# *High-quality Axion in the GUTs Beyond the* $\mathrm{SU}(5)$

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# Background: GUT

* The grand unified theory (GUT) was first proposed by Georgi & Glashow in 1974, with $\mathrm{SU}(5)$ as the minimal simple Lie group to unify $\mathscr{G}_{\mathrm{SM}} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, with $\mathrm{rank}(\mathscr{G}_{\mathrm{SM}}) = \mathrm{rank}(\mathrm{SU}(5)) = 4$.

* 15 SM complex chiral fermions fit into irreps of $\bar{\mathbf{5}}_{\mathbf{F}}$ and $\mathbf{10}_{\mathbf{F}}$ in the $\mathrm{SU}(5)$, which are required by the gauge anomaly cancellation.

* There was also an $\mathrm{SO}(10)$ GUT by Fritzsch & Minkowski in 1975. This is automatically anomaly-free, and its irrep of $\mathbf{16}_{\mathbf{F}}$ even includes a $\nu_R$.

# Background: GUT

| Topics | Authors | Publications | Date |
|---|---|---|---|
| Fourth color unification | Pati, Salam | [5] | 1974 |
| SU(5) GUT | Georgi, Glashow | [6] | 1974 |
| SO(10) GUT | Fritzsch, Minkowski | [7] | 1975 |
| Peccei-Quinn mechanism | Peccei, Quinn | [8] | 1977 |
| Seesaw mechanism | Yanagida | [9] | 1979 |
| | Gell-Mann, Ramond, Slansky | [10] | 1979 |
| KSVZ axion | Kim | [11] | 1979 |
| | Shifman, Vainshtein, Zakharov | [12] | 1980 |
| $\mathrm{SU(N+4)}$ global symmetry | Dimopoulos, Raby, Susskind | [13] | 1980 |
| SUSY SU(5) GUT | Dimopoulos, Georgi | [14] | 1981 |
| DFSZ axion | Zhitnitsky | [15] | 1980 |
| | Dine, Fischler, Srednicki | [16] | 1981 |
| Axion in SU(5) GUT | Wise, Georgi, Glashow | [17] | 1981 |
| Leptogenesis | Fukugita, Yanagida | [18] | 1986 |
| ... | | | |

**Table 1:** Collections of the major theoretical advances in the GUT and some related BSM physical issues listed in chronological order.

# Background: GUT

* Over 40 yrs since the $\mathrm{SU}(5)$ and $\mathrm{SO}(10)$, there is convincing evidence that the BSM new physics should be put forth to address:

  (1) Strong CP problem, e.g., the Peccei-Quinn (PQ) mechanism and the PQ quality
  (2) neutrino masses through the seesaw mechanism
  (3) Baryon asymmetry through the baryogenesis/leptogenesis
  (4) Dark matter

* The simplest $\mathrm{SU}(5)$, $\mathrm{SO}(10)$ and their varieties, do not seem to include all necessary ingredients for BSM. One often needs to put new physical ingredients into the $\mathrm{SU}(5)$ && $\mathrm{SO}(10)$ GUTs by hand.

# Background: Strong CP

* The strong CP problem, a topological term for the QCD vacuum $\mathscr{L}_\theta = \theta \frac{\alpha_{3c}}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu\, a}$, and experimentally from the neutron EDM: $|\bar{\theta}| \lesssim 10^{-10}$ , with $\bar{\theta} = \theta + \arg\det M_q$, very different from the $\mathscr{O}(1)$ expectation of $\theta$ parameter.

* PQ mechanism: to replace $\theta$ by a periodic pseudo-scalar field $a \to a + 2\pi f_a$ , $f_a$ is known as the axion decay constant. There is a classical window of $10^8\,\mathrm{GeV} \lesssim f_a \lesssim 10^{12}\,\mathrm{GeV}$.

* Axion induced potential: $V_{\rm QCD} = \Lambda^4_{\rm QCD}(1 - \cos(a/f_a))$.

* Invisible axion models such as KSVZ and DFSZ, $a$xion comes from a complex && SM-singlet scalar field $\Phi = \frac{1}{\sqrt{2}}(v_a + \rho_a)\exp(ia/f_a)$.

# Background: PQ quality

* PQ quality: $\mathrm{U(1)_{PQ}}$ symmetry (expressed in terms of $\Phi$) is global and put in by hand, and the gravity does not respect global symmetries. It can induce a general operator of $\mathcal{O}_{\not{PQ}}^{d=2m+n} = k\frac{|\Phi|^{2m}\Phi^n}{M_{\rm pl}^{2m+n-4}} + H.c.$ ['92 Kamionkowski, March-Russell, and etc.] $\Delta\mathrm{PQ} = n$ with $\mathrm{PQ}(\Phi) = 1$.
* The $\mathcal{O}_{\not{PQ}}^{d=2m+n}$ shifts the $V_{\rm QCD}$ minima $|\bar{\theta}| = |\langle a\rangle/f_a| \lesssim 10^{-10}$
* if $|k| \sim 10^{-2}$ and $2m+n=5$, $\Rightarrow f_a \lesssim 10\,\mathrm{GeV}$, ruled out,
  else if $f_a \sim 10^{12}\,\mathrm{GeV}$ and $2m+n=5$, $\Rightarrow |k| \lesssim 10^{-55}$, very fine-tuned.
* NB, the renormalizable operators with $2m+n \leq 4$ are in principle possible. The discussion above considered a general SM-singlet $\Phi$.

