_L^\mu \Phi_1 \mu_R + \text{h.c.}$$

$$V_\Phi = m_{11}^2 \Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left[m_{12}^2 \Phi_1^\dagger\Phi_2 + \text{h.c.}\right] + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+ \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left[\frac{\lambda_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right]$$

CP sources, relative phase is physical

Cμ2HDM

Muon magnetic & electric dipole moments

g-2

$$\Delta a_\mu \simeq \frac{m_\mu^4}{8\pi^2 v^2}\frac{\Delta m_H^2}{m_H^4} t_\beta^2 c_{2\theta} \log\left(\frac{m_H^2}{m_\mu^2}\right)$$

EDM

$$d_\mu \simeq -\frac{e\, m_\mu^3}{32\pi^2 v^2}\frac{\Delta m_H^2}{m_H^4} t_\beta^2 s_{2\theta} \log\left(\frac{m_H^2}{m_\mu^2}\right)$$

- ✓ The dominant contributions to Δa_μ and d_μ are provided by the heavy neutral Higgs bosons $H_{1,2}$
- ✓ Only the neutral scalar couplings contain CP violation
- ✓ Two-loop Barr-Zee type diagrams contribute to the muon EDM but is one order of magnitude smaller than the one-loop contribution

Cμ2HDM

Muon magnetic & electric dipole moments

g-2

$$\Delta a_\mu \simeq \frac{m_\mu^4}{8\pi^2 v^2}\frac{\Delta m_H^2}{m_H^4} t_\beta^2 c_{2\theta} \log\left(\frac{m_H^2}{m_\mu^2}\right)$$

EDM

$$d_\mu \simeq -\frac{e\, m_\mu^3}{32\pi^2 v^2}\frac{\Delta m_H^2}{m_H^4} t_\beta^2 s_{2\theta} \log\left(\frac{m_H^2}{m_\mu^2}\right)$$

✓ Both are enhanced for large tan β and $\Delta m_H^2 = m_{H_2}^2 - m_{H_1}^2$

✓ s_θ : mixing between CP-odd and CP-even heavy Higgses

- **θ → 0 (or π/2)** : Δa_μ ⤴, d_μ ⤵ (CP conserving limit)
- **θ → π/4** : Δa_μ ⤵, d_μ ⤴ (maximal CP violation)

θ → π/8 (s_θ → 0.35) will be important both for Δa_μ and d_μ

Cμ2HDM

Constraints

- ✓ Vacuum stability
- ✓ Perturbative unitarity

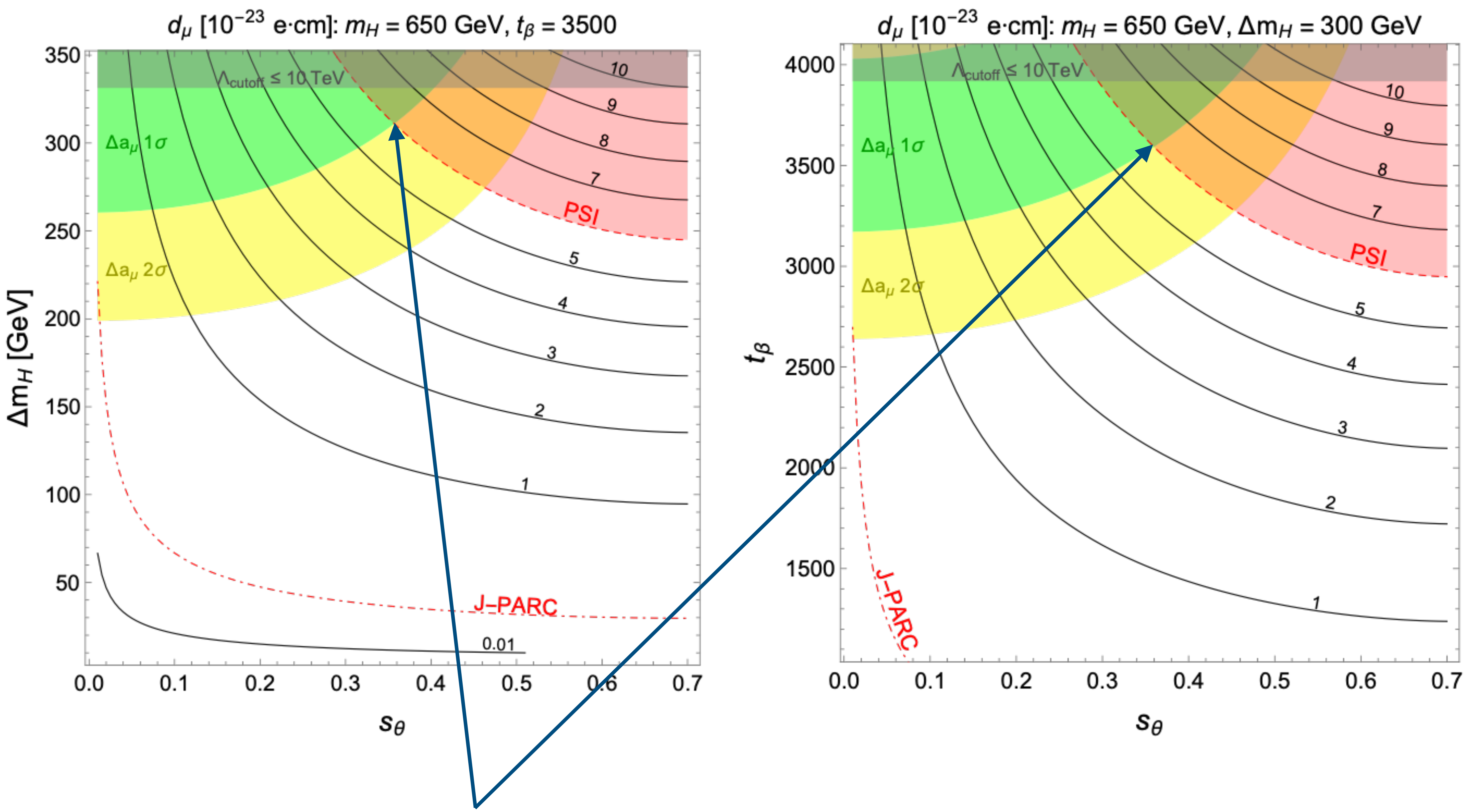
→ Quartic couplings λ_i should be O(1)

