Inert Double Model at future lepton colliders

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based on 2009.03250 and 2204.05237, collaborated with

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Outline

Inert Doublet Model (IDM) [Deshpande and Ma (1978)]

$$H_1 = \begin{pmatrix} G^{\pm} \\ \frac{1}{\sqrt{2}}(v + h^0 + iG^0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^{\pm} \\ \frac{1}{\sqrt{2}}(H^0 + iA^0) \end{pmatrix}$$

$$V = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1^\dagger H_2|^2 + \frac{\lambda_5}{2}\left\{(H_1^\dagger H_2)^2 + \text{h.c.}\right\}$$

$$m_{h^0}^2 = -2\mu_1^2 = 2\lambda_1 v^2, \quad m_{H^0}^2 = \mu_2^2 + \lambda_L v^2, \quad m_{A^0}^2 = \mu_2^2 + \lambda_S v^2,$$

$$m_{H^\pm}^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2, \quad \lambda_{L,S} \equiv \frac{1}{2}(\lambda_3 + \lambda_4 \pm \lambda_5)$$

- obtained by adding an extra doublet to the SM (special case of 2HDM)
- retain an exact Z_2 symmetry ($H_1 \to H_1,\ H_2 \to -H_2$)
  (no directly coupling between H_2 and fermions$\to$ inert)
- parametrization: $\{v, \mu_2^2, \lambda_2, m_{h^0}, m_{H^\pm}, m_{H^0}, m_{A^0}\}$ (five new parameters)
- can provide a DM candidate (H^0 or A^0 according to the sign of λ_5)
- λ_2 only appears in quartic scalar couplings

Scan of IDM parameter space

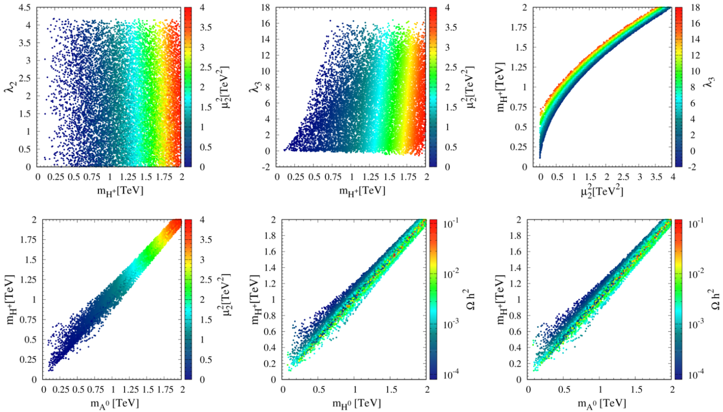
Theoretical constraints:

- Perturbativity: $|\lambda_i| < 8\pi$
- Vacuum stability:
  $\lambda_{1,2} > 0, \quad \lambda_3 + \lambda_4 - |\lambda_5| + 2\sqrt{\lambda_1\lambda_2} > 0, \quad \lambda_3 + 2\sqrt{\lambda_1\lambda_2} > 0$
- Charge-breaking minima: $\lambda_4 - |\lambda_5| \leq 0 \Longleftrightarrow m_{H^\pm} \geq m_{A^0}/m_{H^0}$
- Inert vacuum: $m^2_{h^0}, m^2_{H^0}, m^2_{A^0}, m^2_{H^\pm} > 0, \quad \mu_1^2/\sqrt{\lambda_1} < \mu_2^2/\sqrt{\lambda_2}$
- Tree-level unitarity: $e_i \leq 8\pi \;, \forall\; i = 1, ..., 12$

Experimental constraints:

- Higgs data from LHC: $h^0 \to \gamma\gamma$, $h^0 \to$invisible
- Direct search from LEP: $m_{H^\pm} > 80$ GeV, max$(m_{A^0}, m_{H^0}) > 100$ GeV,
  $m_{A^0} + m_{H^0/H^\pm} > m_{Z/W}$
- Electroweak Precision Tests (EWPT): $S = 0.02 \pm 0.07$, $T = 0.06 \pm 0.06$
- DM relic density ($\Omega h^2 \leq 0.12$), micrOMEGAs (direct, indirect and collider searches), monojet

