BSM opportunities in Triangle Singularity

--- a kinematic perspective

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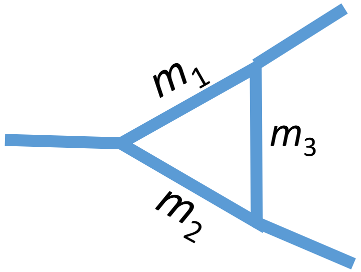

2211.12920
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HKUST, 2023/02/14

Outline

- Triangle/`Landau’ singularity
- T.S. in *t*-channel scattering
- T.S. at collider: beyond a petty threshold?

Landau Singularity

Karplus, Sommerfield, Wichmann, Phys.Rev. 111 (1958)
Landau, Nucl. Phys. 13, 181–192 (1959).

- A situation when *all internal particles go on shell inside a loop*

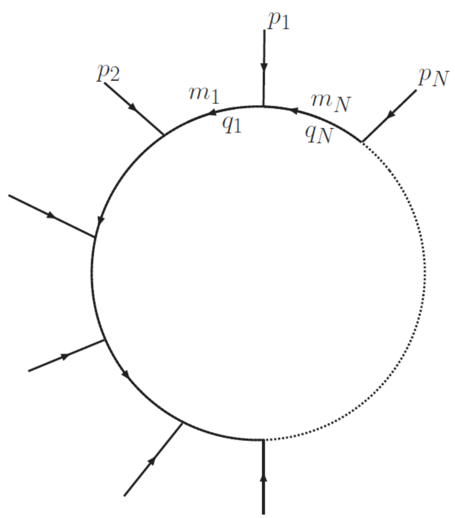


figure from 0806.1498

$$\int \frac{d^D q}{(2\pi)^D i} \frac{1}{D_1 D_2 \cdots D_N}$$

$$\sim \int_0^\infty d\alpha_1 \ldots d\alpha_N \, \delta\left(\sum_i \alpha_i - 1\right) \int \frac{d^D q}{(2\pi)^D i (\alpha_1 D_1 + \cdots + \alpha_N D_N)^N}$$

\`Landau Conditions' at leading singularity

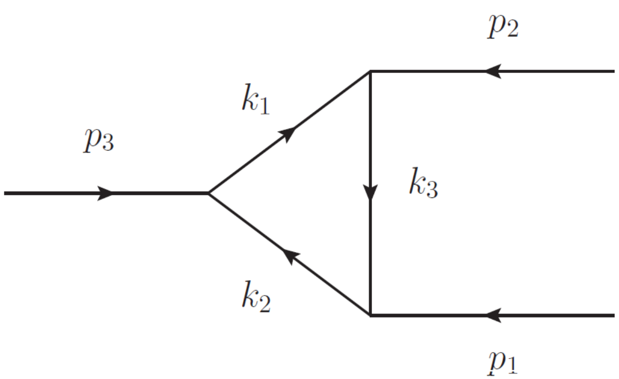
$\sum_i \alpha_i$=1 and α_i>0; $\sum_i \alpha_i \, q_i^\mu = 0$;

$\alpha_i (q_i^2 - m_i^2) = 0$

L.D.Landau 1958'

@ NLO: a.k.a. the \`anomalous threshold'

N=3: triangle singularity



$$\begin{cases} \alpha_1 k_1^\mu + \alpha_2 k_2^\mu + \alpha_3 k_3^\mu = 0 \\ k_1^2 - m_1^2 = k_2^2 - m_2^2 = k_3^2 - m_3^2 = 0. \end{cases}$$

$$\alpha_{1,2,3} \in (0,1)$$

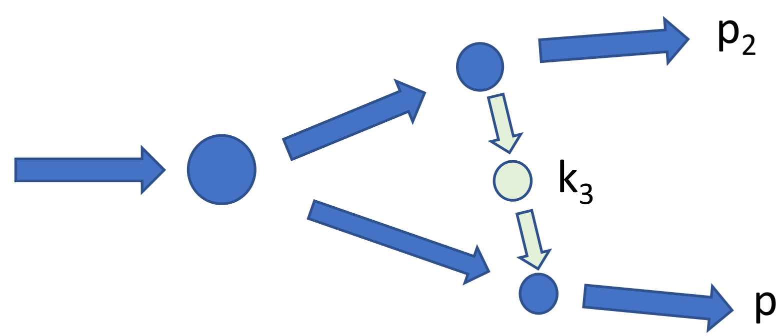
For fixed *k* and *p*, the Eq. Sys. with the internal momentum l^μ is solvable for $N \leq 4$

A well-known T.S. situation in *s*-channel

Two produced particles p_1,p_2 (in decay, etc.) **exchange another physical particle** (k_3)

* Singularity emerges in amplitude if the (re)scattering with k_3 is **inelastic**.

