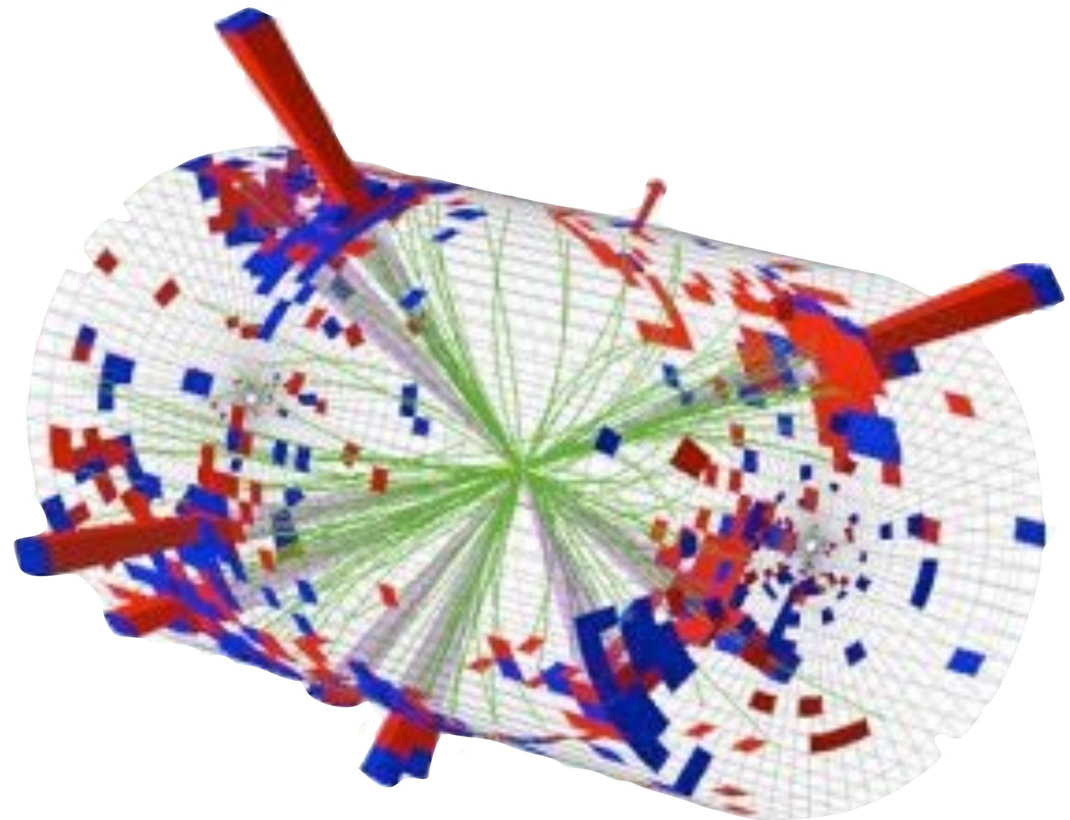

Sub-Leading and Super-Leading Logarithms in Jet Cross Sections

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Zürich Phenomenology Workshop 2023, January 13, 2023

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FOR FUNDAMENTAL PHYSICS

Large logarithms

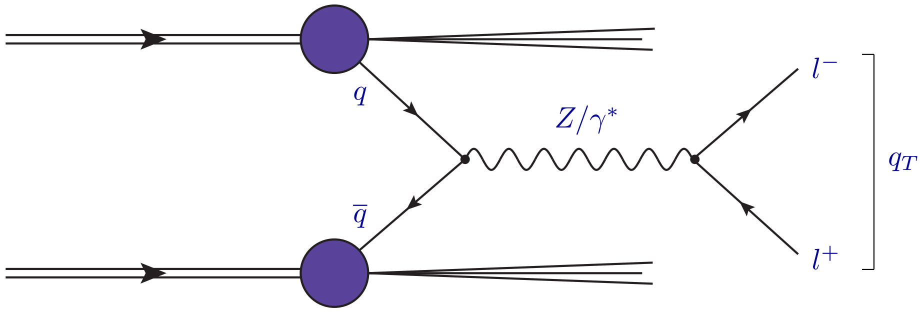
For processes involving scale hierarchies $Q_0 \ll Q$, fixed-order perturbation theory breaks down due to logarithmically enhanced corrections

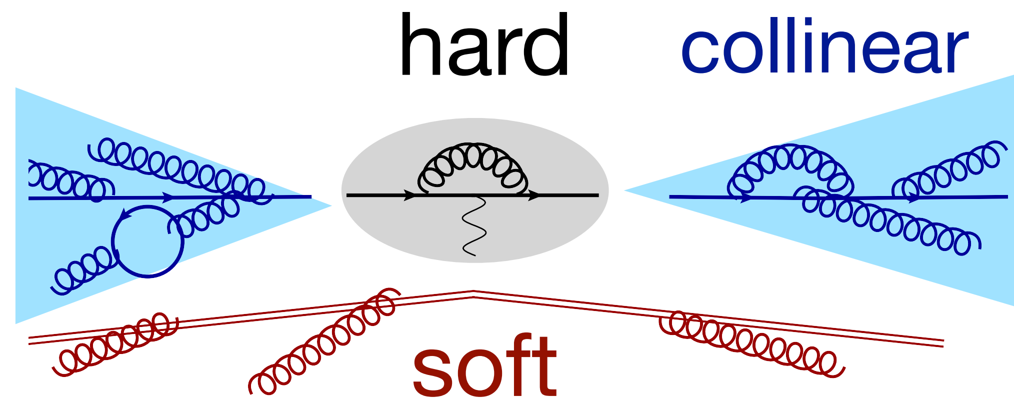
$$\alpha_s^n L^m \qquad \text{with} \qquad L = \ln(Q/Q_0)$$

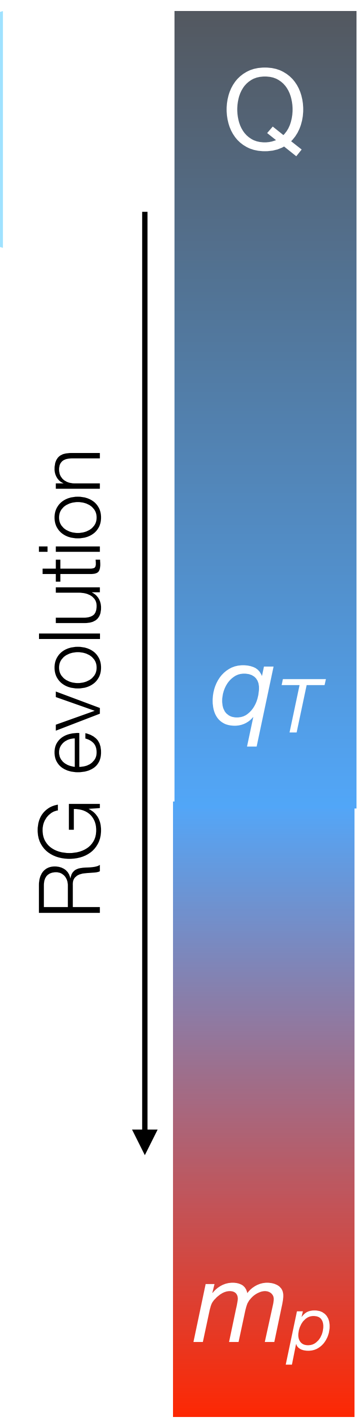
which must be summed to all orders:

- Factorization
- (Soft-Collinear) Effective theory
- (RG) evolution equations

q_T resummation for Z production





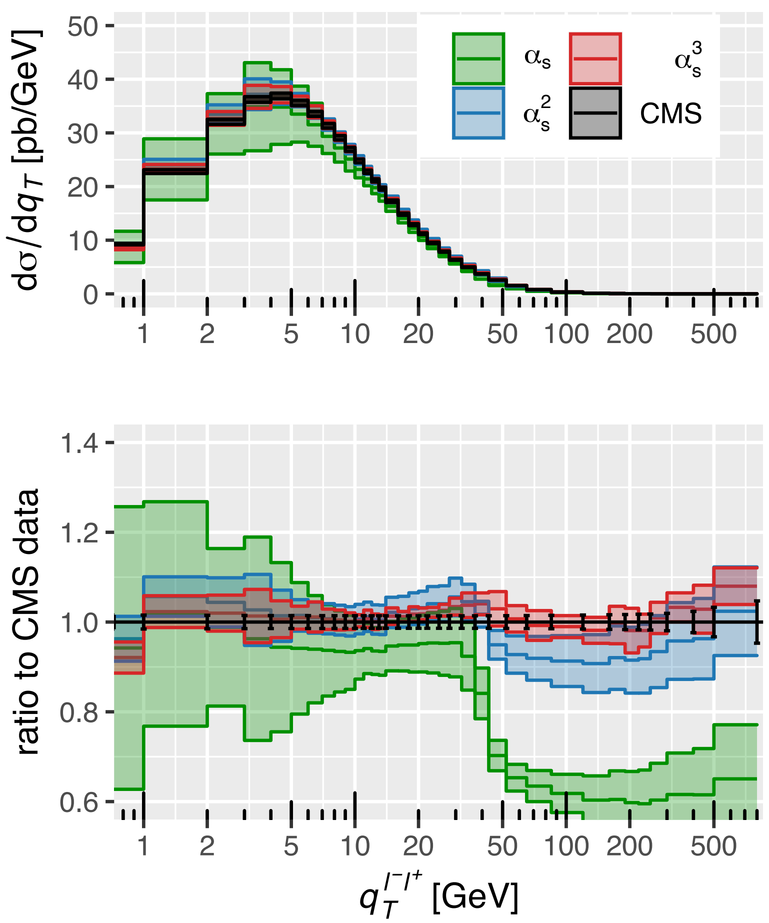


- Factorization theorem

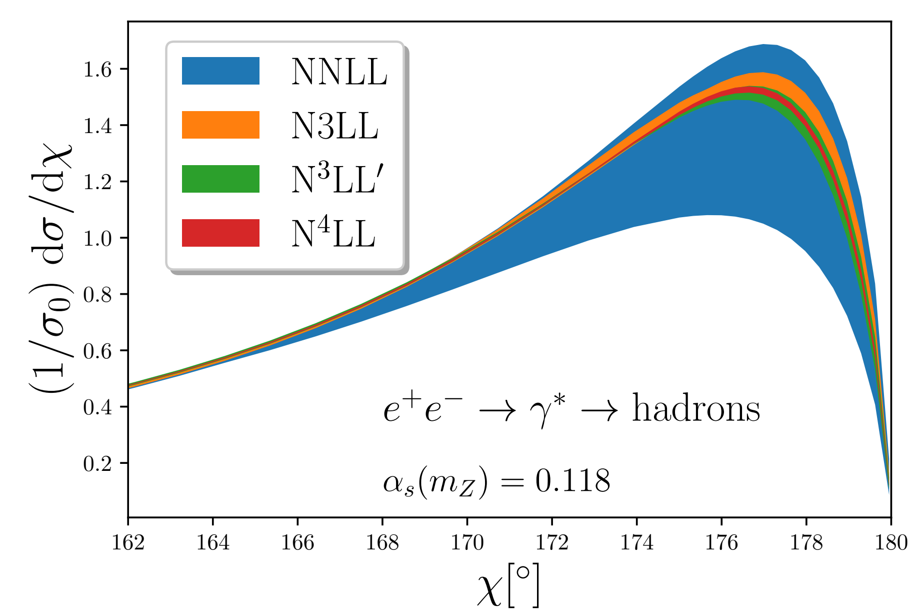
$$\frac{d\sigma}{dq_T dy} = \sum_{ab=q,\bar{q}} \int d^2x_\perp \, e^{-iq_\perp \cdot x_\perp} H_{ab}(Q^2,\mu) \mathcal{B}_a(\xi_1, x_T^2, \mu) \, \mathcal{B}_b(\xi_2, x_T^2, \mu) \mathcal{S}(x_T^2, \mu)$$

- Ingredients fulfil RG evolution equations
- Soft function **S** is simple: corresponds to emissions off initial hard $q\bar{q}$-pair.

Z-boson q_T



Energy-energy correlator (EEC)



Duhr, Mistlberger, Vita, 2205.02242

Neumann, Campbell 2207.07056 CuTe-MCFM

see also: Chen et. al., 2203.01565 Radish+NNLOJet

Resummations up to N^{4}LL (3-loop matching, 4-loop anomalous dimensions!), but only for very inclusive observables (`global event shapes”)

Traditional resummation methods (such as SCET) restricted to global observables which do not involve angular cuts on hadronic radiation.

