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Macroscopic Dark Matter from Cosmic Phase Transitions

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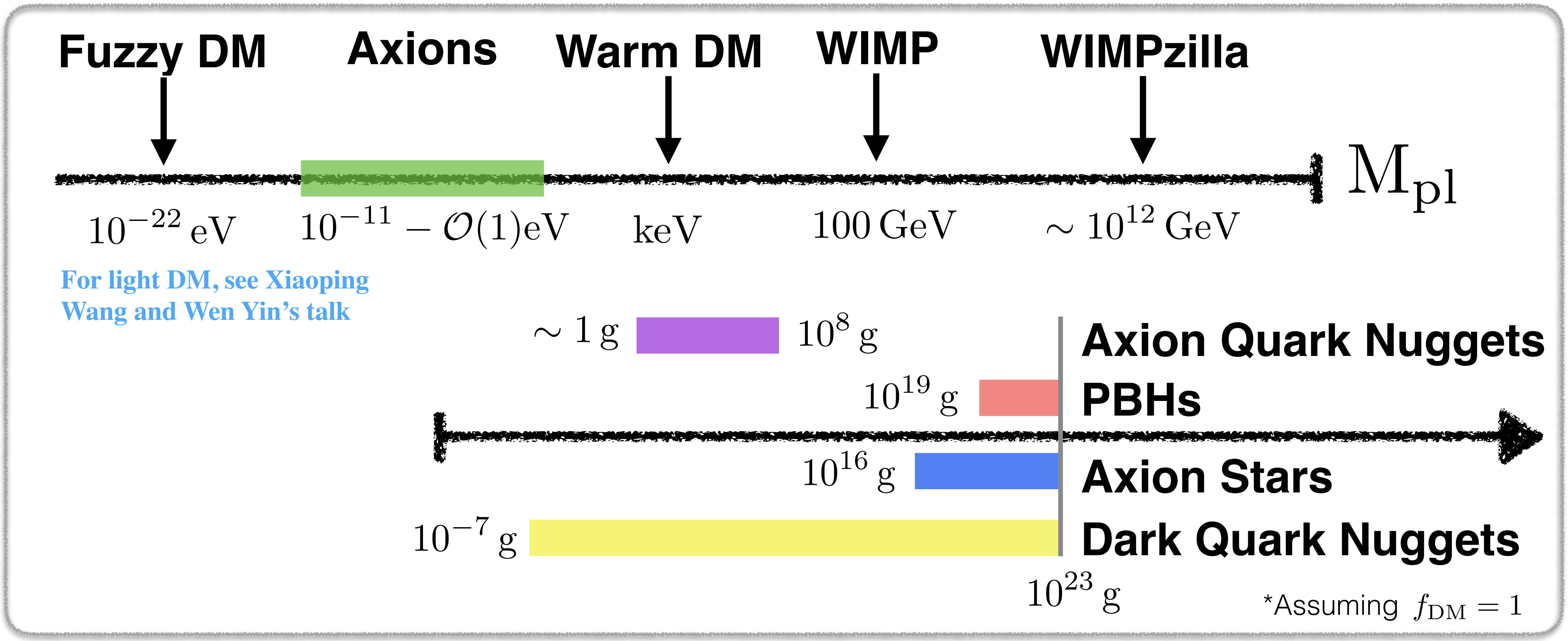
based on works w/ Yang Bai, Andrew J. Long and Nicholas Orlofsky

DM Zoo



- Macroscopic DM candidates may come from phase transitions
- Naturally contained in many theories

Outline

- Brief introduction to macroscopic DM formation
- Models containing macroscopic DM
- Evolution after formation: solitosynthesis
- Model independent signatures and detections

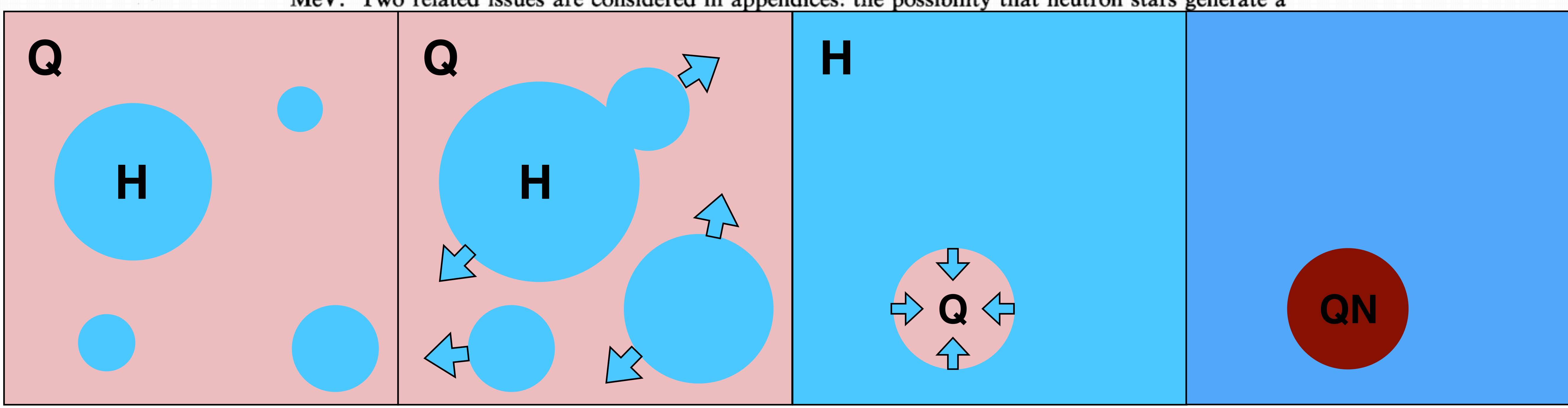
What Witten Proposed Before

PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

Cosmic separation of phases

Edward Witten*
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 9 April 1984)

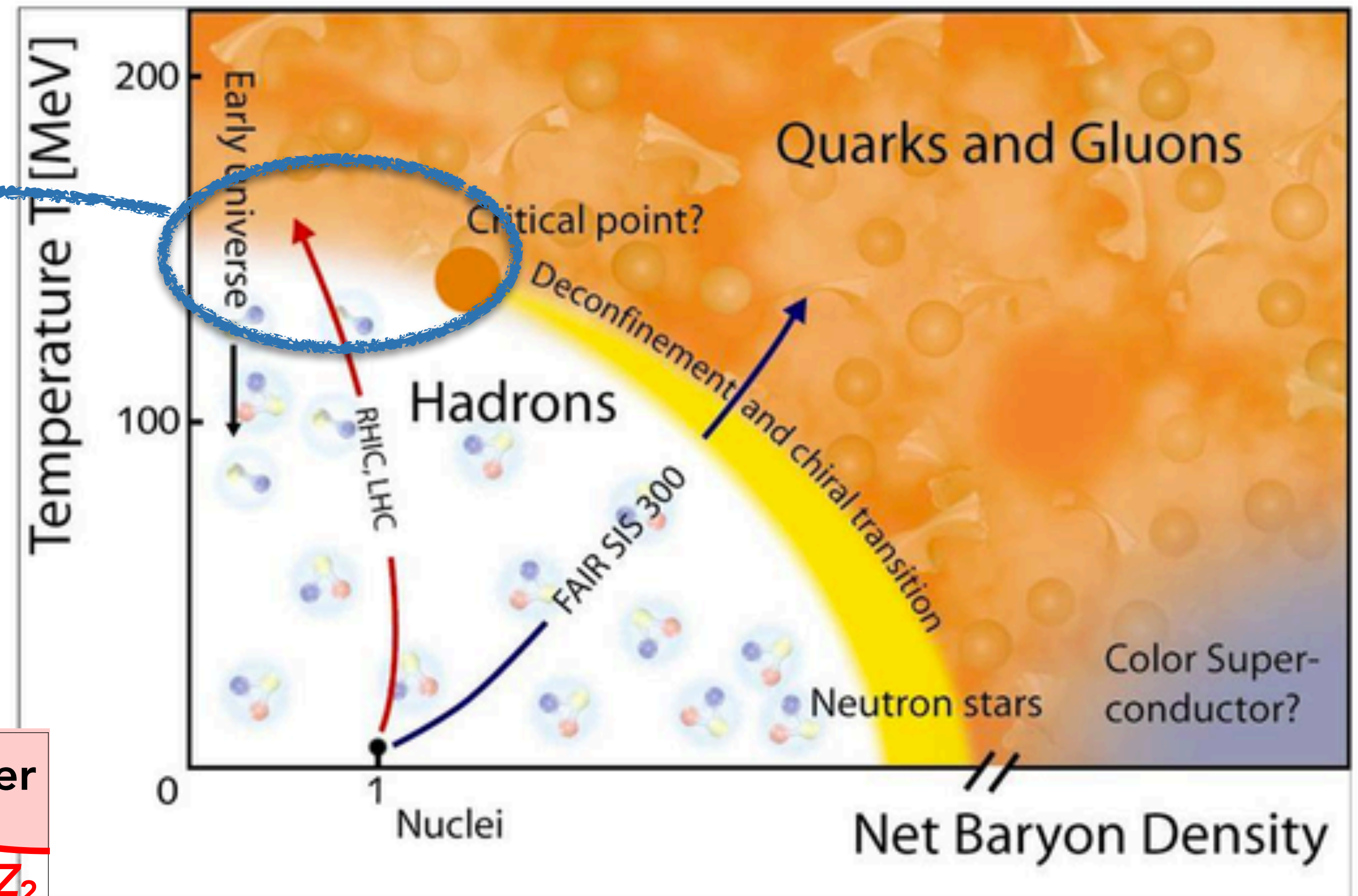
A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a



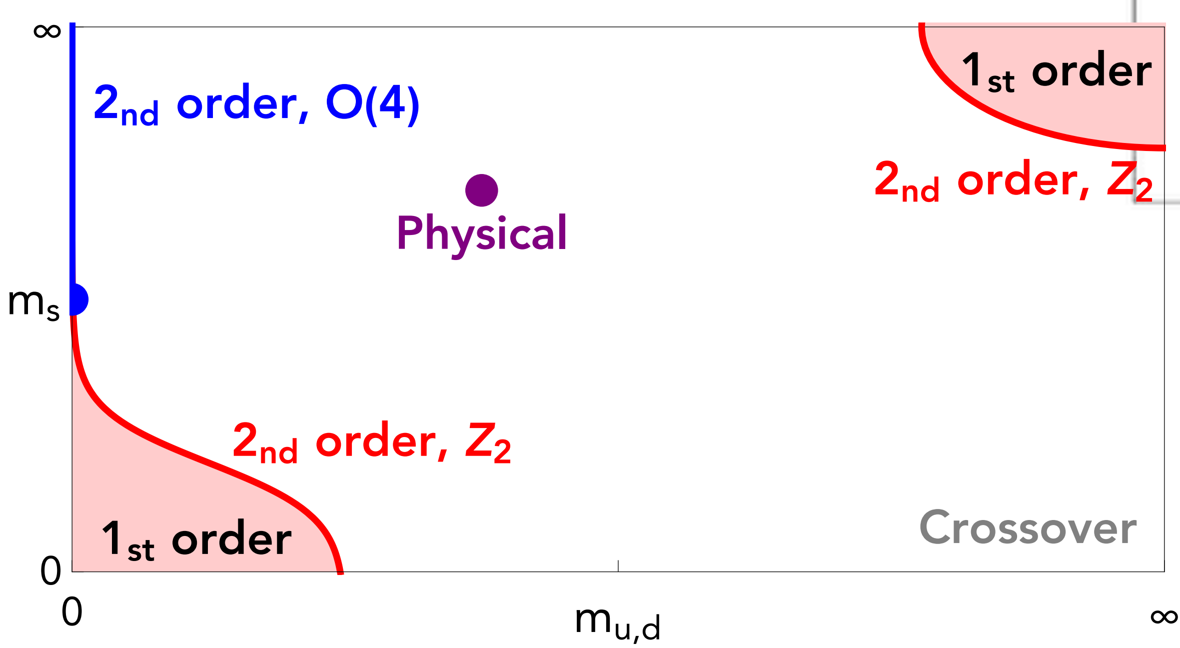
Q: quarks H: hadrons QN: quark nuggets

Degeneracy pressure balancing vacuum pressure

QCD is Upsetting…



Continuous Crossover!



But this doesn't necessarily mean that QCD quark matter doesn't exist, as there is enough room to tune the EOS. See e.g. J. Ren and C. Zhang 2211.12043

…While the Dark Sector is Still Fine

- **First-order phase transition can still come from the dark sector:**
  - ➡ Composite DM
  - ➡ Twin Higgs
  - ➡ SIMP
  - ➡ Scalar extended EW

- **A generic feature of a large class of models!**

Outline

- ✤ **Brief introduction to macroscopic DM formation**
- ✤ **Models containing macroscopic DM**
- ✤ **Evolution after formation: solitosynthesis**
- ✤ **Model independent signatures and detections**

Focus on bosonic models.

For fermonic models, see e.g.
Y. Bai, A. J. Long, *SL*, Phys. Rev. D99 (2019)
K. Kawana, K. Xie, Phys. Lett. B 824 (2022)

Non-topological Solitons, a.k.a. Q-balls

- **Stable macroscopic bound states may exist in a theory with reasonable amount of nonlinearity**
  - ➡ A (global) symmetry to protect the stability
  - ➡ A scalar potential providing an attractive force

[See T. D. Lee and Y. Pang, Phys. Rept. 221 (1992) 251-350 for a review]

- **Examples**
  - ➡ Coleman's *Q-ball* [S. Coleman, Nucl. Phys. B 262 (1985) 2 263]
  - ➡ Baryon-ball/lepton-ball in the MSSM [A. Kusenko, Phys. Lett. B 405 (1997) 108]

Non-topological Solitons, a.k.a. Q-balls

- **Let's use Q-ball with a *global* U(1) as an example**

[See e.g. 2103.06905 for examples of gauged Q-balls]

➡ Let $\Phi = \phi(r)\, e^{-i\omega t}/\sqrt{2}$

➡ From the Lagrangian we immediately have

$$E = \int d^3r \left[\frac{1}{2}\omega^2\phi^2 + \frac{1}{2}(\nabla\phi)^2 + U(\phi)\right], \; Q = \omega \int d^3r\, \phi^2$$

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + \underbrace{\omega^2\phi - \frac{dU'}{d\phi}} = 0$$