# Global Symmetries

* The usual wisdom of a high-quality PQ is to have the $\mathrm{U}(1)_{\mathrm{PQ}}$ as an emergent global symmetry.

* The chiral gauge theory w.o. Unification: to put another confining theory with the SM, e.g. $\mathrm{SU}(5) \otimes \mathcal{G}_{\mathrm{SM}}$ by Gavela, Ibe, Quilez, and Yanagida [1812.08174].

* In 1980, Dimopoulos-Raby-Susskind (DRS) studied a strongly-interacting theory: an anomaly-free $\mathrm{SU}(N+4)$ chiral gauge theory with $N$ anti-fundamental fermions and one rank-2 anti-symmetric fermion, and it has $\mathcal{G}_{\mathrm{DRS}} = \mathrm{SU}(N) \otimes \mathrm{U}(1)$ , $N \geq 2$ . The DRS symmetry is determined by the anomaly-free condition.

# Our results:

* We start from the minimal $\mathrm{SU}(6)$, and identify $\mathcal{G}_{\mathrm{DRS}} = \mathrm{SU}(2)_F \otimes \mathrm{U}(1)_{\mathrm{PQ}}$.

* Our finding is that a *non-minimal* SUSY $\mathrm{SU}(6)$ GUT with its minimal fermion && Higgs setup can lead to:

  (1) Automatic high-quality PQ symmetry breaking @ $10^8\,\mathrm{GeV} \lesssim f_a \lesssim 10^{10}\,\mathrm{GeV}$, with an extended symmetry of $\mathcal{G}_{331} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$
  (2) Automatic KSVZ vector-like quarks $m_D \sim f_a$, with fixed electric charge of $-1/3$.
  (3) A cosmological-safe axion model, no DW formation

# The SU(6) model

# The SU(6) model

* The minimal anomaly-free $\mathrm{SU}(6)$ has fermions of: $2 \times \bar{\mathbf{6}}_{\mathbf{F}} \oplus \mathbf{15}_{\mathbf{F}}$

* How to break the $\mathrm{SU}(6)$ to the $\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$? The Georgi-Glashow $\mathrm{SU}(5)$ is a subgroup of the $\mathrm{SU}(6)$, while the direct breaking to the $\mathrm{SU}(5)$ is unwelcome. The sequential breaking of $\mathrm{SU}(5) \to \mathcal{G}_{\mathrm{SM}}$ leads to the proton decays with lower mass scale, hence faster decay rate.

* Alternative pattern is: $\mathrm{SU}(6) \to \mathcal{G}_{331} \to \mathcal{G}_{\mathrm{SM}}$, with $\mathcal{G}_{331} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$. This is achievable with an adjoint Higgs of $\mathbf{35}_{\mathbf{H}}$ at the GUT scale (1974 Ling-Fong Li).

# The SU(6) Higgs sector

* An adjoint Higgs of $\mathbf{35_H}$ at the GUT scale.

* There is a brute-force method: to perform the tensor products of all $SU(6)$ fermions $\bar{\mathbf{6}} \otimes \bar{\mathbf{6}} = \overline{\mathbf{15}} \oplus \overline{\mathbf{21}}$ , $\bar{\mathbf{6}} \otimes \mathbf{15} = \mathbf{6} \oplus \mathbf{84}$ , and $\mathbf{15} \otimes \mathbf{15} = \overline{\mathbf{15}} \oplus \overline{\mathbf{105}} \oplus \overline{\mathbf{105}}'$, and include all possible Higgs fields to form gauge-invariant Yukawa couplings.

* Physical requirements: all SM Yukawa couplings should be reproduced $\Rightarrow \bar{\mathbf{6}}_{\mathbf{H}}$ (for $d^i$ and $\ell^i$) and $\mathbf{15_H}$ (for $u^i$)

* Two $\bar{\mathbf{6}}_{\mathbf{H}}^{\mathrm{I},\mathrm{II}}$ are needed to respect the $SU(2)_F$.

* A $\mathbf{21_H}$ is introduced for the sterile neutrino Yukawa couplings.

# The SU(6) Higgs sector

* The minimal Higgs sector in the SUSY model: $\bar{\mathbf{6}}_{\mathbf{H}}^{\alpha=\mathrm{I},\mathrm{II}}$, $\mathbf{15_H}$, $\mathbf{21_H}$, $\overline{\mathbf{21}}_{\mathbf{H}}$, $\mathbf{35_H}$

* Hierarchies of Higgs VEVs: $\langle \mathbf{35_H} \rangle \sim \Lambda_{\mathrm{GUT}}$,
$\langle \bar{\mathbf{6}}_{\mathbf{H}}^{\mathrm{II}} \rangle = v_3\,, \quad \langle \mathbf{21_H} \rangle = v_6\,, \quad v_3 \sim v_6 \sim v_{331}$
$\langle \bar{\mathbf{6}}_{\mathbf{H}}^{\mathrm{I}} \rangle = v_d = v_{\mathrm{EW}} \sin\beta\,, \quad \langle \mathbf{15_H} \rangle = v_u = v_{\mathrm{EW}} \cos\beta$
$\Lambda_{\mathrm{GUT}} \gg v_{331} \gg v_{\mathrm{EW}} = (\sqrt{2} G_F)^{-1/2} \simeq 246\,\mathrm{GeV}$

* $\bar{\mathbf{6}}_{\mathbf{H}}^{\mathrm{II}}$ and $\mathbf{21_H}$ are responsible for the $\mathcal{G}_{331} \to \mathcal{G}_{\mathrm{SM}}$ breaking.

