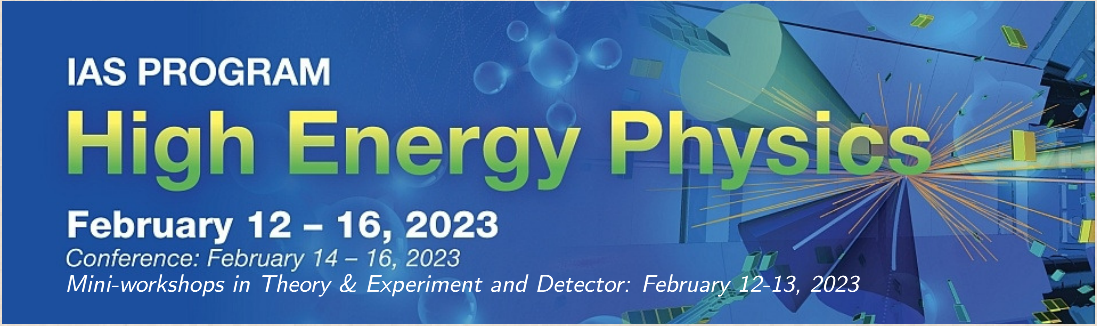

Measurement of $|V_{us}|$ with Tau Decays to Test the First Row CKM Matrix Unitarity

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Introduction

- Standard Model CKM matrix unitary $\Rightarrow$ (first row) $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- since $\sim$2020, first row unitarity test off by 3$-$5 σ
- most precise $|V_{ud}|$ with superallowed $0+ \rightarrow 0+$ nuclear beta decays
- most precise $|V_{us}|$ with $\mathcal{B}(K\ell 2)$, $\mathcal{B}(K\ell 3)$ and lattice QCD form factors and decay constants ($K\ell 2 = K \rightarrow \ell\nu$, $K\ell 3 = K \rightarrow \pi\ell\nu$)
- sub-leading precision $|V_{us}|$ using tau branching fractions

CKM 1st row unitarity test: PDG 2018 $|V_{ud}|$-$|V_{us}|$ review, HFLAV 2018

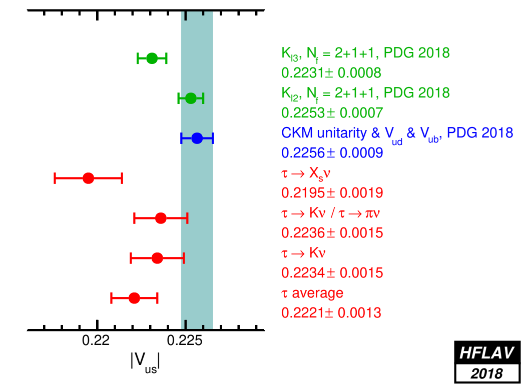
- PDG 2018 $|V_{ud}|$- $|V_{us}|$ review
  - $|V_{us}|$ from kaons ~consistent with unitarity
- HFLAV 2018 report, PRL 93 (2004) 231803
  - $|V_{us}|$ from tau inclusive $>3\sigma$ discrepancy
  - $|V_{us}|$ from tau exclusive ~consistent

HFLAV tau branching fractions fit

- using tau branching fractions of HFLAV-tau fit

note

- there are alternative $|V_{us}|$ determinations from $\tau \to X_s\nu$, which are more consistent with kaons and CKM unitarity
  - Hudspith, Lewis, Maltman & Zanotti 2018
  - Boyle *et al.* 2018



- $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ ~consistent with unitarity except when using $|V_{us}|$ from tau inclusive

CKM 1st row unitarity test: PDG 2020 $|V_{ud}|$-$|V_{us}|$ review, HFLAV 2021 prelim.