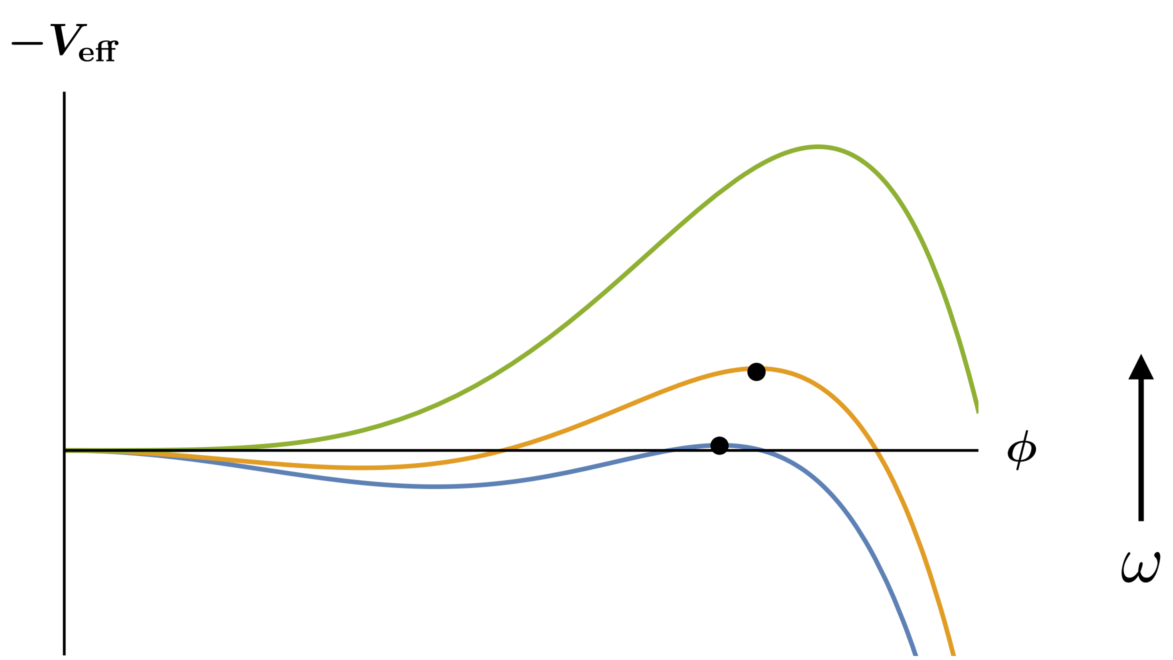
effective potential $V_{\text{eff}} = U - \frac{1}{2}\omega^2\phi^2$

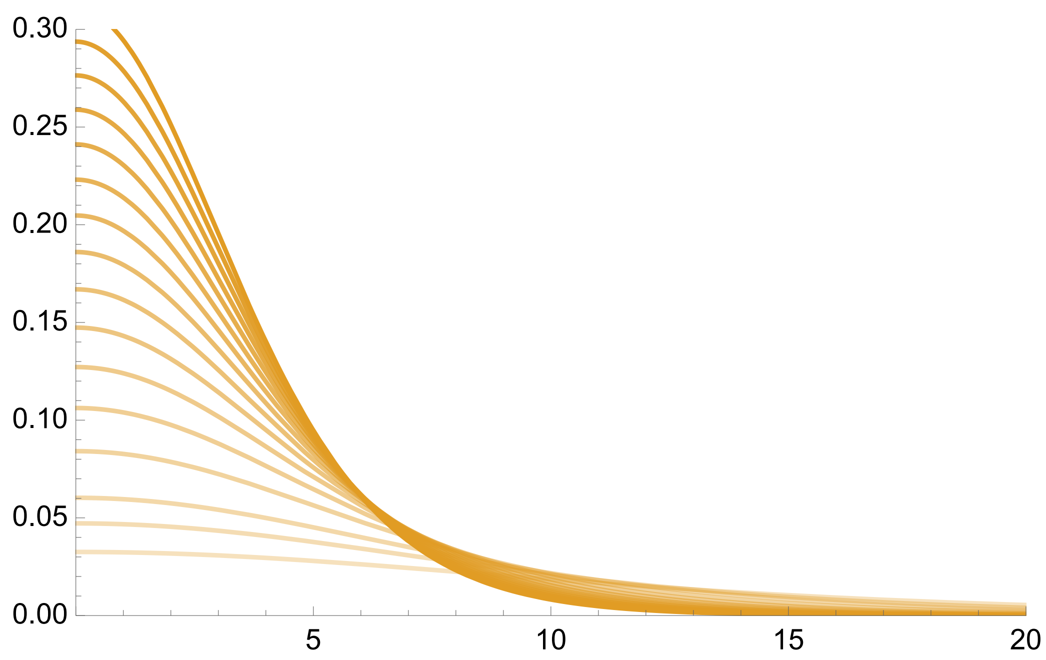
➡ By defining an effective potential, the EOM has a Newtonian interpretation if we take $r \to t,\; \phi \to x$

Non-topological Solitons, a.k.a. Q-balls

- **A particle moving along -V**

$$\phi(r=0)=\phi_0,\ \phi(r=\infty)=0 \to x(t=0)=x_0,\ x(t=\infty)=0$$



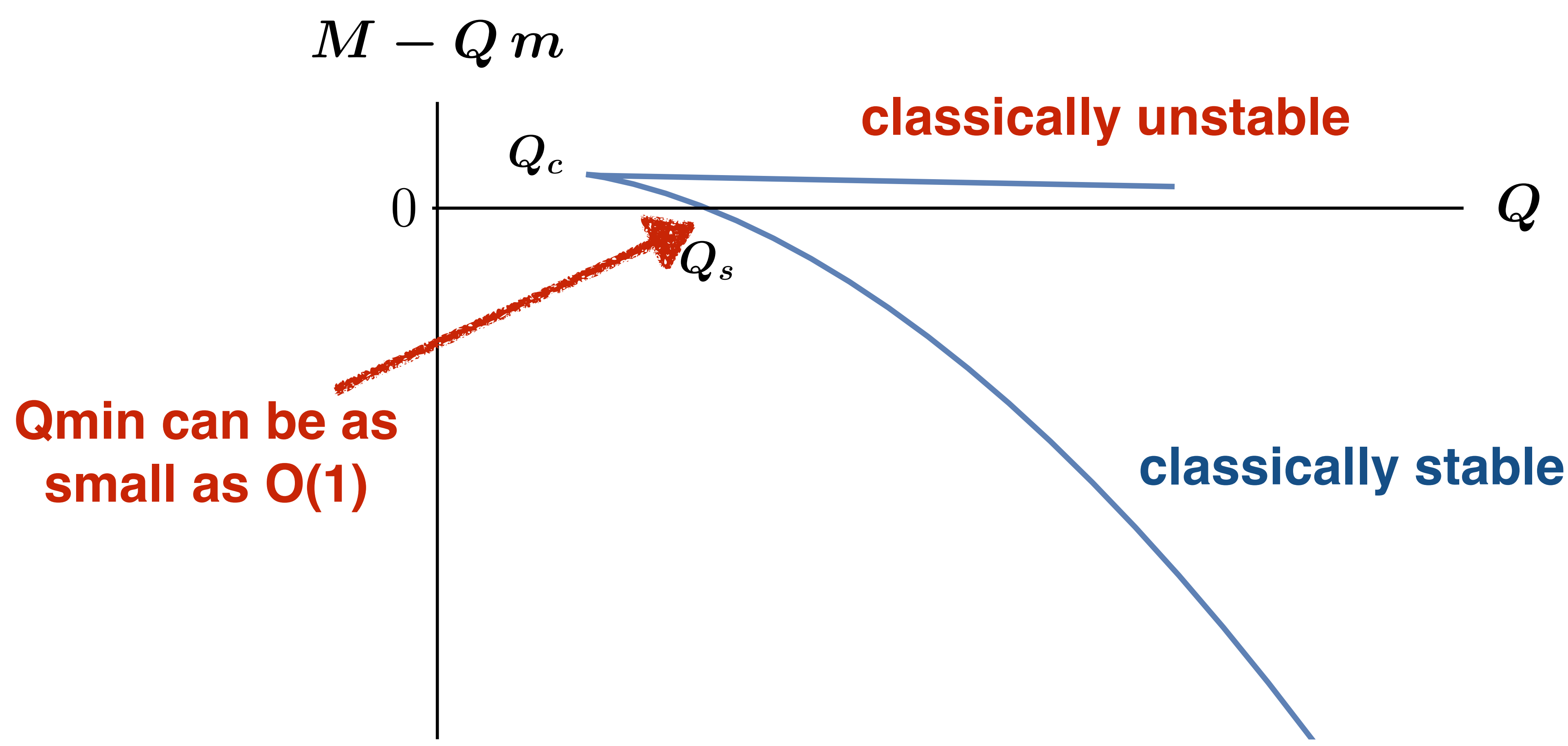


- ➡ There must be a local minimum and a local maxima
- ➡ The local maxima must be greater than zero

so cannot be achieved with a single field at quartic order

Non-topological Solitons, a.k.a. Q-balls

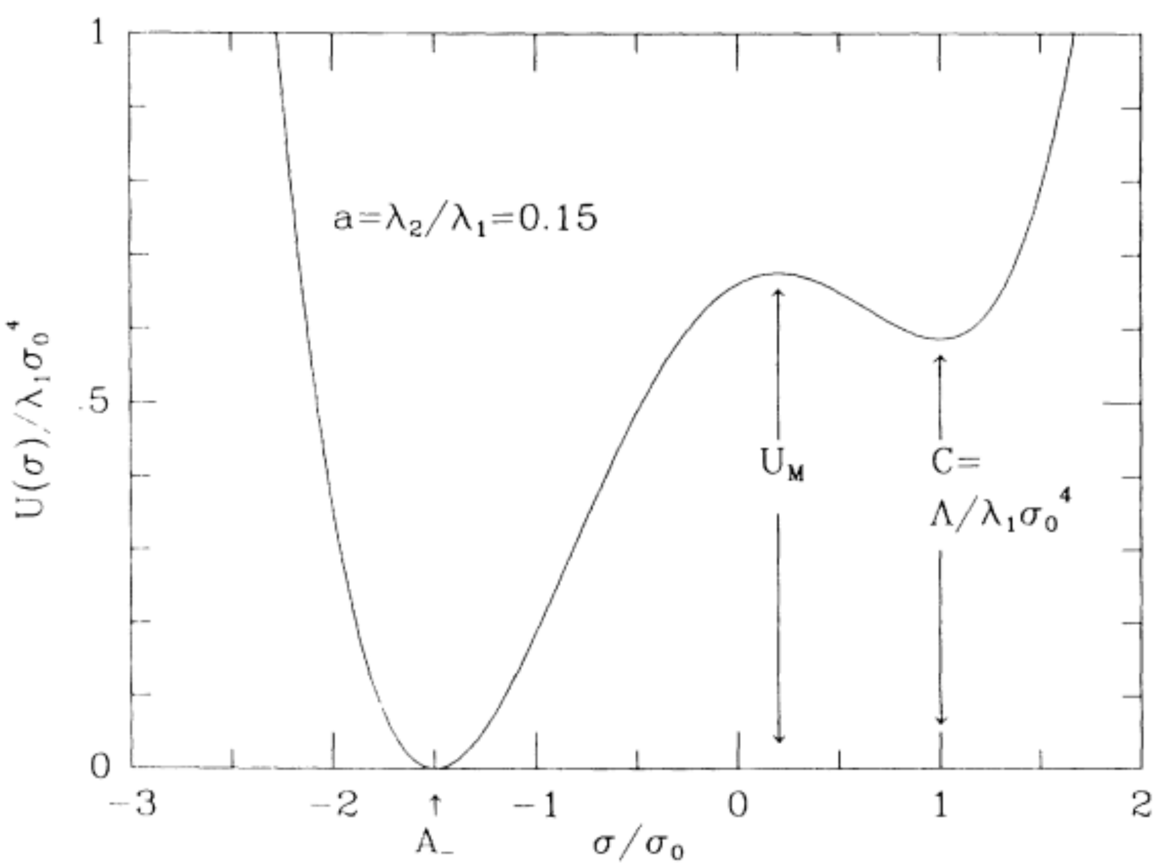
- **Two branches of solutions**
  - Can a Q-ball with charge Q and mass M may decay into Q free U(1) quanta (with mass m)?



Some Q-ball Models

- **The properties of Q-balls depends on the scalar potential and interaction**
  - ➡ Benchmark model: vanishing quartic coupling

$$V(S,\sigma) = \frac{1}{8}\lambda\,(\sigma^2 - \sigma_0^2)^2 + \frac{1}{3}\lambda_2\,\sigma_0(\sigma - \sigma_0)^3 + \frac{m_S^2}{(\sigma_- - \sigma_0)^2}|S|^2(\sigma - \sigma_0)^2 + \Lambda\,,$$



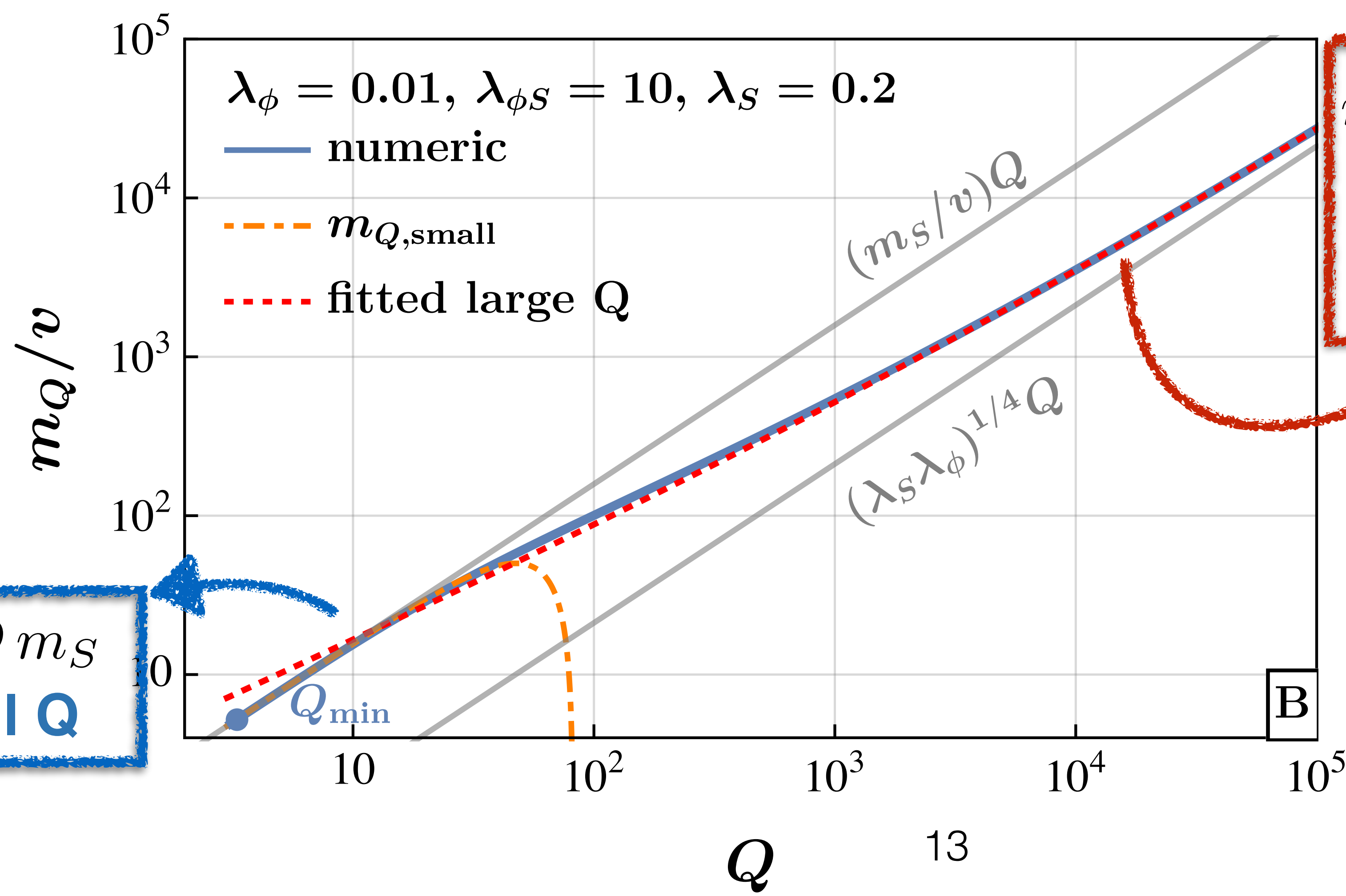
$$m_Q \propto \sigma_0 Q^{3/4}\,, \quad R_Q \propto \sigma_0^{-1} Q^{1/4}$$

[K. Griest, E. Kolb, Phys. Rev. D40 (1989)]