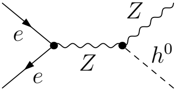
IDM parameter space in different planes



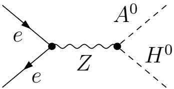
symmetric under $m_{H^0} \leftrightarrow m_{A^0}$

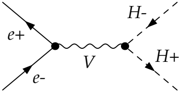
IDM at future lepton colliders

- Higgs strahlung production: pure SM at LO, NP effects at NLO



- pair production of IDM scalar: neutral and charged





- $\mathcal{O}(m_e)$ contribution neglected (no eeS vertices)

- straightforward at LO:

$$\sigma^0(Zh^0) = \frac{4\pi\alpha^2}{3s} g_A^2 \left(g_A^2 + g_H^2\right) \kappa_{Zh^0} \frac{12m_Z^2/s + \kappa_{Zh^0}^2}{(1-m_Z^2/s)^2}$$

$$\sigma^0(H^0A^0) = \frac{4\pi\alpha^2}{3s} g_A^2 \left(g_A^2 + g_H^2\right) \frac{\kappa_{A^0H^0}^3}{(1-m_Z^2/s)^2}$$

$$\sigma^0(H^+H^-) = \frac{\pi\alpha^2}{3s} \kappa_{H^+H^-}^3 \left(1 + g_H^2 \frac{g_V^2 + g_A^2}{(1-m_Z^2/s)^2} - \frac{2g_H g_V}{1-m_Z^2/s}\right)$$

Calculation at NLO

Example: results of $e^+e^- \to Zh^0$ in SM with $\sqrt{s} = 250$ GeV

scheme	$1/\alpha$	σ^0	$\sigma^{1,\mathrm{weak}}$	σ^1	$\sigma^0 + \sigma^1$	Δ
$\alpha(0)$	137.036	223.12(0)	6.09(0)	7.13(2)	230.25(2)	3.20%
$\alpha(m_Z)$	128.943	252.00(0)	-24.33(0)	-23.07(2)	228.93(2)	-9.15%
$\alpha(\sqrt{s})$	127.515	257.68(0)	-30.92(0)	-29.63(2)	228.05(2)	-11.50%

- Renormalizaion of charge ($\overline{\mathrm{MS}}$-like, but called "on shell") [Denner (1993)]

$$\alpha(\mu) \equiv \frac{\alpha(0)}{1 - (\Delta\alpha(\mu))_{f \neq \mathrm{top}}}, \quad \Delta\alpha(\mu) \equiv \Pi(0) - \mathrm{Re}\Pi(\mu^2), \quad \Pi(s) \equiv \frac{\sum_T^{AA}(s)}{s}$$

- Structure Function approach (for $\log m_e$ terms from ISR) [Kuraev and Fadin (1985)]

$$d\overline{\sigma}_{e^+e^-}(p_1, p_2) = dx_1 dx_2 f_{ee}(x_1, Q^2) f_{ee}(x_2, Q^2) d\sigma_{e^+e^-}(x_1 p_1, x_2 p_2)$$

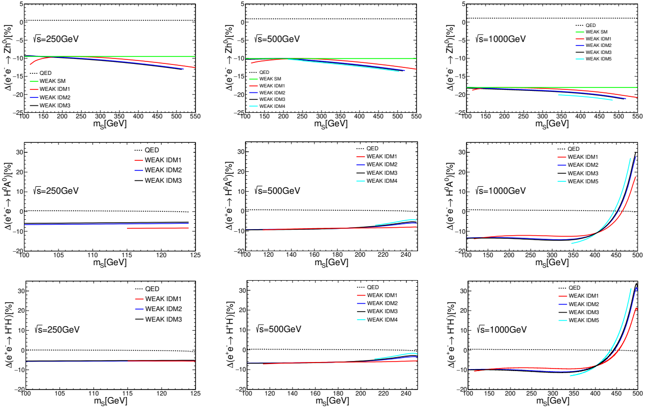
$$f_{ee}(x, Q^2) = \delta(1-x) + \int_{m_e^2}^{Q^2} \frac{\alpha(Q_1^2)}{2\pi} \frac{dQ_1^2}{Q_1^2} \int_x^1 \frac{dz}{z} P_{ee}^+(z) f_{ee}\left(\frac{x}{z}, Q_1^2\right) + \mathcal{O}(\alpha^2)$$