- ✓ Landau pole → We optimize the parameters to maximize $\Lambda_{\rm cutoff}$ in numerical analyses and require $\Lambda_{\rm cutoff} \geqq 10$ TeV
- ✓ Heavy Higgs search at the LHC → $m_H \geqq 650$ GeV
- ✓ T parameter ($|\Delta T| < 0.2$) → Satisfied in the focused parameter space
- ✓ $h \to \mu^+\mu^-$ decay → Modification of the Higgs coupling to the muon $\kappa_\mu \simeq 1 + O(0.1)$ satisfies the current constraint

Cμ2HDM

Results $\Delta m_H \equiv \sqrt{m_{H_2}^2 - m_{H_1}^2}$

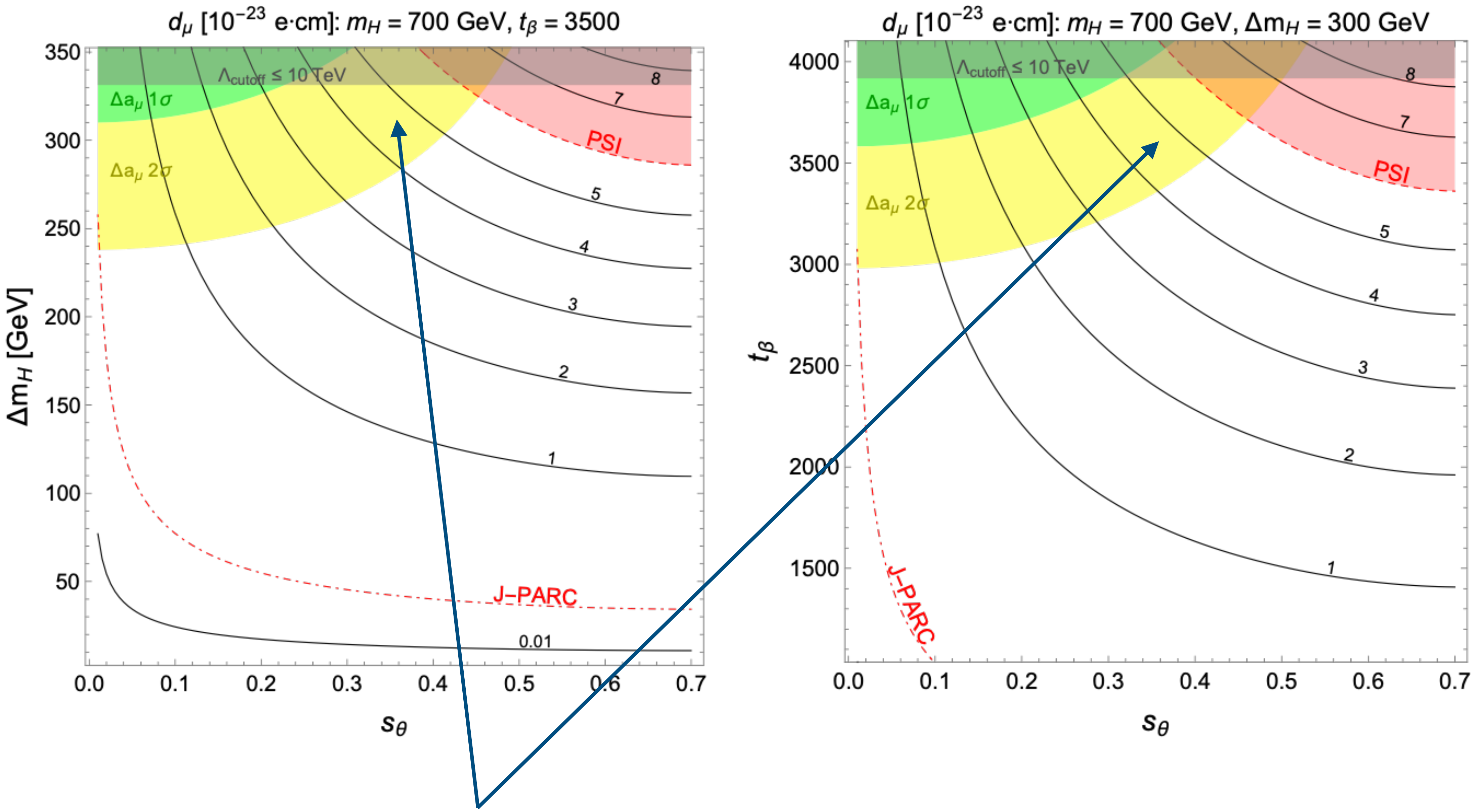
m_H = 650 GeV



Interesting parameter region that Δa_μ is within 1σ deviation and d_μ is within the sensitivity of the PSI experiment at the same time !

Cμ2HDM

Results $\Delta m_H \equiv \sqrt{m_{H_2}^2 - m_{H_1}^2}$ m_H = 700 GeV



There is no region that Δa_μ is within 1σ deviation and
d_μ is within the sensitivity of the PSI experiment at the same time

Radiative m_μ Model

Radiative muon mass model

