EFT Matching with Functional Methods

Anders Eller Thomsen

Based on work with J. Fuentes-Martín, M. König, J. Pagès, and F. Wilsch

UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

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Matching EFTs

The why and the how of it

EFT matching

EFTs are used to interpret experiments and quantify observations

$$\mathcal{L}_{\text{EFT}}(\eta_{\text{L}}) = \mathcal{L}^{d=4}(\eta_{\text{L}}) + \sum_{n=5}^{\infty} \frac{C_{n,i}}{\Lambda^{n-4}} \mathcal{O}_{n,i}(\eta_{\text{L}}) \quad \longrightarrow \quad \text{UV physics}$$

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NP models have to be analyzed one by one

$$\mathcal{L}_{\text{UV}}(\eta_{\text{H}},\, \eta_{\text{L}}) \quad \xrightarrow{\text{Matching}} \quad \mathcal{L}_{\text{EFT}}(\eta_{\text{L}})$$

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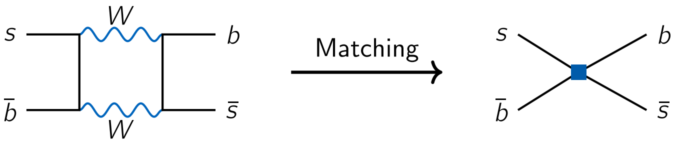
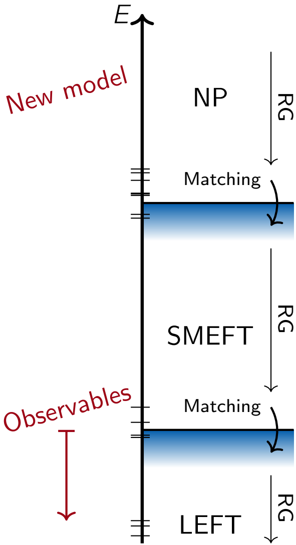
Loop-level matching is required for many processes, e.g., in the SM



EFT workflow



Jenkins, Manohar, Trott [1308.2627]
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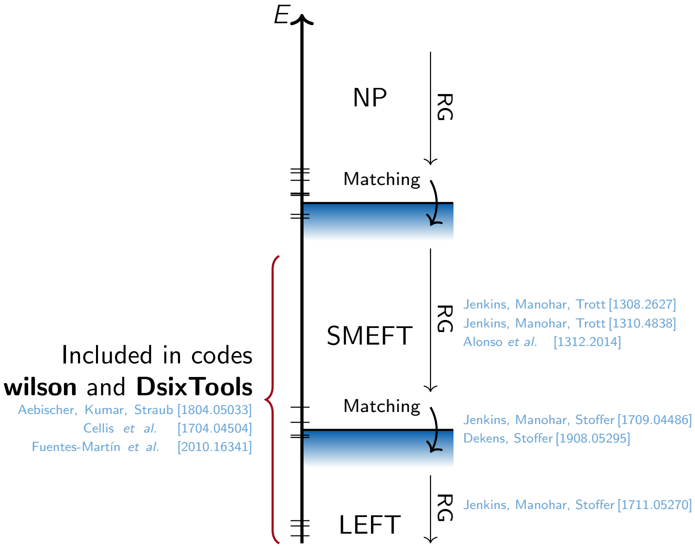

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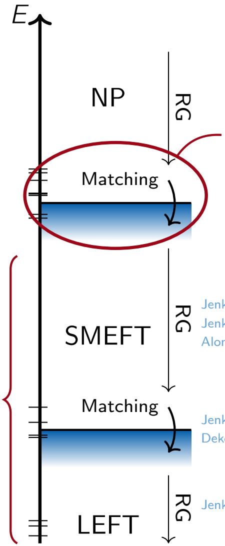

Recent progress: **Matchmakereft** and **Matchete**

Criado [1710.06445]; Das Bakshi *et al.* [1808.04403] Cohen, Lu, Zhang [2012.07851]; Carmona *et al.* [2112.10787]; Fuentes-Martín, König, Pagès, AET, Wilsch [2212.04510]

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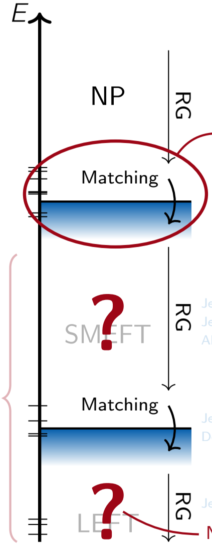

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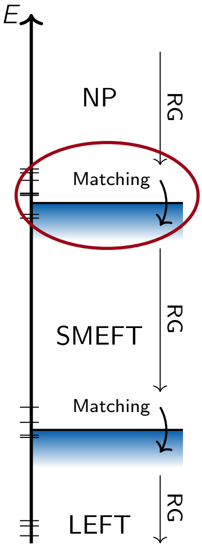
New light states?