- Separation of NLO corrections: QED+weak [Beenakker et al. (1991)]

"QED": 1) all the diagrams with an extra photon attached to the LO diagrams (all real, part of loop)
2) photonic contribution in field renormalization of external particles
a gauge-invariant subgroup of whole QED corrections

"weak": all the others (IR finite)

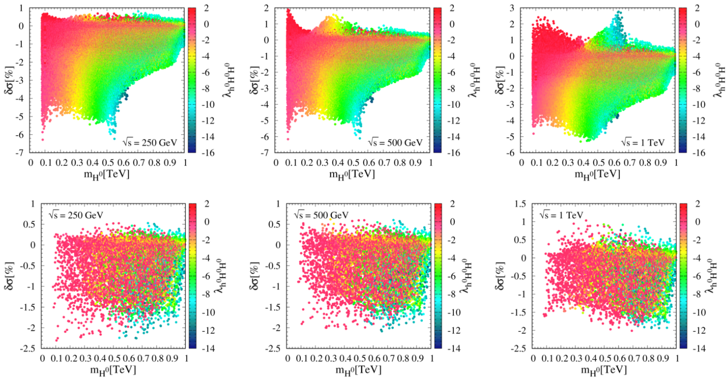
The degenerate case (much simpler)



Relative corrections for all three processes in degenerate case

- degenerate: $m_{H^0} = m_{A^0} = m_{H^\pm} \equiv m_S \to \{\mu_2^2, \lambda_2, m_S\}$ ($\lambda_2 = 2$ everywhere)
- unphysical, theoretical constraints (to get some general information first)
- IDM1-5: five different values (40k, 6k, 0, -10k, -30k) of μ_2^2 (in unit of GeV^2)
- "QED" part: independent (Zh); variation negligible (HA), (HH after resummation)

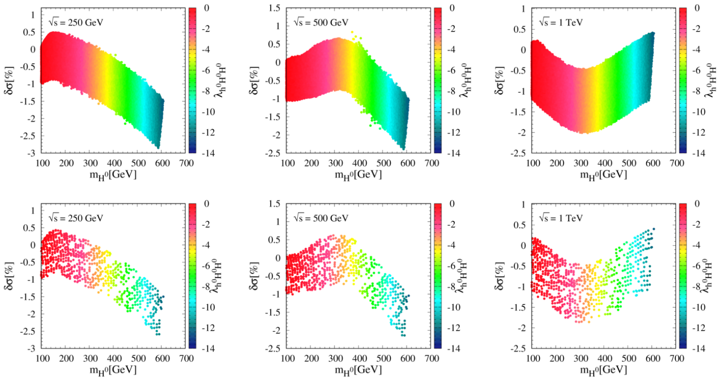
New physics effects in Higgs strahlung $\left[\delta = \left(\sigma^{IDM}_{Zh^0} - \sigma^{SM}_{Zh^0}\right)/\sigma^{SM}_{Zh^0}\right]$



New physics effects in $e^+e^- \to Zh^0$ before and after DM constraints

- δ: all quantities at NLO, only "weak" part survived in the numerator
- h^0 invisible decay closed
- only 1% points survived after DM constraints (micrOMEGAs+monojet)
- $\delta \in [-2.5\%, +1.0\%]$ for any value of λ

An extra piece (A^0 supposed lightest)