The model contains a single Dirac fermion ψ and two scalars ϕ, η.

	L_L^μ	μ_R	H	ψ_L	ψ_R	ϕ	η
$SU(2)_L$	**2**	**1**	**2**	**1**	**1**	**2**	**1**
Y	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1
L_μ	$-$	$-$	$+$	$+$	$+$	$-$	$-$
X	$+$	$+$	$+$	$-$	$-$	$-$	$-$
S_a	$+$	$-$	$+$	$+$	$+$	$+$	$-$

Assign three Z_2 symmetries:

- L_μ … muon number
  → avoid LFV constraints
- X … exotic number
  → DM stability
- S_a … softly broken
  → forbid Higgs Yukawa

$$\mathcal{L} \supset \left(-y_\phi \bar{L}_L \phi^\dagger \psi_R - y_\eta \bar{\psi}_L \eta \mu_R - m_D \bar{\psi}_L \psi_R - \frac{m_{LL}}{2} \bar{\psi}_L \psi_L^c - \frac{m_{RR}}{2} \overline{\psi_R^c} \psi_R + \text{h.c.}\right) - V_{\text{scl}}$$

$$V_{\text{scl}} = \sum_{s=H,\phi,\eta} \left[m_s^2 s^\dagger s + \frac{\lambda_s}{2}(s^\dagger s)^2 \right] + \lambda_{H\phi}(H^\dagger H)(\phi^\dagger \phi) + \lambda_{H\eta}(H^\dagger H)(\eta^\dagger \eta) + \lambda_{\phi\eta}(\phi^\dagger \phi)(\eta^\dagger \eta)$$