Matching weakly coupled theories

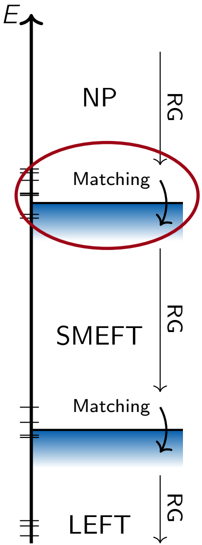


$\mathcal{L}_{\text{EFT}}$ should reproduce the physics of $\mathcal{L}_{\text{UV}}$ at energies $E \ll \Lambda$:

$$\mathcal{L}_{\text{UV}}(\eta_{\text{H}}, \eta_{\text{L}}) \xrightarrow{\text{Matching}} \mathcal{L}_{\text{EFT}}(\eta_{\text{L}})$$

$$\mathcal{L}_{\text{EFT}}(\eta_{\text{L}}) = \mathcal{L}_{\text{kin}}(\eta_{\text{L}}) + \sum_{n=2}^{\infty} \sum_{\ell=0}^{\infty} \frac{C_{n,i}^{(\ell)}}{(4\pi)^{2\ell}\, \Lambda^{n-4}} \mathcal{O}_{n,i}^{(\ell)}(\eta_{\text{L}})$$

Matching weakly coupled theories



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$$\begin{array}{ccc} \mathcal{L}_{\text{UV}}(\eta_{\text{H}}, \eta_{\text{L}}) & \xrightarrow{\text{Matching}} & \mathcal{L}_{\text{EFT}}(\eta_{\text{L}}) \\ \downarrow & & \downarrow \\ \Gamma_{\text{UV}}\big[\hat{\eta}_{\text{H}}(\eta_{\text{L}}), \eta_{\text{L}}\big] & \simeq & \Gamma_{\text{EFT}}[\eta_{\text{L}}] \end{array}$$

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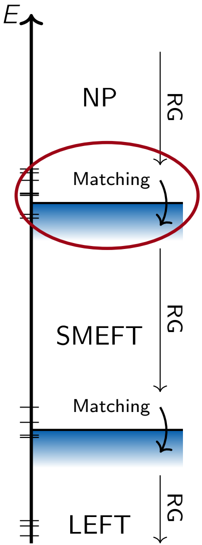

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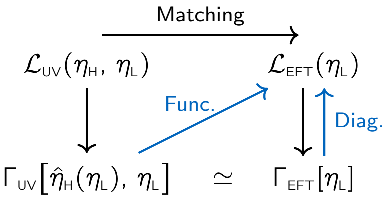

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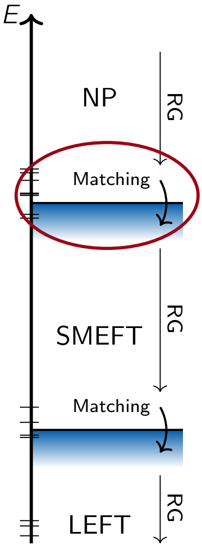

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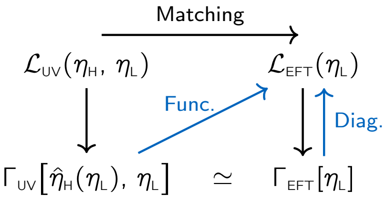

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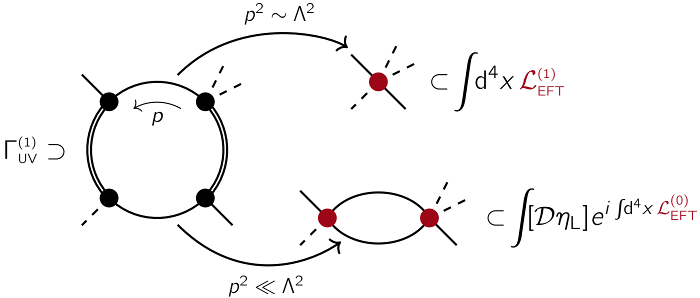
Advantages of functional matching:

- Does not require knowledge of EFT basis
- Well-suited for algorithmic approach
- Computations are manifestly gauge covariant

Mixed (heavy–light) loop example:



Separation of scales

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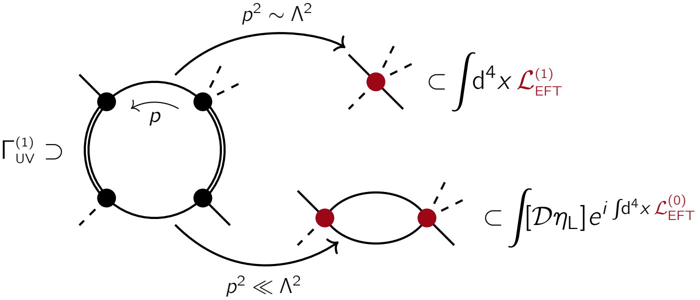

Expansion by regions allows for separating scales in dimensional regularization:

Beneke, Smirnov [hep-ph/9711391]; Jantzen [1111.2589]

$$\Gamma_{\mathrm{UV}}^{(1)} = \Gamma_{\mathrm{UV}}^{(1)}\Big|_{\mathrm{hard}} + \Gamma_{\mathrm{UV}}^{(1)}\Big|_{\mathrm{soft}}$$

Heavy field loops → $\Gamma_{\mathrm{UV}}^{(1)}\big|_{\mathrm{hard}}$

Mixed field loops → $\Gamma_{\mathrm{UV}}^{(1)}\big|_{\mathrm{hard}}$, $\Gamma_{\mathrm{UV}}^{(1)}\big|_{\mathrm{soft}}$

Light field loops → $\Gamma_{\mathrm{UV}}^{(1)}\big|_{\mathrm{soft}}$

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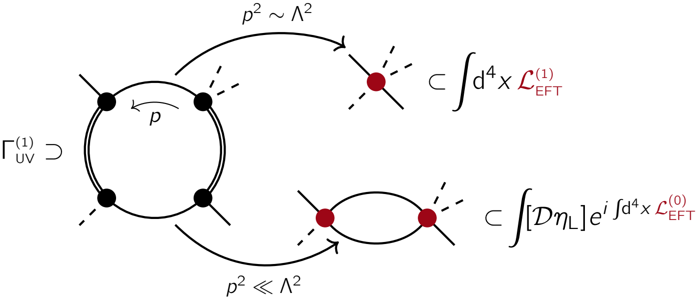