Some Q-ball Models

- **The properties of Q-balls depends on the scalar potential and interaction**
  - ➡ Benchmark model: a Z_2 symmetric potential w/ SSB

$$V(S,\phi) = \frac{1}{4}\lambda_\phi(|\phi|^2 - v^2)^2 + \frac{1}{4}\lambda_{\phi S}|S|^2|\phi|^2 + \lambda_S|S|^4 + m_{S,0}^2|S|^2$$



$m_Q \lesssim Q\, m_S$

at small Q

$m_Q \approx v\,Q\left[(\lambda_S\lambda_\phi)^{1/4} + c_2 Q^{-1/3}\right]$

$R \propto Q^{1/3}$

thin wall approx. at large Q

[Y. Bai, *SL*, N. Orlofsky, *JHEP* 10 (2022) 181]

Some Q-ball Models

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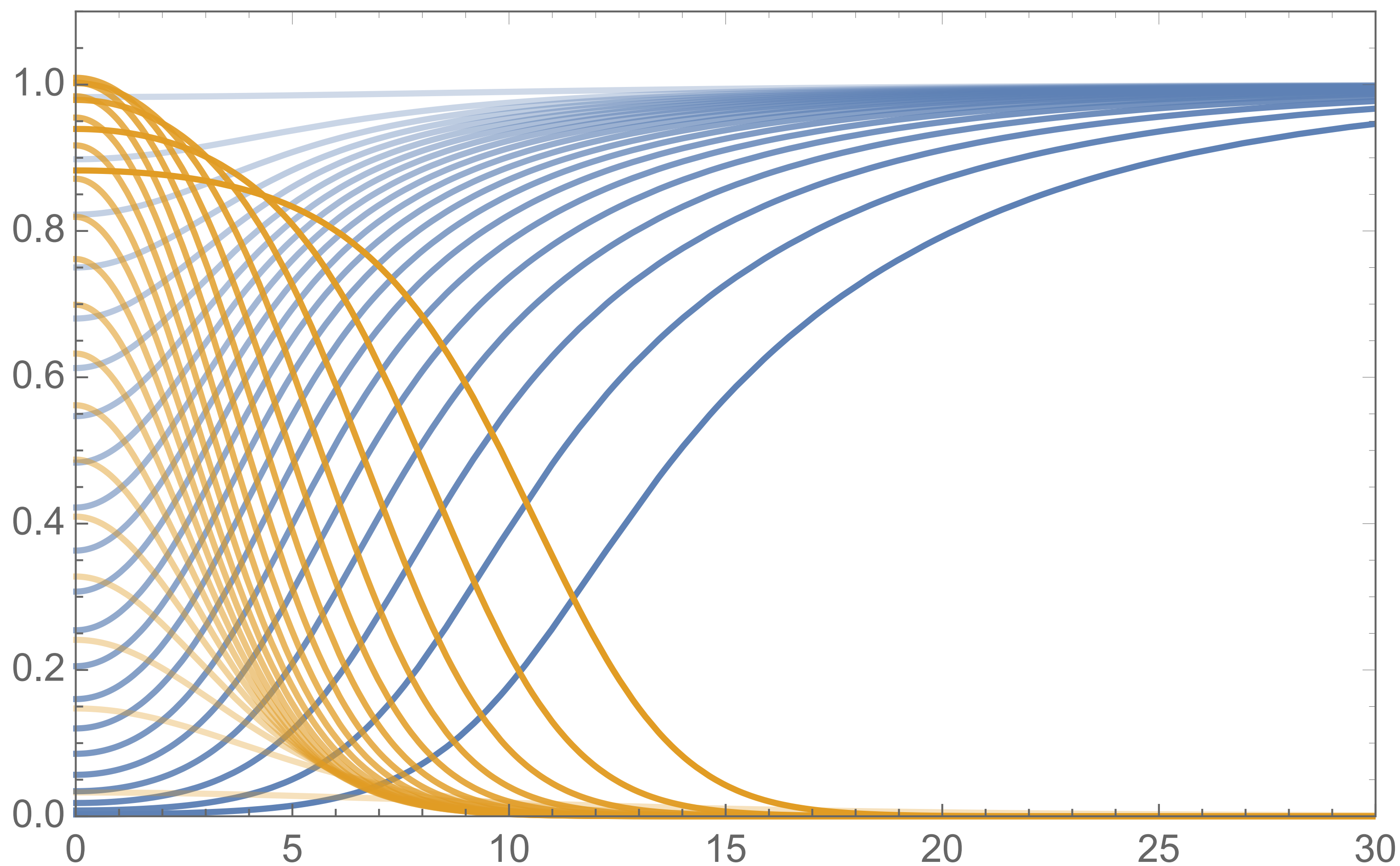
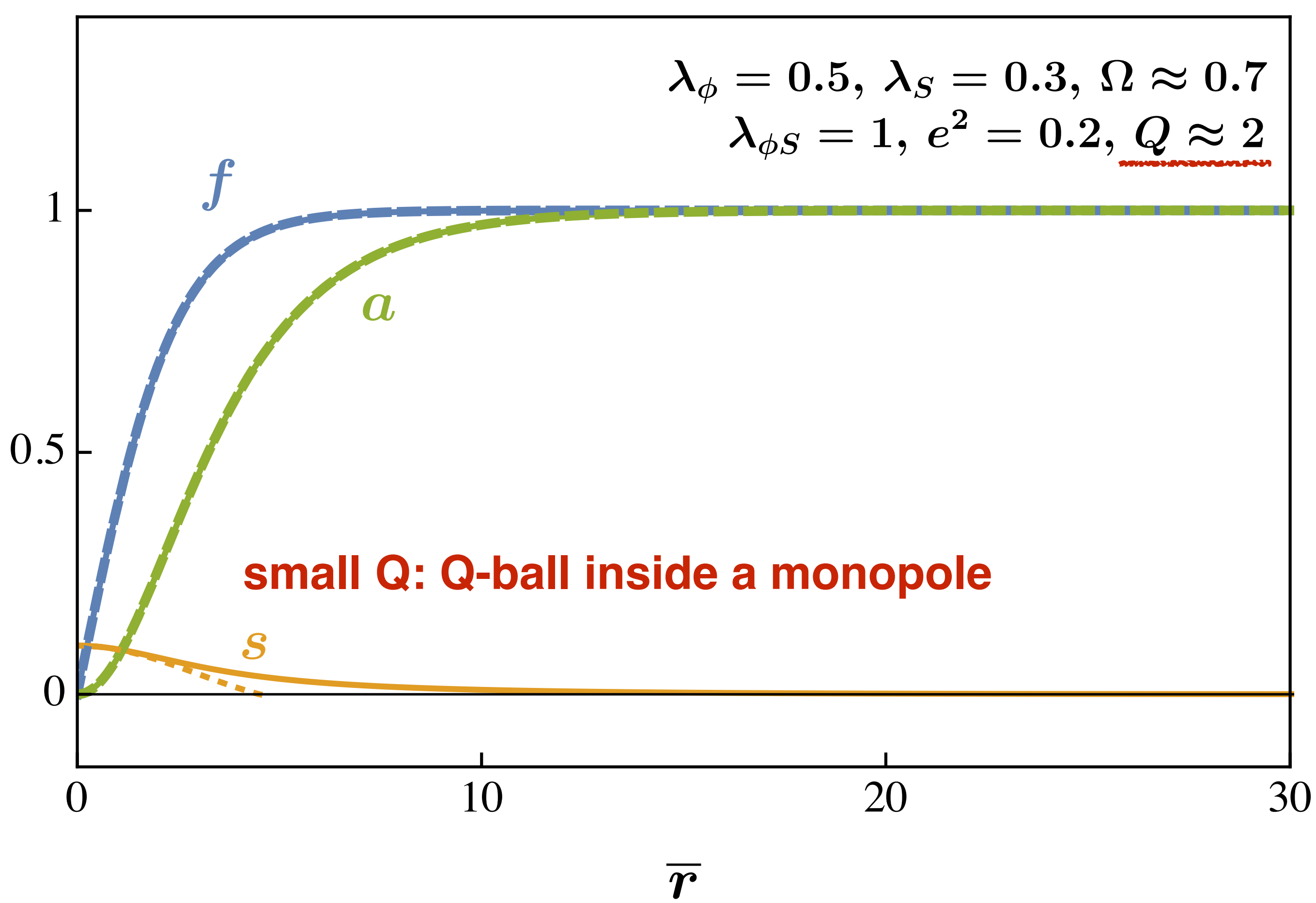
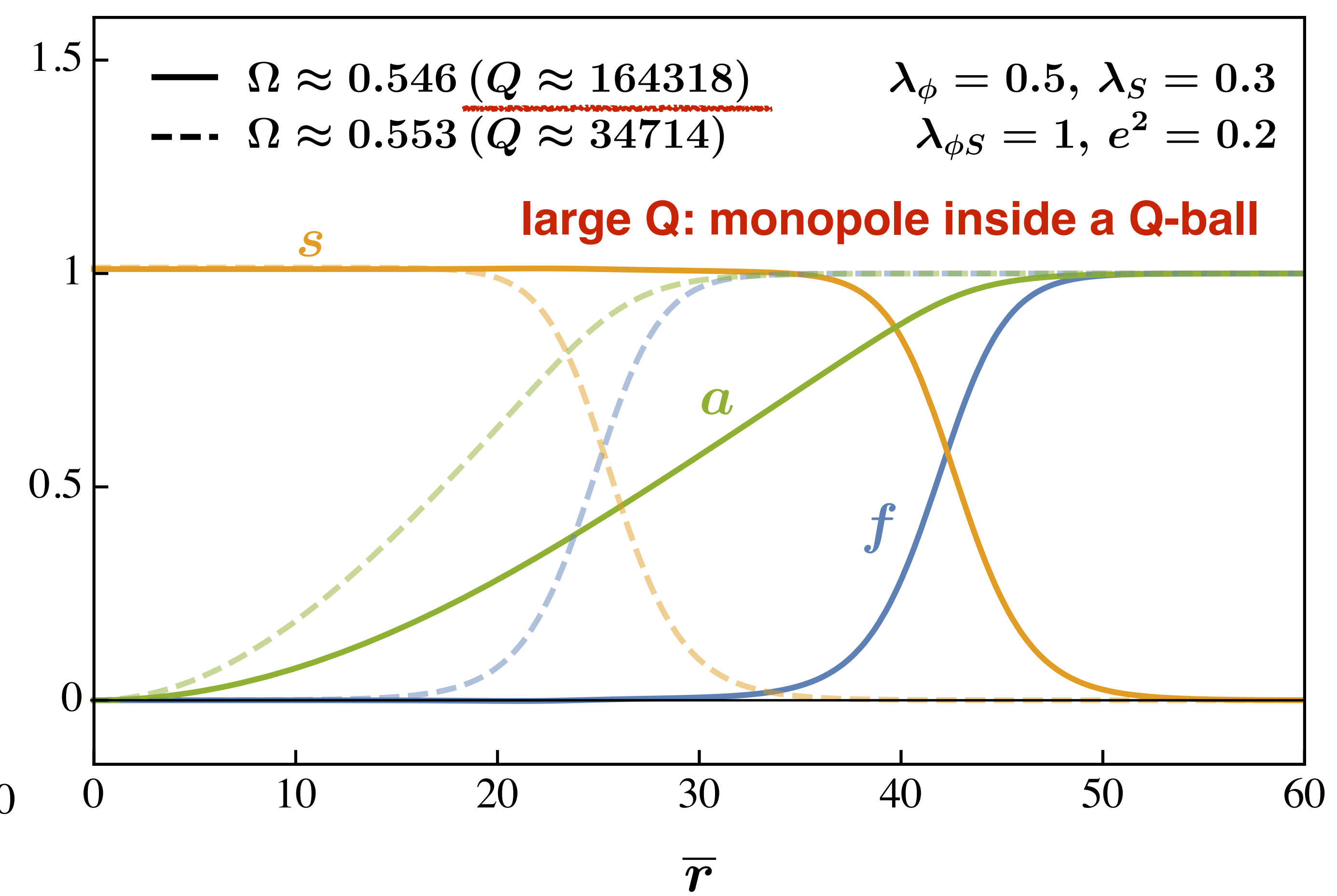


symmetry "restoration"

Some Q-ball Models

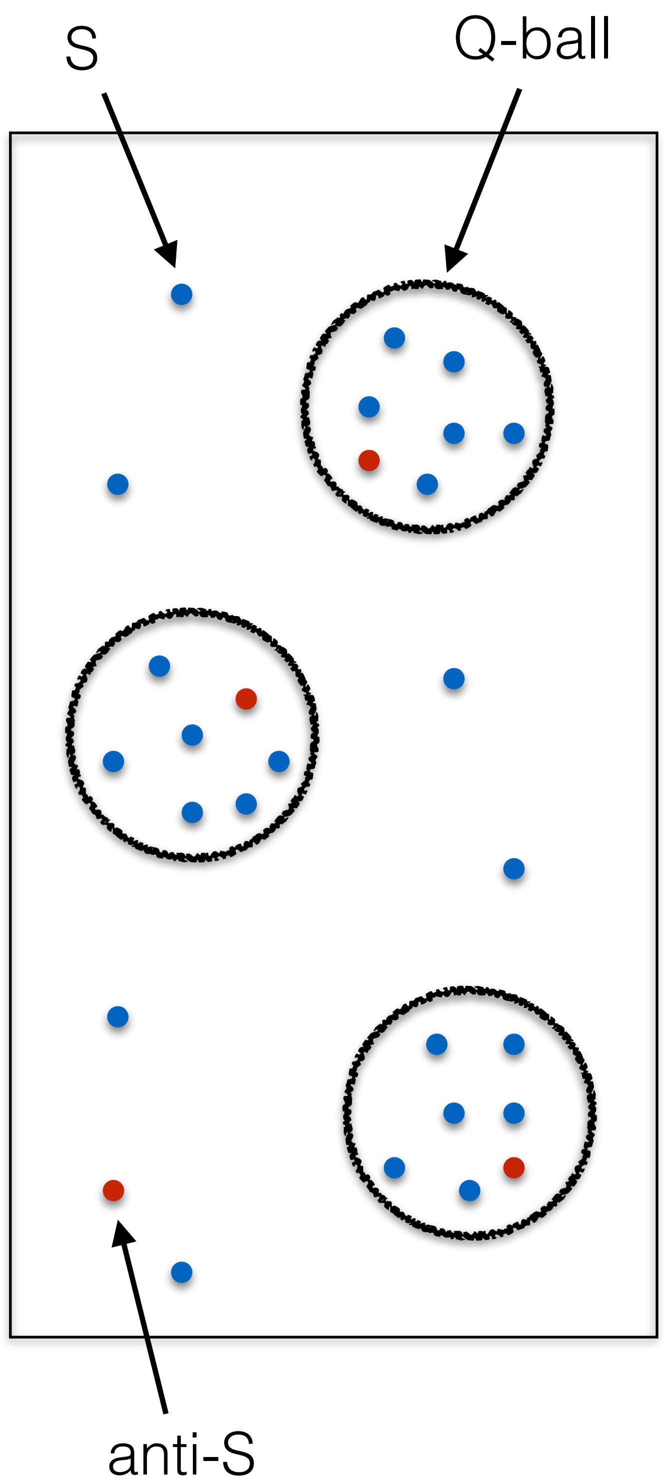
- **The properties of Q-balls depends on the scalar potential and interaction**
  - ➡ sth. fancy: a *nontopological* soliton w/ a *topological* charge
  - ➡ Consider gauged SU(2) × global U(1)





[Y. Bai, *SL*, N. Orlofsky, JHEP 01 (2022)]