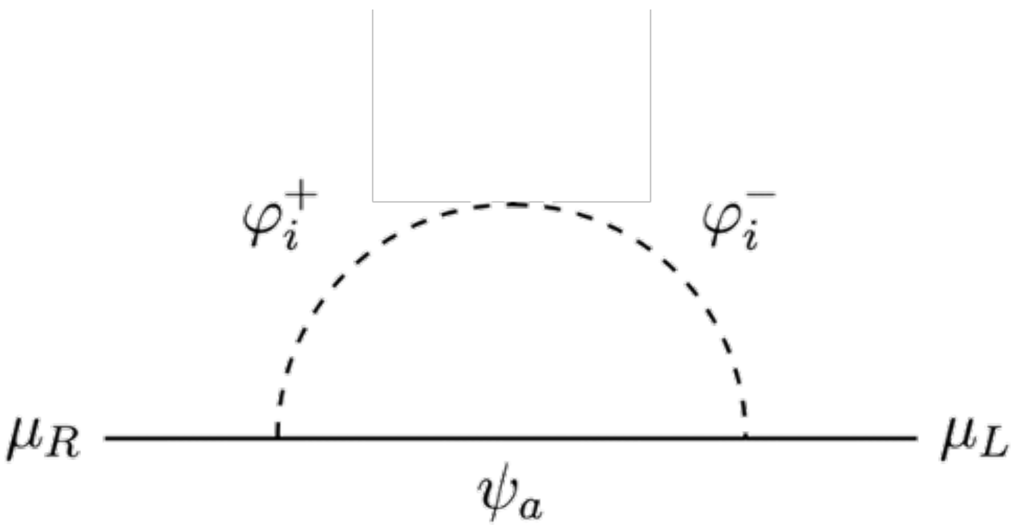
$$+ \lambda'_{H\phi}(H^\dagger \phi)(\phi^\dagger H) + \left(a H \eta^\dagger \phi + \frac{\lambda''_{H\phi}}{2}(H^\dagger \phi)^2 + \text{h.c.} \right)$$

One phase of (m_D, m_{LL}, m_{RR}) cannot be removed ➡ CP violation

Radiative m_μ Model

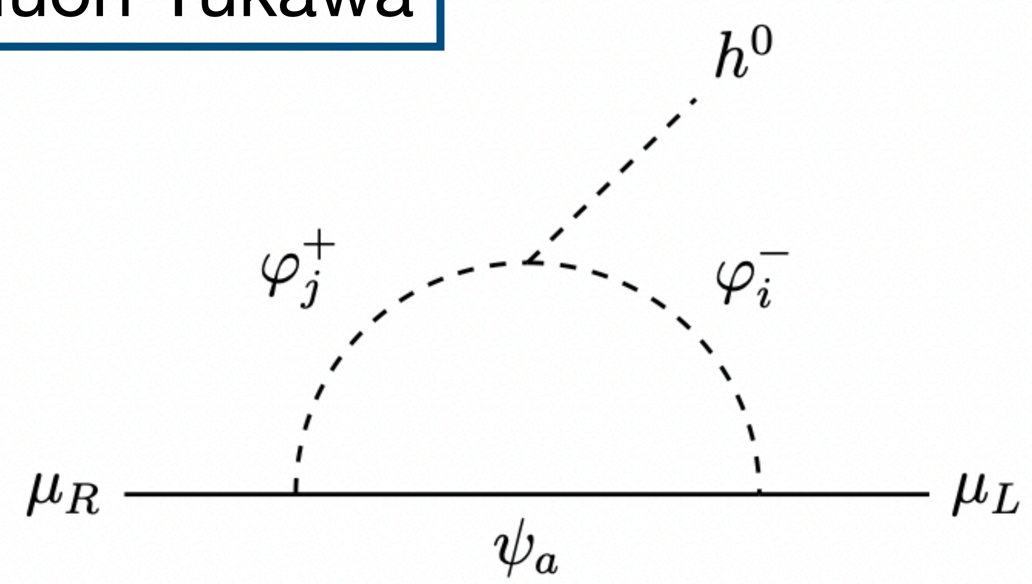
Radiative mass & coupling of the muon

Muon mass



$\psi_{1,2}, \varphi^{\pm}_{1,2}$: mass eigenstates

Muon Yukawa



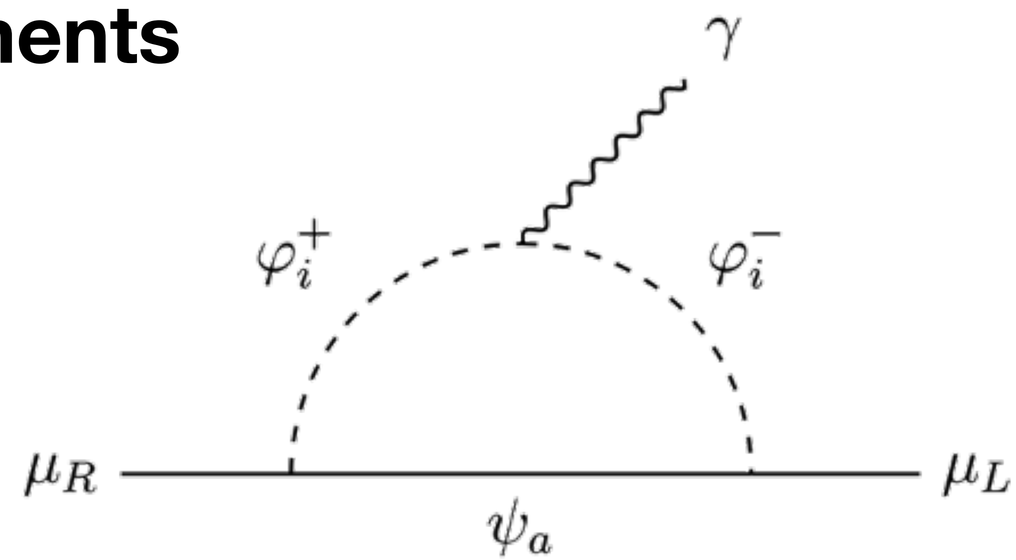
✓ $|m_\mu^{\rm rad}| \neq \frac{|y_\mu^{\rm eff}|}{\sqrt{2}} v_H$ → Constraint will come from $h \to \mu^+\mu^-$ decay