- $\Gamma^{(1)}_{\mathrm{UV}}\big|_{\mathrm{soft}}$: long-distance contributions included in 1-loop matrix elements of tree-level EFT operators

$$\Gamma^{(1)}_{\mathrm{UV}}\big|_{\mathrm{soft}} = \Gamma^{(1)}_{\mathrm{EFT}}$$

- $\Gamma^{(1)}_{\mathrm{UV}}\big|_{\mathrm{hard}}$: short-distance contributions going into the EFT operators

Fuentes *et al.* [1607.02142]

$$\int \mathrm{d}^d x\, \mathcal{L}^{(1)}_{\mathrm{EFT}} = \Gamma^{(1)}_{\mathrm{UV}}\big|_{\mathrm{hard}}$$

Functional matching (abridged)

The theory is expanded around the classical fields, $\hat{\eta}$:

$$\mathcal{L}_{\text{UV}}[\eta+\hat{\eta}] = \underbrace{\mathcal{L}_{\text{UV}}[\hat{\eta}]}_{\text{classical piece}} \;+\; \eta_i \underbrace{\frac{\delta \mathcal{L}_{\text{UV}}}{\delta \eta_i}[\hat{\eta}]}_{\text{EOM}\,\to\,0} \;+\; \tfrac{1}{2}\eta_i\eta_j \underbrace{\frac{\delta^2 \mathcal{L}_{\text{UV}}}{\delta\eta_i\,\delta\eta_j}[\hat{\eta}]}_{\text{fluctuation operator } \mathcal{Q}_{ij}[\hat{\eta}]} + \ldots$$

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By saddle point approximation, the effective action is

$$e^{i\,\Gamma_{\text{UV}}[\hat{\eta}]} = e^{iS_{\text{UV}}[\hat{\eta}]} \int \mathcal{D}\eta \; \exp\left(i \int \mathrm{d}^d x \; \tfrac{1}{2} \eta_i \mathcal{Q}_{ij}[\hat{\eta}] \eta_j + \ldots \right)$$

$$\implies \; \Gamma_{\text{UV}}[\hat{\eta}] = S_{\text{UV}}[\hat{\eta}] + \frac{i}{2} \mathrm{STr} \log \mathcal{Q}[\hat{\eta}] + \ldots$$

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In matching the STr can be expanded around the heavy scale Λ

$$\mathcal{Q} = \Delta^{-1}(P,\, M) - X(P,\, \hat{\eta}), \qquad \Lambda^2 \sim \Delta^{-1} \gg X$$

The master formula for 1-loop matching: Cohen, Lu, Zhang [2011.02484]

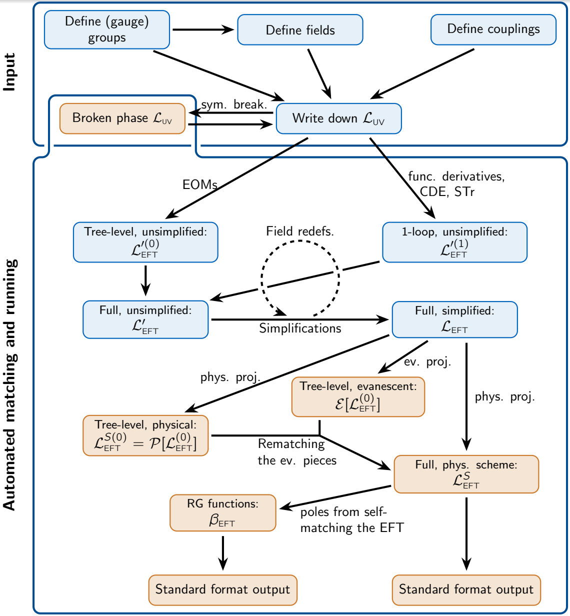
$$\int \mathrm{d}^d x \, \mathcal{L}^{(1)}_{\text{EFT}} = \frac{i}{2} \mathrm{STr}\, \ln \Delta^{-1}\Big|_{\text{hard}} - \frac{i}{2}\sum_{n=1}^{\infty} \frac{1}{n} \mathrm{STr}\big[(\Delta X)^n\big]\Big|_{\text{hard}}$$

Loop integrals evaluated covariantly with CDE

MATCHETE

To make your way through the BSM jungle

Automated EFT matching



Fuentes-Martín, König, Pagès, AET, Wilsch [2212.04510]

- **Matchete v0.1** is a Mathematica package
- Matching of *any* model with heavy scalars/fermions
- Simple and intuitive input/output
- Handles all group theory
- Simplifies to EFT basis*

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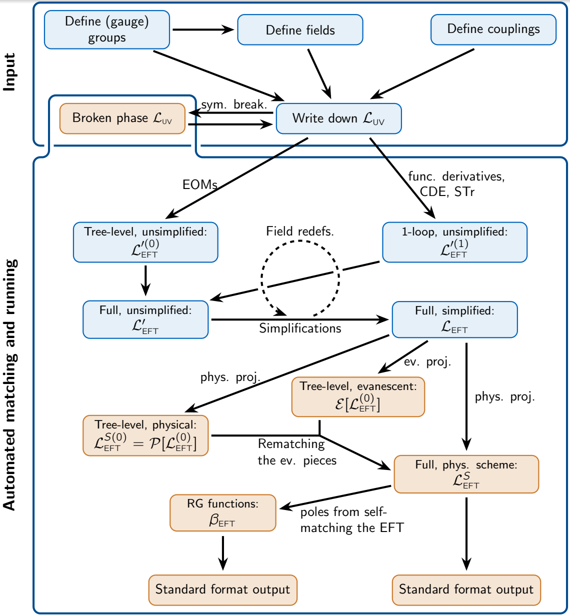