* Two Higgs doublets from the $\bar{\mathbf{6}}_{\mathbf{H}}^{\mathrm{I}}$ and $\mathbf{15_H}$ are responsible for the EWSB.

# The SU(6) fermions

| SU(6) | $\mathcal{G}_{331}$ | $\mathcal{G}_{\mathrm{SM}}$ |
|---|---|---|
| $\bar{\mathbf{6}}^{\mathrm{I}}_{\mathbf{F}}$ | $(\bar{\mathbf{3}}\,,\mathbf{1}\,,+\frac{1}{3})^{\mathrm{I}}_{\mathbf{F}}$ | $(\bar{\mathbf{3}}\,,\mathbf{1}\,,+\frac{2}{3})^{\mathrm{I}}_{\mathbf{F}}\ :\ \underline{d_R^c}$ |
| | $(\mathbf{1}\,,\bar{\mathbf{3}}\,,-\frac{1}{3})^{\mathrm{I}}_{\mathbf{F}}$ | $(\mathbf{1}\,,\mathbf{2}\,,-1)^{\mathrm{I}}_{\mathbf{F}}\ :\ \underline{(e_L\,,-\nu_L)}$ |
| | | $(\mathbf{1}\,,\mathbf{1}\,,0)^{\mathrm{I}}_{\mathbf{F}}\ :\ N$ |
| $\bar{\mathbf{6}}^{\mathrm{II}}_{\mathbf{F}}$ | $(\bar{\mathbf{3}}\,,\mathbf{1}\,,+\frac{1}{3})^{\mathrm{II}}_{\mathbf{F}}$ | $(\bar{\mathbf{3}}\,,\mathbf{1}\,,+\frac{2}{3})^{\mathrm{II}}_{\mathbf{F}}\ :\ D_R^c$ |
| | $(\mathbf{1}\,,\bar{\mathbf{3}}\,,-\frac{1}{3})^{\mathrm{II}}_{\mathbf{F}}$ | $(\mathbf{1}\,,\mathbf{2}\,,-1)^{\mathrm{II}}_{\mathbf{F}}\ :\ (e'_L\,,-\nu'_L)$ |
| | | $(\mathbf{1}\,,\mathbf{1}\,,0)^{\mathrm{II}}_{\mathbf{F}}\ :\ N'$ |
| $\mathbf{15}_{\mathbf{F}}$ | $(\bar{\mathbf{3}}\,,\mathbf{1}\,,-\frac{2}{3})_{\mathbf{F}}$ | $(\bar{\mathbf{3}}\,,\mathbf{1}\,,-\frac{4}{3})_{\mathbf{F}}\ :\ \underline{u_R^c}$ |
| | $(\mathbf{1}\,,\bar{\mathbf{3}}\,,+\frac{2}{3})_{\mathbf{F}}$ | $(\mathbf{1}\,,\mathbf{2}\,,+1)_{\mathbf{F}}\ :\ (\nu'^c_R\,,e'^c_R)$ |
| | | $(\mathbf{1}\,,\mathbf{1}\,,+2)_{\mathbf{F}}\ :\ \underline{e_R^c}$ |
| | $(\mathbf{3}\,,\mathbf{3}\,,0)_{\mathbf{F}}$ | $(\mathbf{3}\,,\mathbf{2}\,,+\frac{1}{3})_{\mathbf{F}}\ :\ \underline{(u_L\,,d_L)}$ |
| | | $(\mathbf{3}\,,\mathbf{1}\,,-\frac{2}{3})_{\mathbf{F}}\ :\ D_L$ |

# The SU(6) Yukawa

* The most general Yukawa in the superpotential:
  $W_Y = \mathbf{15_F}\bar{\mathbf{6}}^{\rho}_{\mathbf{F}}\bar{\mathbf{6}}^{\rho}_{\mathbf{H}} + \mathbf{15_F}\mathbf{15_F}\mathbf{15_H} + \bar{\mathbf{6}}^{\rho}_{\mathbf{F}}(i\sigma_2)_{\rho\sigma}\bar{\mathbf{6}}^{\sigma}_{\mathbf{F}}(\mathbf{15_H} + \mathbf{21_H})$