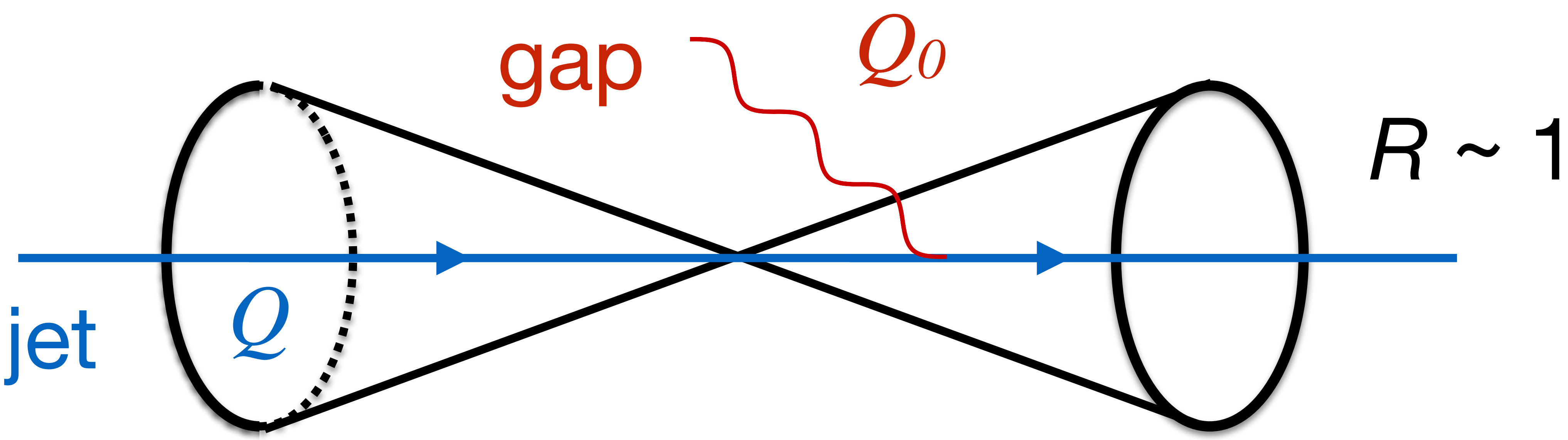
Non-global observables such as

- jet cross sections or isolation-cone cross sections (relevant for γ production)

involve very intricate structure of soft radiation

- secondary emissions: non-global logarithms (NGLs) Dasgupta, Salam ’02
- hadronic collisions: complex phases & breakdown of color coherence: super-leading logarithms SLL Forshaw, Kyrieleis, Seymour ’06

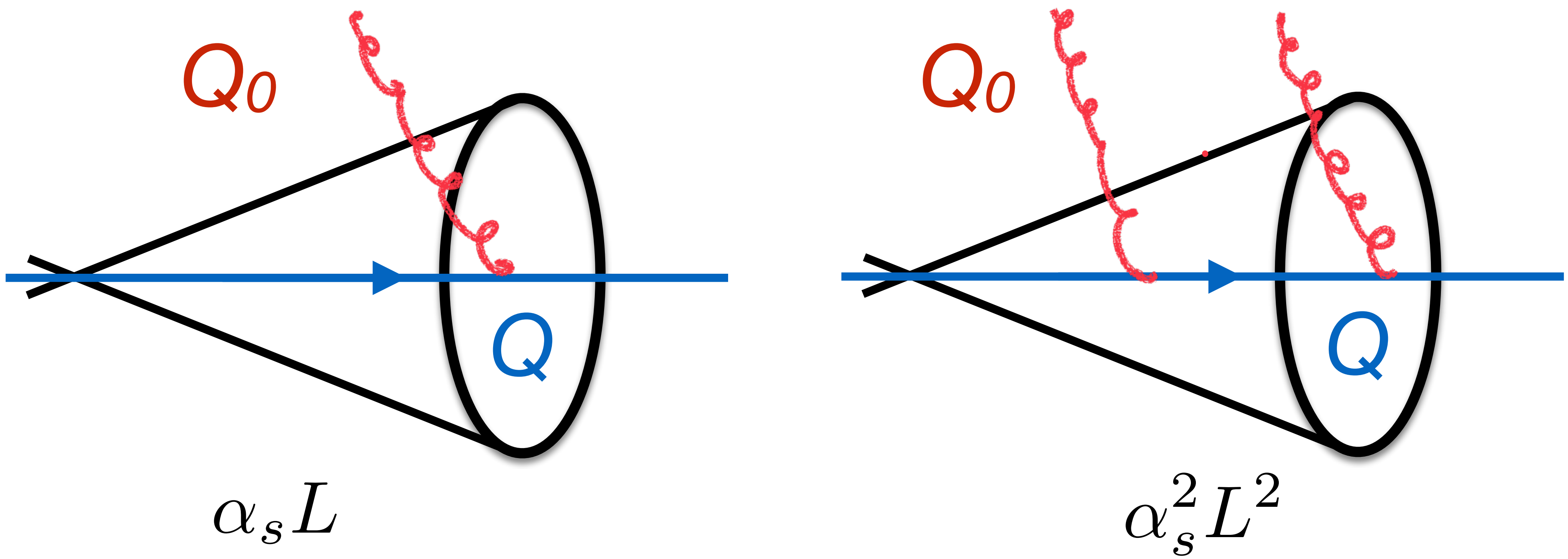
Simplest example of non-global observable: gap between jets aka interjet energy flow aka rapidity slice



→ large logarithms $\alpha_s^n L^m$ with $L = \ln(Q/Q_0)$

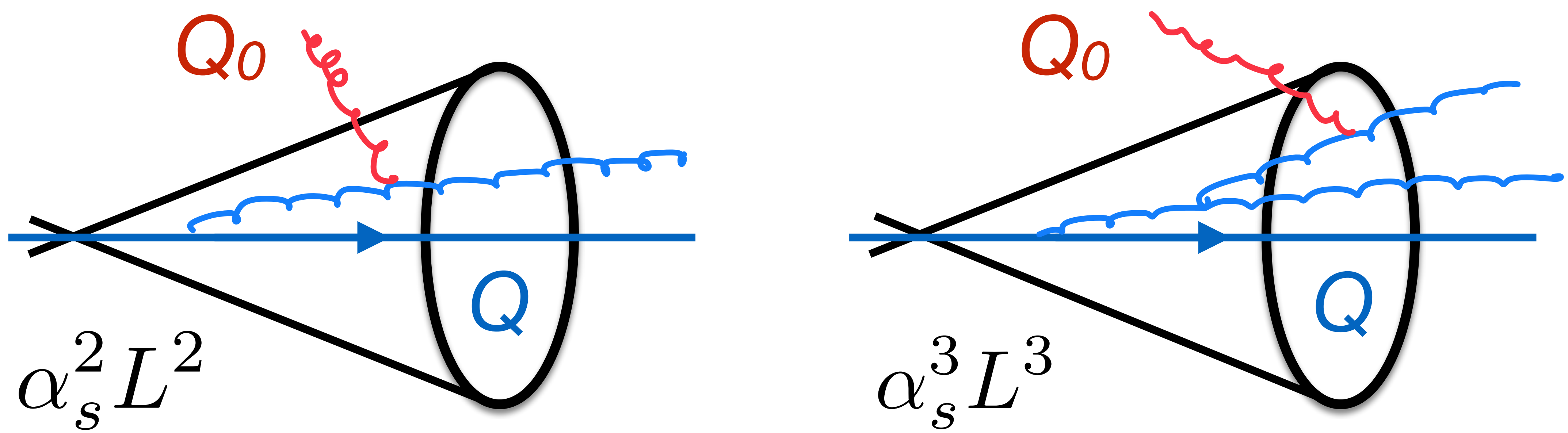
Will discuss case of large cone radius $R \sim 1$.

Global Logarithms



- In (massive) QED, logarithmic terms would exponentiate: full result is exponential of one loop!
- For global observables in QCD, non-abelian higher-order corrections ("non-abelian exponentiation")

Non-global logarithms (NGLs)



- Soft gluons from secondary emissions inside the jets
- Not captured by standard resummation methods. Even leading NGLs $(\alpha_s L)^n$ do not simply exponentiate!
- At large N_c leading NGLs can be obtained with parton shower Dasgupta, Salam '02 or by solving a non-linear integral equation Banfi, Marchesini, Smye '02, the BMS equation

Progress on NGLs

- PanScales, a general-purpose shower, which correctly resums leading large-N_c NGLs (and global logs!) Dasgupta, Dreyer, Hamilton, Monni, Salam and Soyez '20, + … ,'21 Alaric Herren, Höche, Krauss, Reichelt, Schoenherr '22

- Finite-N_c results for leading NGLs in e^+e^- Hatta, Ueda '13 + Hagiwara '15 based on Weigert '03; De Angelis, Forshaw and Plätzer '20

- First NLL numerical results in the large-N_c limit

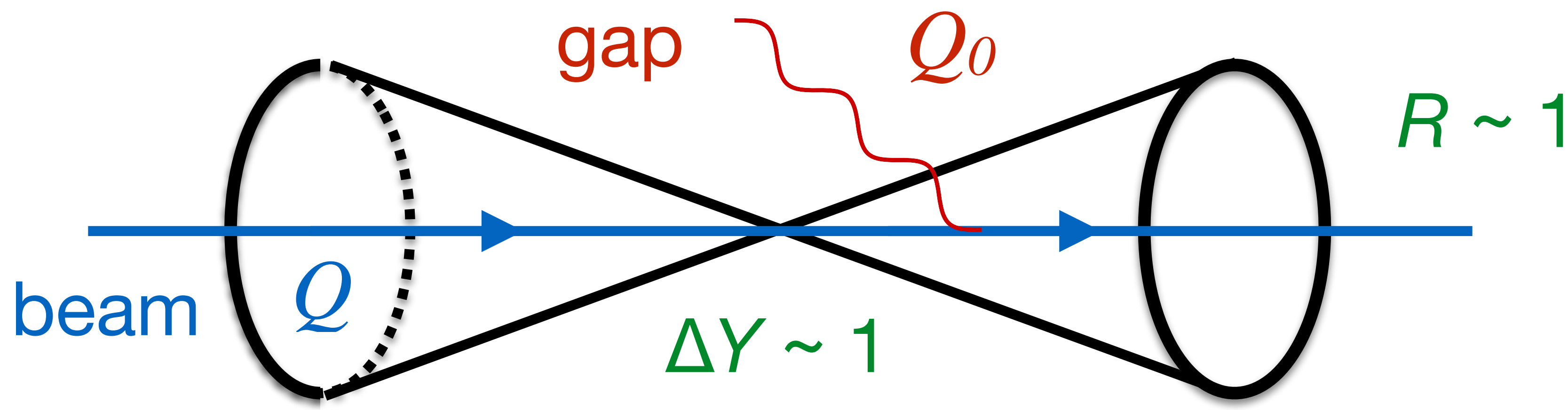
  - Extension of BMS framework to NLL (2104.06416) and numerical implementation in MC code Gnole (2111.02413) Banfi, Dreyer, Monni

  - Two-loop anomalous dimension in factorization framework TB, Rauh, Xu, 2112.02108; implementation into shower code TB, Schalch, Xu, in preparation → numerical results later in the talk

Super-Leading Logs (SLLs)