Relic Abundance: from PT

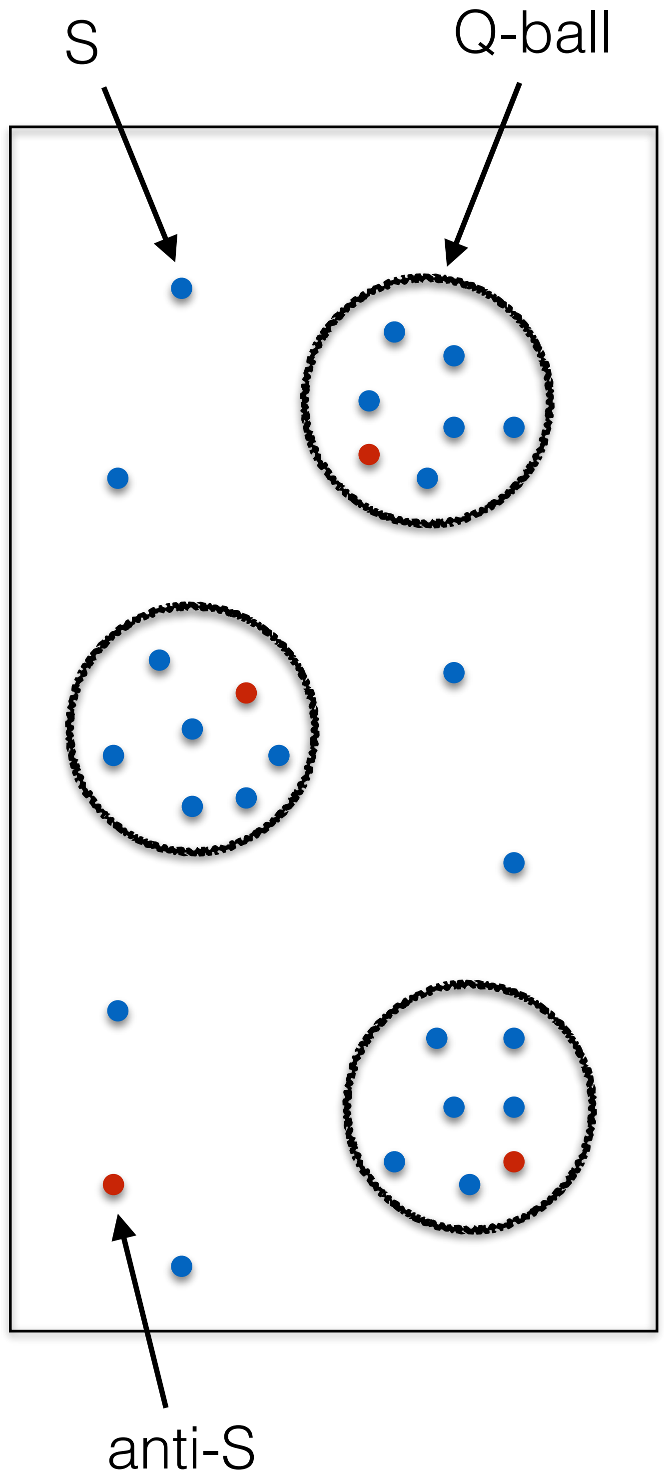


- **Typical M predicted from typical Q**
  - ➡ e.g. $m_Q \approx v\, Q \left[(\lambda_S \lambda_\phi)^{1/4} + c_2 Q^{-1/3} \right]$
- **Typical Q and number density can be inferred from PT**
  - ➡ fraction of the charges in Q-balls, combined w/ charge asymmetry

$$N_S^{\text{Q-ball}} \sim f_{\text{in}}\, n_S / n_{\text{Q-ball}}$$

$$\langle Q \rangle \sim \max \left[\eta N_S^{\text{Q-ball}},\ (N_S^{\text{Q-ball}})^{1/2} \right]$$

Relic Abundance: from PT



- **Typical M predicted from typical Q**
  - ➡ e.g. $m_Q \approx v\, Q \left[(\lambda_S \lambda_\phi)^{1/4} + c_2 Q^{-1/3} \right]$
- **Typical Q and number density can be inferred from PT**
  - ➡ comparable numbers of Q-ball vs. nucleation sites

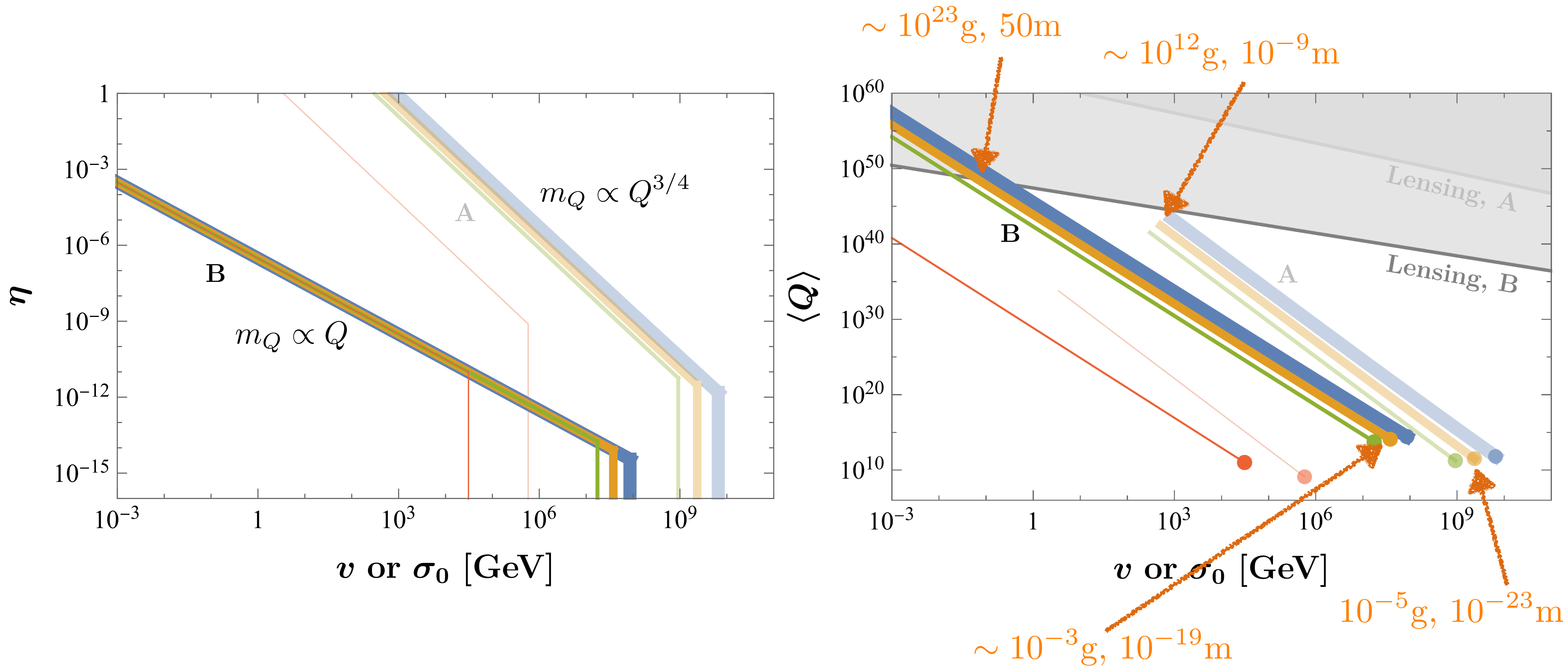
$$\gamma \approx T^4 \left(\frac{S_3}{2\pi T} \right)^{3/2} e^{-S_3/T}$$

$$h(t) = \exp\left[-\frac{4\pi}{3} \int_{t_c}^{t} dt'\, v_{\rm wall}^3 \,(t - t')^3 \,\gamma(t') \right]$$

$$n \approx n_{\rm nuc} = \int_{t_c}^{t_n} dt'\, \gamma(t')\, h(t')$$

[See K. Xie et. al., Phys.Rev.D 105 (2022) for an *ab initio* calculation]

Parameter Space: PT



Assuming $Q_{\rm min} = 4,\ f_{\rm in} = 1,\ v_{\rm wall} = 1$

For Completeness

- **Vector bosons are also bosons, so…?**
  - Yes there are also soliton states of vector bosons

[M. Jain, *Phys.Rev.D* 106 (2022) 8, 8]

[H. Zhang, M. Jain, M. A. Amin, *Phys.Rev.D* 105 (2022) 9, 096037]

  - A higgsed Yang-Mills theory or potential like $V(W^{\mu}W_{\mu})$ is required to provide interactions between the vector bosons
  - Spin-0 configuration has a preferred energy, though

Outline

- Brief introduction to macroscopic DM formation
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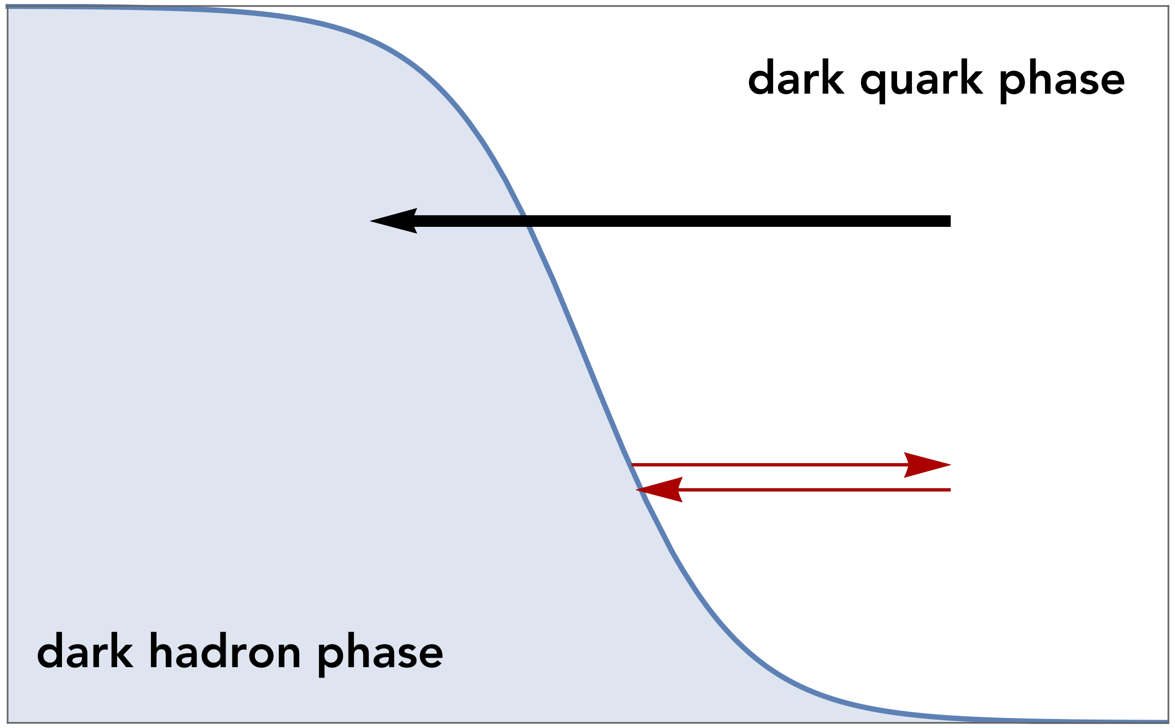
"Snow-Ploughing" the Charges?

- **Having much of the dark sector particles to be kept in the unbroken phase usually requires them to be (unusually) heavy in the broken phase**

$$m_{\text{broken}} > 2\gamma_w p_z, \quad p_z \sim p_{\text{rms}} \sim 3.6 T_c$$

$$\gamma_w \sim 1 \Rightarrow m_{\text{broken}} \gtrsim 7\, T_c \sim 7\, v$$



[D. Bodeker et. al., JCAP 0905 (2009)]

Assembling the Charges

- **Late universe evolution could change the story**
    - ➡ The soliton may absorb or release free particle/anti-particles and change their charges and sizes
    - ➡ Known as "solitosynthesis" [K. Griest, E. Kolb, *Phys.Rev.D* 40 (1989)]

$$
\begin{aligned}
S + S^\dagger &\leftrightarrow \phi + \phi^\dagger \ , \\
(Q) + S &\leftrightarrow (Q+1) + X \ , \\
(Q) + S^\dagger &\leftrightarrow (Q-1) + X \ , \\
(Q_{\rm min}) + S^\dagger &\leftrightarrow \underbrace{S + S + \cdots + S}_{Q_{\rm min}-1} + X \ . \\
(Q_1) + (Q_2) &\leftrightarrow (Q_1 + Q_2) + X \ , \\
(Q_1) + (-Q_2) &\leftrightarrow \begin{cases} (Q_1 - Q_2) + X & \text{for } Q_1 - Q_2 \geq Q_{\rm min} \ , \\ \underbrace{S + S + \cdots + S}_{Q_1 - Q_2} + X & \text{for } Q_{\rm min} > Q_1 - Q_2 \geq 0 \ . \end{cases}
\end{aligned}
$$

A Big Thermalized System

- **We consider the thermalization of free scalars w/ Q-ball charging from Qmin to Qmax within a Hubble time**

  recall that Qmin can be as small as O(1)

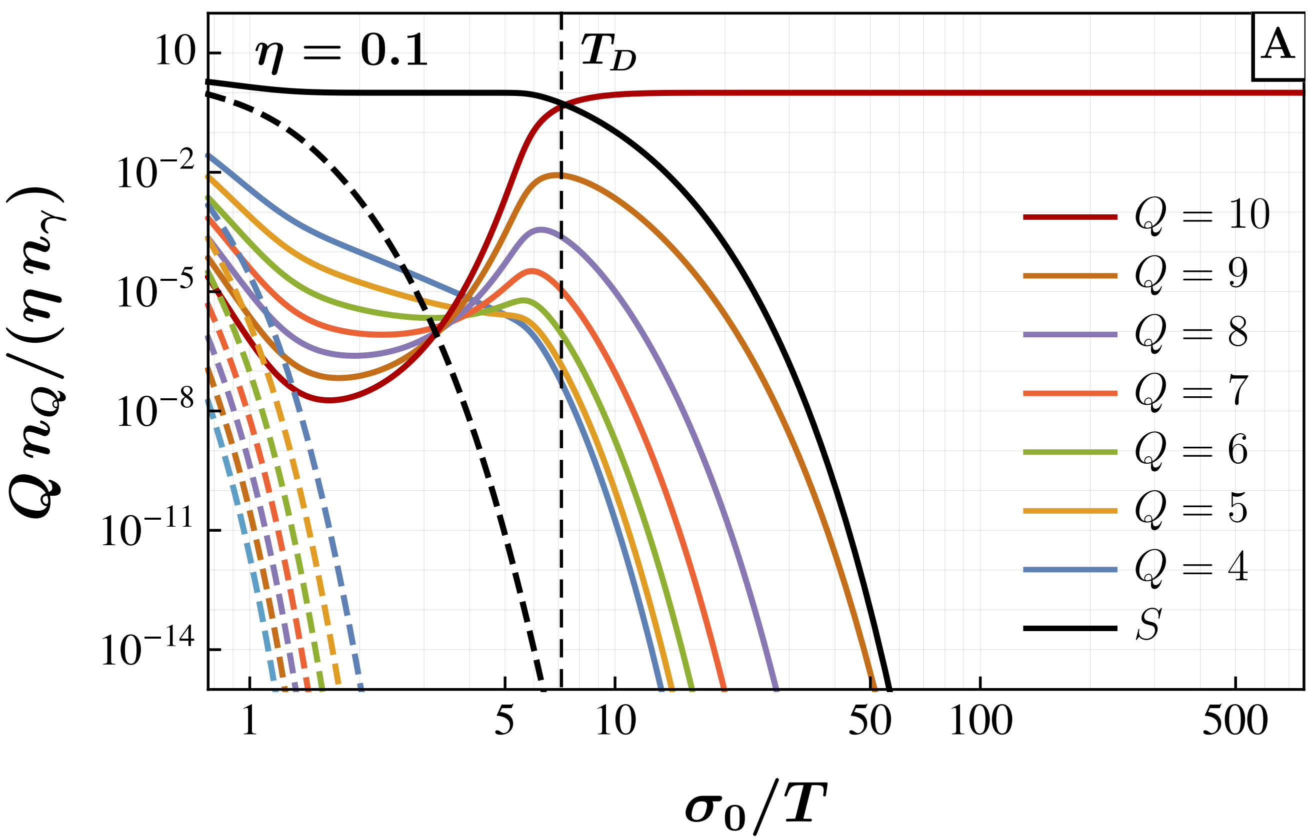
  - ➡ A system in thermal equilibrium

$$\tau_{Q_{\min}\to Q_{\max}} = \sum_{Q=Q_{\min}}^{Q_{\max}} \frac{1}{n_S\,(\sigma v_{\rm rel})_Q} \lesssim 1/H$$