$|V_{ud}|$

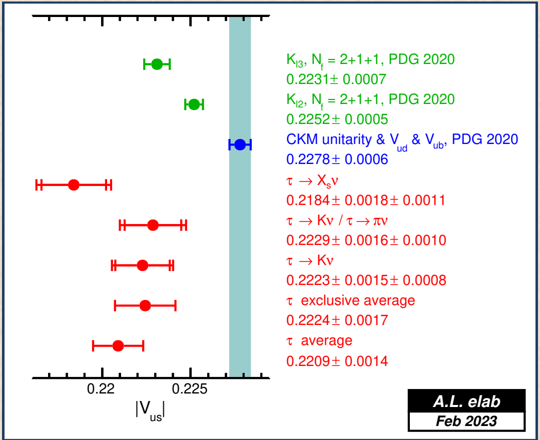
- new dispersive calculation of Δ_R^V inner or universal electroweak radiative corrections (RC) to superallowed nuclear beta decays
  Seng, Gorchtein & Ramsey-Musolf, Phys. Rev. D 100, 013001 (2019)
  - $\sim 2\times$ more precise
  - significant shift

$|V_{us}|$ from kaons

- updated more precise lattice QCD constants

$|V_{us}|$ from tau

- HFLAV 2021 prelim., PDG 2021 $|V_{ud}|$



- $|V_{ud}|$ - $|V_{us}|_K$ anomaly $\sim 3\,\sigma$

  (PDG uncertainty scale factor = 2 on $|V_{us}|_K$ because $|V_{us}|_{K\ell 3}$ and $|V_{us}|_{K\ell 2}$ are inconsistent)

CKM 1st row unitarity test: PDG 2021 $|V_{ud}|$-$|V_{us}|$ review, HFLAV 2021 prelim.

$|V_{ud}|$

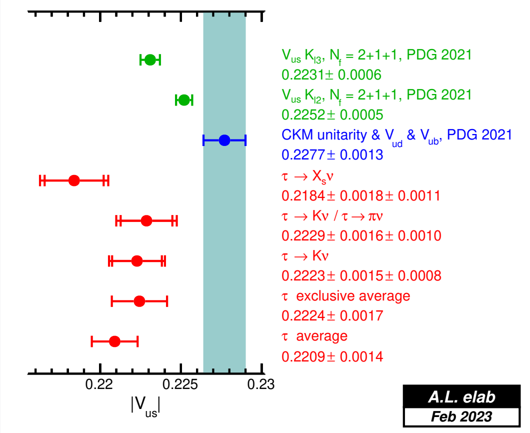
- J.C.Hardy & Ii.S.Towner, PRC 102, 045501 (2020)
  - revised experimental inputs
Marciano and Sirlin 2006	2.361 ± 0.038
Seng et al. 2018/2019	2.467 ± 0.022
Czarnecki, Marciano and Sirlin 2019	2.426 ± 0.032
Adopted value for Δ_R^V	2.454 ± 0.019
  - increased systematic uncertainty (new nuclear corrections)

$|V_{us}|$ from kaons

- improved K_{e3} radiative corrections, Seng, Gorchtein & Ramsey-Musolf, arXiv:2103.04843 [hep-ph]
- new calculation of $|V_{us}|_{K\ell 3}$ Seng, Galviz, Marciano, Meißner, arXiv:2107.14708 [hep-ph]

$|V_{us}|$ from tau

- HFLAV 2021 prelim., PDG 2021 $|V_{ud}|$



- $|V_{ud}|$ - $|V_{us}|_K$ anomaly $\sim 3\,\sigma$
  - no scale factor on $|V_{us}|_K$
  - $\sim 5\,\sigma$ without increased $|V_{ud}|$ systematics

$|V_{us}|$ determinations fron kaons

- $\Gamma(K \to \pi\ell\bar{\nu}_\ell[\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}^K \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\ell \left(1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)^2$ $\quad K_{\ell 3}$
- $\frac{\Gamma(K^- \to \ell^- \bar{\nu}_\ell)}{\Gamma(\pi^- \to \ell^- \bar{\nu}_\ell)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \frac{m_K(1 - m_\ell^2/m_K^2)^2}{m_\pi(1 - m_\ell^2/m_\pi^2)^2} (1 + \delta_{EM})$ $\quad K_{\ell 2}$
- $I_K^\ell =$ phase-space form factor integrals

$|V_{us}|$ calculation using tau branching fractions

$|V_{us}|$ from tau "inclusive"