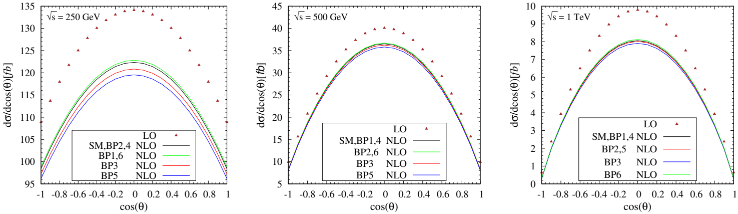
New physics effects in $e^+e^- \to Zh^0$ before and after DM constraints

- h^0 invisible decay open: $\mathrm{Br}(h^0 \to A^0A^0) < 11\%$
- $\sim 10\%$ points survived after DM constraints, shapes almost unchanged
- λ @lowest bounds (larger coupling leads to larger effects, but...)
- overall: δ can be sizable, within the detection potential of future lepton colliders

Benchmark points (H^0 supposed lightest)

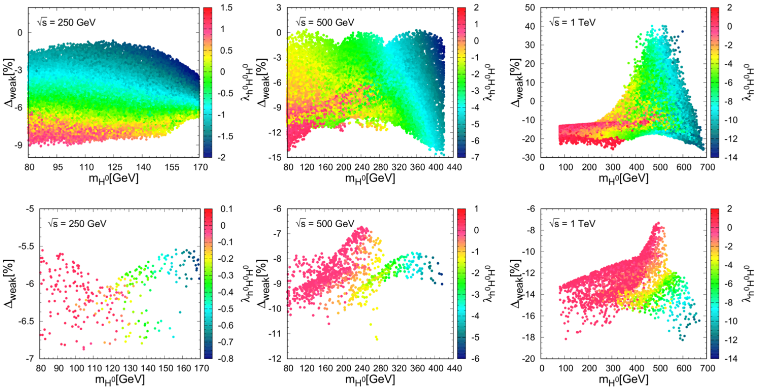
Benchmark Points	BP1	BP2	BP3	BP4	BP5	BP6
m_{H^0} (GeV)	59.3	62.0	57.2	94.1	105.6	514.2
m_{A^0} (GeV)	170.5	339.3	307.9	101.9	576.9	740.3
$m_{H^\pm}$ (GeV)	145.1	327.5	342.0	110.4	593.6	516.2
μ_2^2 (GeV^2)	3642.7	3733.9	3514.3	9059.5	10945.8	265875.8
δ@250GeV(%)	0.414	-0.093	-1.221	-0.098	-2.294	0.525
δ@500GeV(%)	0.191	0.583	-0.675	-0.046	-1.889	0.594
δ@1TeV(%)	-0.072	-0.456	-1.824	0.098	-0.715	0.850

- BP3&BP5: relatively large NP effects
- BP6: sizable NP effects with all masses heavy, suitable for indirect search
- BP2: prefer higher energies
- BP1&BP4: hard to detect via Zh^0



Angular distribution for Higgs strahlung

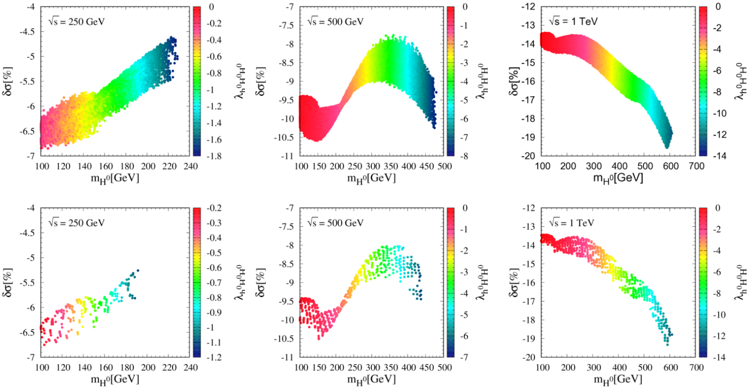
Neutral scalar pair production (h^0 invisible decay closed)