Forshaw, Kyrieleis, Seymour ’06 ’08

Analyze gap between jets at hadron collider, cone around beam direction



Large logarithms $\alpha_s^n L^m$ with $L = \ln(Q/Q_0)$

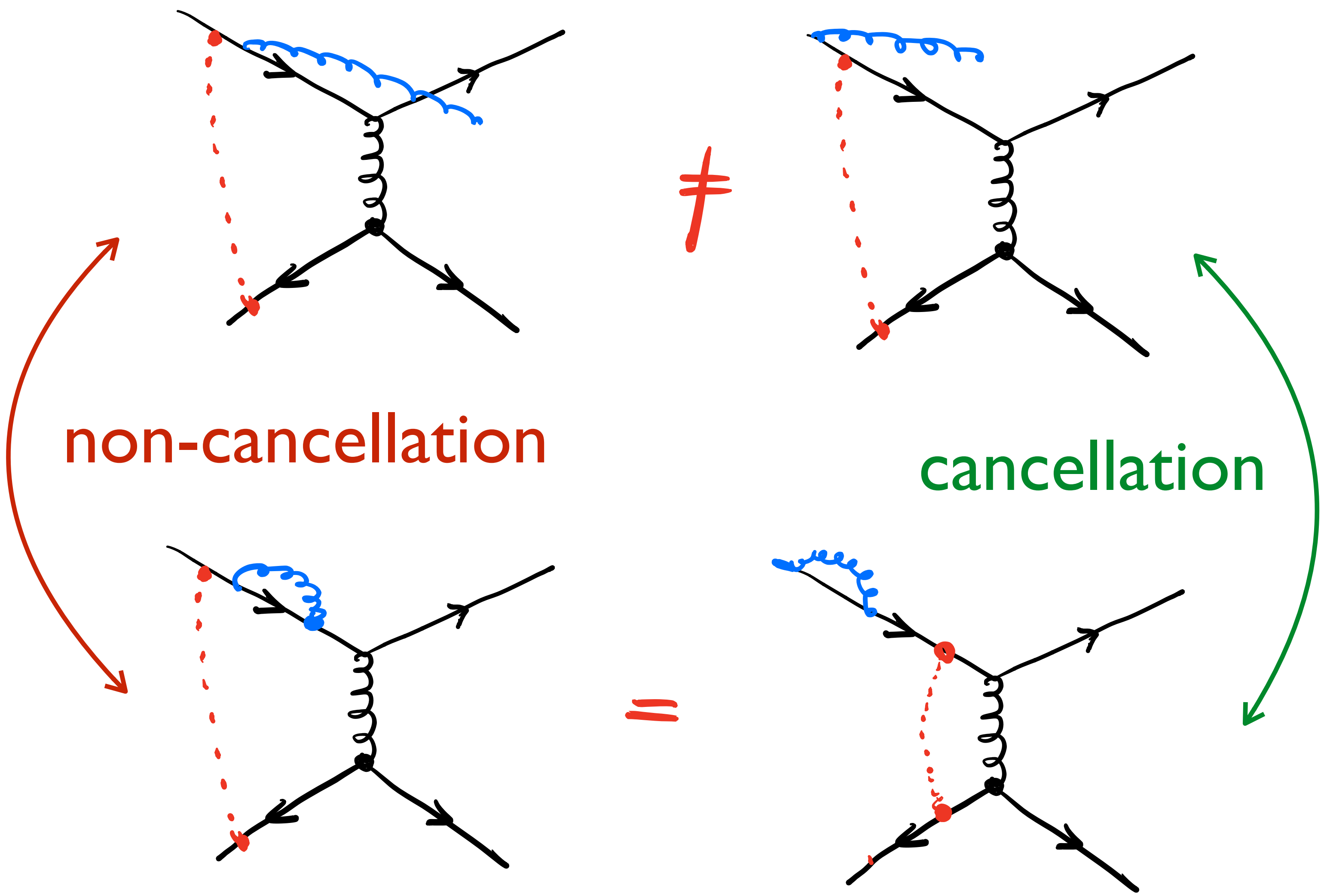
- e^+e^- : $m \leq n$, leading logs $m = n$
- pp : $\alpha_s L,\ \alpha_s^2 L^2,\ \alpha_s^3 L^3\ ,\alpha_s^4 L^5\ \ldots\ ,\alpha_s^{3+n} L^{3+2n}$

missing in large-N_c parton showers!
(Deductor? Soper and Nagy … ’19)

Non-cancellation of collinear logs

Forshaw, Kyrieleis, Seymour ’06 ’08; Catani, de Florian, Rodrigo ’11, …

Double logarithms due to soft+collinear configurations.



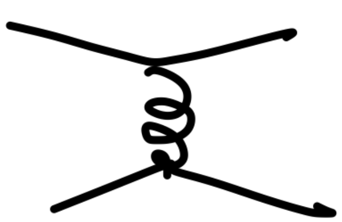
Blue: collinear emission. Red: Glauber/Coulomb phase

Note: Glauber phases cancel in e^+e^- and in large-N_c limit

Earlier results on SLLs

Since effect first arises at $O(\alpha_s^4)$, only few results

- Discovery of effect, computation of first SLL in gaps between jets for $qq \to qq$ Forshaw, Kyrieleis, Seymour '06
- Colour space calculation of leading SLL Forshaw, Kyrieleis, Seymour '08
    - Note that SLLs vanish in the large-N_c limit.
- Diagrammatic calculation, first *two* orders, different channels *qq, qg, gg* Keates and Seymour '09



$$S_O^{(4)} = \left(\frac{\alpha_s}{4\pi}\right)^4 L_Q^5 \, \Delta Y \, \pi^2 \, \frac{8}{15} \left(3N_c^2 - 4\right) \, \sigma_0 \, ,$$

$$S_O^{(5)} = \left(\frac{\alpha_s}{4\pi}\right)^5 L_Q^7 \, \Delta Y \, \pi^2 \, \frac{4}{315} \, N_c \left(-27N_c^2 + 44\right) \, \sigma_0$$

Glauber $(i\pi)^2$

New: resummed leading SLLs

TB, Neubert, Shao '21

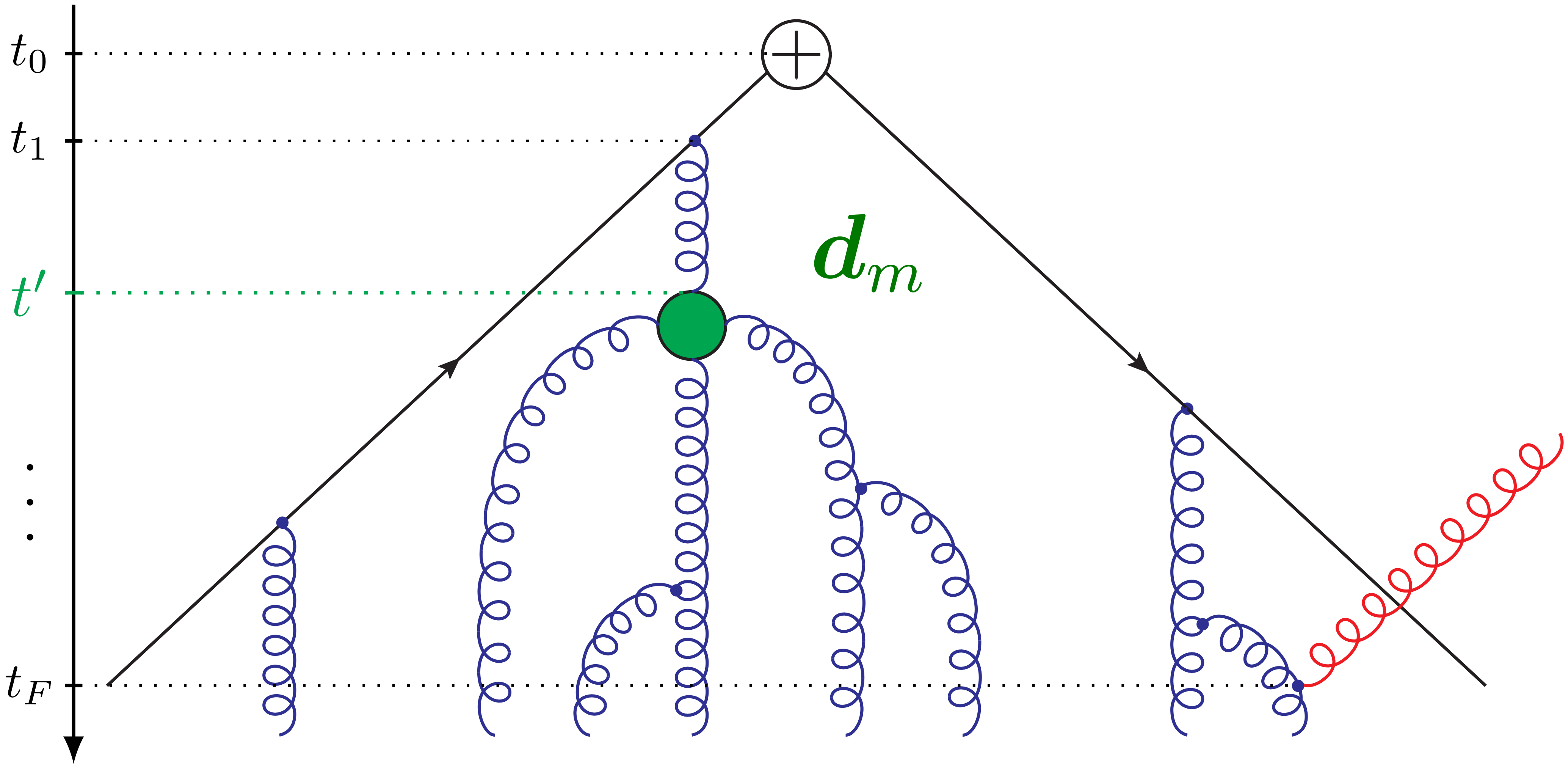
Based on factorization and RG evolution in effective field theory.

Simplest case: $qq \to qq$ scattering with photon exchange

$$\Delta\hat{\sigma}^{(S)} = -\hat{\sigma}_B \frac{4C_F}{3\pi} \alpha_s^3 L^3 \Delta Y \, {}_2F_2\left(1, 1; 2, \tfrac{5}{2}; -w\right)$$

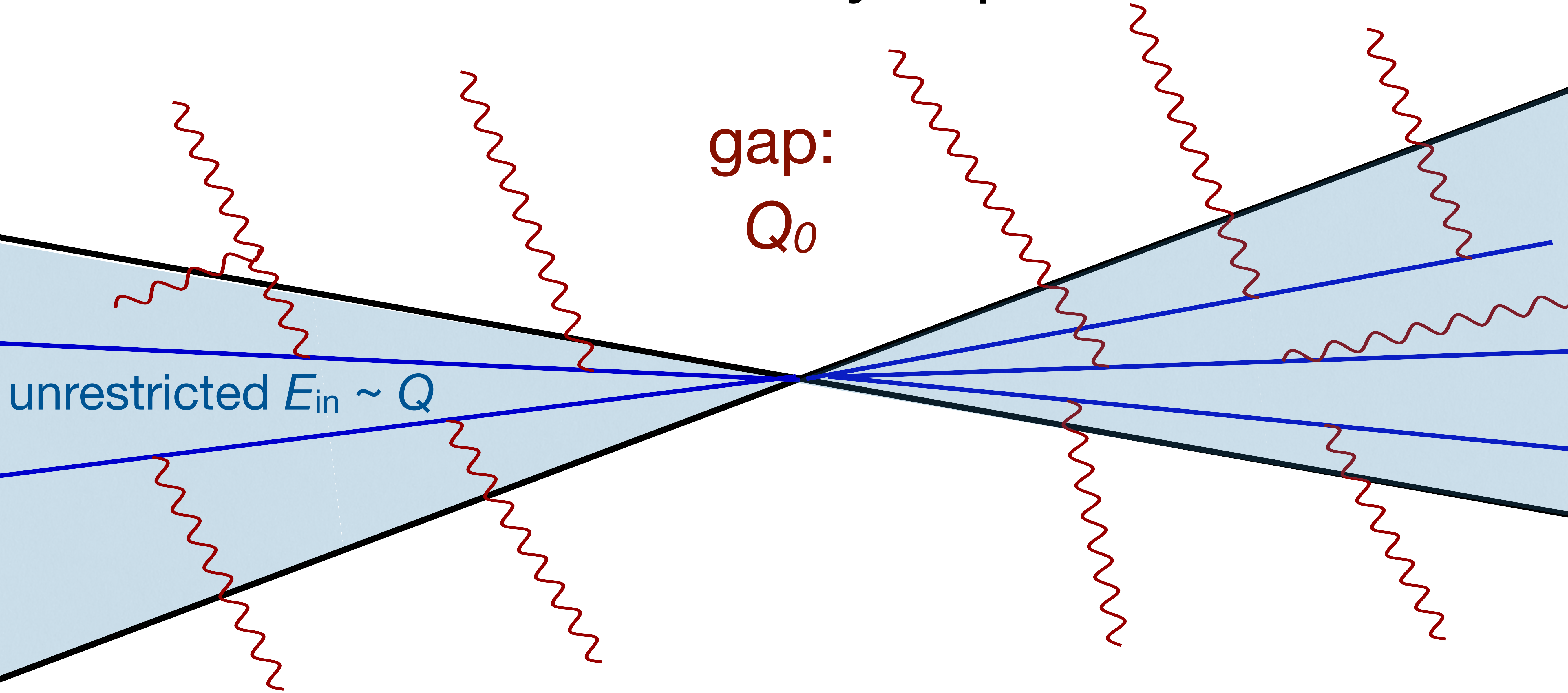
with $w = \frac{N_c \alpha_s}{\pi} L^2 .$ $\qquad \sim \frac{\ln w}{w}$ for large w

Note: Standard Sudakov has form e^{-cw}



Resummation of subleading NGLs

Soft radiation in jet processes



Hard partons (quarks and gluons) inside jets act as sources: soft radiation pattern depends on color-charges and directions of all hard partons!

Factorization for gap between jets in e^+e^-

TB, Neubert, Rothen, Shao ’15 ’16, see also Caron-Huot ‘15

Hard function
m hard partons along
fixed directions $\{n_1, \ldots, n_m\}$
$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

Soft function
squared amplitude
with m Wilson lines

$$\sigma(Q, Q_0) = \sum_{m=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu) \big\rangle$$

color trace

integration over directions

Have factorization formulas for a variety of non-global observables.