- $$|V_{us}|_{\tau s} = \sqrt{R(\tau \to X_{\text{strange}}\,\nu) / \left[\frac{R(\tau \to X_{\text{non-strange}}\,\nu)}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}} \right]}$$ $\tau \to X_s \nu$
  - $R(\tau \to X\,\nu) = \mathcal{B}(\tau \to X\,\nu) / \mathcal{B}(\tau \to e\bar{\nu}\nu)_{\text{universality improved}}$
  - $\delta R_{\tau,\text{SU3 breaking}}$ computed with perturbative QCD (CIPT, FOPT) OPE + WA m_s estimate
  - E. Gamiz *et al.*, JHEP 01 (2003) 060, PRL 94 (2005) 011803, Nucl. Phys .Proc. Suppl. 169 (2007) 85, PoS KAON (2008) 008

$|V_{us}|$ from tau "exclusive"

- $$\frac{\mathcal{B}(\tau^- \to K^- \nu_\tau)}{\mathcal{B}(\tau^- \to \pi^- \nu_\tau)} = \left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \frac{|V_{us}|^2_{\tau K/\pi}}{|V_{ud}|^2} \frac{\left(m_\tau^2 - m_K^2\right)^2}{\left(m_\tau^2 - m_\pi^2\right)^2} (1 + \delta R_{\tau K/\tau\pi})$$ $\tau \to K / \tau \to \pi$
- $$\mathcal{B}(\tau^- \to K^- \nu_\tau) = \frac{1}{16\pi} \left(\frac{G_F}{\hbar^3 c^3}\right)^2 |V_{us}|^2_{\tau K} f_{K\pm}^2 \frac{\tau_\tau}{\hbar} m_\tau^3 c^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}^{m_\tau} (1 + \delta R_{\tau K})$$ $\tau \to K$

$|V_{us}|$ calculation notes

tau branching fractions from HFLAV tau branching fractions fit

- HVLAV 2021 report (prelim.): arXiv:2206.07501

Decay constants and form factors from Lattice QCD FLAG averages

- FLAG review 2021, EPJC 82, 869 (2022)

radiative corrections definitions

- $\Gamma(\tau^- \to \pi^- \nu_\tau) = \Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau) S_{EW}^{m_\tau}(1 + \delta R_{\tau\pi})$
- $\Gamma(\tau^- \to K^- \nu_\tau) = \Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau) S_{EW}^{m_\tau}(1 + \delta R_{\tau K})$
- $\dfrac{\Gamma(\tau^- \to K^- \nu_\tau)}{\Gamma(\tau^- \to \pi^- \nu_\tau)} = \dfrac{\Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau)}{\Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau)} \dfrac{S_{EW}^{m_\tau}(1 + \delta R_{\tau K})}{S_{EW}^{m_\tau}(1 + \delta R_{\tau\pi})} = \dfrac{\Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau)}{\Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau)}(1 + \delta R_{\tau K/\tau\pi})$

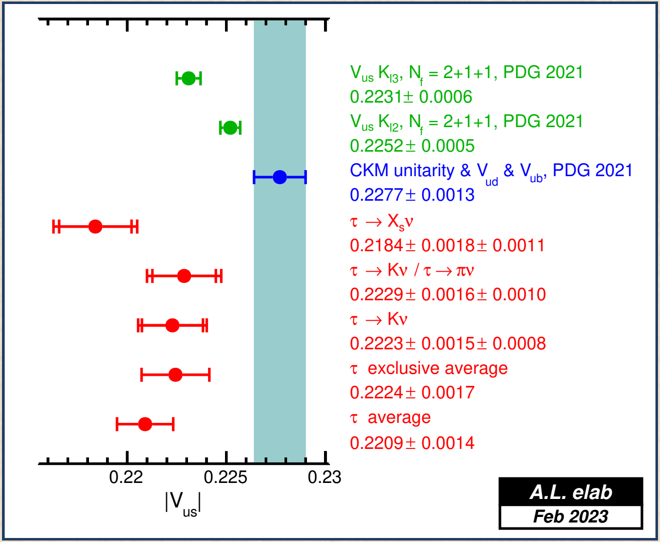
recent new radiative corrections for tau hadronic decays

- [M.A.Arroyo-Ureña, G.Hernández-Tomé, G.López-Castro, P.Roig, I.Rosell, PRD 104 (2021) L091502]
- $\delta R_{\tau\pi} = (-0.24 \pm 0.56)\%$, $\delta R_{\tau K} = (-0.15 \pm 0.57)\%$, $\delta R_{\tau K/\tau\pi} = (0.10 \pm 0.80)\%$
  - $\delta R_{\tau\pi}$ and $\delta R_{\tau K}$ ~uncorrelated
- larger uncertainties than previous ones [*] but more reliable
  [*] R.Decker, M.Finkemeier, PLB 334 (1994) 199; NPB 438 (1995) 17; NPB 40 (1995) 453 (P.S.)