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Future plans:

- Handling of evanescent contribution
- Symmetry breaking and heavy vectors
- Interface with EFT tool chain
- 1-loop RG computations

Simplification and basis reduction

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2} m^2 \phi^2 - \frac{\lambda}{24}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3 \partial^2 \phi + \frac{C_3}{\Lambda^2}\phi^2 (\partial_\mu \phi)^2$$

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Exact simplification (linear):

IBP, Dirac identities, group identities, commutation relations...

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On-shell equivalence (non-linear):

Field redefinition: $\phi \longrightarrow \phi + \dfrac{3C_2 - C_3}{3\Lambda^2}\phi^3$

$$\mathcal{L} \longrightarrow \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2} m^2 \phi^2 - \left(\frac{\lambda}{24} + \frac{(3C_2 - C_3)m^2}{3\Lambda^2}\right)\phi^4 + \frac{18C_1 - \lambda(3C_2 - C_3)}{18\Lambda^2}\phi^6$$

Matchete contains routines performing both kinds of simplifications

Example: SM + Vector-like lepton

Setup

SM Lagrangian

In[3]:=
```
LSM = LoadModel["SM"];
```

Define new field

In[4]:=
```
DefineField[EE, Fermion, Charges → {U1Y[-1]}, Mass → {Heavy, ME}]
```

Define new coupling

In[5]:=
```
DefineCoupling[yE, EFTOrder → 0, Indices → {Flavor}]
```

Write interactions

In[6]:=
```
Lint = -yE[p] × Bar@l[i, p] ** PR ** EE[] × H[i] // PlusHc;
Lint // NiceForm
```

Out[7]//NiceForm=

$$-\overline{yE}^{p}\,\bar{H}_{i}\,\left(\overline{EE}\cdot P_{L}\cdot l^{ip}\right)-yE^{p}\,H^{i}\,\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot EE\right)$$

Define full UV Lagrangian

In[8]:=
```
LUV = LSM + FreeLag[EE] + Lint;
LUV // NiceForm
```

Out[9]//NiceForm=

$$-\frac{1}{4}B^{\mu\nu 2}-\frac{1}{4}G^{\mu\nu A2}-\frac{1}{4}W^{\mu\nu I2}+D_{\mu}\bar{H}_{i}\,D_{\mu}H^{i}+\mu^{2}\,\bar{H}_{i}\,H^{i}+i\,\left(\bar{d}^{p}_{a}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}d^{ap}\right)+i\,\left(\bar{e}^{p}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}e^{p}\right)+$$
$$i\,\left(\overline{EE}\cdot\gamma_{\mu}\cdot D_{\mu}EE\right)-ME\,\left(\overline{EE}\cdot EE\right)+i\,\left(\bar{l}^{p}_{i}\cdot\gamma_{\mu}\,P_{L}\cdot D_{\mu}l^{ip}\right)+i\,\left(\bar{q}^{p}_{ai}\cdot\gamma_{\mu}\,P_{L}\cdot D_{\mu}q^{aip}\right)+i\,\left(\bar{u}^{p}_{a}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}u^{ap}\right)-$$
$$\frac{1}{2}\lambda\,\bar{H}_{i}\,\bar{H}_{j}\,H^{i}\,H^{j}-\overline{Yd}^{pr}\,\bar{H}_{i}\,\left(\bar{d}^{r}_{a}\cdot P_{L}\cdot q^{aip}\right)-\overline{Ye}^{pr}\,\bar{H}_{i}\,\left(\bar{e}^{r}\cdot P_{L}\cdot l^{ip}\right)-Ye^{pr}\,H^{i}\,\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot e^{r}\right)-Yd^{pr}\,H^{i}\,\left(\bar{q}^{p}_{ai}\cdot P_{R}\cdot d^{ar}\right)-$$
$$Yu^{pr}\,\bar{H}_{i}\,\left(\bar{q}^{p}_{aj}\cdot P_{R}\cdot u^{ar}\right)\varepsilon^{ji}-\overline{Yu}^{pr}\,H^{j}\,\left(\bar{u}^{r}_{a}\cdot P_{L}\cdot q^{aip}\right)\bar{\varepsilon}_{ij}-\overline{yE}^{p}\,\bar{H}_{i}\,\left(\overline{EE}\cdot P_{L}\cdot l^{ip}\right)-yE^{p}\,H^{i}\,\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot EE\right)$$

Example: SM + Vector-like lepton

Matching

In[10]:=
```
LEFT = Match[LUV, LoopOrder → 1, EFTOrder → 6] /. ϵ^-1 → 0;
```

In[11]:=
```
LEFTOnShell = LEFT // EOMSimplify;
Length@%
```

EOMSimplify: The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these terms to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.