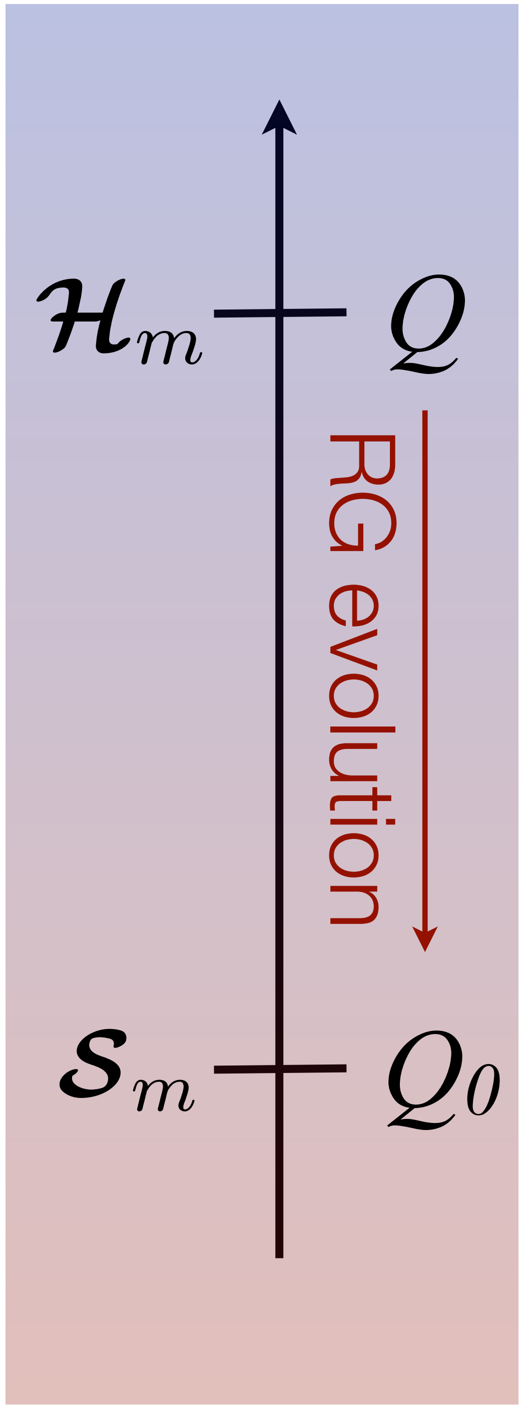
Resummation by RG evolution

Wilson coefficients fulfill RG equations

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(Q,\mu) = -\sum_{l=2}^{m}\boldsymbol{\mathcal{H}}_l(Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(Q,\mu)$$

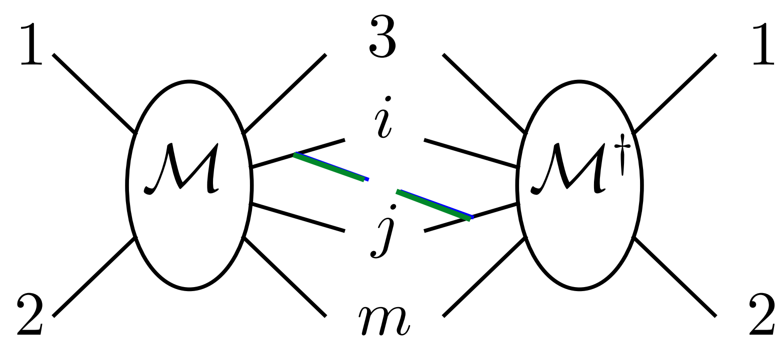
1. Compute $\mathcal{H}_m$ at a characteristic high scale $\mu_h \sim Q$
2. Evolve $\mathcal{H}_m$ to the scale of low energy physics $\mu_s \sim Q_0$
3. Evaluate $\mathcal{S}_m$ at low scale $\mu_s \sim Q_0$

Avoids large logarithms $\alpha_s^n \ln^n(Q/Q_0)$ of scale ratios which spoil convergence of perturbation theory.



One-loop anomalous dimension $\boldsymbol{\Gamma}^{(1)}$

$$\boldsymbol{\mathcal{H}}_m \boldsymbol{R}_m = \sum_{(ij)}$$

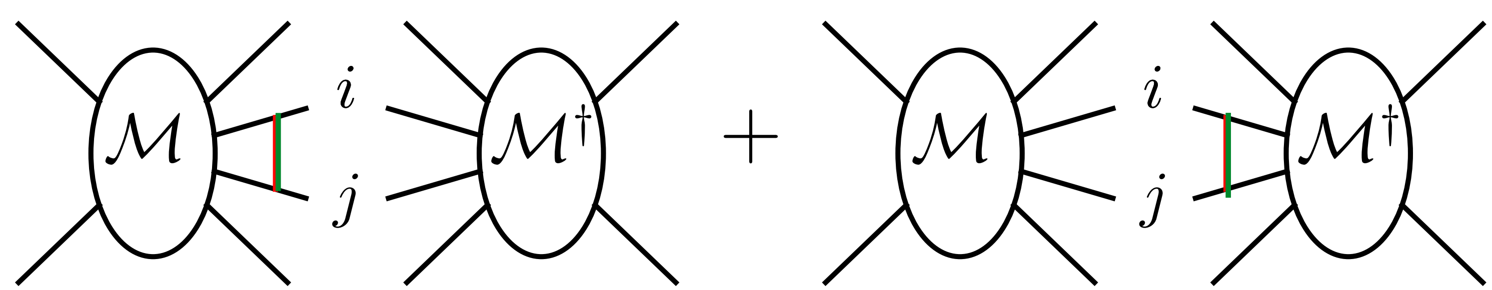


$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\boldsymbol{R}_m = -4 \sum_{(ij)} \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R}\, W_{ij}^{m+1}\, \Theta_{\mathrm{hard}}(n_{m+1})$$

extra hard parton!

$$\boldsymbol{\mathcal{H}}_m \boldsymbol{V}_m = \sum_{(ij)}$$



$$\boldsymbol{V}_m = 2 \sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}\right) \int \frac{d\Omega(n_k)}{4\pi}\, W_{ij}^k$$

soft dipole

$$W_{ij}^q = \frac{n_i \cdot n_j}{n_i \cdot n_q\, n_j \cdot n_q}$$

RG = Parton Shower

- Ingredients for LL

$$\boldsymbol{\mathcal{H}}_2(\mu = Q) = \sigma_0$$
$$\boldsymbol{\mathcal{H}}_m(\mu = Q) = 0 \text{ for } m > 2$$
$$\boldsymbol{\mathcal{S}}_m(\mu = Q_0) = 1$$

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- RG

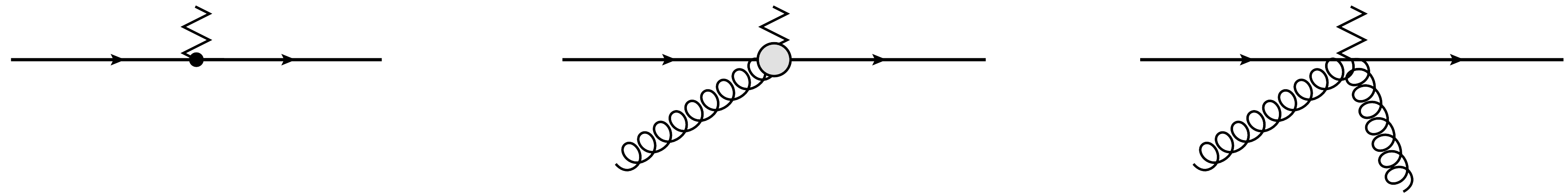
$$\frac{d}{dt}\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\boldsymbol{R}_{m-1}\,.$$

shower evolution time

$$t \equiv t(\mu_h, \mu_s) = \int_{\alpha_s(\mu_s)}^{\alpha_s(\mu_h)} \frac{d\alpha}{\beta(\alpha)}\,\frac{\alpha}{4\pi}$$

- equivalent to parton shower equation

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_1)e^{(t-t_1)V_n} + \int_{t_1}^{t} dt' \boldsymbol{\mathcal{H}}_{m-1}(t')\boldsymbol{R}_{m-1}e^{(t-t')\boldsymbol{V}_n}$$



$$\boldsymbol{\mathcal{S}}_m^{(0)} = \mathbf{1}$$

$$\boldsymbol{\mathcal{H}}_2(t_0) \longrightarrow \boldsymbol{\mathcal{H}}_3(t_1) \longrightarrow \boldsymbol{\mathcal{H}}_4(t_2)$$

$$\sigma_{\mathrm{LL}}(Q, Q_0) = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_2^{(0)} \otimes \boldsymbol{U}_{2m} \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m^{(0)} \rangle$$

$$= \langle \boldsymbol{\mathcal{H}}_2^{(0)}(t) + \int \frac{d\Omega_3}{4\pi} \boldsymbol{\mathcal{H}}_3^{\mathrm{LL}} + \int \frac{d\Omega_3}{4\pi} \int \frac{d\Omega_4}{4\pi} \boldsymbol{\mathcal{H}}_4^{\mathrm{LL}} + \dots \rangle$$

LL shower at large N_c equivalent to Dasgupta Salam '01. Have flexible implementation for general *k*-jet processes

- uses LHE event files from Madgraph for LO $\mathcal{H}_k$
- studied gap fractions and photon isolation cones

Balsiger, TB, Shao, '18; Balsiger, TB, Ferroglia '20

TB, Favrod, Xu JHEP 01 (2023) 005

Subleading logarithms

- Factorization theorem is not limited to LL accuracy.
- For NLL we need one-loop matching + two-loop running

$\boldsymbol{\mathcal{H}}_2(\mu_h) = \sigma_0 \, |C_V(Q^2, \mu_h)|^2 \, \mathbf{1}$ one-loop virtual

$\boldsymbol{\mathcal{H}}_3(\mu_h)$ hard real emission corrections

$\boldsymbol{\mathcal{S}}_m^{(1)}(\mu_s)$ JHEP 04 (2019) 020 with Marcel Balsiger, Ding Yu Shao

$\boldsymbol{\Gamma}_{lm}^{(2)}$ two-loop anomalous dim.

2112.02108 with Thomas Rauh and Xiaofeng Xu

Two-loop anomalous dimension $\boldsymbol{\Gamma}^{(2)}$

2112.02108 with TB, Rauh, Xu; see also Caron-Huot 1501.03754

$$\boldsymbol{\Gamma}^{(2)} = \begin{pmatrix} \boldsymbol{v}_2 & \boldsymbol{r}_2 & \boldsymbol{d}_2 & 0 & \dots \\ 0 & \boldsymbol{v}_3 & \boldsymbol{r}_3 & \boldsymbol{d}_3 & \dots \\ 0 & 0 & \boldsymbol{v}_4 & \boldsymbol{r}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{v}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$\boldsymbol{d}_m$: double real emission

$\boldsymbol{r}_m$: real-virtual corr.

$\boldsymbol{v}_m$: two-loop virtual

$$\begin{aligned} \boldsymbol{d}_m = & \sum_{(ij)} \sum_{k} i f^{abc} \left(\boldsymbol{T}^a_{i,L} \boldsymbol{T}^b_{j,L} \boldsymbol{T}^c_{k,R} - \boldsymbol{T}^a_{i,R} \boldsymbol{T}^b_{j,R} \boldsymbol{T}^c_{k,L} \right) K_{ijk;qr} \theta_{\text{in}}(n_q) \theta_{\text{in}}(n_r) \\ & - 2 \sum_{(ij)} \boldsymbol{T}^c_{i,L} \boldsymbol{T}^c_{j,R} K_{ij;qr} \theta_{\text{in}}(n_q) \theta_{\text{in}}(n_r) \,, \\ \boldsymbol{r}_m = & - 2 \sum_{i} \sum_{(jk)} i f^{abc} (\boldsymbol{T}^a_{i,L} \boldsymbol{T}^b_{j,R} \boldsymbol{T}^c_{k,R} - \boldsymbol{T}^a_{i,R} \boldsymbol{T}^b_{j,L} \boldsymbol{T}^c_{k,L}) \int \left[d^2 \Omega_r \right] K_{ijk;qr} \theta_{\text{in}}(n_q) \\ & - \sum_{(ij)} \boldsymbol{T}^a_{i,L} \boldsymbol{T}^a_{j,R} \left\{ W^q_{ij} \left[4 \beta_0 \ln(2 W^q_{ij}) + \gamma^{\text{cusp}}_1 \right] - 2 \int \left[d^2 \Omega_r \right] K_{ij;qr} \right\} \theta_{\text{in}}(n_q) \\ & + 8 i \pi \sum_{i} \sum_{(jk)} i f^{abc} \left(\boldsymbol{T}^a_{i,L} \boldsymbol{T}^b_{j,R} \boldsymbol{T}^c_{k,R} + \boldsymbol{T}^a_{i,R} \boldsymbol{T}^b_{j,L} \boldsymbol{T}^c_{k,L} \right) W^q_{ij} \ln W^q_{jk} \, \theta_{\text{in}}(n_q) \,, \\ \boldsymbol{v}_m = & \sum_{(ijk)} i f^{abc} \left(\boldsymbol{T}^a_{i,L} \boldsymbol{T}^b_{j,L} \boldsymbol{T}^c_{k,L} - \boldsymbol{T}^a_{i,R} \boldsymbol{T}^b_{j,R} \boldsymbol{T}^c_{k,R} \right) \int \left[d^2 \Omega_q \right] \int \left[d^2 \Omega_r \right] K_{ijk;qr} \\ & + \sum_{(ij)} \frac{1}{2} (\boldsymbol{T}^a_{i,L} \boldsymbol{T}^a_{j,L} + \boldsymbol{T}^a_{i,R} \boldsymbol{T}^a_{j,R}) \int \left[d^2 \Omega_q \right] W^q_{ij} \left[4 \beta_0 \ln(2 W^q_{ij}) + \gamma^{\text{cusp}}_1 \right] \\ & - i \pi \sum_{(ij)} \frac{1}{2} (\boldsymbol{T}^a_{i,L} \boldsymbol{T}^a_{j,L} - \boldsymbol{T}^a_{i,R} \boldsymbol{T}^a_{j,R}) \, \Pi_{ij} \, \gamma^{\text{cusp}}_1 \,. \end{aligned}$$

+ additional terms from converting to angular integrals in d=4

$\boldsymbol{\Gamma}^{(2)}$ depends on angular functions

$$K_{ij;qr} \qquad K_{ijk;qr}$$

for two- and three-leg terms.