* The PQ charge and a discrete $\mathbb{Z}_{4\,\mathscr{R}}$ symmetry:

| | $\bar{\mathbf{6}}^{\rho}_{\mathbf{F}}$ | $\mathbf{15_F}$ | $\bar{\mathbf{6}}_{\mathbf{H}\,\rho}$ | $\mathbf{15_H}$ | $\mathbf{21_H}$ | $\overline{\mathbf{21}}_{\mathbf{H}}$ | $\mathbf{35_H}$ |
|---|---|---|---|---|---|---|---|
| $\mathrm{SU}(2)_F$ | $\Box$ | 1 | $\overline{\Box}$ | 1 | 1 | 1 | 1 |
| $\mathrm{U}(1)_{\mathrm{PQ}}$ | 1 | 1 | $-2$ | $-2$ | $-2$ | 0 | 0 |
| $\mathbb{Z}_{4\mathcal{R}}$ | 0 | 0 | 2 | 2 | 2 | 0 | 0 |

At the UV the global $\mathrm{U}(1)_{\mathrm{PQ}}[\mathrm{SU}(6)]^2$ anomaly: $N_{\mathrm{SU}(6)} = 9$

# The SU(6) Yukawa

* At the $\mathscr{G}_{331} \to \mathscr{G}_{\rm SM}$ breaking:

  $\mathbf{15_F \bar{6}_F^{II} \bar{6}_H^{II}} + H.c. \supset (\mathbf{3}, \mathbf{3}, 0)_{\mathbf{F}} \otimes (\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}^{\rm II} \otimes (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{H}}^{\rm II} + H.c.$

  $(\mathbf{1}, \bar{\mathbf{3}}, +\frac{2}{3})_{\mathbf{F}} \otimes (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{\rm II} \otimes (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{H}}^{\rm II} + H.c.$

  $\Rightarrow m_D \sim m_{e'} \sim m_{\nu'} \simeq \mathcal{O}(v_{331})$

  $D$-hadron lifetime: $\tau_D \sim m_D^{-1} \sim \mathcal{O}(10^{-36}) - \mathcal{O}(10^{-34})$ sec, Vs. the BBN constraint of $\tau_Q \lesssim 10^{-2}$ sec.

* $\mathbf{\bar{6}_F^{[I} \bar{6}_F^{II]} 21_H} + H.c. \supset$

  $(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{[\rm I} \otimes (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{\rm II]} \otimes (\mathbf{1}, \mathbf{6}, +\frac{2}{3})_{\mathbf{H}} + H.c.$

  $\Rightarrow m_{N,N'} \simeq \mathcal{O}(v_{331})$

# The SU(6) Yukawa

* $\mathbf{15_F}\bar{\mathbf{6}}^{\mathrm{I}}_{\mathbf{F}}\bar{\mathbf{6}}^{\mathrm{I}}_{\mathbf{H}} + H.c. \supset$
$$(\mathbf{3}, \mathbf{2}, +\frac{1}{3})_{\mathbf{F}} \otimes (\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3})^{\mathrm{I}}_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{2}, -1)^{\mathrm{I}}_{\mathbf{H}} + H.c.$$
$$(\mathbf{1}, \mathbf{2}, -1)_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{1}, +2)^{\mathrm{I}}_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{2}, -1)^{\mathrm{I}}_{\mathbf{H}} + H.c.$$
$$\Rightarrow m_{d,\ell} \simeq \mathcal{O}(v_{\mathrm{EW}})$$

$\mathbf{15_F}\mathbf{15_F}\mathbf{15_H} + H.c. \supset$
$$(\mathbf{3}, \mathbf{2}, +\frac{1}{3})_{\mathbf{F}} \otimes (\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3})^{\mathrm{I}}_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{2}, +1)_{\mathbf{H}} + H.c.$$
$$\Rightarrow m_u \simeq \mathcal{O}(v_{\mathrm{EW}})$$

# The SU(6) Axion

# The SU(6) Axion

* The physical axion field comes from: $\mathbf{6}_{\mathbf{H}}^{\mathrm{II}} \supset (\mathbf{1}\,, \mathbf{3}\,, +\frac{1}{3})_{\mathbf{H}}^{\mathrm{II}} \supset \frac{v_3}{\sqrt{2}} \exp(i a_3 / v_3)$ and $\mathbf{21}_{\mathbf{H}} \supset (\mathbf{1}\,, \mathbf{6}\,, +\frac{2}{3})_{\mathbf{H}} \supset \frac{v_6}{\sqrt{2}} \exp(i a_6 / v_6)$

* To impose an orthogonality condition between the $\mathrm{U}(1)_{\mathrm{PQ}}$ $J^\mu_{\mathrm{PQ}} = q_3 v_3 (\partial^\mu a_3) + q_6 v_6 (\partial^\mu a_6)$ and the $\mathrm{U}(1)_N$ $J^\mu_N = \frac{1}{3} v_3 (\partial^\mu a_3) + \frac{2}{3} v_6 (\partial^\mu a_6)$ currents. Physical charge: $q \equiv c_1 \, \mathrm{PQ} + c_2 \, N$.

* ’t Hooft global anomaly matching: $N_{\mathrm{SU}(3)_c} = N_{\mathrm{SU}(6)} \Rightarrow c_1 = 1$.

* $a_{\mathrm{phys}} = \cos\phi \, a_3 + \sin\phi \, a_6, \tan\phi = \frac{v_3}{2 v_6}$.