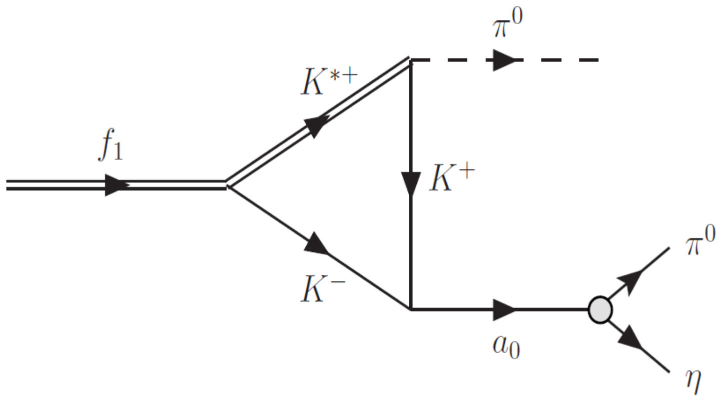
C.Schmid,1967
I. Aitchison, C. Kacser,1968



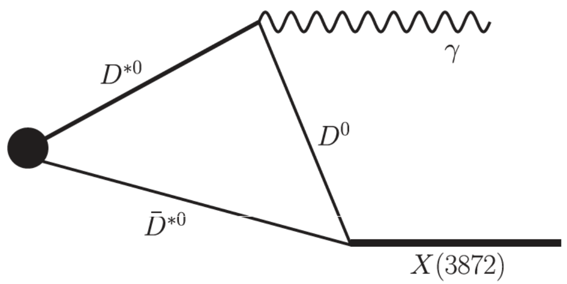
Hadron interactions

Hadron spectroscopy review:
Guo, Liu, Sakai, 1912.07030

Singularity may be mis-identified as new resonances.



One TS diagram in f(1285) decay
Aceti, Dias, Oset, 1501.06505

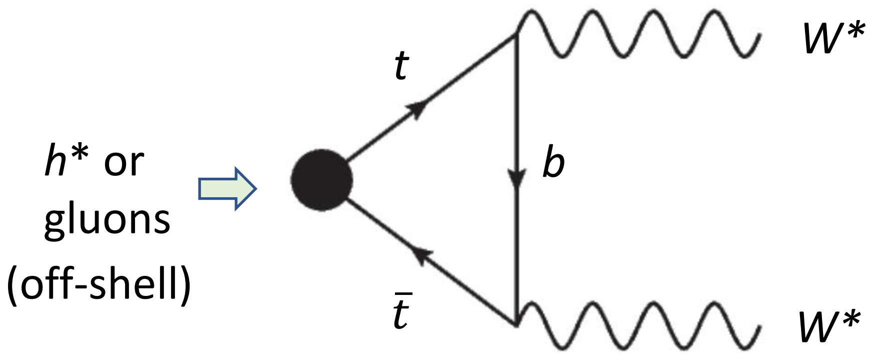


X(3872) production with a TS diagram
F.K.Guo, 1902.11221

- Large number of hadronic states opt to meet Landau conditions
- Mild (compared to HE) virtuality suppression for external momenta

@ weak / higher energy scale?

- Known example in the SM: tt -> X*X*



Virtual final-state W bosons:
Can trigger T.S. for $350 < \sqrt{s} < 750$ GeV
No physical solution for two on-shell Ws.

leads to an anomalous threshold:
finite correction in xsec.

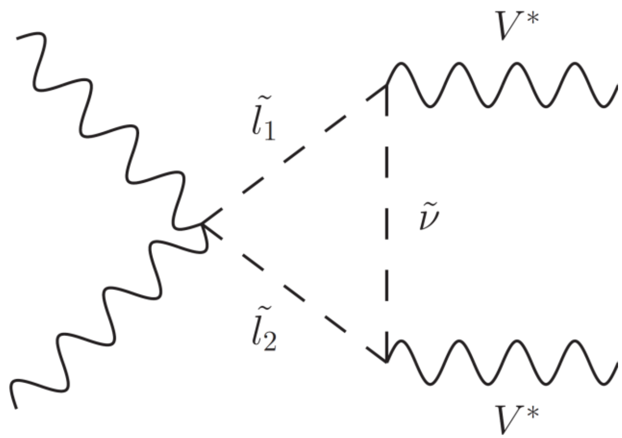
Other discussions in (NLO) SM, see:

Landau singularity in Z Z-> Z Z, Denner,Dittmaier,Hahn, 9612390
N=4 (box) Landau singularity in gg-> h bb, Boudjema and Duc Ninh, 0806.1498