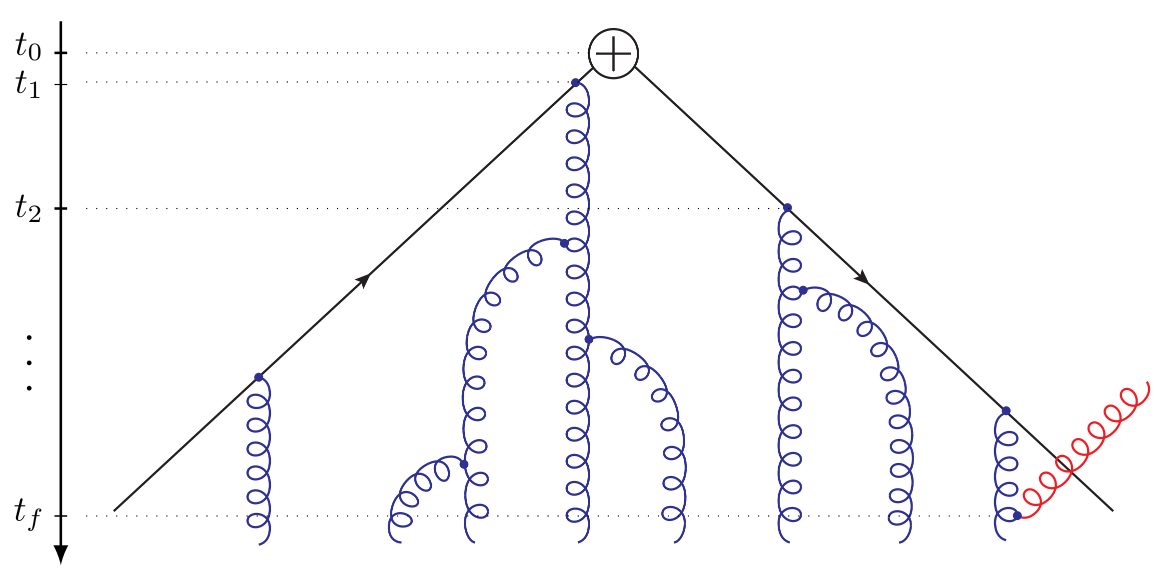
NLL shower

TB, Schalch, Xu in preparation

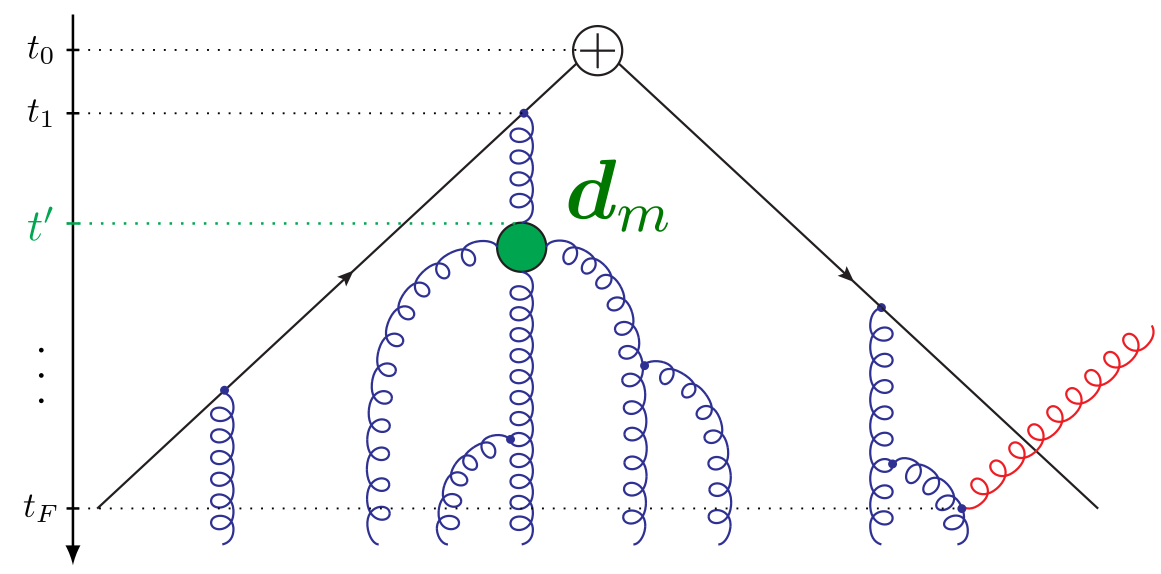
Contribution $\Delta\boldsymbol{\mathcal{H}}_m$ from two-loop running is suppressed by α_s. Expand

$$\begin{aligned}\Delta\boldsymbol{\mathcal{H}}_m(t) &= \boldsymbol{\mathcal{H}}_k(t_0)\Delta\boldsymbol{U}_{km}(t,t_0)\\ &= \boldsymbol{\mathcal{H}}_k(t_0)\int_{t_0}^{t} dt'\, \boldsymbol{U}^{\mathrm{LL}}_{kl}(t'-t_0)\cdot \frac{\alpha(t')}{4\pi}\left(\boldsymbol{\Gamma}^{(2)}_{ll'} - \frac{\beta_1}{\beta_0}\boldsymbol{\Gamma}^{(1)}_{ll'}\right)\cdot \boldsymbol{U}^{\mathrm{LL}}_{l'm}(t-t')\end{aligned}$$

LL shower



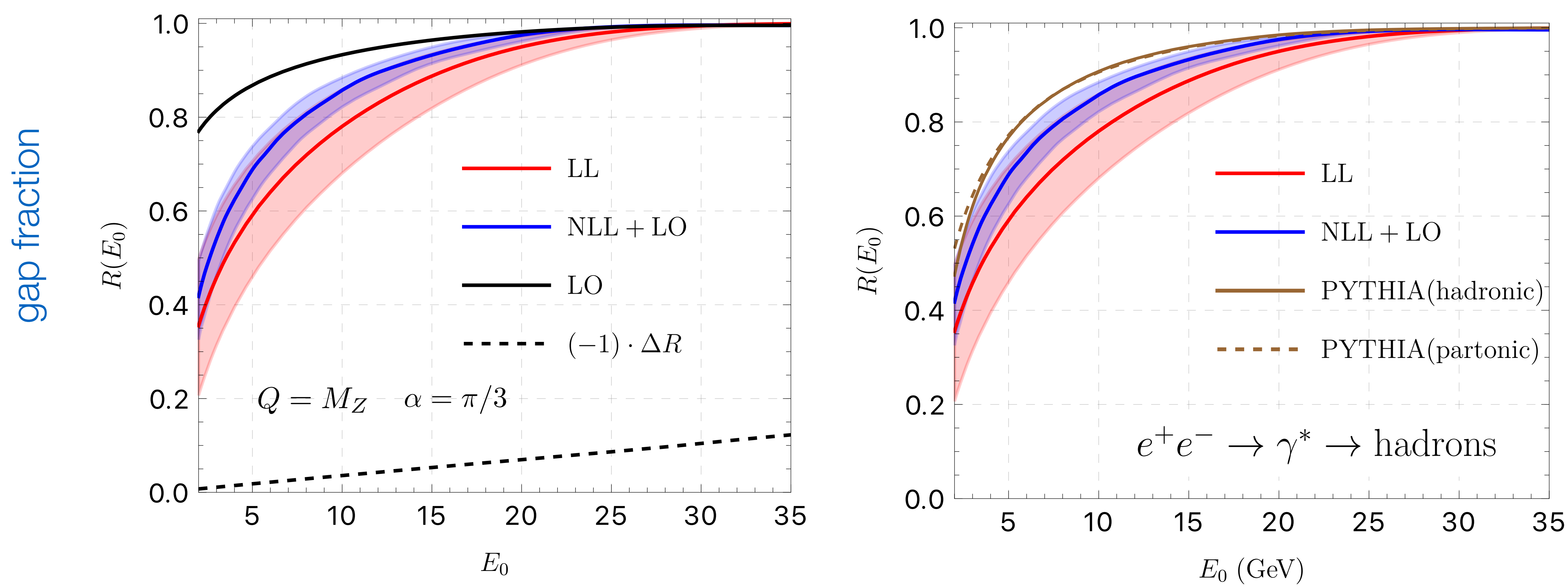
NLL correction



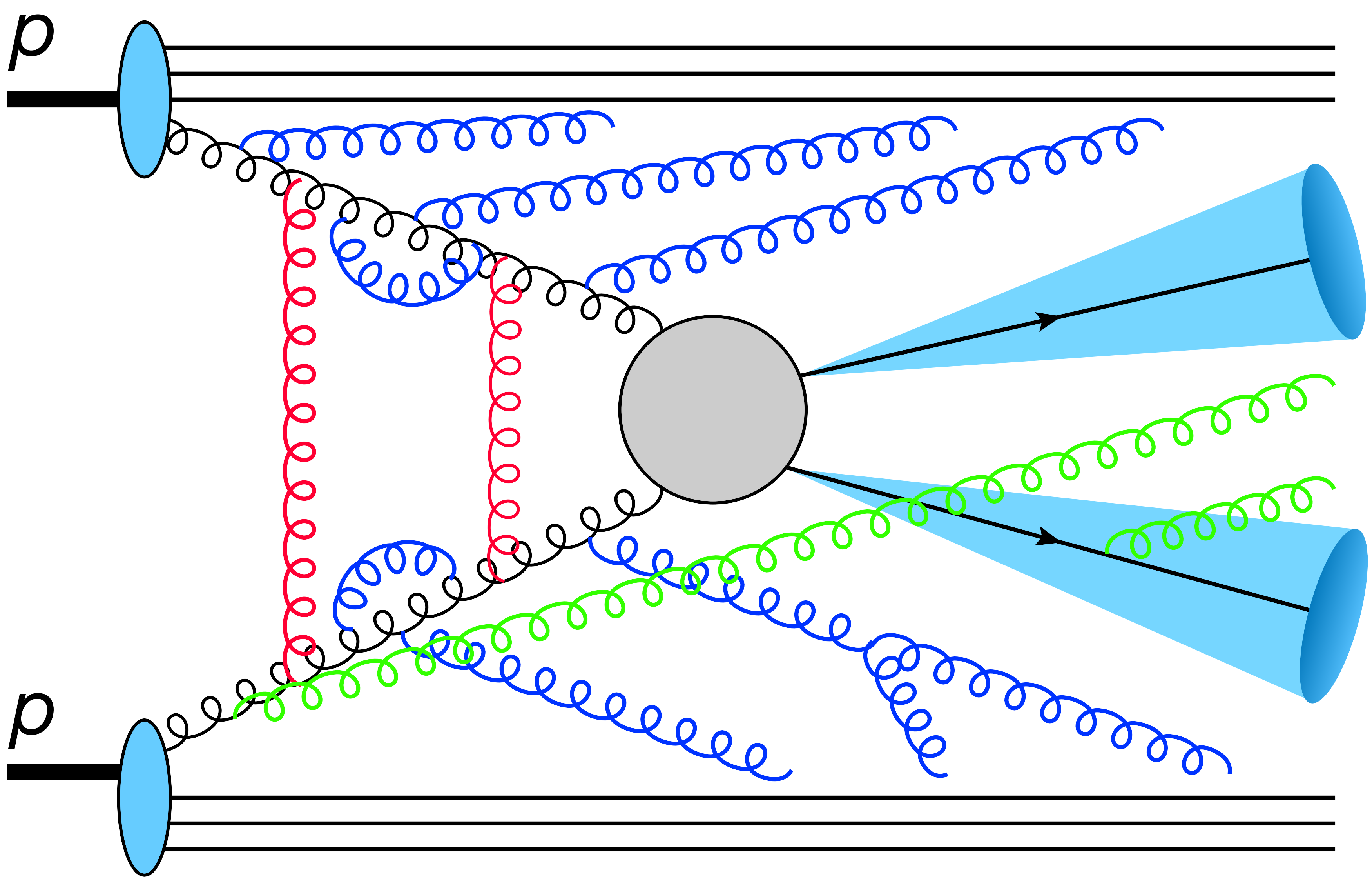
analogous for $\boldsymbol{r}_m$ and $\boldsymbol{v}_m$

First numerical results

TB, Schalch, Xu in preparation



- Matched to O(α_s) fixed order
- Switch off resummation at $E_0 = E_0^{\max} = 35\,\mathrm{GeV}$
- Qualitative agreement with `Gnole` Banfi, Dreyer, Monni '21; detailed comparison in progress