» Added new CG cg1 with indices {Bar[SU2L[fund]], SU2L[adj], Bar[SU2L[fund]]}

Out[12]= 67

In[13]:=
```
SelectOperatorClass[LEFTOnShell, {e, Bar@e, H, Bar@H}, 1] // GreensSimplify // NiceForm
```

Out[13]//NiceForm=

$$\frac{i}{360}\hbar\frac{1}{\mathrm{ME}^2}\left(48\,\mathrm{gY}^4\,\delta^{pr}+5\,\overline{\mathrm{yE}}^{s}\left(3\,\mathrm{yE}^{t}\,\overline{\mathrm{Ye}}^{tr}\,\mathrm{Ye}^{sp}\left(1+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{\mathrm{ME}^2}\right]\right)-2\,\mathrm{yE}^{s}\,\mathrm{gY}^2\left(13+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{\mathrm{ME}^2}\right]\right)\delta^{pr}\right)\right)$$
$$\left(-D_\mu\bar{H}_i\,H^i\left(\bar{e}^r\cdot\gamma_\mu P_R\cdot e^p\right)+\bar{H}_i\,D_\mu H^i\left(\bar{e}^r\cdot\gamma_\mu P_R\cdot e^p\right)\right)$$

Example: SM + Vector-like lepton

```
LEFTOnShell // NiceForm
```

liceForm=

$-\frac{1}{4} G^{\mu\nu A 2} - \frac{1}{4} W^{\mu\nu I 2} + \left(-\frac{1}{4} - \frac{1}{3}\hbar\, gY^2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\right) B^{\mu\nu 2} + D_\mu \bar{H}_i\, D_\mu H^i +$

$\Big(cHH + \frac{1}{6}\hbar\, \overline{yE}^p\, yE^p\, cHH\, \frac{1}{ME^2} \Big(2\, cHH - 3\, ME^2 \Big(1 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\, \bar{H}_i H^i + i\, (\bar{d}^r_a \cdot \gamma_\mu P_R \cdot D_\mu d^{ap})\, \delta^{pr} +$

$i\, (\bar{e}^r \cdot \gamma_\mu P_R \cdot D_\mu e^p)\, \delta^{pr} + i\, (\bar{l}^r_i \cdot \gamma_\mu P_L \cdot D_\mu l^{ip})\, \delta^{pr} + i\, (\bar{q}^r_{ai} \cdot \gamma_\mu P_L \cdot D_\mu q^{aip})\, \delta^{pr} + i\, (\bar{u}^r_a \cdot \gamma_\mu P_R \cdot D_\mu u^{ap})\, \delta^{pr} +$

$\Big(-\frac{1}{2}\lambda + \hbar \Big(-\frac{1}{2}\overline{yE}^p \Big(4\, yE^r\, \overline{Ye}^{rs}\, Ye^{ps} \Big(1 + \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big) - yE^p \Big(-2\, \overline{yE}^r\, yE^r\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right] + \lambda \Big(1 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big) -$

$\quad \frac{1}{180}\, cHH\, \frac{1}{ME^2} \Big(12\, gY^4 - 5\, \overline{yE}^p\, yE^p\, gY^2 \Big(13 + 6\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big) +$

$\quad\quad 5\, \overline{yE}^p \Big(-12\, (\overline{yE}^r\, yE^p\, yE^r + 6\, yE^r\, \overline{Ye}^{rs}\, Ye^{ps} - 2\, yE^p \lambda) + yE^p\, gL^2 \Big(5 + 6\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\Big)\Big)\, \bar{H}_i \bar{H}_j H^i H^j +$

$\Big(-\overline{Yd}^{pr} + \frac{1}{12}\hbar\, \overline{yE}^s\, yE^s\, \overline{Yd}^{pr}\, \frac{1}{ME^2} \Big(-4\, cHH + 3\, ME^2 \Big(1 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\, \bar{H}_i\, (\bar{d}^r_a \cdot P_L \cdot q^{aip}) +$

$\Big(-\overline{Ye}^{pr} + \frac{1}{24}\hbar\, yE^s\, \frac{1}{ME^2} \Big(-3\, \overline{yE}^p\, \overline{Ye}^{sr}\, (2\, cHH - ME^2) \Big(3 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big) + 2\, \overline{yE}^s\, \overline{Ye}^{pr} \Big(-4\, cHH + 3\, ME^2 \Big(1 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\Big)$

$\bar{H}_i\, (\bar{e}^r \cdot P_L \cdot l^{ip}) +$

$\Big(-Ye^{rp} + \frac{1}{24}\hbar\, \overline{yE}^s\, \frac{1}{ME^2} \Big(3\, ME^2 \Big(2\, yE^s\, Ye^{rp} \Big(1 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big) + yE^r\, Ye^{sp} \Big(3 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big) -$

$\quad 2\, cHH \Big(4\, yE^s\, Ye^{rp} + 3\, yE^r\, Ye^{sp} \Big(3 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\Big)\, H^i\, (\bar{l}^r_i \cdot P_R \cdot e^p) +$

$\Big(-Yd^{rp} + \frac{1}{12}\hbar\, \overline{yE}^s\, yE^s\, Yd^{rp}\, \frac{1}{ME^2} \Big(-4\, cHH + 3\, ME^2 \Big(1 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\, H^i\, (\bar{q}^r_{ai} \cdot P_R \cdot d^{ap}) +$

$\Big(-Yu^{rp} + \frac{1}{12}\hbar\, \overline{yE}^s\, yE^s\, Yu^{rp}\, \frac{1}{ME^2} \Big(-4\, cHH + 3\, ME^2 \Big(1 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\, \bar{H}_i\, (\bar{q}^r_{aj} \cdot P_R \cdot u^{ap})\, \varepsilon^{ji} +$

$\Big(-\overline{Yu}^{pr} + \frac{1}{12}\hbar\, \overline{yE}^s\, yE^s\, \overline{Yu}^{pr}\, \frac{1}{ME^2} \Big(-4\, cHH + 3\, ME^2 \Big(1 + 2\, \text{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\, H^j\, (\bar{u}^r_a \cdot P_L \cdot q^{aip})\, \bar{\varepsilon}_{ij} +$

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Evanescent Operators

Why can't QFT just play nice?