$\mathcal{B}(\tau \to X_s \nu)$ from HFLAV 2021 prelim. fit

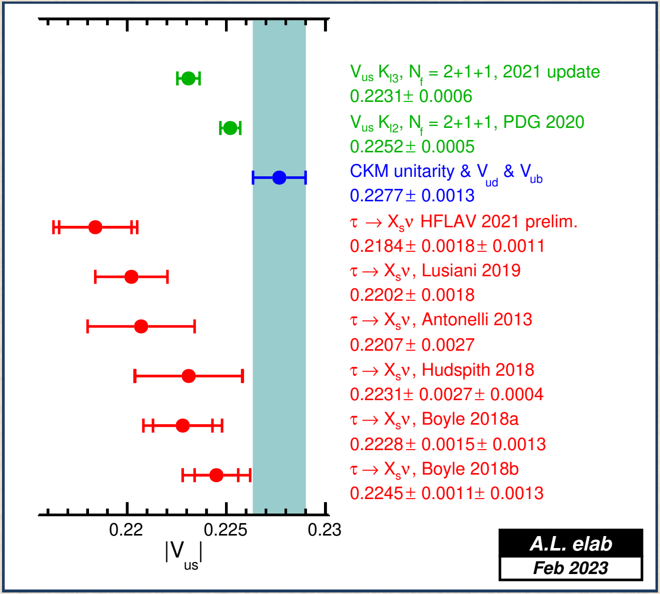
Tau decay mode	Branching fraction (%)
$K^- \nu_\tau$	0.6957 ± 0.0096
$K^- \pi^0 \nu_\tau$	0.4322 ± 0.0148
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0634 ± 0.0219
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0465 ± 0.0213
$\pi^- \bar{K}^0 \nu_\tau$	0.8375 ± 0.0139
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.3810 ± 0.0129
$\pi^- \bar{K}^0 2\pi^0 \nu_\tau$ (ex. K^0)	0.0234 ± 0.0231
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.0222 ± 0.0202
$K^- \eta \nu_\tau$	0.0155 ± 0.0008
$K^- \pi^0 \eta \nu_\tau$	0.0048 ± 0.0012
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0094 ± 0.0015
$K^- \omega \nu_\tau$	0.0410 ± 0.0092
$K^- \phi(K^+ K^-) \nu_\tau$	0.0022 ± 0.0008
$K^- \phi(K^0_S K^0_L) \nu_\tau$	0.0015 ± 0.0006
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.2924 ± 0.0068
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.0387 ± 0.0142
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$X_s^- \nu_\tau$	2.9076 ± 0.0478

$|V_{us}|$ from tau branching fractions



- HFLAV 2021 fit prelim., takes into account correlations between tau BR measurements

$|V_{us}|$ from tau inclusive, alternative determinations



$|V_{us}|$ from tau inclusive, alternative determinations

A. Pich *et al.* group

- original simple formulation
  - easily re-used
- $\delta R_{\tau,\text{SU3 breaking}}$ calculations
- simple $|V_{us}|$ calculation, easily used with updated tau branching fractions
- procedure used by HFLAV 2021 prelim., Lusiani 2019, Antonelli 2013

K. Maltman *et al.* group

- more elaborate procedures
  - cannot be used without collaborating with authors
- uses experimental measurements to suppress perturbative QCD uncertainties
- published Hudspith 2018 and Boyle 2018a/b, more details in the following

- so far there is no general consensus about recommended $|V_{us}|$ calculation
- HFLAV reports $|V_{us}|$ with Gamiz *et al.* calculation because it can be easily updated, while K.Maltman *et al.* calculations are much more complex and would require assistance of the authors