Resummation of Super-Leading Logs

TB, Neubert, Shao ’21

TB, Neubert, Stillger, Shao, in preparation

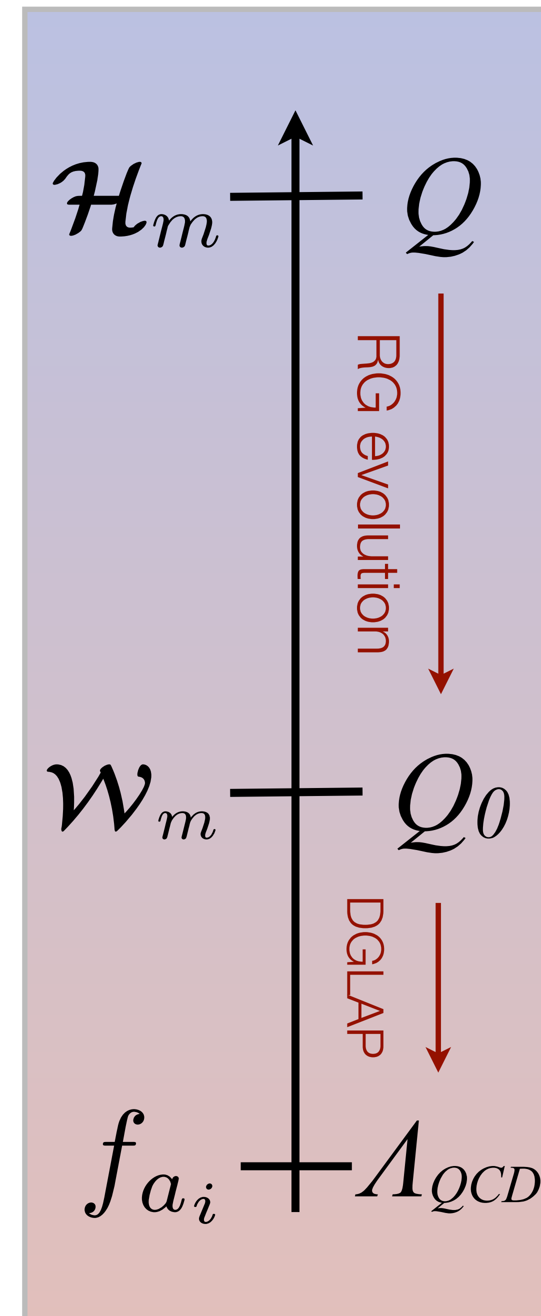
Hadronic collisions

$$\sigma(Q_0) = \sum_{a_1,a_2=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=4}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\},Q_0,x_1,x_2,\mu) \rangle$$

- $\boldsymbol{\mathcal{H}}_m$ analogous to e^+e^- but low energy matrix element $\boldsymbol{\mathcal{W}}_m$ in SCET contains soft Wilson lines + collinear fields for incoming partons
- Leading order matrix element

$$\boldsymbol{\mathcal{W}}_m(\{\underline{n}\},Q_0,x_1,x_2,\mu_s) = f_{a_1}(x_1)\, f_{a_2}(x_2)\, \mathbf{1}$$

- Low-E theory at $\mu \sim Q_0$ involves Glauber gluons, which mediate soft-collinear interactions. Rothstein, Stewart ’16
  - Rapidity divs. generate single logs of Q



One-loop anomalous dimension

$$\mathbf{\Gamma}^H\left(\{\underline{n}\},\xi_1,\xi_2,s,\mu\right) = \frac{\alpha_s}{4\pi}\mathbf{\Gamma}^{(1)} = \frac{\alpha_s}{4\pi}\begin{pmatrix} \boldsymbol{V}_k & \boldsymbol{R}_k & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_{k+1} & \boldsymbol{R}_{k+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_{k+2} & \boldsymbol{R}_{k+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_{k+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

k: number of partons at Born-level

- Split into soft(+collinear) and purely collinear

$$\mathbf{\Gamma}^{(1)}(\xi_1,\xi_2) = \mathbf{\Gamma}_1^C(\xi_1)\delta(1-\xi_2) + \delta(1-\xi_1)\mathbf{\Gamma}_2^C(\xi_2) + \delta(1-\xi_1)\,\delta(1-\xi_2)\,\mathbf{\Gamma}^S$$

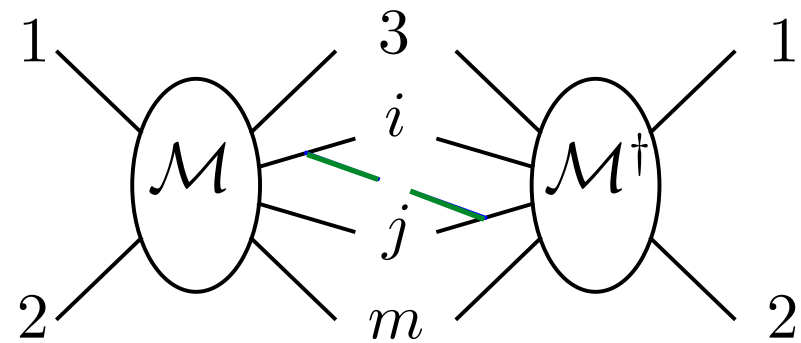
- Split soft part

$$\mathbf{\Gamma}^S = \overline{\mathbf{\Gamma}} + \mathbf{\Gamma}^G + \mathbf{\Gamma}^c \ln\frac{\mu^2}{\hat{s}}$$

wide-angle soft — Glauber — cusp: soft+collinear

Soft wide-angle emissions $\overline{\boldsymbol{\Gamma}}$

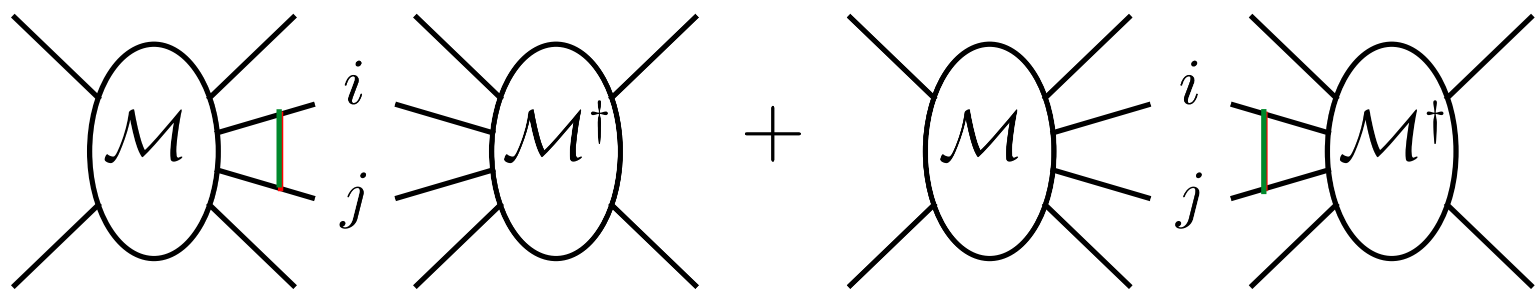
$$\boldsymbol{\mathcal{H}}_m \overline{\boldsymbol{R}}_m = \sum_{(ij)}$$



$$\overline{\boldsymbol{R}}_m = -4 \sum_{(ij)} \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R}\, \overline{W}_{ij}^{m+1}\, \Theta_{\mathrm{hard}}(n_{m+1})$$

extra hard parton!

$$\boldsymbol{\mathcal{H}}_m \overline{\boldsymbol{V}}_m = \sum_{(ij)}$$



$$\overline{\boldsymbol{V}}_m = 2 \sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}\right) \int \frac{d\Omega(n_k)}{4\pi}\, \overline{W}_{ij}^{k}$$

soft dipole

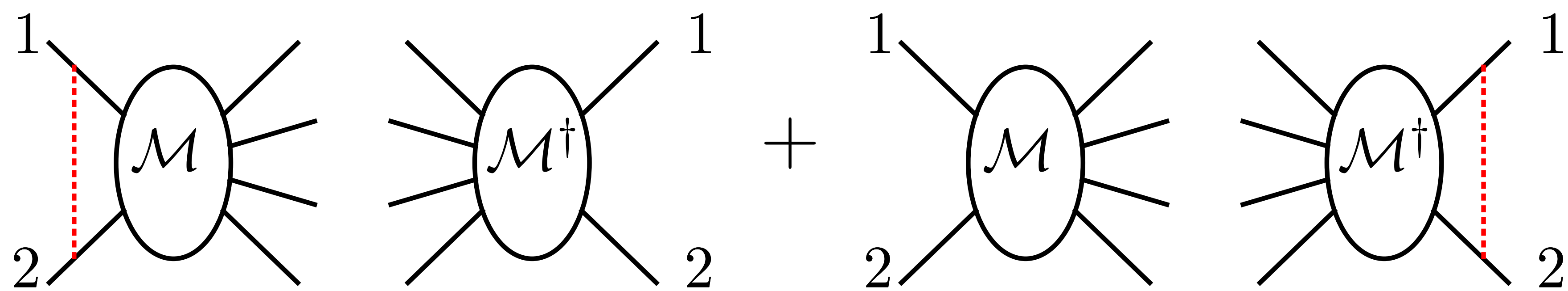
$$W_{ij}^{q} = \frac{n_i \cdot n_j}{n_i \cdot n_q\, n_j \cdot n_q}$$

soft dipole with collinear subtraction

$$\overline{W}_{ij}^{q} = W_{ij}^{q} - \frac{1}{n_i \cdot n_q}\delta(n_i - n_q) - \frac{1}{n_j \cdot n_q}\delta(n_j - n_q)$$

Glauber term Γ^G

$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}^G \; =$$



$$\boldsymbol{V}^G = -8i\pi \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right)$$

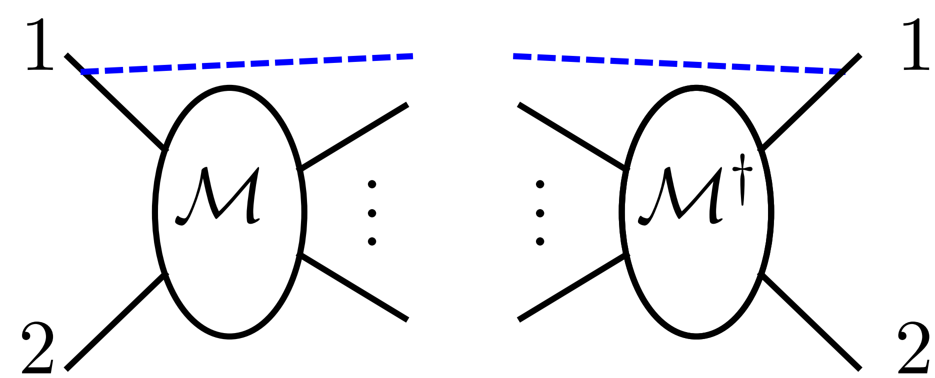
Used color conservation $\sum_i \boldsymbol{T}_i = 0$ to simplify Glauber terms in 1 + 2 → 3 + … + *m*

Π_{ij} = 1 if both inc./out.

$$\sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) \Pi_{ij} = 4 \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right)$$

(Soft+)Collinear Cusp Term $\boldsymbol{\Gamma}^c$

$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{R}_1^c \; =$$



$$\boldsymbol{R}_i^c = -4\boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R} \, \delta(n_{m+1} - n_i)$$
$$\boldsymbol{V}_i^c = 4C_i \, \mathbf{1}$$

- Only present for initial-state partons i=1,2. Final state terms cancel!
- Multiplied by $\ln \frac{\mu^2}{\hat{s}}$ → double logarithms!

Computation of SLLs

Cannot use large N_c: compute order by order

$$\left\langle \boldsymbol{\mathcal{H}}_4\, \boldsymbol{U}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1} \right\rangle = \left\langle \boldsymbol{\mathcal{H}}_4\, \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}(\{\underline{n}\},\mu)\right] \hat{\otimes}\,\mathbf{1} \right\rangle$$

$$= \langle \boldsymbol{\mathcal{H}}_4 \rangle + \int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \langle \boldsymbol{\mathcal{H}}_4\, \boldsymbol{\Gamma}(Q,\mu)\hat{\otimes}\mathbf{1} \rangle + \int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \int_{\mu}^{\mu_h} \frac{d\mu'}{\mu'} \langle \boldsymbol{\mathcal{H}}_4 \boldsymbol{\Gamma}(Q,\mu)\, \boldsymbol{\Gamma}(Q,\mu')\hat{\otimes}\,\mathbf{1} \rangle + \dots$$

$\hat{\sigma}_{\mathrm{LO}}$ $\alpha_s L$ $\alpha_s^2 L^2$

Need products of anomalous dimensions. Each μ integral produces single log ($\overline{\boldsymbol{\Gamma}}$, $\boldsymbol{\Gamma}^G$) or double logs ($\boldsymbol{\Gamma}^c$), i.e. SLLs!