EFT from a 2HDM

Example: take SM + leptophilic Higgs, $\Phi \sim (\mathbf{1}, \mathbf{2})_{1/2}$:

$$\mathcal{L} \supset \mathcal{L}_{\text{SM}} + D_\mu \Phi^\dagger D^\mu \Phi - M_\Phi^2 \, \Phi^\dagger \Phi - \left(y_{\Phi e}^{pr} \, \bar{\ell}_p \Phi e_r + \text{h.c.} \right) + \ldots$$

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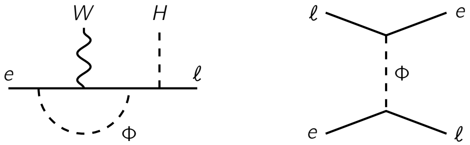

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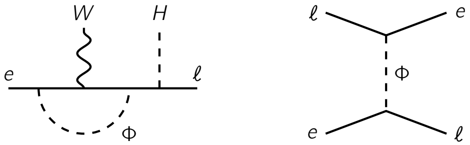

Below the scale $M_\Phi \gg v_{\text{EW}}$

$$\mathcal{L}_{\text{EFT}} \supset C_{eW}^{pr} Q_{eW}^{pr} + C_{\ell e}^{prst} R_{\ell e}^{prst}$$

But the tree-level operator $R_{\ell e}$ is not part of the Warsaw basis

Changing basis in an EFT

In 4D, $\mathcal{L}_{\text{EFT}} = \widetilde{\mathcal{L}}_{\text{EFT}}$, where

$$\mathcal{L}_{\text{EFT}} \supset C_{eW}^{pr} Q_{eW}^{pr} + C_{\ell e}^{prst} R_{\ell e}^{prst} \qquad R_{\ell e}^{prst} = (\bar{\ell}_p e_r)(\bar{e}_s \ell_t)$$

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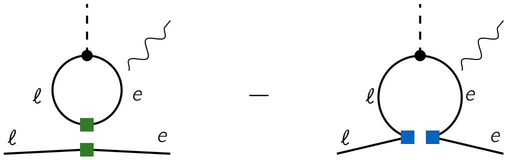
But the 1-loop EFT amplitudes are different!

$$i\big(\mathcal{A}_{eH\to\ell W} - \widetilde{\mathcal{A}}_{eH\to\ell W}\big) = \frac{g_2}{64\pi^2} [C_{\ell e}]^{prst} y_e^{ts} \big(\bar{u}\tau^I \sigma_{\mu\nu} P_R u\big) q^\mu \varepsilon^{*\nu}$$



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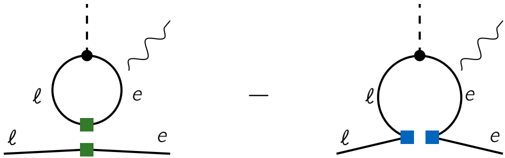

In $d \neq 4$, there is an *evanescent operator:*

$$R_{\ell e}^{prst} = -\tfrac{1}{2} Q_{\ell e}^{ptsr} + E_{\ell e}^{prst}, \qquad E_{\ell e}^{prst} \xrightarrow{d\to 4} 0$$

Evanescent operators

An *evanescent operator* E is an operator satisfying

$$E = \text{rank}(\epsilon) \xrightarrow{d\to4} 0$$

Evanescent contributions have long been accounted for in the LEFT (Weak Effective Hamiltonian). Not so much in BSM context

Buras, Weisz '90; Dugan, Grinstein '91; Herrlich, Nierste [hep-ph/9412375];...

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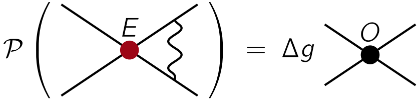
The physical contributions from evanescent operators are *finite and local*



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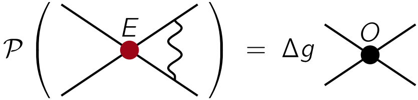

e.g., in the 2HDM example

$$E_{\ell e}^{prst} \longrightarrow -\frac{g_L y_e^{ts}}{128\pi^2} Q_{eW}^{pr} + [\text{many other contributions}]$$

The physical projector

Reduction of Dirac structures for 4-fermion operators, e.g.,

Compatibility with NDR

$$(\gamma^{\mu}\gamma^{\nu}\gamma^{\lambda}P_{\mathrm{L}})\otimes[\gamma_{\lambda}\gamma_{\nu}\gamma_{\mu}P_{\mathrm{L}}] = 4(1-2\epsilon)\,(\gamma^{\mu}P_{\mathrm{L}})\otimes[\gamma_{\mu}P_{\mathrm{L}}] + E^{[3]}_{LL}$$

Fierz identities for 4-fermion operators, e.g.,

$$(P_{\mathrm{R}})\otimes[P_{\mathrm{L}}] = -\tfrac{1}{2}(\gamma_{\mu}P_{\mathrm{L}}]\otimes[\gamma_{\mu}P_{\mathrm{R}}) + E_{\mathrm{Fierz}}(P_{\mathrm{R}}, P_{\mathrm{L}})$$