$|V_{us}|$ from $\tau \to X_s \nu$ determinations

M. Antonelli *et al.*, JHEP 10 (2013) 76

- predict tau BRs $\mathcal{B}(\tau \to K\nu)$, $\mathcal{B}(\tau \to K\pi^0\nu)$, $\mathcal{B}(\tau \to K^0_s\pi\nu)$ from kaon BRs $\mathcal{B}(K \to \ell\nu)$, $\mathcal{B}(K \to \ell\pi^0\nu)$
- replace measurements of above tau branching fractions their predictions
- compute $|V_{us}|$ with Gamiz *et al.* 2003 technique
- other tau branching fractions from HFLAV 2012

A.L., SciPost Phys. Proc. 1 (2019) 1

- use Antonelli 2013 predictions of 3 tau branching fractions, but rather than replacing the respective tau measurements, statistically combine predictions and measurements in modified HFLAV tau BRs fit
- compute $|V_{us}|$ with Gamiz *et al.* 2003 technique
- other tau branching fractions from:
  - HFLAV Spring 2017
  - *BaBar* ICHEP 2018 results (6 channels)

$|V_{us}|$ from $\tau \to X_s \nu$ determinations

J. Hudspith *et al.*, PLB 781 (2018) 206

- uses tau spectral functions

  "a combination of continuum and lattice results is shown to suggest a new implementation of the flavor-breaking sum rule approach in which not only |Vus|, but also D>4 effective condensates, are fit to data."
- replace tau BRs $\mathcal{B}(\tau \to K\pi^0\nu)$, $\mathcal{B}(\tau \to K^0_s\pi\nu)$ with Antonelli 2013 predictions
- other tau branching fractions from HFLAV Spring 2017

P. Boyle *et al.*, PRL 121 (2018) 202003

- compute $|V_{us}|$ from tau inclusive with a nove technique using
  - tau spectral functions
  - lattice QCD
- capitalizes on LQCD work for muon $g-2$ hadronic contribution
- two $|V_{us}|$ results:

  Boyle 2018a: using HFLAV Spring 2017 results

  Boyle 2018b: HFLAV Spring 2017 replacing $\mathcal{B}(\tau \to K\pi^0\nu)$, $\mathcal{B}(\tau \to K^0_s\pi\nu)$ with Antonelli 2013

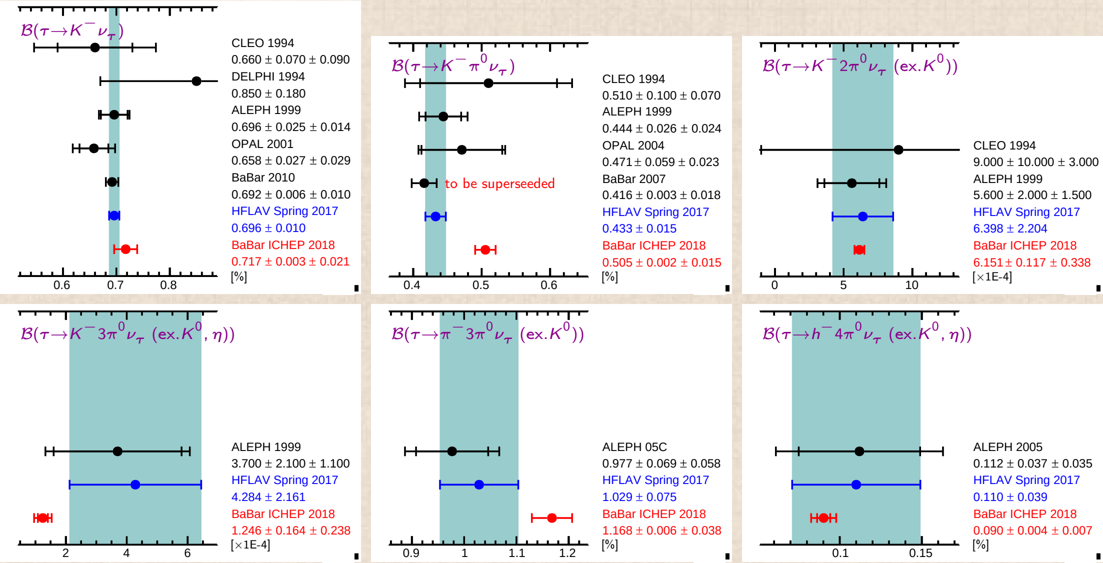
Uncertainties

method	experiment [%]	theory [%]	lattice QCD [%]	rad.corr. [%]
$\tau \to X_s \nu$	0.84	0.49		
$\tau \to K / \tau \to \pi$	0.72		0.18	0.40
$\tau \to K$	0.69		0.19	0.29