✓ Generated muon mass generally has a phase : $m_\mu^{\rm rad} = m_\mu e^{i\theta_\mu}$

Radiative m_μ Model

Muon magnetic & electric dipole moments

$$\mathcal{L}_{\rm dipole} = -\frac{e}{2}C_T(q^2)\left(\bar{\mu}\sigma^{\alpha\beta}\mu\right)F_{\alpha\beta} - \frac{e}{2}C_{T'}(q^2)\left(\bar{\mu}i\sigma^{\alpha\beta}\gamma_5\mu\right)F_{\alpha\beta}$$



Remove the phase of muon mass : $\mu \to e^{-i\theta_\mu\gamma_5/2}\mu$

$$\Rightarrow \quad -\frac{e}{4m_\mu}a_\mu\left(\bar{\mu}\sigma^{\alpha\beta}\mu\right)F_{\alpha\beta} - \frac{i}{2}d_\mu\left(\bar{\mu}\sigma^{\alpha\beta}\gamma_5\mu\right)F_{\alpha\beta}$$

$$\begin{cases} g-2 : a_\mu = 2m_\mu\left(C_T(0)\cos\theta_\mu + C_{T'}(0)\sin\theta_\mu\right) \\ \text{EDM} : d_\mu = e\left(C_{T'}(0)\cos\theta_\mu - C_T(0)\sin\theta_\mu\right) \end{cases}$$

Loop factor is replaced by m_μ

$$m_\mu^{\rm rad}, C_T(0), C_{T'}(0) \propto \frac{y_\phi y_\eta}{16\pi^2}$$

Radiative m_μ Model

Constraints

- ✓ Vacuum stability
- ✓ Perturbative unitarity
- ✓ *T* parameter ($|\Delta T| < 0.2$)

→ Trilinear (a) and quartic (λ_i) couplings are constrained

- ✓ LHC searches for exotic particles

$$pp \to \varphi_1\varphi_1 \to \mu\mu + \psi_1\psi_1$$

Two muons plus a large missing energy

The signal is similar to that of a pair production of sleptons decaying into leptons and a missing energy

→ The lower bound is smaller than the focused mass range

Radiative m_μ Model

Constraints

✓ $h \to \mu^+\mu^-$ decay (LHC)

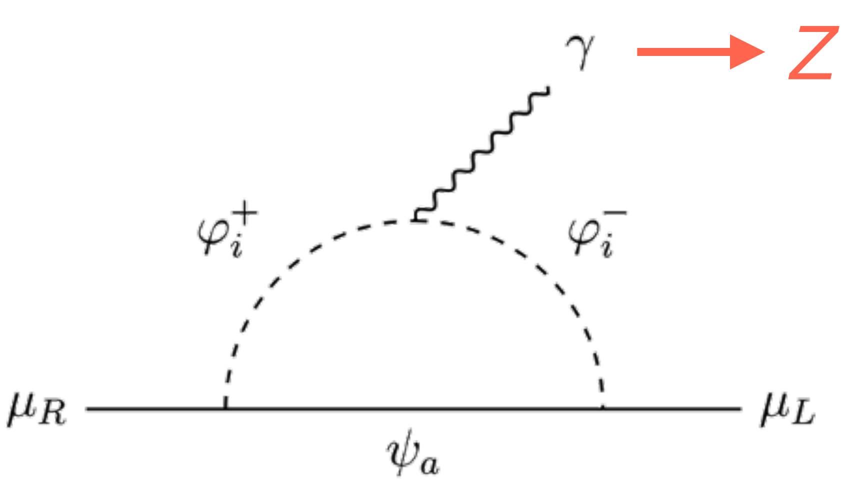
$$\kappa_\mu = \sqrt{\frac{\Gamma(h \to \mu^+\mu^-)|_{\mathrm{SM+NP}}}{\Gamma(h \to \mu^+\mu^-)|_{\mathrm{SM}}}} \qquad \begin{cases} |\kappa_\mu| < 1.47 & \text{(ATLAS)} \\ 0.61 < |\kappa_\mu| < 1.44 & \text{(CMS)} \end{cases}$$