Will set μ_h=Q and μ_s=Q_0 and ignore running of α_s.

Leading SLLs

1. Want maximum number of $\boldsymbol{\Gamma}^c$'s at given order.
2. Important properties of anomalous dimensions

$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{\Gamma}^c \, \overline{\boldsymbol{\Gamma}} = \boldsymbol{\mathcal{H}}_m \, \overline{\boldsymbol{\Gamma}} \, \boldsymbol{\Gamma}^c$$

"color coherence"

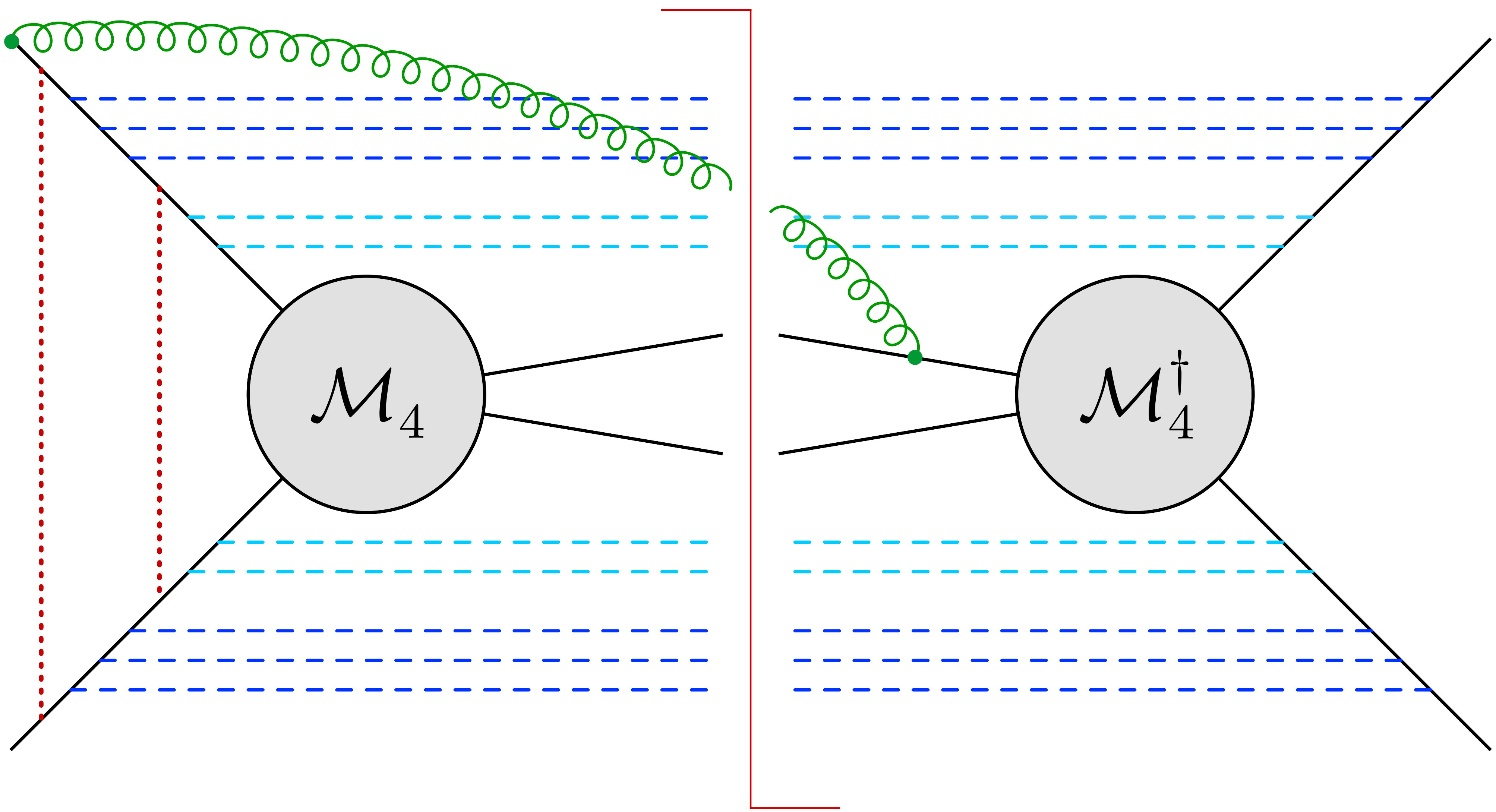
$$\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0$$

$$\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}^G \otimes \mathbf{1} \rangle = 0$$

3. Properties imply that the leading SLLs at (*n*+3)-rd order arise from matrix elements

$$C_{rn} = \langle \boldsymbol{\mathcal{H}}_4 \, (\boldsymbol{\Gamma}^c)^r \, \boldsymbol{V}^G \, (\boldsymbol{\Gamma}^c)^{n-r} \, \boldsymbol{V}^G \, \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle \qquad 0 \leq r \leq n$$

$$C_{4,10} = \langle \boldsymbol{\mathcal{H}}_4 \, (\mathbf{\Gamma}^c)^4 \, \mathbf{\Gamma}^G (\mathbf{\Gamma}^c)^6 \, \mathbf{\Gamma}^G \, \overline{\mathbf{\Gamma}} \otimes \mathbf{1} \rangle$$



+ many more diagrams: Glauber(s) on the right side, different attachents for wide-angle soft, virtuals …

Evaluation of C_{rn}

- Basic strategy: commute $\boldsymbol{\Gamma}^c$'s and $\boldsymbol{\Gamma}^G$ to the right where they vanish.

- After a lot of color algebra, one finds

$$C_{rn} = -16\,(4\pi)^2\,(4N_c)^n \sum_{i=1}^{7} v_i^r \,\langle \boldsymbol{\mathcal{H}}_{2\to M}\,\boldsymbol{Q}_i \rangle$$

- Eigenvalues

power-like *n* and *r* dependence

$$v_1 = 0\,, \quad v_2 = \frac{1}{2}\,, \quad v_3 = 1\,, \quad v_4 = \frac{3N_c - 2}{2N_c}\,, \quad v_5 = \frac{3N_c + 2}{2N_c}\,, \quad v_6 = \frac{2\,(N_c - 1)}{N_c}\,, \quad v_7 = \frac{2\,(N_c + 1)}{N_c}$$

- Eigenoperators are $\boldsymbol{Q}_i$ are combinations color 10 basic structures.

Eigenoperators, color structures O_i and S_i

$$\boldsymbol{Q}_1 = J_{12}\left[\frac{4N_c}{N_c^2-1}\,C_1C_2\,\boldsymbol{S}_6\right],$$

$$\boldsymbol{Q}_2 = \sum_{j=3}^{M+2} J_j\left[-\frac{N_c}{N_c^2-1}\,\boldsymbol{O}_4^{(j)}\right] + J_{12}\left[\frac{2N_c}{N_c^2-1}\,(C_1+C_2)\,\boldsymbol{S}_5 - \frac{4N_c}{N_c^2-1}\,C_1C_2\,\boldsymbol{S}_6\right],$$

$$\boldsymbol{Q}_3 = \sum_{j=3}^{M+2} J_j\left[-\frac{N_c^2}{2(N_c^2-4)}\,\boldsymbol{O}_2^{(j)}\right] + J_{12}\left[\frac{N_c^2}{N_c^2-4}\,\boldsymbol{S}_3 - \frac{N_c^2}{3}\,\boldsymbol{S}_5\right]$$

$$\begin{aligned}\boldsymbol{Q}_4 = \sum_{j=3}^{M+2} J_j&\left[\frac{1}{2}\,\boldsymbol{O}_1^{(j)} + \frac{N_c}{4(N_c-2)}\,\boldsymbol{O}_2^{(j)} - \frac{1}{2}\,\boldsymbol{O}_3^{(j)} + \frac{1}{2(N_c-1)}\,\boldsymbol{O}_4^{(j)}\right]\\ &+ J_{12}\left[\frac{1}{2}\,\boldsymbol{S}_1 + \frac{N_c}{4(N_c-2)}\,\boldsymbol{S}_2 - \frac{N_c}{2(N_c-2)}\,\boldsymbol{S}_3 - \frac{1}{2}\,\boldsymbol{S}_4\right.\\ &\qquad\left. + \left((C_1+C_2)\,\frac{N_c-2}{N_c-1} + \frac{N_c(N_c-4)}{6}\right)\boldsymbol{S}_5 + \frac{2C_1C_2}{N_c-1}\,\boldsymbol{S}_6\right],\end{aligned}$$

$$\begin{aligned}\boldsymbol{Q}_5 = \sum_{j=3}^{M+2} J_j&\left[\frac{1}{2}\,\boldsymbol{O}_1^{(j)} + \frac{N_c}{4(N_c+2)}\,\boldsymbol{O}_2^{(j)} + \frac{1}{2}\,\boldsymbol{O}_3^{(j)} + \frac{1}{2(N_c+1)}\,\boldsymbol{O}_4^{(j)}\right]\\ &+ J_{12}\left[\frac{1}{2}\,\boldsymbol{S}_1 + \frac{N_c}{4(N_c+2)}\,\boldsymbol{S}_2 - \frac{N_c}{2(N_c+2)}\,\boldsymbol{S}_3 + \frac{1}{2}\,\boldsymbol{S}_4\right.\\ &\qquad\left. + \left(-(C_1+C_2)\,\frac{N_c+2}{N_c+1} + \frac{N_c(N_c+4)}{6}\right)\boldsymbol{S}_5 + \frac{2C_1C_2}{N_c+1}\,\boldsymbol{S}_6\right],\end{aligned}$$

$$\boldsymbol{Q}_6 = -J_{12}\left[\frac{1}{2}\,\boldsymbol{S}_1 + \frac{N_c}{4(N_c-2)}\,\boldsymbol{S}_2 - \frac{1}{2}\,\boldsymbol{S}_4 + \frac{2C_1C_2}{N_c-1}\,\boldsymbol{S}_6\right],$$

$$\boldsymbol{Q}_7 = -J_{12}\left[\frac{1}{2}\,\boldsymbol{S}_1 + \frac{N_c}{4(N_c+2)}\,\boldsymbol{S}_2 + \frac{1}{2}\,\boldsymbol{S}_4 + \frac{2C_1C_2}{N_c+1}\,\boldsymbol{S}_6\right].$$

$$\begin{aligned}\boldsymbol{S}_1 &= f_{abe}f_{cde}\,\{\boldsymbol{T}_1^b,\boldsymbol{T}_1^c\}\,\{\boldsymbol{T}_2^a,\boldsymbol{T}_2^d\},\\ \boldsymbol{S}_2 &= d_{ade}d_{bce}\,\{\boldsymbol{T}_1^b,\boldsymbol{T}_1^c\}\,\{\boldsymbol{T}_2^a,\boldsymbol{T}_2^d\},\\ \boldsymbol{S}_3 &= d_{ade}d_{bce}\left[\boldsymbol{T}_2^a\left(\boldsymbol{T}_1^b\boldsymbol{T}_1^c\boldsymbol{T}_1^d\right)_+ + (1\leftrightarrow 2)\right],\\ \boldsymbol{S}_4 &= \{\boldsymbol{T}_1^a,\boldsymbol{T}_1^b\}\,\{\boldsymbol{T}_2^a,\boldsymbol{T}_2^b\},\\ \boldsymbol{S}_5 &= \boldsymbol{T}_1\cdot\boldsymbol{T}_2\,,\\ \boldsymbol{S}_6 &= \boldsymbol{1}\,.\end{aligned}$$

$$\begin{aligned}\boldsymbol{O}_1^{(j)} &= f_{abe}f_{cde}\,\boldsymbol{T}_2^a\,\{\boldsymbol{T}_1^b,\boldsymbol{T}_1^c\}\,\boldsymbol{T}_j^d - (1\leftrightarrow 2)\,,\\ \boldsymbol{O}_2^{(j)} &= d_{ade}d_{bce}\,\boldsymbol{T}_2^a\,\{\boldsymbol{T}_1^b,\boldsymbol{T}_1^c\}\,\boldsymbol{T}_j^d - (1\leftrightarrow 2)\,,\\ \boldsymbol{O}_3^{(j)} &= \boldsymbol{T}_2^a\,\{\boldsymbol{T}_1^a,\boldsymbol{T}_1^b\}\,\boldsymbol{T}_j^b - (1\leftrightarrow 2)\,,\\ \boldsymbol{O}_4^{(j)} &= 2C_1\,\boldsymbol{T}_2\cdot\boldsymbol{T}_j - 2C_2\,\boldsymbol{T}_1\cdot\boldsymbol{T}_j\,.\end{aligned}$$

$$J_j = \int\frac{d\Omega(n_k)}{4\pi}\left(W_{1j}^k - W_{2j}^k\right)\Theta_{\rm gap}(n_k)$$

TB, Neubert, Stillger, Shao, in preparation

Resummed result

Combine C_{rn} with μ integrals and carry out the sums.