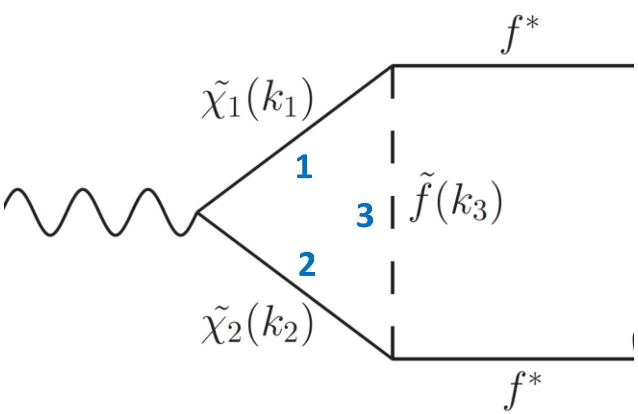
Anomalous threshold in MSSM slepton pair production:

Why not try BSM?

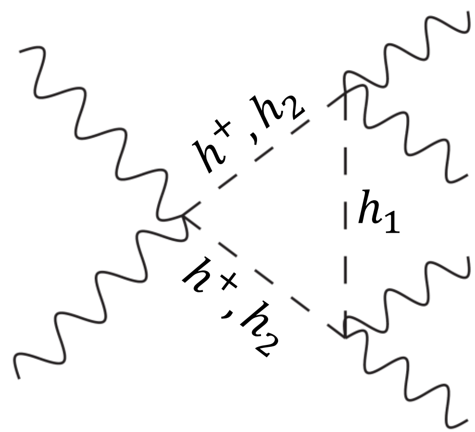
- A kaleidoscope of new particles.
- Extra bosons (a blessing).
- Can have a purely SM final state.



VBF diagram (slepton loop, MSSM)
[SM final state: visible for a `compressed' MSSM spectrum, when $m_{\tilde{l}} \approx m_{\tilde{\nu}}$]



Drell-Yan like (gaugino+sfermion loop, MSSM)
[GIM theorem may apply when k_1 and k_2 assume different particles]



VBF – all bosons (2HDM)
[k_1, k_2 can assume identical particles]

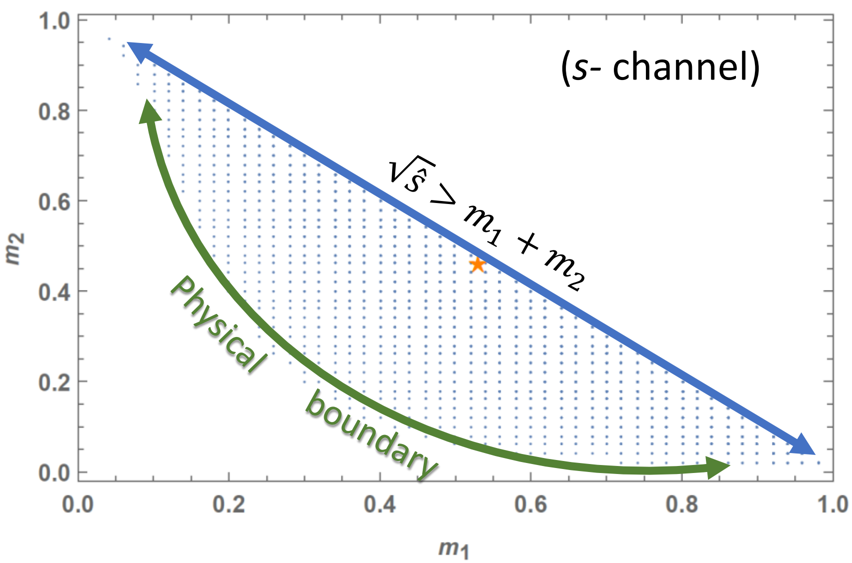
Kinematic region

For given internal line masses & p_B^2, physical solutions require

Assume k_3 direction:
$B \to A$

$$p_C^2 \in \left[(m_1+m_2)^2,\, m_1^2+m_2^2+m_2 m_3+\frac{m_2}{m_3}\left(m_1^2-p_B^2\right)\right]$$ ⇨ init. state inv. m^2

$$p_A^2 \in \left[(m_2+m_3)^2,\, m_2^2+m_3^2+m_1 m_2+\frac{m_2}{m_1}\left(m_3^2-p_B^2\right)\right]$$ ⇨ at least one final-state system is massive