  - ➡ This system can be built both “top-down” (interactions shrinking the sizes of large Q-balls) or “bottom-up” (fusion of free particles)

Q-ball Charge Domination

- **Assuming certain amount of asymmetry within the dark sector**
  - ➡ In equilibrium and with a reasonable* M(Q) vs. Q, the binding energy will push the Q charges into larger Q-balls



$$T_D = \frac{Q_{\max} m_S - m_{Q_{\max}}}{\log\left\{\frac{1}{Q_{\max}}\left[\frac{2}{\eta c_\gamma}\left(\frac{m_S}{2\pi T_D}\right)^{\frac{3}{2}}\right]^{Q_{\max}-1}\left(\frac{m_S}{m_{Q_{\max}}}\right)^{\frac{3}{2}}\right\}}$$

Plots in this section are made with the benchmark model in Griest & Kolb 89', where $M \propto Q^{3/4}$

*Say $M \propto Q^p$, "reasonable" means p<1

The Freeze-out of the System

- **The evolution should finally freeze-out**
  - Solve a set of Boltzmann equations for each component and determine the freeze-out temperature T_F
  - For solitons to be the main DM components (of charge), we should at least expect $T_D > T_F$

geometric Xsec

$$
\begin{aligned}
S + S^\dagger &\leftrightarrow \phi + \phi^\dagger \ , \\
(Q) + S &\leftrightarrow (Q+1) + X \ , \\
(Q) + S^\dagger &\leftrightarrow (Q-1) + X \ , \\
(Q_{\min}) + S^\dagger &\leftrightarrow \underbrace{S + S + \cdots + S}_{Q_{\min}-1} + X \ .
\end{aligned}
$$

not important

$$
\begin{aligned}
(Q_1) + (Q_2) &\leftrightarrow (Q_1 + Q_2) + X \ , \\
(Q_1) + (-Q_2) &\leftrightarrow \begin{cases} (Q_1 - Q_2) + X & \text{for } Q_1 - Q_2 \geq Q_{\min} \ , \\ \underbrace{S + S + \cdots + S}_{Q_1 - Q_2} + X & \text{for } Q_{\min} > Q_1 - Q_2 \geq 0 \ . \end{cases}
\end{aligned}
$$

  - Also they assemble solitons from free particles

The Freeze-out of the System

- **Write down all the Boltzmann equations**

$$\dot{n}_Q + 3Hn_Q = -\,\delta_{Q,Q_{\min}}(\sigma v_{\rm rel})_{Q_{\min}}\left(n_{Q_{\min}}n_{S^\dagger} - n^{\rm eq}_{Q_{\min}}n^{\rm eq}_{S^\dagger}\left(\frac{n_S}{n_S^{\rm eq}}\right)^{Q_{\min}-1}\right)$$
$$-\,(1-\delta_{Q,Q_{\max}})(\sigma v_{\rm rel})_Q\left(n_Q n_S - n_Q^{\rm eq}n_S^{\rm eq}\left(\frac{n_{Q+1}}{n_{Q+1}^{\rm eq}}\right)\right)$$
$$+\,(1-\delta_{Q,Q_{\min}})(\sigma v_{\rm rel})_{Q-1}\left(n_{Q-1}n_S - n_{Q-1}^{\rm eq}n_S^{\rm eq}\left(\frac{n_Q}{n_Q^{\rm eq}}\right)\right)$$
$$-\,(1-\delta_{Q,Q_{\min}})(\sigma v_{\rm rel})_Q\left(n_Q n_{S^\dagger} - n_Q^{\rm eq}n_{S^\dagger}^{\rm eq}\left(\frac{n_{Q-1}}{n_{Q-1}^{\rm eq}}\right)\right)$$
$$+\,(1-\delta_{Q,Q_{\max}})(\sigma v_{\rm rel})_{Q+1}\left(n_{Q+1}n_{S^\dagger} - n_{Q+1}^{\rm eq}n_{S^\dagger}^{\rm eq}\left(\frac{n_Q}{n_Q^{\rm eq}}\right)\right)$$

- **Summing over all Qs**

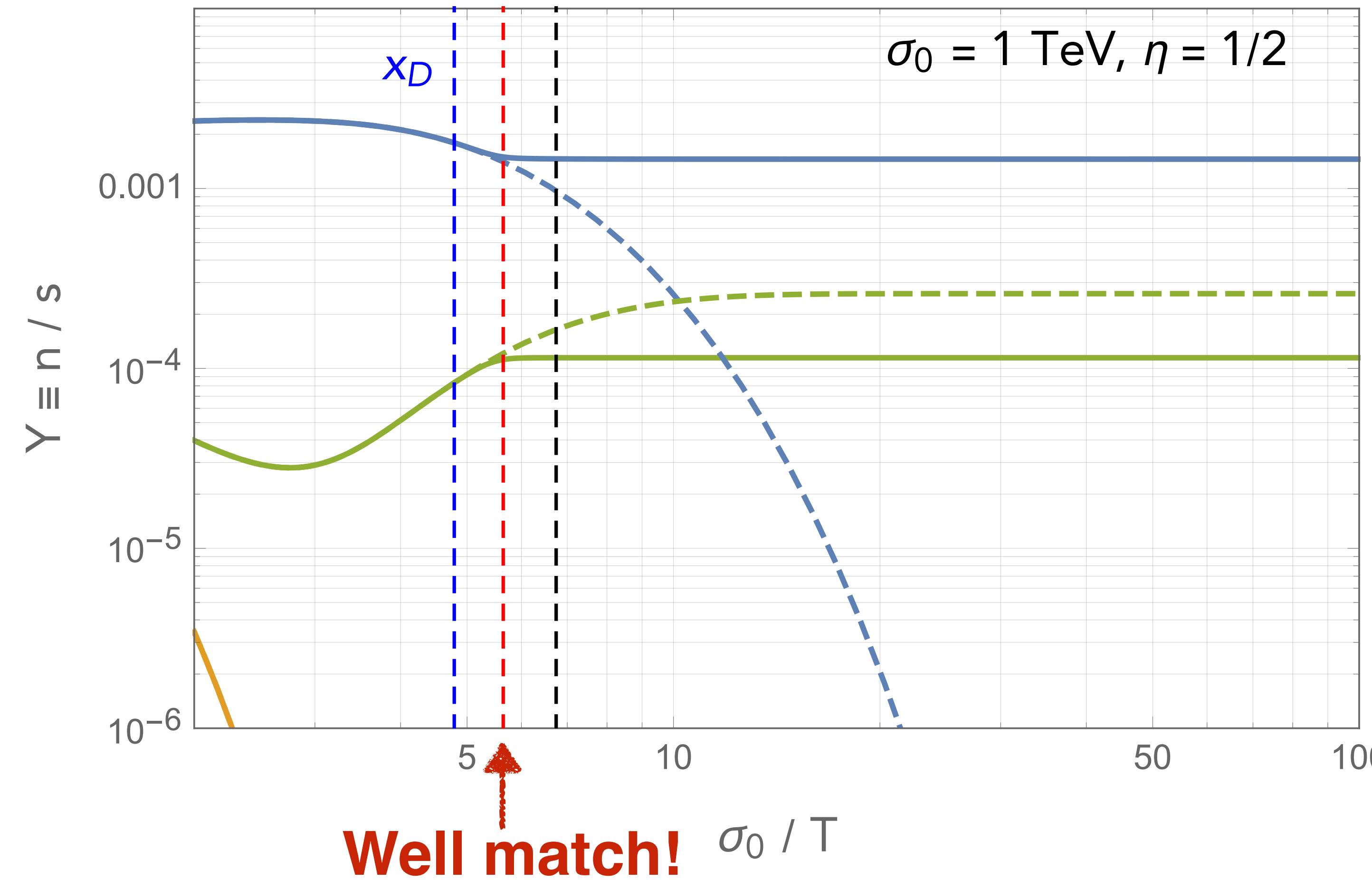
$$\dot{n}_{\rm NTS} + 3Hn_{\rm NTS} = -\sigma v(Q_{\rm min})\left(n_{Q_{\min}}n_{\bar\phi} - n^{\rm eq}_{Q_{\min}}n^{\rm eq}_{\bar\phi}\left(\frac{n_\phi}{n_\phi^{\rm eq}}\right)^{Q_{\min}-1}\right)$$

$$\Rightarrow H n_{\rm NTS} \sim \sigma v(Q_{\rm min})\,n_{Q_{\min}}n_{\bar\phi}\,|_{T=T_F}$$

The Freeze-out of the System

- Write down all the Boltzmann equations

$$T_F = \frac{(Q_{\min} - 1 - Q_{\max})\mu - (m_S + m_{Q_{\min}} - m_{Q_{\max}})}{\log\left[\frac{\pi\, g_*^{1/2}\, T_F^{1/2} [2\pi\, m_{Q_{\max}}/(m_S\, m_{Q_{\min}})]^{3/2})}{\sqrt{90} M_{\rm pl}\, (\sigma v_{\rm rel})_{Q_{\min}}}\right]}$$



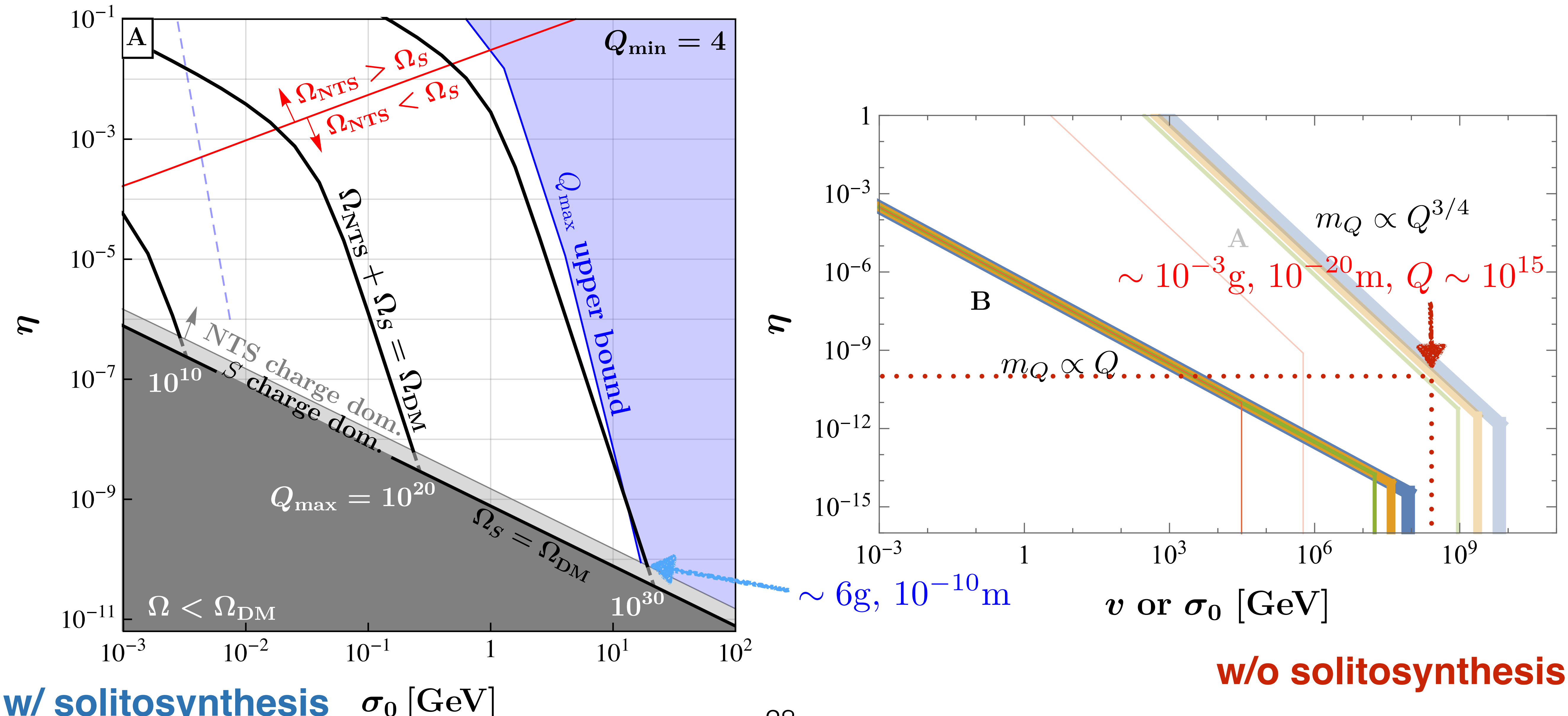
- Summing over all Qs

$$\dot{n}_{\rm NTS} + 3H n_{\rm NTS} = -\sigma v(Q_{\min}) \left(n_{Q_{\min}} n_{\bar\phi} - n^{\rm eq}_{Q_{\min}} n^{\rm eq}_{\bar\phi} \left(\frac{n_\phi}{n_\phi^{\rm eq}}\right)^{Q_{\min}-1} \right)$$

$$\Rightarrow H n_{\rm NTS} \sim \sigma v(Q_{\min}) n_{Q_{\min}} n_{\bar\phi}|_{T=T_F}$$

Parameter Space: Solitosynthesis

- For solitons to be relevant, we require $T_D > T_F$
- Also consider the DM relic abundance