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Choosing a set of identities allows for defining the *physical projector* $\mathcal{P}$:

$$O_d = \underbrace{\mathcal{P}\, O_d}_{\text{phys. part}} + \underbrace{\mathcal{E}_\mathcal{P}\, O_d}_{\text{ev. part}}, \qquad \mathcal{E}_\mathcal{P} = \mathrm{id} - \mathcal{P}$$

Evanescence-free schemes

For an EFT Lagrangian $\mathcal{L} = \bar{g}_a O^a + \bar{\eta}_i E^i$, the 1-loop effective action is

$$\Gamma = \int_x \left(\bar{g}_a O^a + \bar{\eta}_i E^i \right) + \overline{\Gamma}(g, \eta).$$

1-loop diagrams, tree-level couplings

bare couplings

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1-loop diagrams, tree-level couplings

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Scheme	$\overline{\text{MS}}$
Action $\mathcal{P} : O^a$	$\bar{g}_a = g_a + \delta g_a$
Action $\mathcal{E}_{\mathcal{P}} : E^i$	$\bar{\eta}_i = \eta_i + \delta \eta_i$
Eff. action $\mathcal{P}\Gamma$	$\int_x \bar{g}_a O^a + \mathcal{P}\overline{\Gamma}(g, \eta)$

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Scheme		$\overline{\text{MS}}$	Compensated
Action	$\mathcal{P} : O^a$	$\bar{g}_a = g_a + \delta g_a$	$(\bar{g}_a + \Delta g_a) - \Delta g_a$
	$\mathcal{E}_{\mathcal{P}} : E^i$	$\bar{\eta}_i = \eta_i + \delta \eta_i$	$\delta\eta_i + \eta_i$
Eff. action $\mathcal{P}\Gamma$		$\int_x \bar{g}_a O^a + \mathcal{P}\overline{\Gamma}(g, \eta)$	$\int_x (\bar{g}_a + \Delta g_a) O^a + \mathcal{P}\overline{\Gamma}(g, \eta) - \int_x \Delta g_a O^a$

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(bare couplings: $\bar{g}_a$, $\bar{\eta}_i$; 1-loop diagrams, tree-level couplings: $\overline{\Gamma}(g,\eta)$)

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The evanescent contribution is defined by

$$\int_x \Delta g_a O^a \equiv \mathcal{P}\left[\overline{\Gamma}(g, \eta) - \overline{\Gamma}(g, 0) \right]$$

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1-loop diagrams, tree-level couplings (annotation on $\overline{\Gamma}(g,\eta)$)

bare couplings

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1-loop diagrams, tree-level couplings

bare couplings

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1-loop diagrams, tree-level couplings (annotating $\overline{\Gamma}(g, \eta)$); bare couplings (annotating $\bar{g}_a$)

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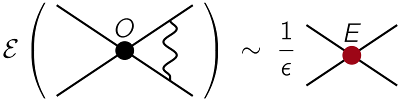
Handling evanescent contributions means computing Δg

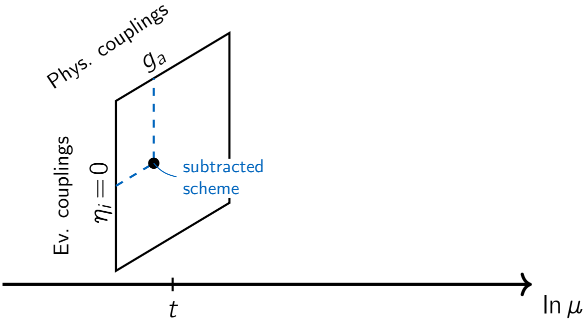
RG in evanescent schemes



$$\implies \delta\eta(g) \neq 0$$

RG in evanescent schemes

$$\mathcal{E}\left(\text{[diagram: } O \text{ vertex with gauge exchange]} \right) \sim \frac{1}{\epsilon} \text{ [diagram: } E \text{ vertex]} \implies \delta\eta(g) \neq 0$$



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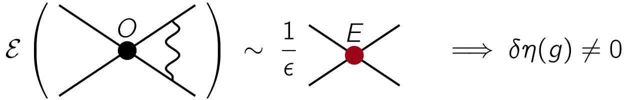

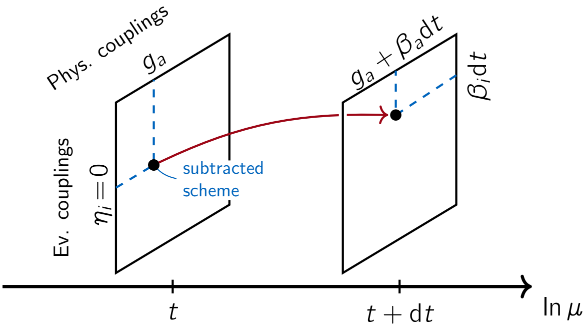

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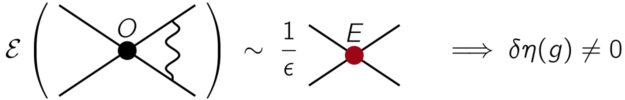
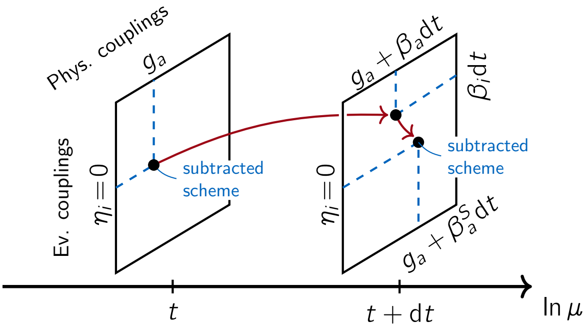
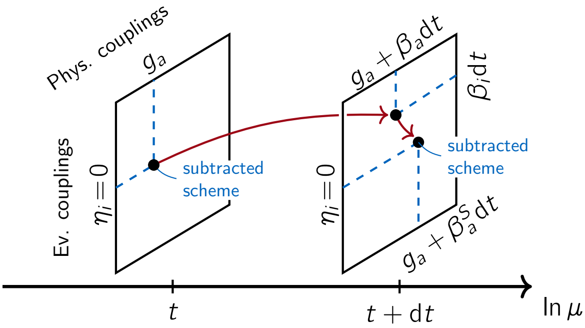