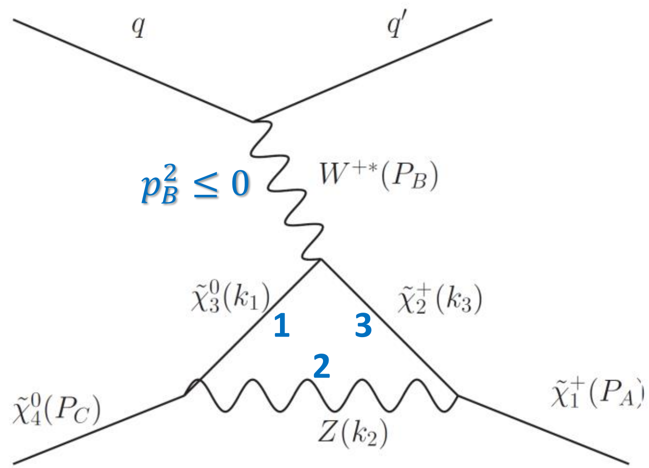
(1) above pair-production threshold
(2) Satisfy physical boundary ($\alpha_i > 0$)

⇦ T.S. region on $\{m_1,m_2\}$ plane at given $\sqrt{p_C^2}$, and $p_A^2, p_B^2 > 0$

A singularity in *t*-channel

- p_A^2, p_C^2 are likely massive when internal lines are at BSM mass scale.
- NOTE: Solutions exist for a negative $p_B^2 \leq 0$ when $p_A^2, p_C^2 > 0$.

A negative p_B^2 stands for a momentum exchange in a scattering process (MSSM)



massive init. state particle (can not be a LSP dark matter candidate)

massive final state particle or particle system

Extendable to (soft) $\sqrt{|p_B^2|} \ll m_{BSM}$ region.

$$\begin{cases} m_1 \to m_3 \\ p_C^2 \to p_A^2 \end{cases} \quad \text{for } p_B^2 \to 0$$

Ideal in case a model contains light boson(s).

Not a spontaneous decay when $p_B^2 < 0$.

Need a massive initial state particle for 2->2 scattering (difficult at collider for a BSM particle)

The massive initial state is unstable (so unlikely a dark matter candidate)

An MSSM benchmark point.

Loop particles	Process	$m_1(\Gamma_1)$ (GeV)	$m_2(\Gamma_2)$ (GeV)	$m_3(\Gamma_3)$ (GeV)	p_A^2 (GeV^2)	p_B^2 (GeV^2)	p_C^2 (GeV^2)
$\{\tilde{\chi}_2^-, \tilde{\chi}_1^+, \tilde{\chi}_1^0\}$	DY-like	1528(13)	359(0.9)	306(0)	700^2	800^2	1943^2
$\{\tilde{l}_1, \tilde{l}_2, \tilde{\nu}\}$	WW fusion	1701(1.56)	1500(0.34)	1499(0.16)	3000^2	100^2	3202^2
$\{\tilde{\chi}_3^0, Z, \tilde{\chi}_2^+\}$	t-channel	1351(3.79)	91(2.45)	1528(13)	1621^2	-2500^2	1528^2

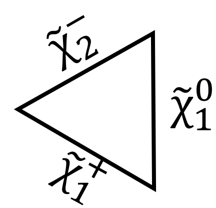
`EWSBMSSM' parameters

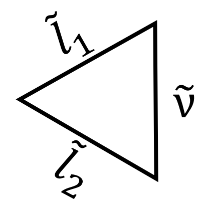
(run with SUSPECT2)

Parameter	Value
$\tan\beta$	10
μ	300 GeV
m_1	1.5 TeV
m_2	1.4 TeV
ml3	1.5 TeV
mr3	1.7 TeV

(modified pars. only)
pass LHC search bounds

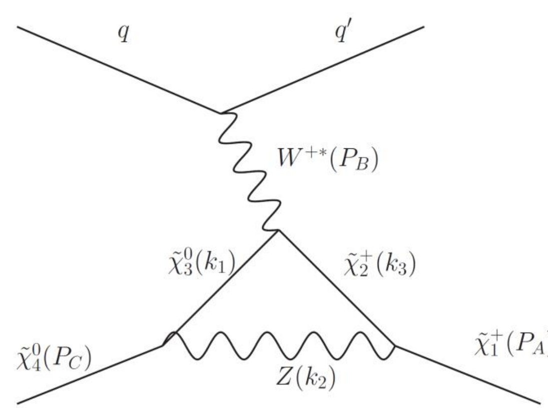
s-channel: Drell-Yan & VBF, into SM V*V* final states.