* Axion decay const: $9 v_{331}^{-2} = \frac{1}{4} v_6^{-2} + v_3^{-2}$ and $f_a = v_{331}/18$.

# The PQ quality

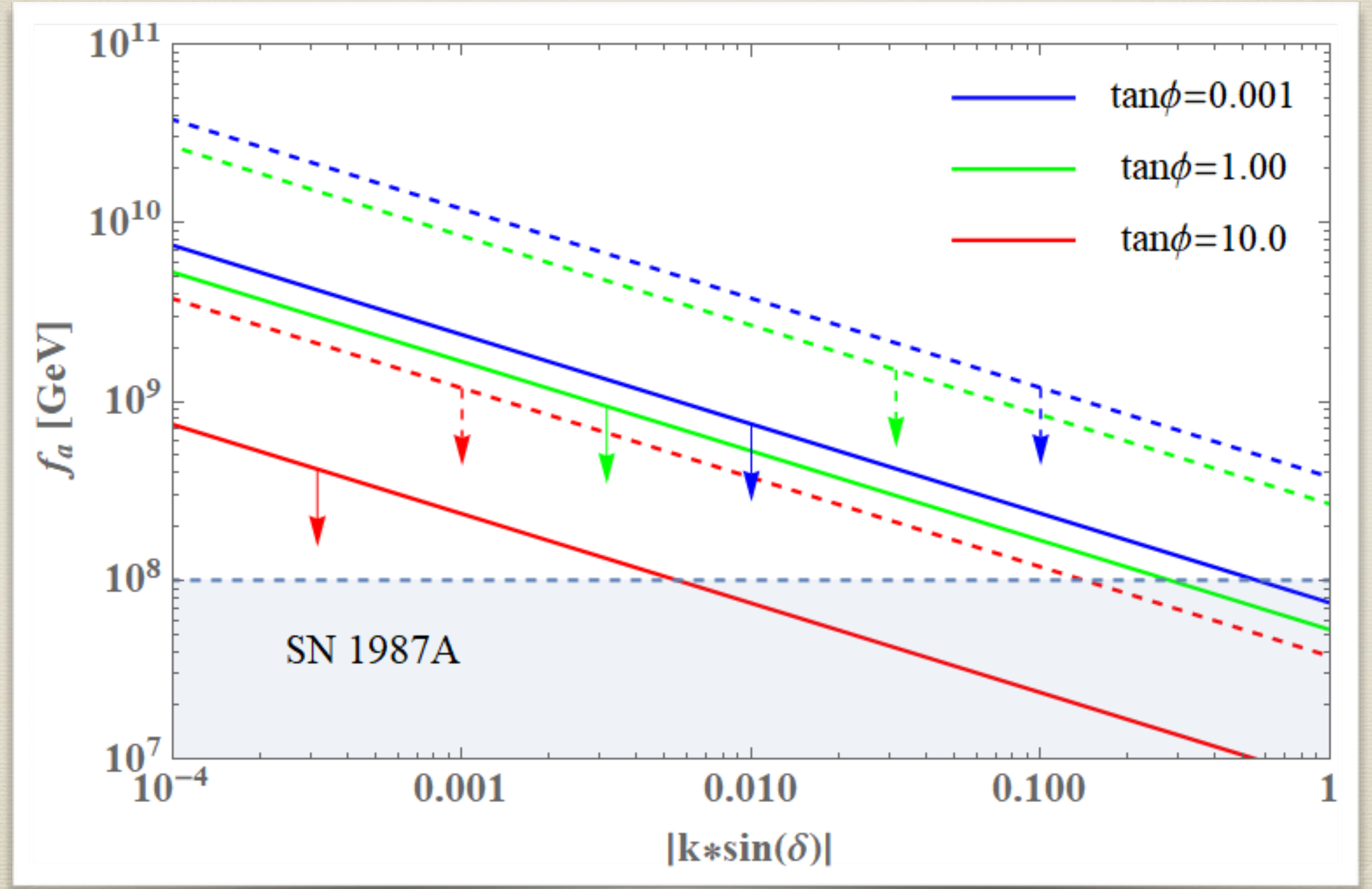
* The leading PQ-breaking operator respecting the $\mathrm{SU}(2)_F$ and $\mathbb{Z}_{4\mathscr{R}}$:
$\mathcal{O}^{d=6}_{\not{PQ}} = \left[\epsilon_{\alpha\beta}\epsilon_{abc}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{a\,,\alpha}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{b\,,\beta}(\mathbf{1}\,,\mathbf{3}\,,-\frac{2}{3})^{c}\right]^2$
if no $\mathbb{Z}_{4\mathscr{R}}$: $\mathcal{O}^{d=3}_{\not{PQ}} = \epsilon_{\alpha\beta}\epsilon_{abc}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{a\,,\alpha}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{b\,,\beta}(\mathbf{1}\,,\mathbf{3}\,,-\frac{2}{3})^{c}$ is dangerous in PQ-quality.