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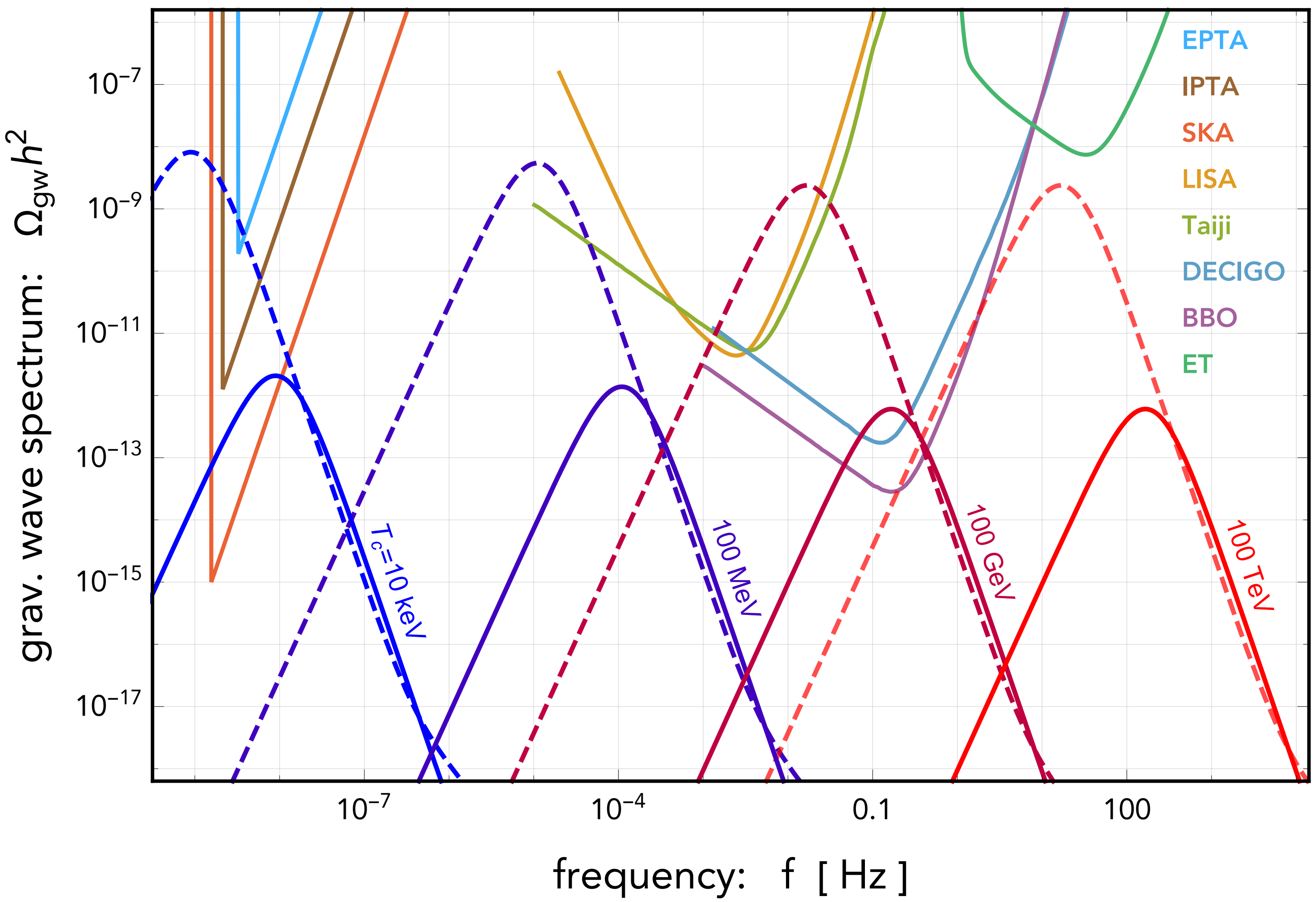
Gravitational Wave

Stay tuned for Haipeng An's talk tomorrow

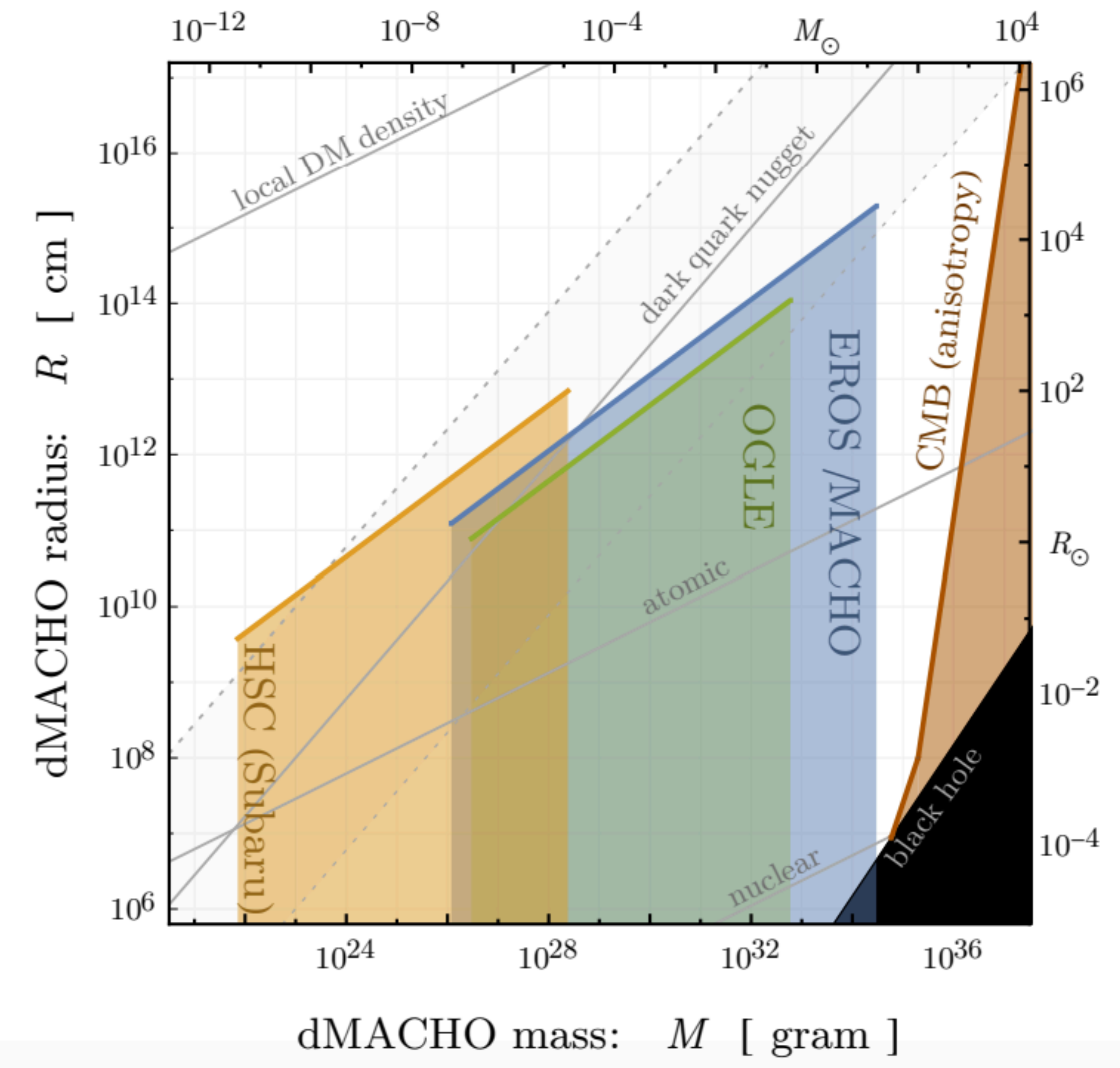
- **GW spectrum:** $\Omega_{\rm gw}\, h^2 = \Omega_\phi\, h^2 + \Omega_{\rm sw}\, h^2 + \Omega_{\rm turb}\, h^2$

$\alpha \equiv \dfrac{\rho_{\rm vac}}{\rho_{\rm rad}}$ energy density ratio $\qquad \dfrac{\beta}{H} \equiv T\,\dfrac{{\rm d}S}{{\rm d}T}$ strength of the phase transition



$(\alpha, \beta/H) = (0.1, 10^4)$ for solid
$= (1, 10^3)$ for dashed

Lensing and Accretion



✤ **If heavy enough:**

➡ The accreted matter around the macroscopic DM can generates radiation influencing CMB

[Y. Bai, A. J. Long, *SL*, JCAP 09 (2020)]

➡ The optical signal may be distinguishable from normal stars and thus can be directly searched for on telescopes like *Gaia*

[D. Curtin et. al., JHEP 07 (2022)]

Direct Detection?

- **There are many recent discussions on the direct detection of heavy DM using an array of sensitive detectors**
  - ➡ Optically levitated microsphere [A. Kawasaki, arXiv:1809.00968]
  - ➡ Array of quantum-limited impulse sensor

    [D. Carney et. al, arXiv:1903.00492]
  - ➡ Assuming a nugget interact with a detector of size L

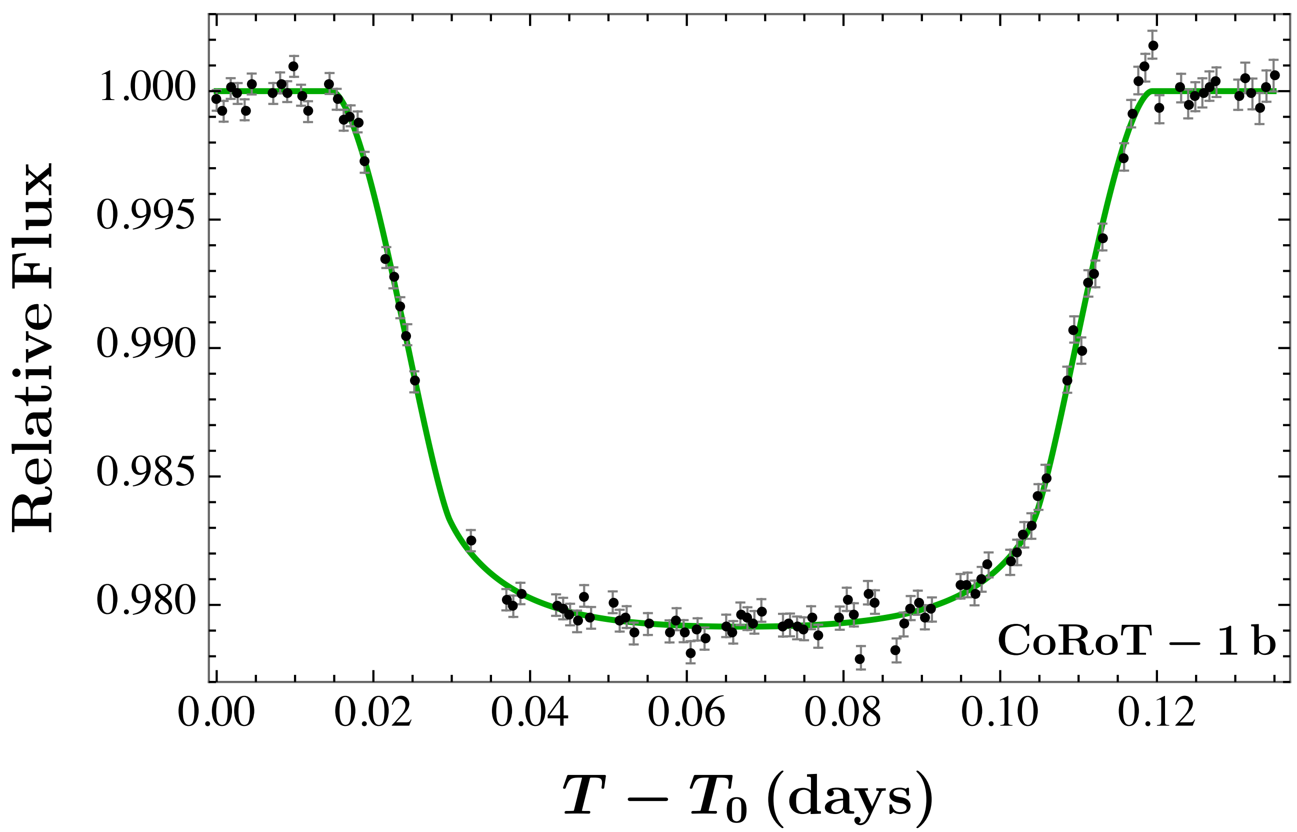
$$N_{\rm int} = n_{\rm dQN}\, v_{\rm dQN} L^2 \Delta t \simeq \left(2.5\times 10^{-15}\right)\left(\frac{T_c}{0.1{\rm GeV}}\right)^3 \left(\frac{\tilde\sigma}{0.1}\right)^{-9/2}\left(\frac{L}{10\,{\rm m}}\right)^2\left(\frac{\Delta t}{1\,{\rm yr}}\right)$$

$$N_{\rm int} > 1 \Rightarrow T_c > \left(7.4\,{\rm TeV}\right)\left(\frac{\tilde\sigma}{0.1}\right)^{3/2}\left(\frac{L}{10\,{\rm m}}\right)^{-2/3}\left(\frac{\Delta t}{1\,{\rm yr}}\right)^{-1/3}$$

Transit Light Curves

- An exoplanet may block its hosting star and lower the observed intensity periodically