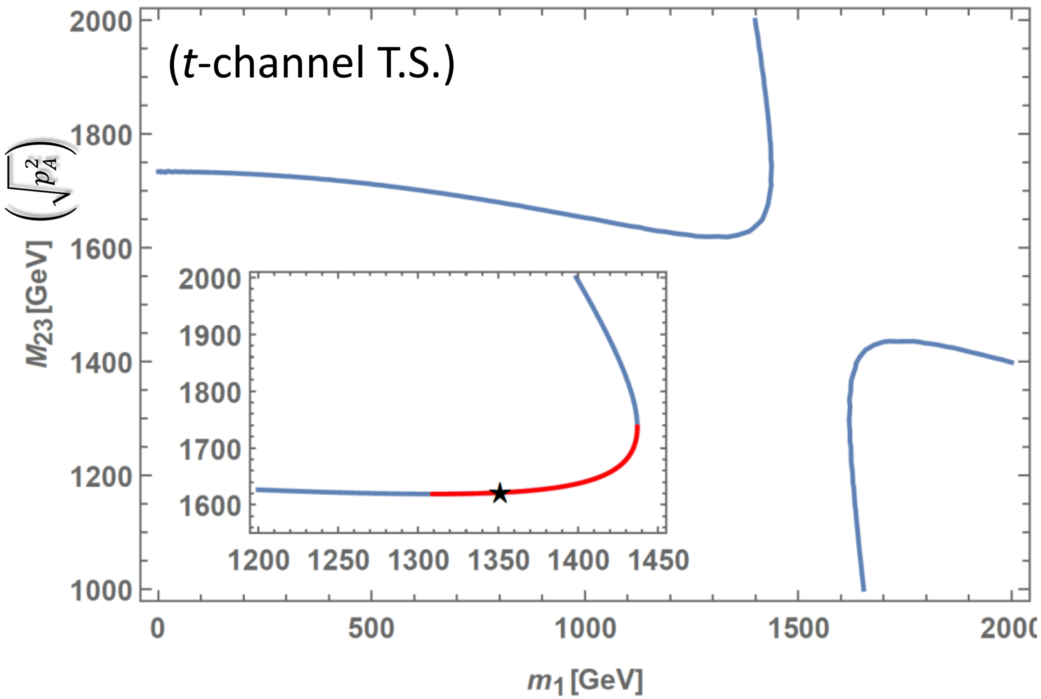


t-channel: $p_A^2 < p_C^2$,

system becomes
(more) excited after
scattering



Dalitz plot



Conventional choice:

fix m_2, m_3 and p_B^2, p_C^2

m1 and as $m_{23} = \sqrt{p_A^2}$ as variables

blue: trajectory of $\det|y_{ij}|=0$ (*t*-channel)

red: physical solutions

asterisk: MSSM benchmark

Landau conditions equiv. as

$$\beta_i + \sum_j^{j\neq i} \beta_j y_{ij} = 0,$$

$$\text{where} \quad y_{ij} \equiv \frac{m_i^2 + m_j^2 - p_k^2}{2 m_i m_j},$$

$$\text{and } \beta_i \equiv \alpha_i m_i$$

Solutions require

$$det \begin{vmatrix} 1 & y_{12} & y_{13} \\ y_{12} & 1 & y_{23} \\ y_{13} & y_{23} & 1 \end{vmatrix} = 0$$

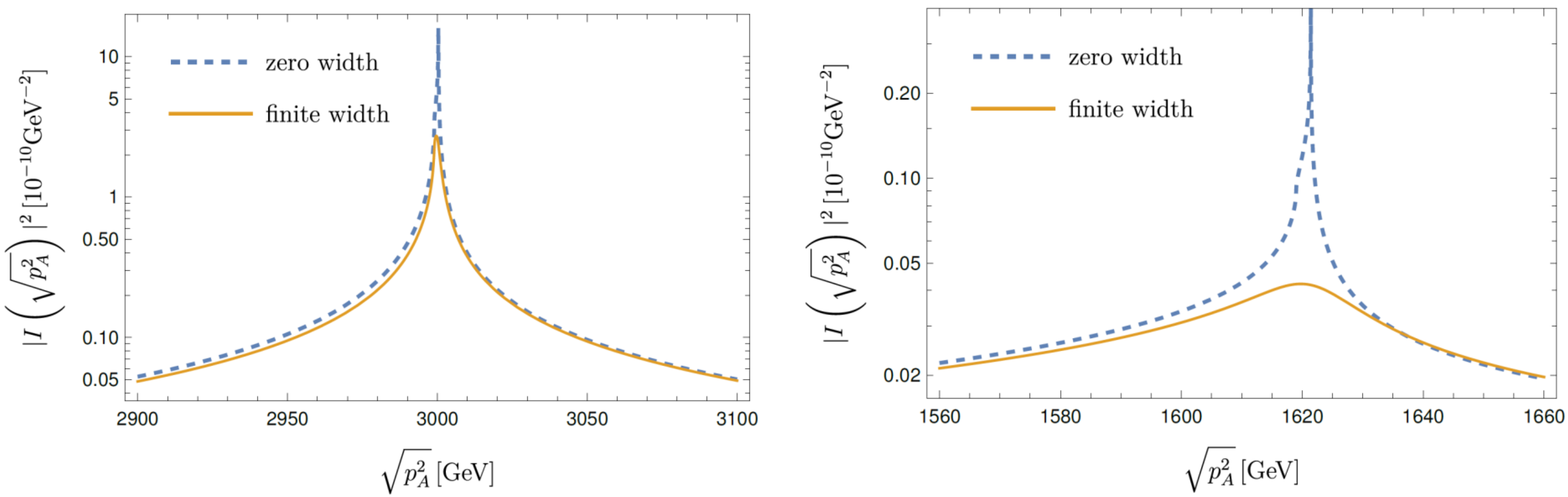
$$1 + 2 y_{12} y_{23} y_{13} - y_{12}^2 - y_{23}^2 - y_{13}^2 = 0$$