Relative weak corrections to $e^+e^- \to H^0A^0$ before and after DM constraints

- cross section drop rapidly due to kinematic suppression
- "weak" part only
- points with even larger corrections excluded by DM constraints

h^0 invisible decay open



- little changes in the shapes
- overall: NLO corrections quite large, always negative
- $Z^{(*)}A^0A^0 \to l^+l^- +$ missing energy

Cross section for BPs

	$\sqrt{s}$ (GeV)	σ^0 (fb)	$\sigma^{1,\text{weak}}$ (fb)	$\sigma^{1,QED}$ (fb)	σ^{NLO} (fb)	Δ(%)
BP1	250	12.080	-0.697	0.031	11.414	-5.513
	500	35.880	-3.513	0.255	32.622	-9.080
	1000	11.879	-1.637	0.104	10.346	-12.905
BP2	500	6.755	-0.539	0.028	6.244	-7.565
	1000	8.947	-1.274	0.064	7.737	-13.524
BP3	500	11.422	-1.014	0.055	10.463	-8.396
	1000	9.611	-1.449	0.073	8.235	-14.317
BP4	250	65.670	-4.475	0.313	61.508	-6.338
	500	43.011	-3.972	0.335	39.374	-8.456
	1000	12.378	-1.642	0.116	10.852	-12.328
BP5	1000	3.516	-0.638	0.018	2.896	-17.634

- cross section: large enough
- BP1 and BP4: small NP effects in Zh^0, can directly produce H^0A^0 at all energies $\rightarrow$ direct search
- BP6: opposite, beyond the threshold of H^0A^0, sizable NP effects $\rightarrow$ indirect search

Resummation of Coulomb singularity

$$\Delta = \Delta^{\rm H}_{\rm weak} + \Delta^{\rm H}_{\rm QED} + \Delta^{\rm C}_{\rm QED}, \quad \Delta^{\rm C}_{\rm QED} = \frac{\alpha\pi}{2\beta} \equiv \frac{X}{2}$$

$$\sigma^{\rm NLO} = \sigma^0 (1 + \Delta^{\rm H}_{\rm weak} + \Delta^{\rm H}_{\rm QED} + \Delta^{\rm C}_{\rm QED})$$

$$\to \sigma^{\rm NLO}_{\rm resum.} = \sigma^0_{\rm resum.} (1 + \Delta^{\rm H}_{\rm weak} + \Delta^{\rm H}_{\rm QED})$$

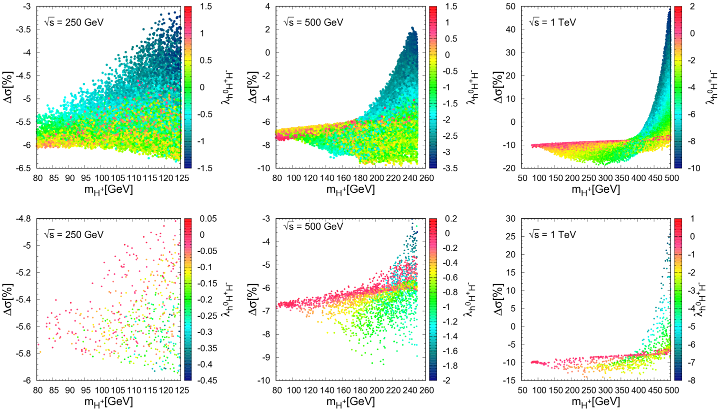
$$\sigma^0_{\rm resum.} = |\psi(0)|^2 \sigma^0, \quad |\psi(0)|^2 = \frac{X}{1 - e^{-X}} = 1 + \frac{X}{2} + \frac{X^2}{12} + \cdots$$

- caused by soft photon exchanging between outgoing charged particles
- well studied in W-pair production at LEP [Fadin:1993kg,Bardin et al. (1993)]
- $|\psi(0)|^2$ originally obtained by Sommerfeld and Sakharov [Sommerfeld (1931); Sakharov (1948)]
- $\Delta_{\rm QED}$ after resummation is quite small in whole region (variation negligible)