* Axion effective potential:
$$V = \Lambda^4_{\mathrm{QCD}}(1-\cos(a_{\mathrm{phys}}/f_a)) + \frac{|k|(v_u v_d v_3)^2}{4M^2_{\mathrm{pl}}}\cos\left(\frac{a_{\mathrm{phys}}}{6f_a}+\delta\right)$$

* $|\bar{\theta}| \equiv \left|\frac{\langle a_{\mathrm{phys}}\rangle}{f_a}\right| \lesssim 10^{-10} \Rightarrow f_a \lesssim \frac{3.7\times 10^7\ \cos\phi}{|k\sin(\delta)|^{1/2}}(\tan\beta + \frac{1}{\tan\beta})\,\mathrm{GeV}$

This is solely determined by the symmetry consideration in a GUT.

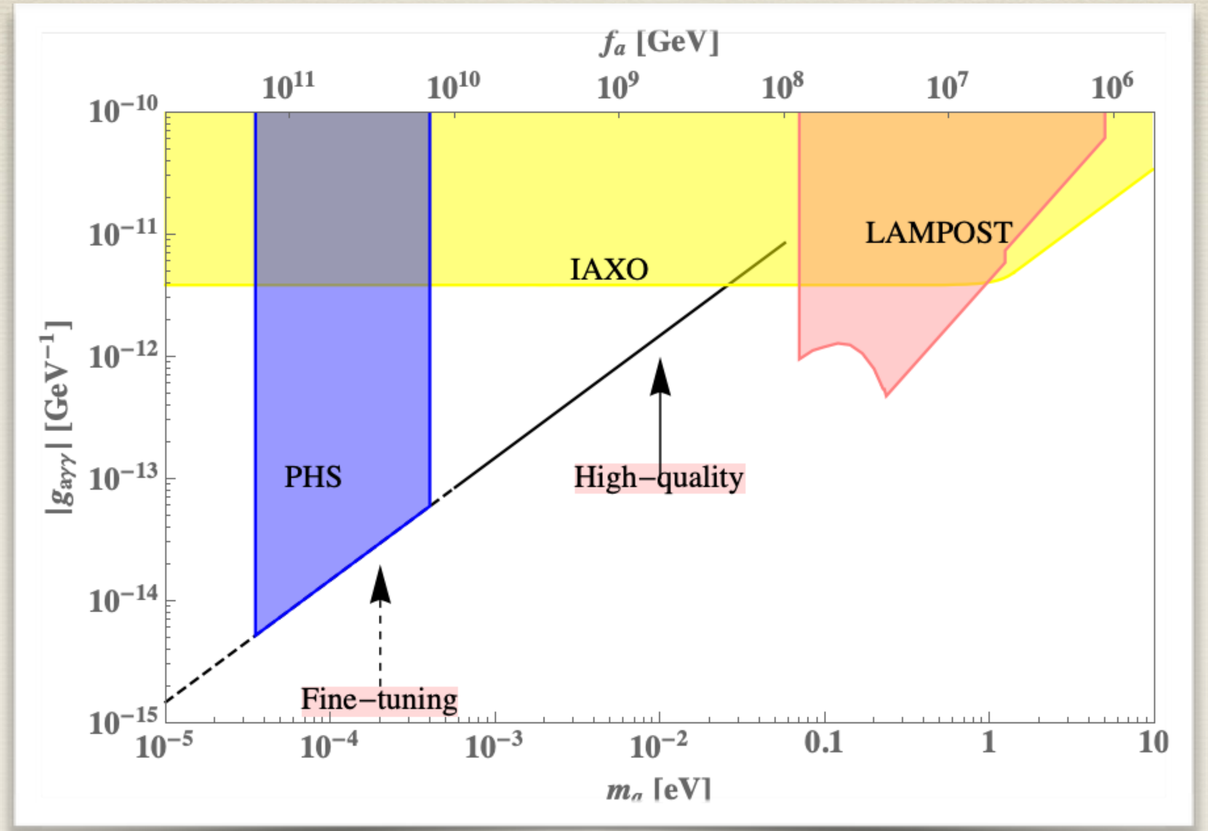
# The PQ quality




$10^8\,\text{GeV} \lesssim f_a \lesssim 10^{10}\,\text{GeV}$

$$m_a = 5.70\left(\frac{10^{12}\,\text{GeV}}{f_a}\right)\mu\text{eV} \sim (10^{-4}, 10^{-2})\text{eV}$$

# The axion searches




$$g_{a\gamma\gamma} = (\frac{E}{N_{\mathrm{SU(3)}_c}} - 1.92)\Big(\frac{1.14\times 10^{-3}\,\mathrm{GeV}}{f_a}\Big)\,\mathrm{GeV}^{-1}$$

$\mathrm{U(1)_{PQ}[U(1)_{em}]^2}$ anomaly factor : $E = \sum_f \mathrm{PQ}_f \dim(\mathscr{C}_f)\mathrm{Tr}q_f^2 = -40/3$

$\mathrm{U(1)_{PQ}[SU(3)_c]^2}$ anomaly factor : $N = \sum_f \mathrm{PQ}_f\, T(\mathscr{R}_f) = -5$

# The Axion domain walls

* Back to the axion effective potential:

$$V = \Lambda_{\rm QCD}^4 (1 - \cos(a_{\rm phys}/f_a)) + \frac{|k|(v_u v_d v_3)^2}{4M_{\rm pl}^2} \cos\left(\frac{a_{\rm phys}}{6f_a} + \delta\right)$$

* The $\cos(\frac{a_{\rm phys}}{f_a})$ term is periodic and has degenerate minima, this leads to the DWs [Kibble-Zurek mechanism].