containing 6 kinematic parameters.

$\alpha_i > 0$ select a small section (red) of physical solutions.

Peak in amplitude (C_0)

Singularity encoded in $$I\left(\sqrt{p_A^2}\right) = \int \frac{\mathrm{d}^4 l}{i\pi^2}\left[\frac{1}{l^2 - m_3^2}\cdot\frac{1}{(l+p_A)^2 - m_2^2}\cdot\frac{1}{(l+p_A+p_C)^2 - m_1^2}\right]$$



(a) WW fusion (b) t-channel

Finite width of internal particles gives a small Im part.

Singularity -> broadened peak

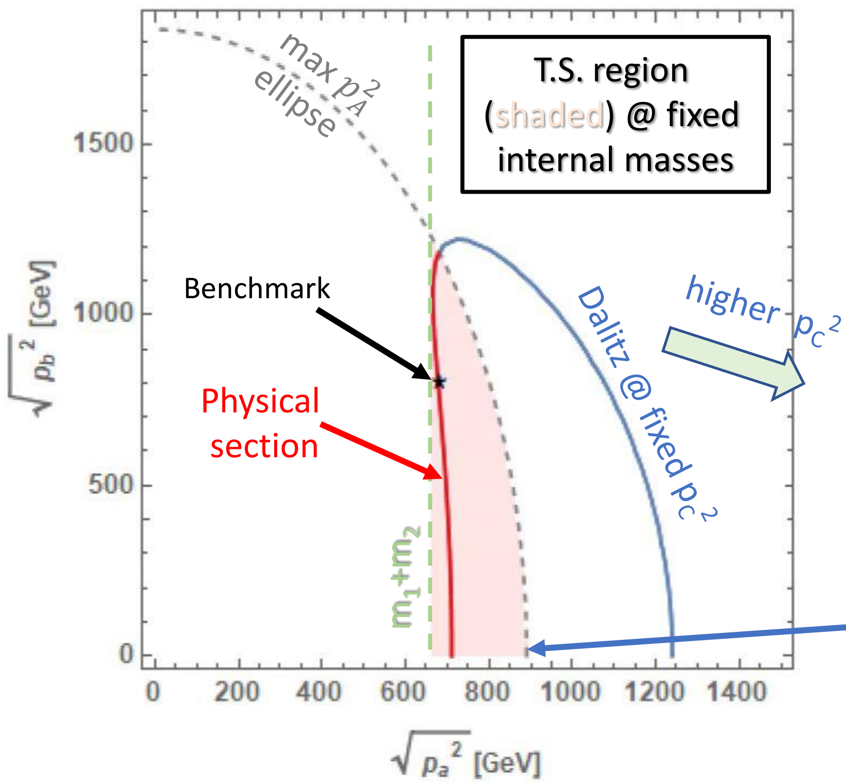
Plots: VBF & t-channel at MSSM benchmark scenarios, with p_C^2 fixed.

Reconstruction of T.S. ?

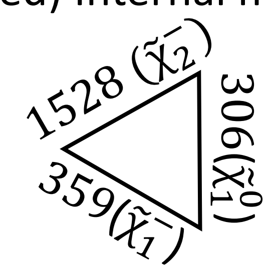
p_A, p_B as particle systems:

(e.g. if particle states lay inside $p^2{}_A$ *and* $p^2{}_B$ physical ranges.)

- Physical range(s) of external inv. masses: peak → threshold
- $\sqrt{s}$ at T.S. shrinks to a peak only if final-state inv. $p^2{}_{A,B}$ become *both fixed*.
- Extension toward higher $\sqrt{s}$ from beam pdf / ISR.



(fixed) Internal mass:



p_C^2 maximizes at max p_A^2, the physical section reduces to one point.