The current constraint is satisfied in the focused parameter region

FCC may be able to measure κ_μ with precision ~ 0.4%

✓ $Z \to \mu^+\mu^-$ decay (LEP)

$$\frac{\Gamma(Z \to \mu^+\mu^-)}{\Gamma(Z \to e^+e^-)} = 1.0009 \pm 0.0028$$



The current constraint is satisfied in the focused parameter region

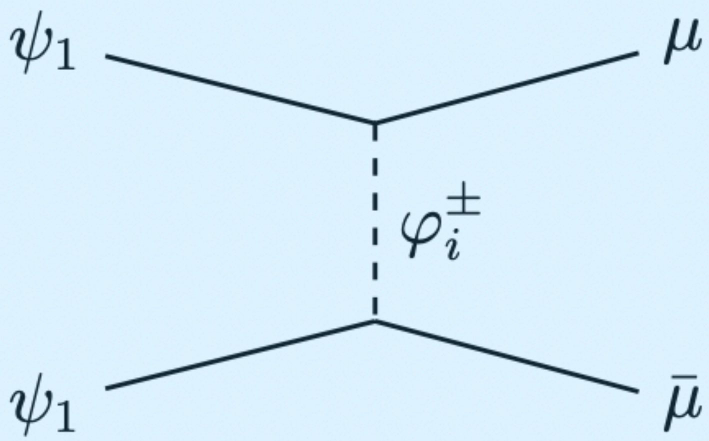
Radiative m_μ Model

Dark matter

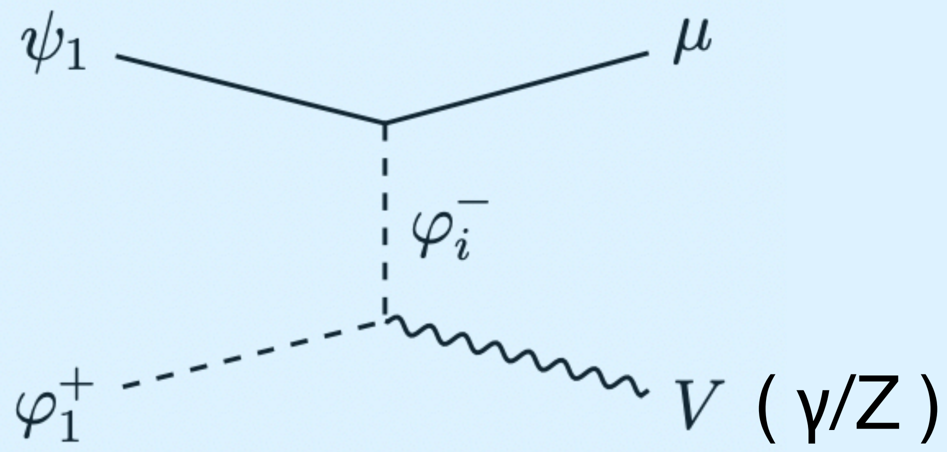
The lightest Majorana fermion becomes a DM candidate.

The correct DM abundance is realized by thermal freeze-out process.

Annihilation modes :



DM self-annihilation



DM-charged scalar coannihilation

We use micrOMEGAs 5.2.13 for calculation.