$m_{H^\pm}$ (GeV)	100	150	200	225	245	249	249.9
β	0.9165	0.8000	0.6000	0.4359	0.1990	0.0894	0.0283
$\lvert\psi(0)\rvert^2$	1.0134	1.0153	1.0204	1.0282	1.0625	1.1425	1.4918
$\Delta_{\rm weak}(\%)$	-6.859	-6.609	-5.695	-4.379	-2.869	-3.058	-3.175
Before Resummation							
σ^0 (fb)	95.320	63.392	26.744	10.254	0.976	0.0883	0.00280
$\sigma^{\rm NLO}$ (fb)	90.344	60.288	25.783	10.085	1.004	0.0970	0.00389
$\Delta_{\rm QED}(\%)$	1.628	1.711	2.101	2.731	5.738	12.911	42.143
After Resummation							
σ^0 (fb)	96.593	64.362	27.291	10.543	1.037	0.1009	0.00418
$\sigma^{\rm NLO}$ (fb)	90.256	60.230	25.756	10.075	1.003	0.0971	0.00401
$\Delta_{\rm QED}(\%)$	0.299	0.189	0.071	-0.064	-0.384	-0.723	-0.933

Example: Cross section and relative corrections before and after resummation.

Relative weak corrections to $e^+e^- \to H^+H^-$



- whole parameter space (no matter h^0 invisible decay is open or closed)
- similar shapes before and after DM constraints, but bounds changed
- a large peak near the threshold with $\sqrt{s} = 1000$ GeV
- caused by $h^0H^+H^-$ coupling

Benchmark points (H^0 supposed lightest)

BP	BP1	BP2	BP3	BP4	BP5	BP6
$m_{H^\pm}$ (GeV)	116.8	123.4	209.5	243.7	295.4	472.9
m_{H^0} (GeV)	57.0	121.9	122.9	59.3	204.1	181.4
m_{A^0} (GeV)	102.3	200.0	125.2	238.3	205.7	473.5
μ_2^2 (GeV^2)	3159.5	14723.8	15037.5	3558.6	41195.9	32220.8

$\sqrt{s}$ (GeV)	BP	σ^0 (fb)	$\Delta_{\rm weak}(\%)$	$\Delta_{\rm QED}(\%)$	$\sigma^{\rm NLO}$ (fb)
250	BP1	23.940	-5.941	-0.138	22.484
	BP2	2.300	-4.825	-0.475	2.178
500	BP1	86.733	-7.267	0.261	80.655
	BP2	82.604	-6.373	0.247	77.543
	BP3	20.593	-9.213	0.033	18.703
	BP4	1.446	-2.842	-0.332	1.400
1000	BP1	28.563	-10.631	0.421	25.647
	BP2	28.280	-9.360	0.409	25.749
	BP3	23.284	-13.896	0.286	20.115
	BP4	20.715	-10.987	0.249	18.491
	BP5	16.365	-14.023	0.194	14.102
	BP6	1.092	11.543	-0.174	1.216

- two points for one certain c.m. energy (away from and near the threshold)
- large enough cross section
- QED (after resummation) vs weak
- near threshold: smaller cross section vs larger positive weak corrections
- $W^{(*)}W^{(*)}(Z^{(*)})(Z^{(*)})+$ missing energy

Conclusion and discussion

- IDM parameter space is scanned with theoretical and experimental constraints (many LO constraints, mismatch at NLO)
- Full one-loop EW corrections to several processes at future lepton colliders are studied:
  - Higgs Strahlung: NP effects is sizable, within the detection potential
  - both pair production: large corrections found, should take into account in future study
  - charged pair production: a peak found near the threshold, from h^0 exchanging
- What we have studied are parton-level cross sections. Convolution with structure functions is needed to include ISR effects.

Thanks!

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