More than a threshold?

p_A, p_B as particle systems:

(e.g. if particle states lay inside $p^2{}_A$ *and* $p^2{}_B$ physical ranges.)

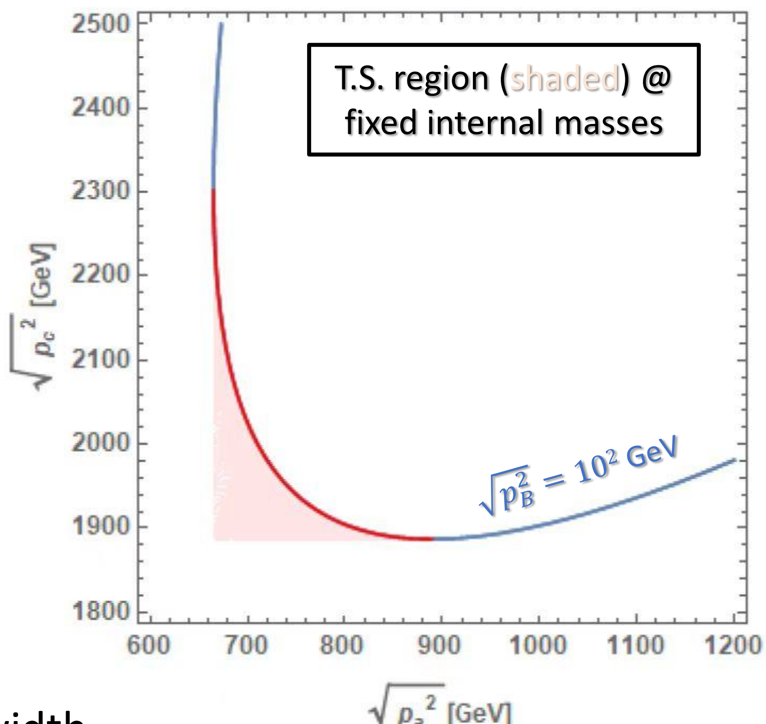
- Physical range(s) of external inv. masses: peak → threshold
- $\sqrt{s}$ at T.S. shrinks to a peak only if final-state inv. $p^2{}_{A,B}$ become *both fixed*.
- Smeared toward higher $\sqrt{s}$ from beam pdf / ISR.

One FS mass extends to lower (EW) range:

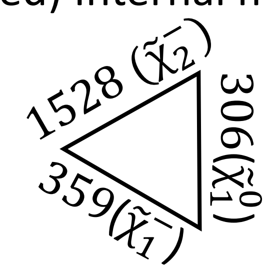
One physical FS particle:

Correlate COM energy and FS par.sys. invariant mass at singularity

* par. sys. should be visible
* F.S. combinatorics
* broadening by external width

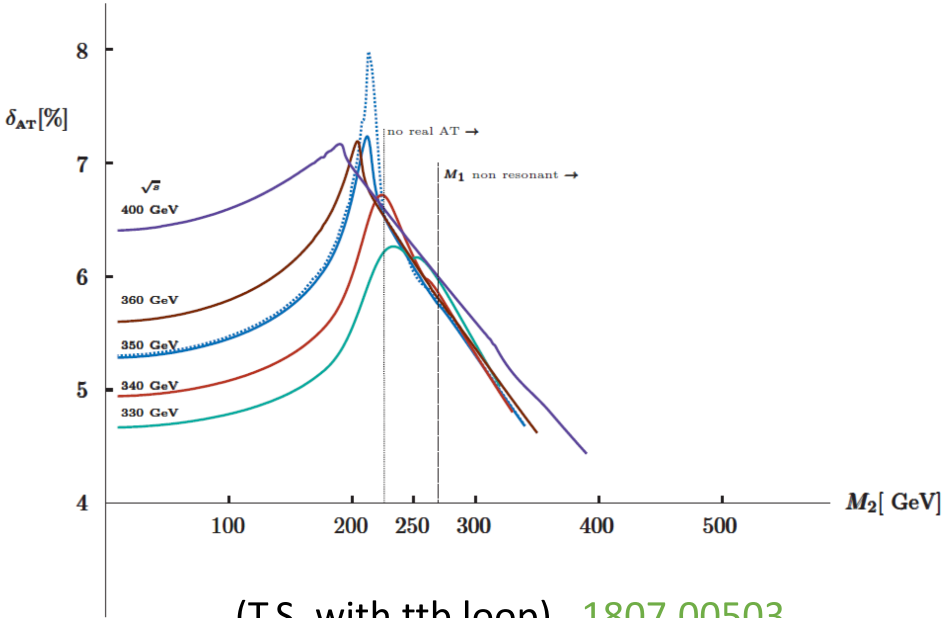


(fixed) Internal mass:

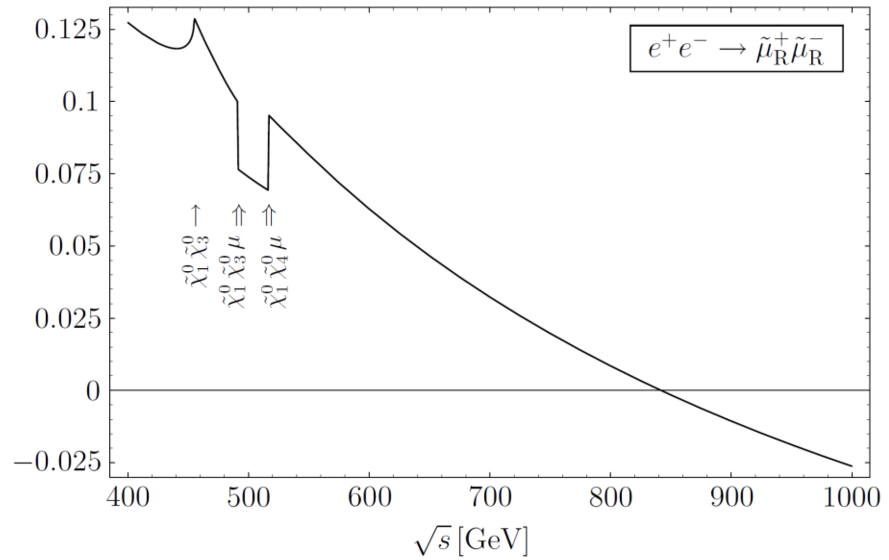


Visibility (threshold)

- EW/BSM scale loop correction to SM-> SM
- Apply to `compressed scenarios'

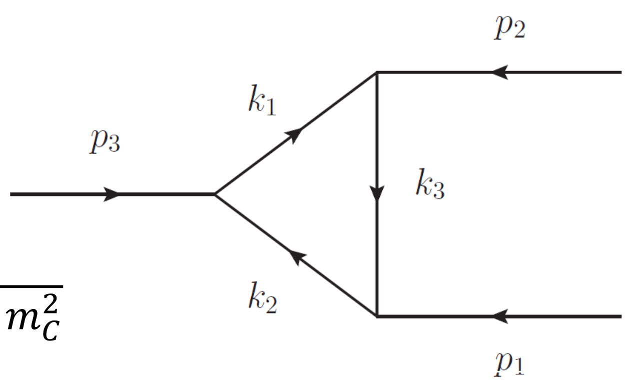


(T.S. with ttb loop) 1807.00503
Correction percentage around threshold for $H^*(Q) \to W^*\, W^* \to \mu^+ e^- \nu\bar{\nu}$



(T.S. with neutralino-muon loop)
hep-ph/0310182: slepton production: correction from multiple thresholds

Less suppressed scenario?



$\sim \frac{1}{p_C^2 - m_C^2}$

$\sim \frac{1}{p_B^2 - m_B^2}$

$\sim \frac{1}{p_A^2 - m_A^2}$

A not small virtuality suppression as

$$\sqrt{p_C^2} > m_1 + m_3$$

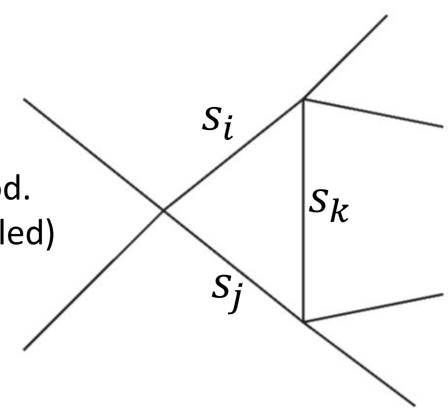
$$\sqrt{p_A^2} > m_2 + m_3$$

when BSM particles are heavy. e.g. in an *s*-channel process: XX > scalar> triangle

IS: 4-particle vertex allows for TS in a *boson-fusion* process, avoids a large 1/s.

FS: 4-particle vertices replaces a highly virtual propagator with two *more identifiable,* on-shell particles.

VBF prod. (if coupled)



~ four boson coupling. w/o heavy prop.

on-shell final particles more identifiable

Summary

- BSM spectrum can, and *often*, satisfy triangle singularity.
- Unlike with hadrons, physical range(s) of external inv. momenta lead to
  * Anomalous thresholds at collide. (no on-shell st.)
  * Kinematic correlations (one on-shell)
  * Peak (both on shell)
- Triangle singularity also occurs in a *t*-channel ($q^2 < 0$) scattering, and can approach to a soft realm (q^2->0).