Radiative m_μ Model

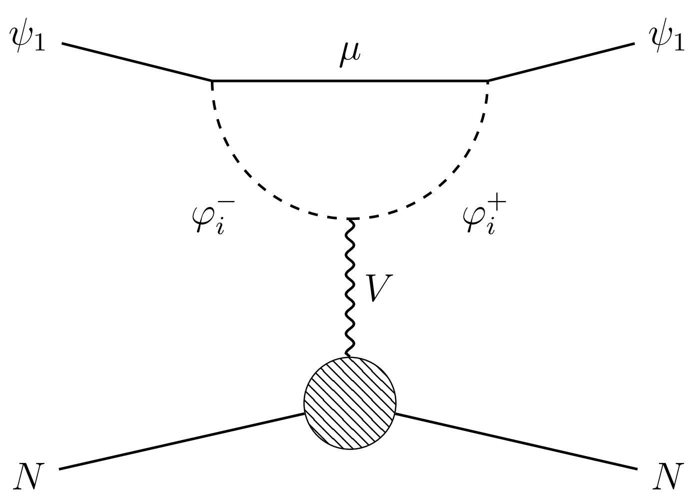
Dark matter

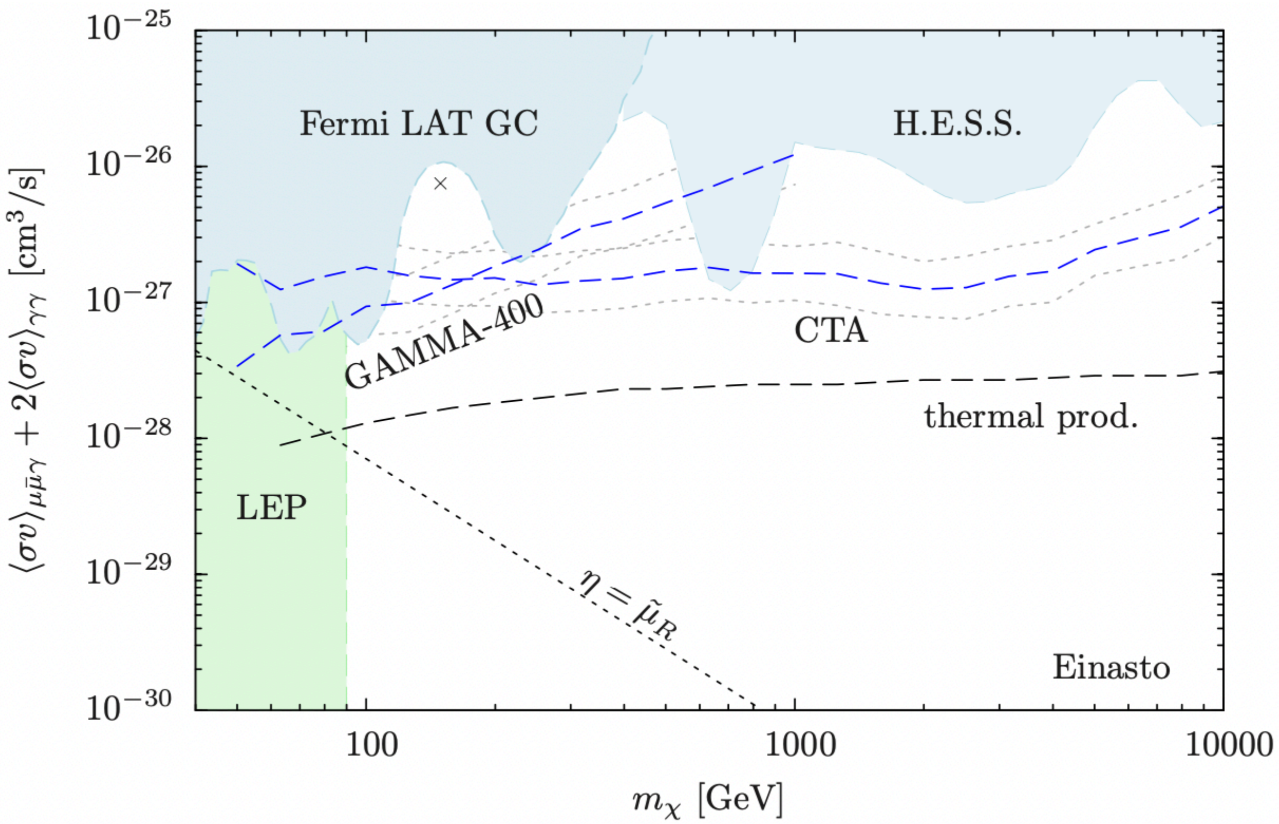
✔ Indirect detection

The annihilation cross section is dominated by $\psi_1\psi_1 \rightarrow \mu\bar{\mu}$

The current constraint on the annihilation cross section is not stringent.