uncertainties prospects

- experiment and rad.corr. uncertainties had minor or no improvements since LEP 1 times
- lattice QCD uncertainties decreased substantially in recent years
- in general, it is hard to precisely measure tau branching fractions
- Belle II, super charm-tau factories and FCC(Z) will help

BABAR ICHEP 2018 preliminary tau BR measurements



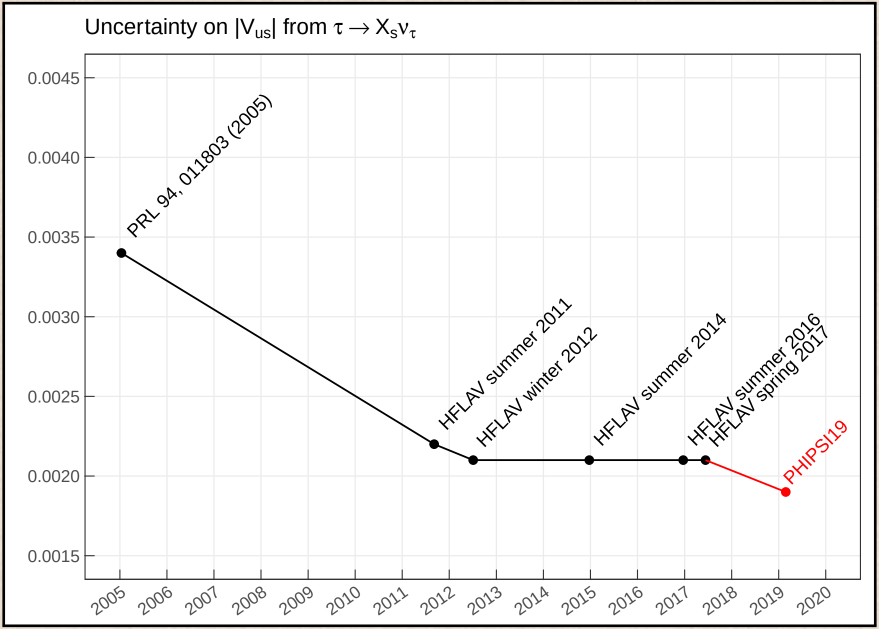
- common fit to 6 tau BR, presented at ICHEP 2018 as preliminary, not yet published

$|V_{us}|$ from $\tau \to X_s \nu_\tau$ uncertainties budget: improvements with BABAR 2018 results

Updated $|V_{us}|$ from $\tau \to X_s \nu_\tau$ uncertainty budget

$\pi^- \bar{K}^0 2\pi^0 \nu_\tau$ (ex. K^0)	0.3933
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0464
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0449
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.3452
$K^- \pi^0 \nu_\tau$	0.1575
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.2438
$\pi^- \bar{K}^0 \nu_\tau$	0.2373
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.2201
$K^- \nu_\tau$	0.1453
$K^- \omega \nu_\tau$	0.1573
$K^- \nu_\tau$	0.1453
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.1148
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0254
$K^- \pi^0 \eta \nu_\tau$	0.0198
$K^- \eta \nu_\tau$	0.0137
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0136
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0094
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0021
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0010
$\tau \to$ non-strange	0.0855
$\mathcal{B}_e^{\text{univ}}$	0.0044
theory	0.4863

Precision of $|V_{us}|$ from $\tau \to X_s\nu$ over time



Conclusions

- $|V_{us}|$ can be calculated using measurements of tau branching fractions and spectral functions
- tau inclusive $|V_{us}|$ measurement does not require lattice QCD form factors and decay constants
  - however, no general consensus on best estimation and suppression of hadronic uncertainties
- more precision requires
  - more precise tau branching fractions (hard)
  - eventually, more precise radiative corrections, form factors, decay constants

Thanks for your attention!

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