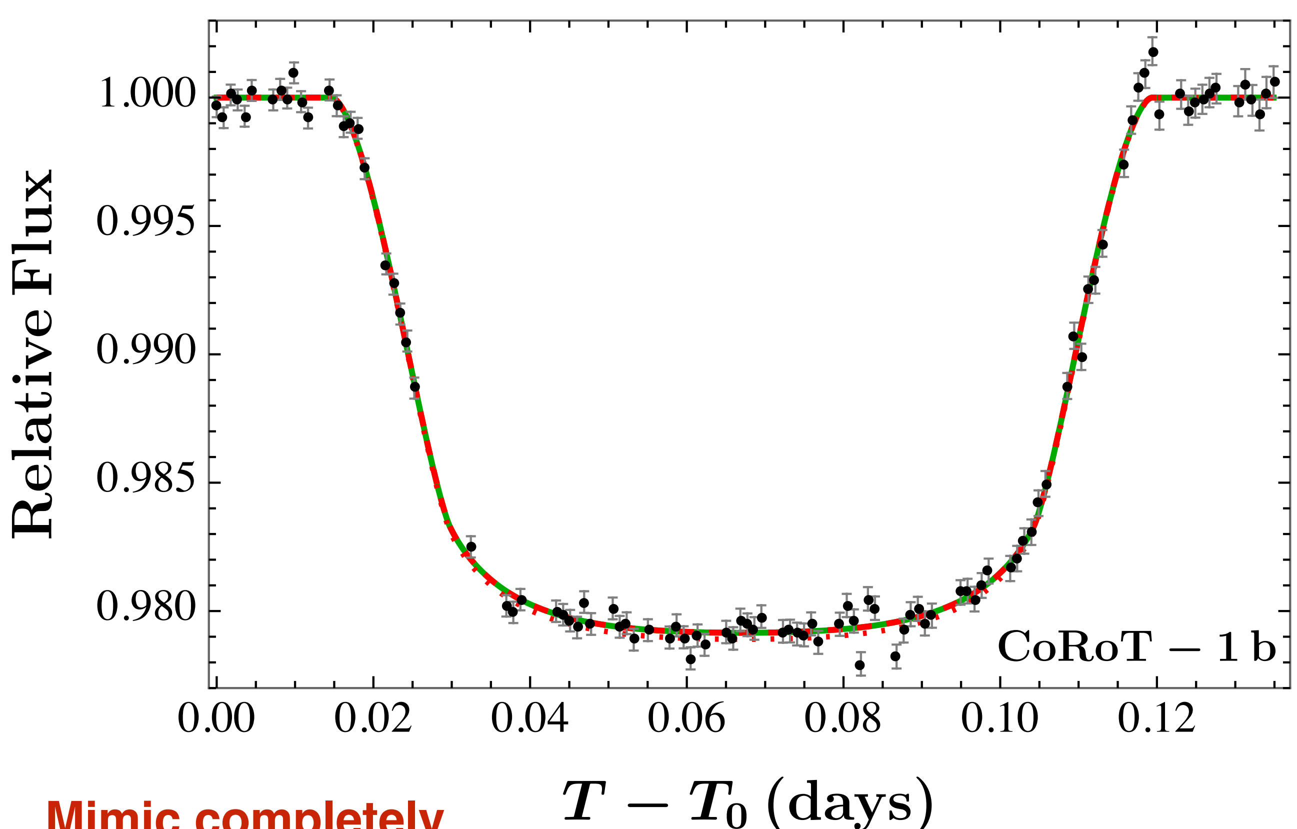
Green: best-fitted ordinary exoplanet

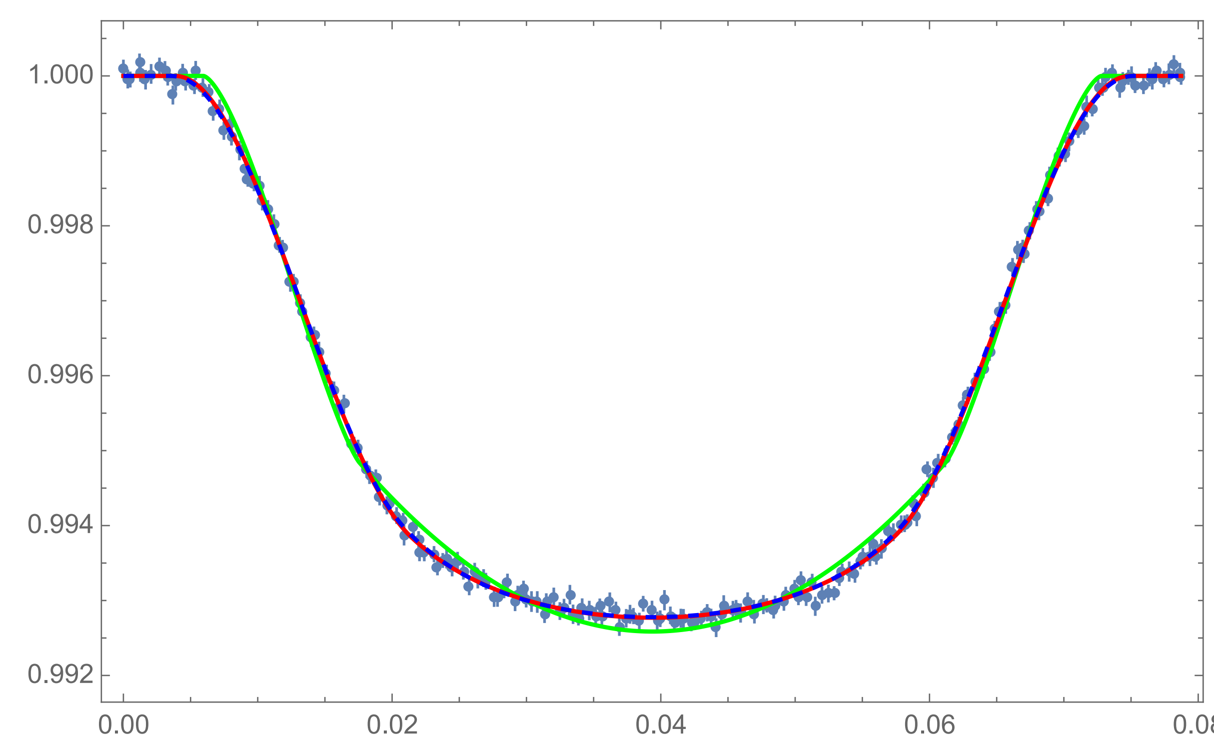


Transit Light Curves

- **An exoplanet may block its hosting star and lower the observed intensity periodically**
- **What if the "exoplanet" is a macroscopic DM?**
  - ➡ Not necessarily completely opaque

Green: best-fitted ordinary exoplanet   Red: best-fitted dark exoplanet





Mimic completely

A dark exoplanet light curve may not be completely mimicked by regular ones

Conclusion

- Macroscopic DM can be naturally formed from cosmic PT, while the late universe evolution is also important
- The candidates can be either fermonic or bosonic. Model building can be fun
- The signatures and constraints are largely model dependent, while it would be very interesting and important to think of model independent/insensitive detection methods

Thank you!

Backup

Collapse into BH?

- **Not in the models I've worked on, but definitely possible**
  - Adding in Yukawa interactions in fermionic theories

[K. Kawana, K. Xie, Phys. Lett. B 824 (2022)]

  - The unbroken phase may be treated as a local overdensity

[J. Liu et. al., Phys. Rev. D 105 (2022) 2, 2]

Fermionic Macroscopic DM

- **Dark QCD: SU(N_d) w/ N_f massless flavors**

$$\mathcal{L} = \sum_{i=1}^{N_f} i\,\bar{\psi}_i \gamma^\mu D_\mu \psi_i - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a}$$

  - ➡ Dark baryon number
  - ➡ The macroscopic states are lumps of dark quark matter, i.e., dark quark nuggets

[Y. Bai, A. J. Long, *SL*, Phys. Rev. D99 (2019)]

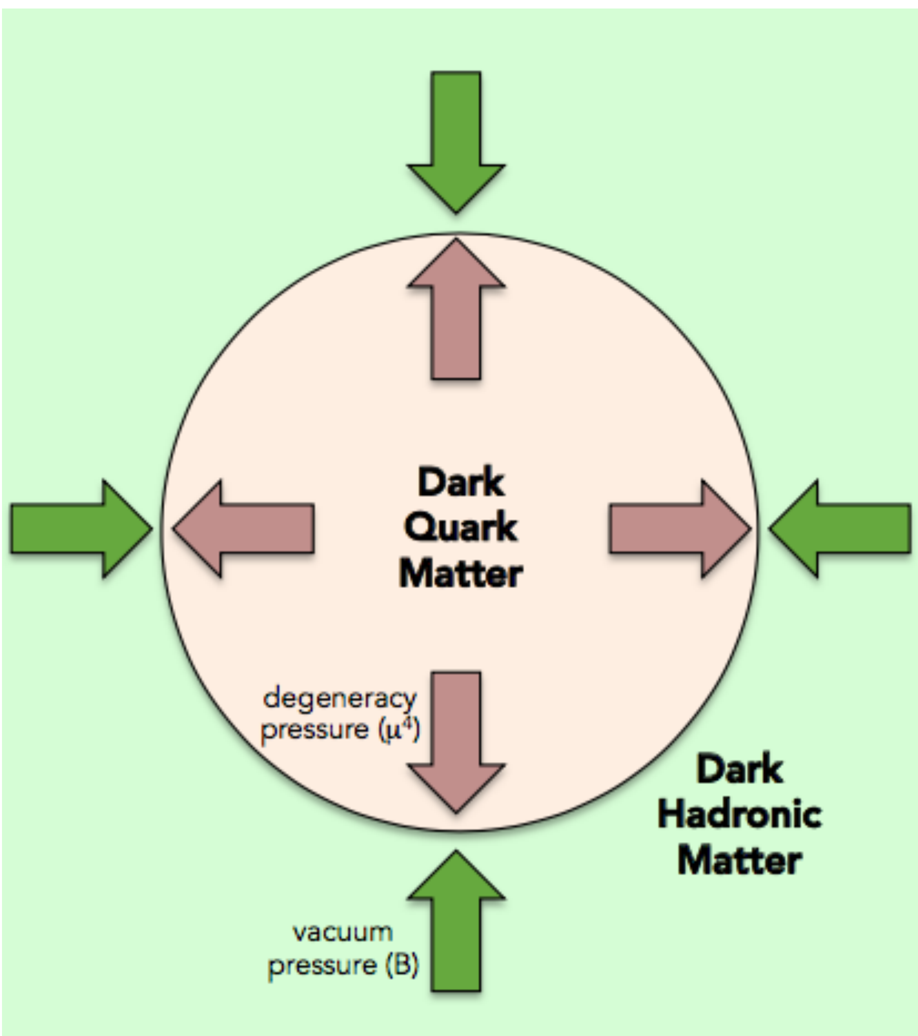
- **Pisarski & Wilczek's argument: N_d>=3, N_f >=3 and a phase transition exists, it will be first order**
  - ➡ Compared w/ lattice studies

[R. Pisarski and F. Wilczek, Phys. Rev. D29 (1984)]

- **The scale in this model:** $T_c \sim \Lambda_d$

Fermionic Macroscopic DM

- **The final state of the system satisfies $T \ll \mu$**
  - ➡ System supported by Fermi degeneracy pressure



- **From thermal dynamics:**

$$n = g\frac{\mu^3}{6\pi^2} \quad \rho = g\frac{\mu^4}{8\pi^2} + B \quad P = g\frac{\mu^4}{24\pi^2} - B$$

$$g = 2N_d N_f \qquad n_{\mathrm{B_d,nug}} = \frac{1}{N_d} n = N_f \frac{\mu^3}{3\pi^2}$$

- **Balancing the pressure**

$$P\Big|_{\mu=\mu_{\mathrm{eq}}} = 0 \Rightarrow \mu_{\mathrm{eq}} = \left(\frac{12\pi^2}{N_d N_f}\right)^{1/4} B^{1/4}$$

$$\Rightarrow \begin{cases} \rho = 4B \\ n_{\mathrm{B_d,nug}} = \left(\frac{64 N_f}{3\pi^2 N_d^3}\right)^{1/4} B^{3/4} \end{cases}$$

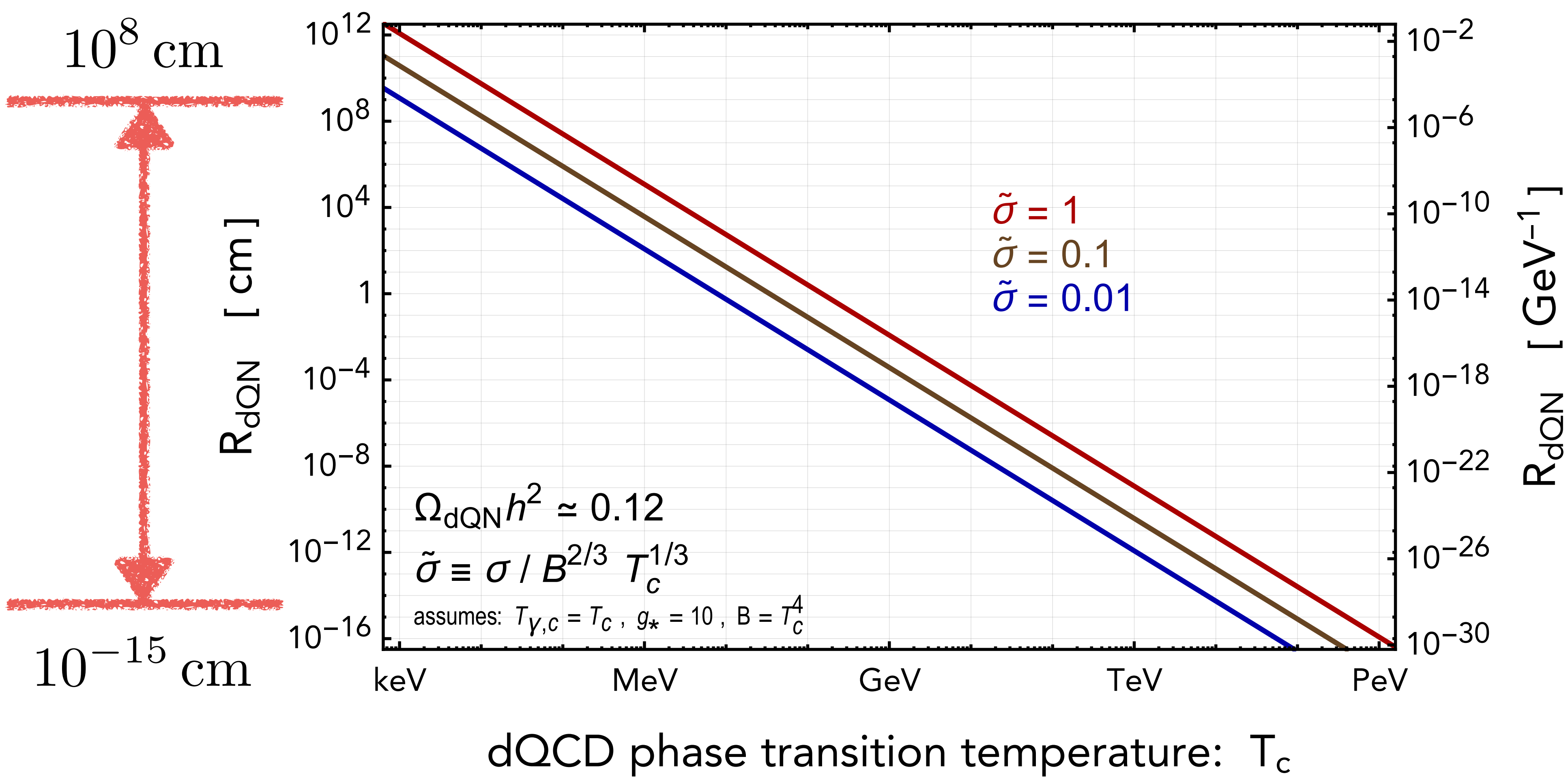
➡ We expect $B \sim \Lambda_d^4$

Fermionic Macroscopic DM

- **Combining the ingredients before**

$$R_{\rm nug} \simeq (0.081\ {\rm cm}) \left[\frac{B}{(0.1\ {\rm GeV})^4}\right]^{-1/3} \left(\frac{T_c}{0.1\ {\rm GeV}}\right)^{-1} \left(\frac{\tilde\sigma}{0.1}\right)^{3/2}$$

$\tilde\sigma \equiv \sigma/(B^{2/3} T_c^{1/3})$

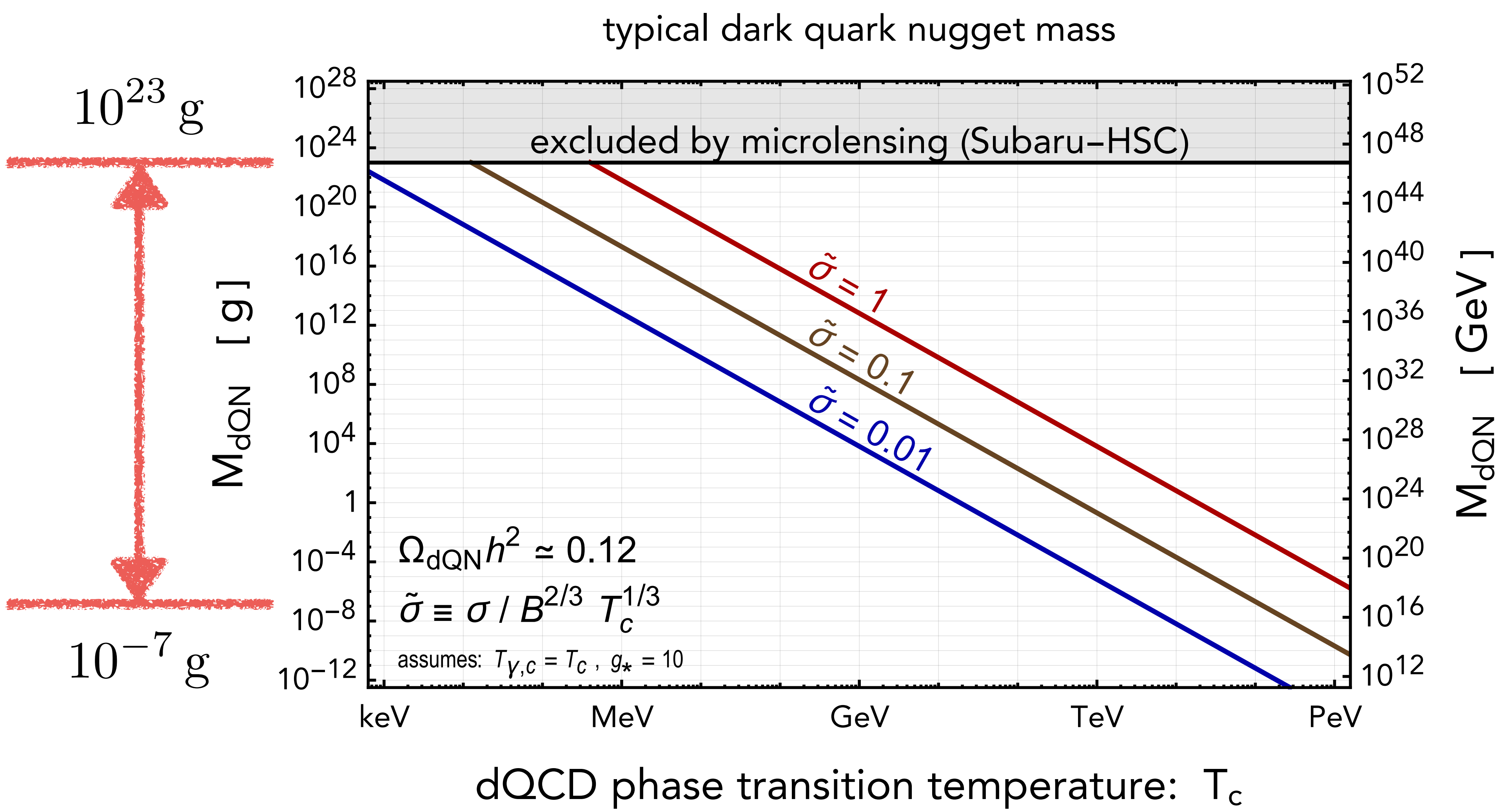


(for $N_f = N_d = 3$)