✔ Direct detection





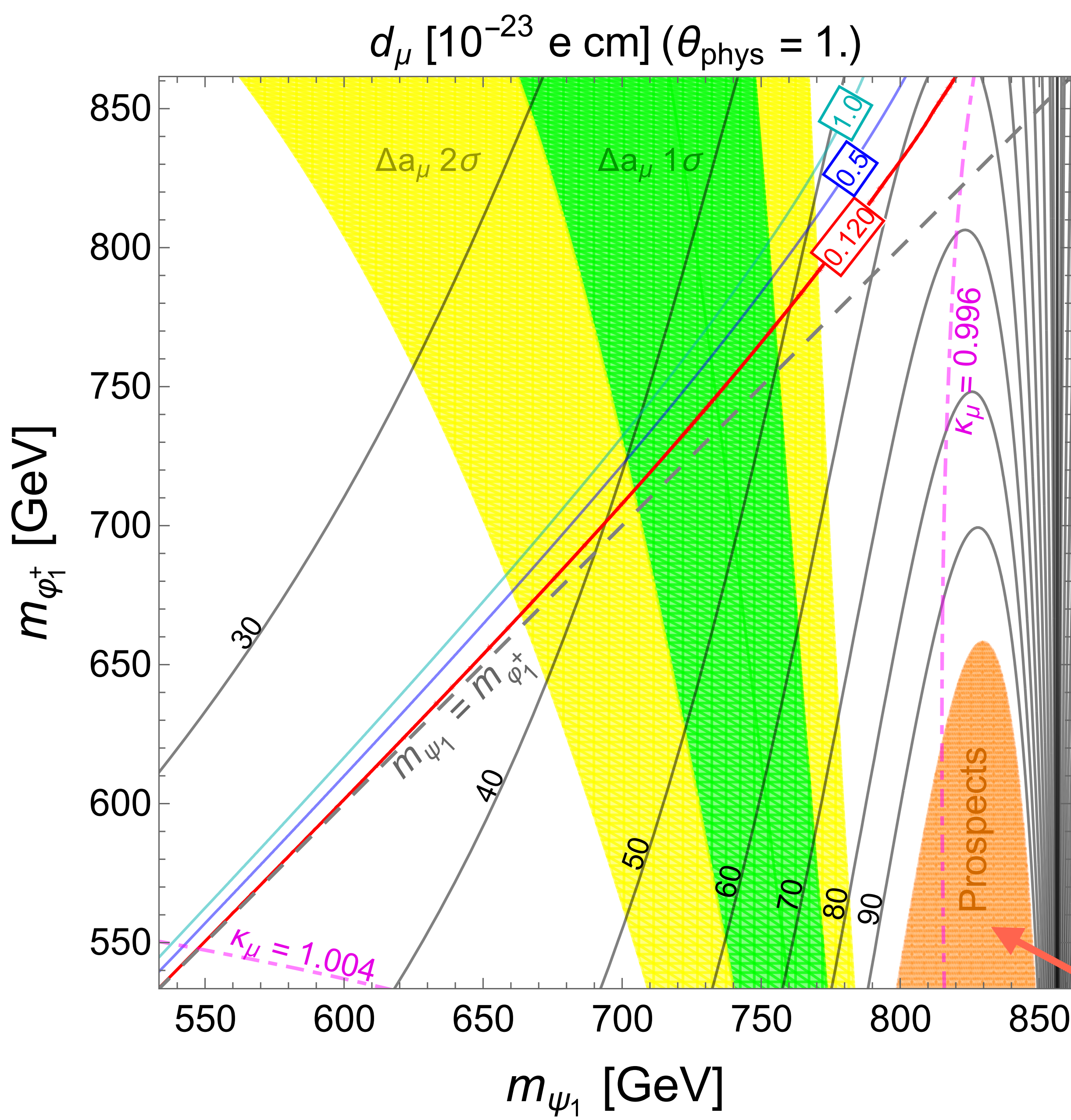
The SI DM-nucleon scattering cross section is smaller than the current limit.

Garny, Ibarra, Pato, Vogl (2013)

Radiative m_μ Model

Results

Inputs: $\begin{cases} y_\phi = 1.2,\ M_\phi^2 = (1000\,\text{GeV})^2,\ a = 900\,\text{GeV}, \\ m_D = 700\,\text{GeV},\ m_{RR} = 1000\,\text{GeV},\ \theta_{\text{phys}} = 1.0 \end{cases}$



- ✓ $d_\mu > 10^{-22}$ e cm !

  Note: PSI sensitivity ∼ 6×10^{-23} e cm
- ✓ Δa_μ: Exotic mass of 700 GeV
- ✓ DM relic density

  → coannihilation is needed
- ✓ Sufficiently small contributions to $h \to \mu^+\mu^-$ and $Z \to \mu^+\mu^-$
- ✓ $d_\mu > 10^{-21}$ e cm is predicted

  … FNAL, J-PARC prospects

Summary

- Two models to predict a large muon EDM

CP-violating muon specific 2HDM

Enhanced by a large tanβ $\sim$ 3500 , $|d_\mu| \sim \mathcal{O}(10^{-23})\, e\, \mathrm{cm}$

Radiative muon mass model

No loop suppression , $|d_\mu| \sim \mathcal{O}(10^{-22})\, e\, \mathrm{cm}$

- Both models can be tested at PSI experiment.

- Our result strongly encourages searches for the muon EDM which might be a key to open the door of new physics !

Thank you.