* DWs are problematic in cosmology, with the energy density $\rho_{\rm DW} \sim \sigma/t$. The energy densities for radiation/matter: $\rho_{\rm rad} \propto t^{-2}$, $\rho_{\rm matt} \propto t^{-3/2}$. DWs can overtake the Universe once they are formed.

# The Axion domain walls

* The explicit PQ-breaking term acts as the biased term to collapse the DWs. [Vilenkin ('81), Gelmini, Gleiser, Kolb, ('89), Larsson, Sarkar, White ('96)]

* To have DWs collapse before formation: $t_{\rm dec} < t_{\rm form}$.

* In our case:

$$t_{\rm form} \sim 10^2\,{\rm sec}\left(\frac{10^{13}\,{\rm GeV}}{v_{331}}\right) \sim \mathcal{O}(10^4) - \mathcal{O}(10^6)\,{\rm sec}$$

$$t_{\rm dec} \approx \frac{\sigma_{\rm DW}}{v_{331}^4} \sim 10^{-66}\,{\rm sec}\left(\frac{M_{\rm pl} v_{331}}{v_u v_d}\right)^2 \left(\frac{10^{13}\,{\rm GeV}}{v_{331}}\right)^3$$

$$\sim \mathcal{O}(10^{-8}) - \mathcal{O}(10^{-6})\,{\rm sec}$$

# The SU(6) Unification

# The SU(6) unification

* The gauge couplings: $(\alpha_{3c}, \alpha_{3L}, \alpha_N)$ for the $\mathcal{G}_{331}$, and $(\alpha_{3c}, \alpha_{2L}, \alpha_Y)$ for the $\mathcal{G}_{\rm SM}$. Use $\alpha_1 = \frac{4}{3}\alpha_N$ for the $\mathcal{G}_{331}$ embedding into the $\rm SU(6)$.

* The RGEs of the $\rm SU(6)$:
$$\alpha_i^{-1}(\mu_2) = \alpha_i^{-1}(\mu_1) - \frac{b_i^{(1)}}{2\pi}\log\left(\frac{\mu_2}{\mu_1}\right) + \delta_i$$
$\delta_i$ to account for higher-order effects: two-loop && mass threshold.

* The matching conditions: $\alpha_{3L}^{-1}(v_{331}) = \alpha_{2L}^{-1}(v_{331})$,
$$\alpha_1^{-1}(v_{331}) = -\frac{1}{4}\alpha_{2L}^{-1}(v_{331}) + \frac{3}{4}\alpha_Y^{-1}(v_{331}).$$

# The SU(6) unification

* non-SUSY: $b_i^{(1)} = -\frac{11}{3}C_2(\mathcal{G}_i) + \frac{2}{3}\sum_f T(\mathcal{R}_f^i) + \frac{1}{3}\sum_s T(\mathcal{R}_s^i)$

  SUSY: $b_i^{(1)} = -3C_2(\mathcal{G}_i) + \sum_\chi T(\mathcal{R}_\chi^i)$

* The $\mathrm{SU}(6)$ SUSY extension can avoid the $\mu$-problem, since $\mathbf{\bar{6}_H^I 15_H}$ is not gauge-invariant.

* non-SUSY: $m_Z \leq \mu \leq v_{331} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(2)_L}, b^{(1)}_{\mathrm{U}(1)_Y}) = (-7, -3, 7)$