Fermionic Macroscopic DM

- **Combining the ingredients before**

$$M_{\rm nug} \simeq (2.1 \times 10^{11}\ {\rm g}) \left(\frac{T_c}{0.1\ {\rm GeV}}\right)^{-3} \left(\frac{\tilde\sigma}{0.1}\right)^{9/2}$$



(for $N_f = N_d = 3$)

Some Q-ball Models

- **The properties of Q-balls depends on the scalar potential**
  - ➡ Benchmark model B: a Z_2 symmetric potential

$$V(S,\phi) = \frac{1}{4}\lambda_\phi(|\phi|^2 - v^2)^2 + \frac{1}{4}\lambda_{\phi S}|S|^2|\phi|^2 + \lambda_S|S|^4 + m_{S,0}^2|S|^2$$

  - ➡ EOM: let $S = e^{-i\omega t}\, v\, s(r)/\sqrt{2}\,,\ \phi = v\, f(r)\,,\ \omega = v\,\Omega$

$$f'' + \frac{2}{r}\, f' - \frac{1}{2}\,\lambda_\phi\, f\,(f^2-1) - \frac{1}{8}\,\lambda_{\phi S}\, s^2 f = 0\,,$$
$$s'' + \frac{2}{r}\, s' + \Omega^2\, s - \frac{1}{4}\,\lambda_{\phi S}\, f^2\, s - \lambda_S\, s^3 - \mu_0^2\, s = 0\,,$$

[Y. Bai, *SL*, N. Orlofsky, 2208.12290]

Basics of QMBs

- **A Q-monopole-ball (QMB) is charged both topologically and non-topologically**
  - ➡ Introduce a quartic coupling between the scalar fields

$$\mathcal{L} = |\partial_\mu S|^2 + \frac{1}{2}(D_\mu \phi^a)^2 - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - V(S,\phi)\,,$$

$$V(S,\phi) = \frac{1}{8}\lambda_\phi(\phi^a\phi^a - v^2)^2 + \frac{1}{2}\lambda_{\phi S}|S|^2(\phi^a\phi^a) + \lambda_s|S|^4 + \underline{m^2_{s,0}|S|^2}$$

taken to be zero

$$m_S = v\sqrt{\lambda_{\phi S}/2}$$

$$\phi^a = \hat{r}^a\, v\, f(r)\,, \quad S = e^{-i\omega t}\frac{v}{\sqrt{2}}\, s(r)\,, \quad A_0 = 0\,, \quad A_i^a = \epsilon^{aij}\frac{\hat{r}^j}{e\, r}\, a(r)$$

$$\omega = \Omega/v\,, \quad m_{S,0} = \mu_0\, v\,, \quad r = \overline{r}/v$$

Some Q-ball Models

- **The properties of Q-balls depends on the scalar potential**
  - ➡ sth fancy: a *nontopological* soliton w/ a *topological* charge
    - the “Q-monopole-ball”
  - ➡ Consider gauged SU(2) × global U(1)

$$a'' - \frac{1}{\bar{r}^2}\, a(1-a)(2-a) + e^2\,(1-a)\, f^2 = 0\,,$$

$$f'' + \frac{2}{\bar{r}}\, f' - \frac{2}{\bar{r}^2}\,(1-a)^2\, f - \frac{1}{2}\,\lambda_\phi\, f\,(f^2-1) - \frac{1}{2}\,\lambda_{\phi S}\, s^2 f = 0\,,$$

$$s'' + \frac{2}{\bar{r}}\, s' + \Omega^2\, s - \frac{1}{2}\,\lambda_{\phi S}\, f^2\, s - \lambda_S\, s^3 - \mu_0^2\, s = 0\,,$$

Q-ball Charge Domination

- **Assuming certain amount of asymmetry within the dark sector**
  - In equilibrium and with a reasonable M(Q) vs. Q, the binding energy will push the Q charges into larger Q-balls
  - The temperature T_D when the Q-balls dominate the charge abundance is determined by solving:

$$\eta = [n_s - \bar{n}_s + \sum_Q Q(n_Q - \bar{n}_Q)]/n_\gamma$$

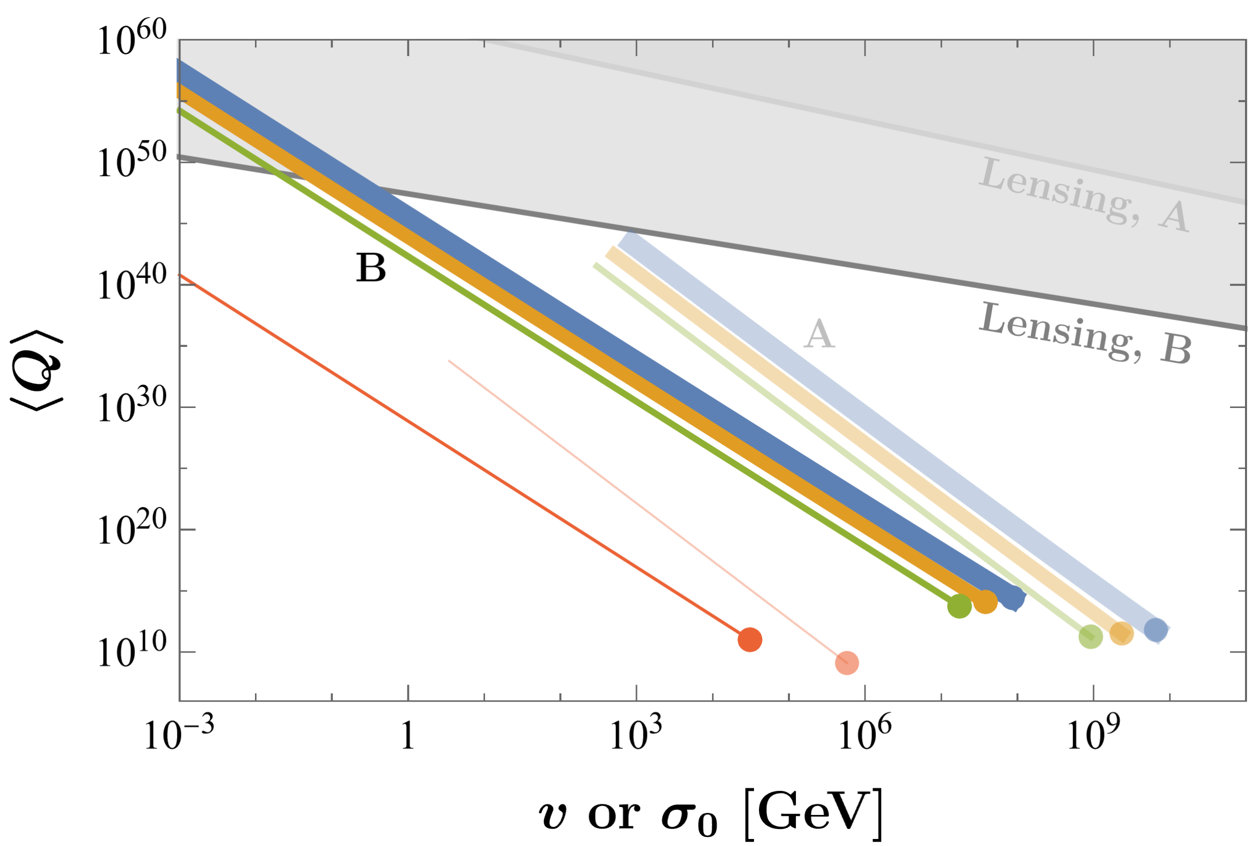
$$\left(\frac{m_S T_D}{2\pi}\right)^{3/2} \exp\left(\frac{\mu - m_S}{T_D}\right) = Q_{\max}\left(\frac{m_{Q_{\max}} T_D}{2\pi}\right)^{3/2} \exp\left(\frac{Q_{\max}\mu - m_{Q_{\max}}}{T_D}\right) = \frac{1}{2}\eta c_\gamma T_D^3$$

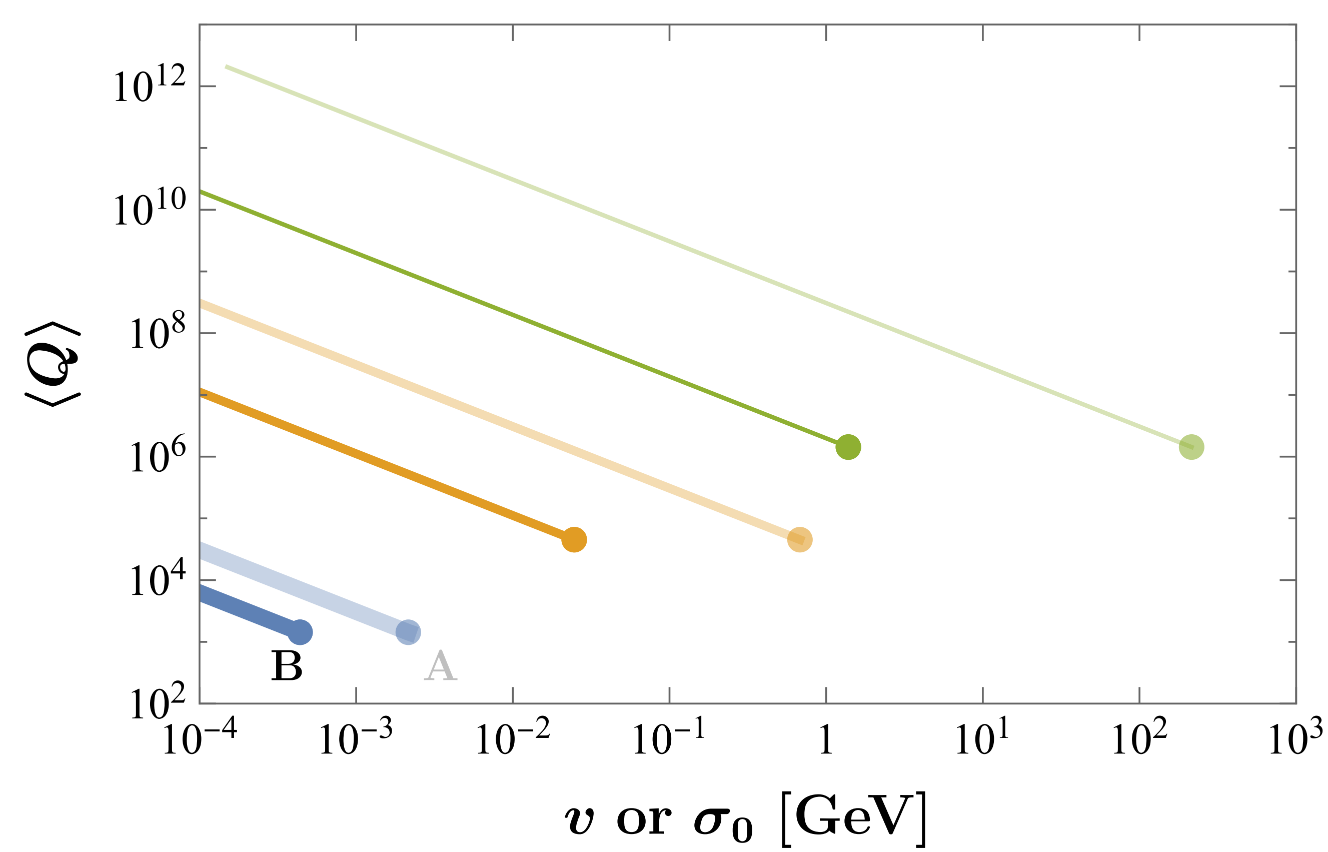
  - After T_D the chemical potential:

$$\mu \simeq \frac{1}{Q_{\max}}\left(m_{Q_{\max}} + T\log\left[\frac{\eta\, c_\gamma}{Q_{\max}}\left(\frac{2\pi T}{m_{Q_{\max}}}\right)^{3/2}\right]\right)$$

Beyond FOPT

- **SOPT may also be a viable approach**
  - ➡ There's not a "snowplow", and the number density of Q-balls is determined by the correlation length



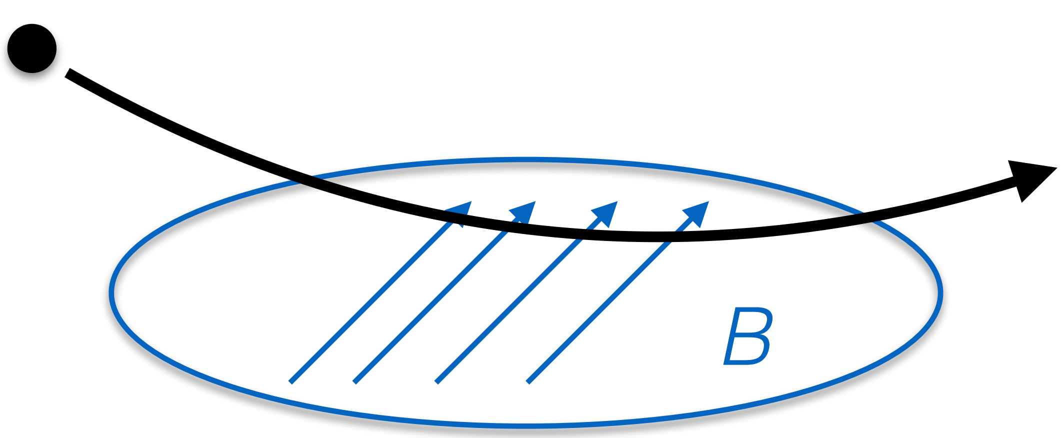


typical charge is much smaller compared to the FOPT case

Model Dependent Constraints

- **The Parker bound**
  - galactic magnetic accelerate magnetic objects, which extract magnetic energy from the galaxy
  - energy drained through this process should not deplete galactic magnetic field within a regeneration time

$$\Delta E \times F \times (\pi\,\ell_c^2) \times (4\pi\,\mathrm{sr}) \times t_{\mathrm{reg}} \lesssim \frac{B^2}{2}\,\frac{4\pi\,\ell_c^3}{3}, \qquad \Delta E \sim M\,\Delta v^2/2$$



[E. Parker, Astrophys. J. 160 (1970) 383;
M. Turner, E. Parker, T. Bogdan, Phys. Rev. D 26 (1982) 1296]