Simplest case is $qq{\rightarrow}qq$ scattering with photon exchange

$J_i = \pm\Delta Y$

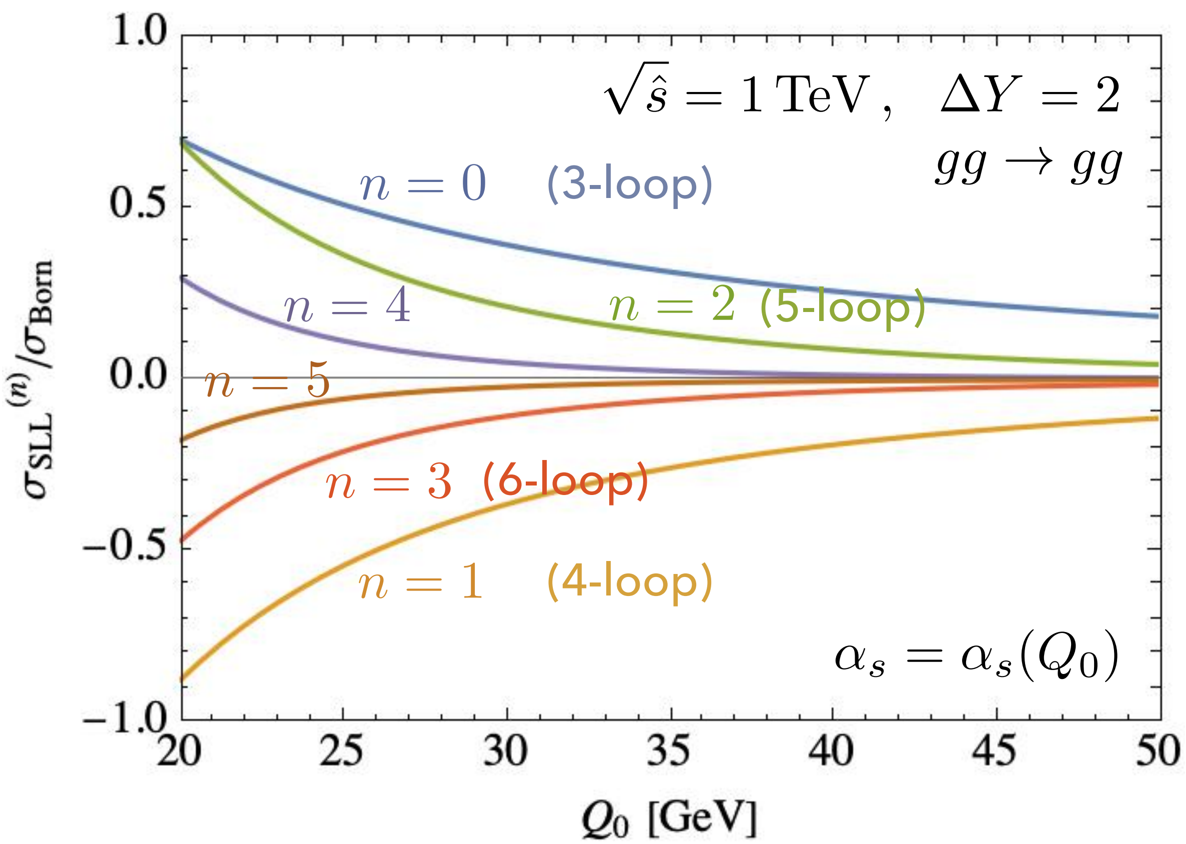
$$\boxed{\Delta\hat{\sigma}^{(S)} = -\hat{\sigma}_B \, \frac{4C_F}{3\pi} \, \alpha_s^3 \, L^3 \, \Delta Y \; {}_2F_2\!\left(1,1;2,\tfrac{5}{2};-w\right)}$$

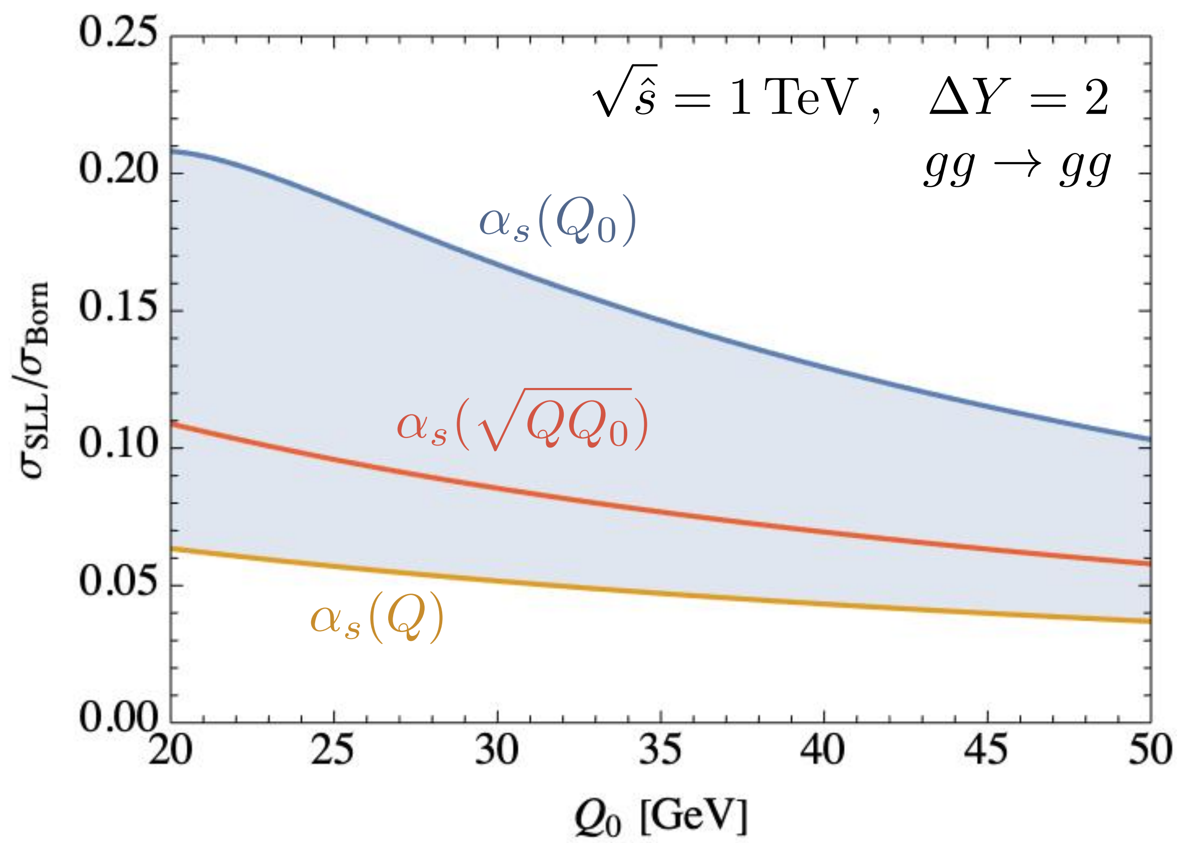
with $w = \dfrac{N_c \alpha_s}{\pi} L^2$.

$\sim \dfrac{\ln w}{w}$ for large w

Note: Standard Sudakov has form e^{-cw}.

Forward gluon-gluon scattering





- Slow convergence: necessary to include eight terms (10 loops!) to converge to resummed result
- *Very sensitive* to choice of μ in α_s: should include running!

Summary & Outlook

- Non-global logarithms
  - First subleading resummations (in large-N_c) become available
  - Next steps:
    - extension to hadronic collisions, more complicated observables
    - inclusion of finite-N_c effects
- Super-leading logarithms
  - Resummation of leading superleading logs now available
  - Next steps:
    - full phenomenological analysis
    - analysis of low-energy matrix elements, Glauber contributions
    - single logarithmic resummation? NGL × SLL?

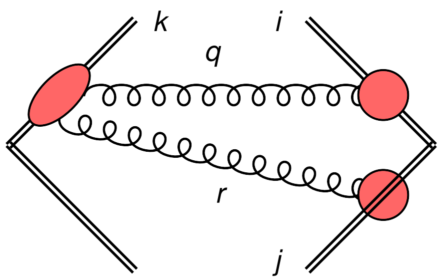
Extra slides

Angular functions

Caron-Huot ‘15

Three-leg correlations

$$K_{ijk;qr} = 8\left(W_{ik}^{q}W_{jk}^{r} - W_{ik}^{q}W_{jq}^{r} - W_{ir}^{q}W_{jk}^{r} + W_{ij}^{q}W_{jq}^{r}\right)\ln\left(\frac{n_{kq}}{n_{kr}}\right)$$



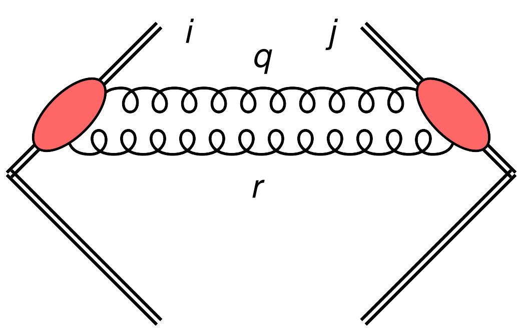
Two-leg correlations

$$K_{ij;qr} = C_A K_{ij;qr}^{(a)} + \left[n_F T_F - 2C_A\right] K_{ij;qr}^{(b)} + \left[C_A - 2n_F T_F + n_S T_S\right] K_{ij;qr}^{(c)}$$

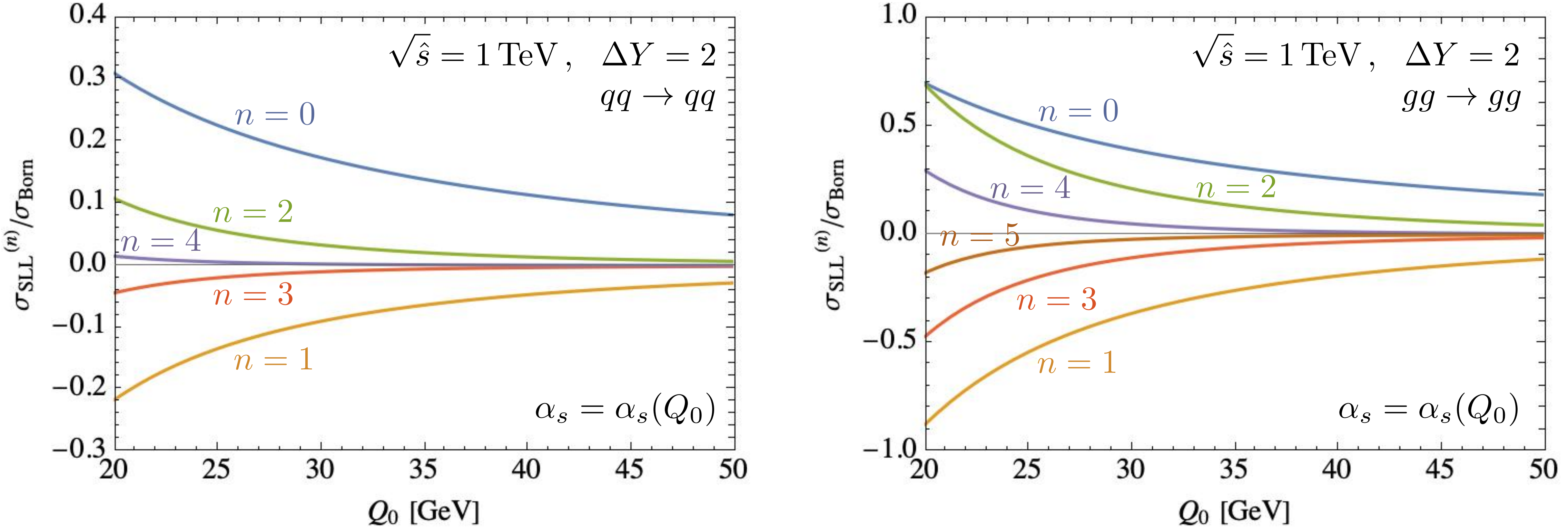
$$K_{ij;qr}^{(a)} = \frac{4n_{ij}}{n_{iq}n_{qr}n_{jr}}\left[1 + \frac{n_{ij}n_{qr}}{n_{iq}n_{jr} - n_{ir}n_{jq}}\right]\ln\frac{n_{iq}n_{jr}}{n_{ir}n_{jq}}\,,$$

$$K_{ij;qr}^{(b)} = \frac{8n_{ij}}{n_{qr}(n_{iq}n_{jr} - n_{ir}n_{jq})}\ln\frac{n_{iq}n_{jr}}{n_{ir}n_{jq}}\,,$$

$$K_{ij;qr}^{(c)} = \frac{4}{n_{qr}^2}\left(\frac{n_{iq}n_{jr} + n_{ir}n_{jq}}{n_{iq}n_{jr} - n_{ir}n_{jq}}\ln\frac{n_{iq}n_{jr}}{n_{ir}n_{jq}} - 2\right).$$



Terms of order $\alpha_s^{n+3} L^{2n+3}$



- Hard function for gluon exchange in t-channel.
- *n*=0 term is not SLL, but missing in large N_c limit.