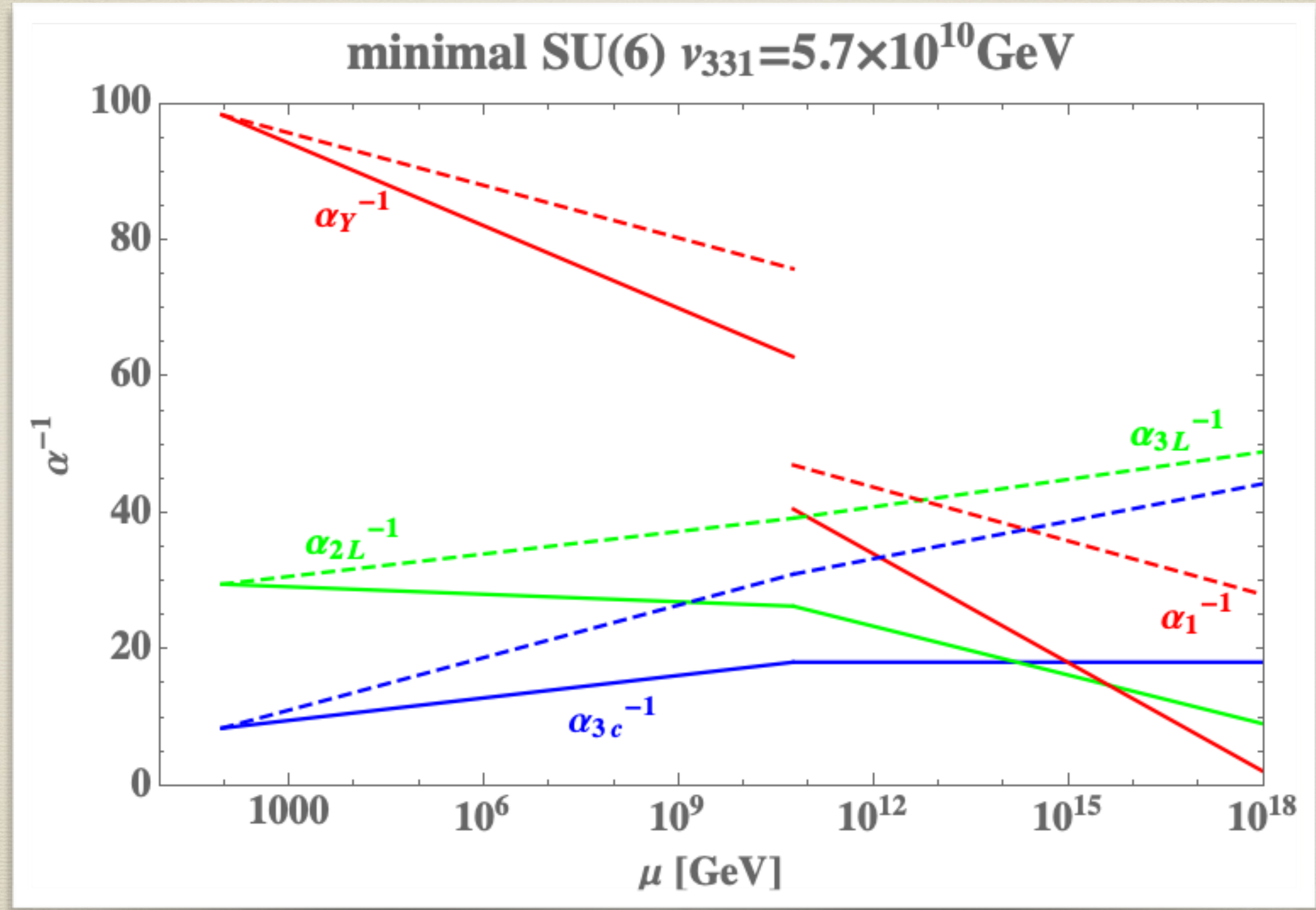
  $v_{331} \leq \mu \leq \Lambda_{\mathrm{GUT}} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(3)_L}, b^{(1)}_{\mathrm{U}(1)_1}) = (-5, -\frac{11}{3}, \frac{43}{6})$

* SUSY:

  $m_Z \leq \mu \leq v_{331} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(2)_L}, b^{(1)}_{\mathrm{U}(1)_Y}) = (-3, 1, 11)$

  $v_{331} \leq \mu \leq \Lambda_{\mathrm{GUT}} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(3)_L}, b^{(1)}_{\mathrm{U}(1)_1}) = (0, \frac{13}{2}, \frac{29}{2})$

# The SU(6) unification




$\alpha_{3c}(m_Z)\,,\alpha_{\rm em}(m_Z)\,,\sin^2\theta_W(m_Z)$ as inputs

# The SU(6) unification

* To impose the unification condition at the UV: $\alpha_{3c}^{-1}(\Lambda_{\rm GUT}) = \alpha_{3L}^{-1}(\Lambda_{\rm GUT}) = \alpha_1^{-1}(\Lambda_{\rm GUT}) = \alpha_{\rm GUT}^{-1}(\Lambda_{\rm GUT})$
* Benchmark $v_{331} = 5.7 \times 10^{10}\,{\rm GeV}$, we find:
  $\Lambda_{\rm GUT} \approx 7.8 \times 10^{15}\,{\rm GeV}\,, \quad \alpha_{\rm GUT}^{-1}(\Lambda_{\rm GUT}) = 18.14$
  $\sin^2\theta_W(m_Z) = 0.22923$
  PDG: $\sin^2\theta_W(m_Z) = 0.23117$
* Proton lifetime:
  $$\tau[p \to e^+\pi^0] \sim 10^{36}\,{\rm yrs}\left(\frac{\alpha_{\rm GUT}^{-1}}{35}\right)^2\left(\frac{\Lambda_{\rm GUT}}{10^{16}\,{\rm GeV}}\right)^4$$
  $$\approx 9.8 \times 10^{34}\,{\rm yrs}$$
  Super-Kamionkande: $\tau_p \gtrsim 2.4 \times 10^{34}\,{\rm yrs}$

# Summary

* We showed a *non-minimal* $SU(6)$ SUSY GUT model with the minimal setup to achieve a high-quality axion by identifying the $U(1)_{PQ}$ as the Abelian component of the emergent global symmetries.

* The axion decay constant: $10^8\,\mathrm{GeV} \lesssim f_a \lesssim 10^{10}\,\mathrm{GeV}$ w.o. much fine-tuning of the EFT parameter.

* The GUT spectrum contains vector-like KSVZ $D$-quarks, and heavy leptons && singlet neutrinos, type-I seesaw.

* Safe from the cosmological constraints.

* Higher quality with extended GUT symmetry?