RG in evanescent schemes

$$\mathcal{E}\left(\text{[diagram: } O \text{ vertex with loop]} \right) \sim \frac{1}{\epsilon}\, \text{[diagram: } E \text{ vertex]} \quad \Longrightarrow \quad \delta\eta(g) \neq 0$$



In the subtracted evanescent scheme

$$\frac{\mathrm{d}g_a}{\mathrm{d}t} = \beta_a^S = \beta_a + \overbrace{\beta_i \left. \frac{\partial \Delta g_a}{\partial \eta_i} \right|_{\eta=0}}^{\text{2-loop}}$$

Application in the SMEFT

Tree-level BSM matching to the SMEFT can produce 49 different, redundant four-fermion operators, which will result in non-trivial evanescent contribution at 1-loop order, e.g.,

$$R_{\ell e} = (\bar{\ell} e)(\bar{e} \ell) \qquad R_{qu}^{(8)} = (\bar{q} T^A u)(\bar{u} T^a q) \qquad R_{u^c e l q^c} = (\bar{u}^c e)(\bar{l} q^c)$$

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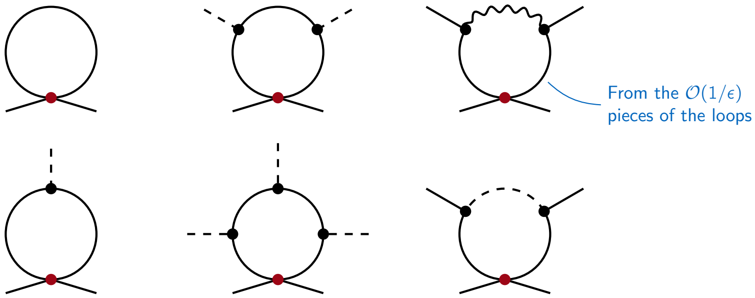
$$R_{\ell e} = (\bar{\ell} e)(\bar{e} \ell) \qquad R_{qu}^{(8)} = (\bar{q} T^A u)(\bar{u} T^a q) \qquad R_{u^c e l q^c} = (\bar{u}^c e)(\bar{l} q^c)$$

For dimension-6 SMEFT, evanescent operators contribute through 6 covariant trace topologies



Fuentes-Martín, König, Pagès, AET, Wilsch [2211.09144]

Matematica interface

Filter: Redundant | SMEFT | All

$R_{\ell e}^{prst}$ $R_{\ell u}^{prst}$ $R_{\ell d}^{prst}$ R_{qe}^{prst} $R_{qu}^{(1)prst}$ $R_{qu}^{(8)prst}$ $R_{qd}^{(1)prst}$ $R_{qd}^{(8)prst}$ $R_{\ell uqe}^{prst}$ $R_{\ell^c\ell}^{prst}$ $R_{q^cq}^{prst}$ $R_{q^cq}^{\prime prst}$ $R_{q^c\ell}^{prst}$ $R_{q^c\ell}^{\prime prst}$ $R_{e^ce}^{prst}$ $R_{u^cu}^{prst}$ $R_{d^cd}^{prst}$

$R_{e^cu}^{prst}$ $R_{e^cd}^{prst}$ $R_{u^cd}^{prst}$ $R_{u^cd}^{\prime prst}$ $R_{u^cdqq^c}^{prst}$ $R_{u^ce\ell q^c}^{prst}$ $R_{q^cqq^c\ell}^{prst}$ $R_{u^cud^ce}^{prst}$ $R_{\ell\ell}^{(3)prst}$ $R_{qq}^{(1,8)prst}$ $R_{qq}^{(3,8)prst}$ $R_{\ell q}^{(1)prst}$ $R_{\ell q}^{(3)prst}$ $R_{uu}^{(8)prst}$ $R_{dd}^{(8)prst}$

R_{eu}^{prst} R_{ed}^{prst} $R_{ud}^{(1)prst}$ $R_{ud}^{(8)prst}$ $R_{\ell qde}^{prst}$ $R_{\ell^ce}^{prst}$ $R_{\ell^cu}^{prst}$ $R_{\ell^cd}^{prst}$ $R_{q^ced\ell^c}^{prst}$ $R_{q^ce}^{prst}$ $R_{q^cu}^{prst}$ $R_{q^cu}^{\prime prst}$ $R_{q^cd}^{prst}$ $R_{q^cd}^{\prime prst}$ $R_{d^c\ell q^cu}^{prst}$ $R_{u^c\ell q^cd}^{prst}$ $R_{q^ceu^cq}^{prst}$

Operator definition:

$$R_{\ell qde}^{prst} = (\bar{\ell}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu e_t)$$

Reduces to:

$Q_{\ell edq}^{prst}$, $Q_{quqd}^{(1)prst}$, Q_{dW}^{pr}, Q_{dB}^{pr}, Q_{dH}^{pr}, Q_{yd}^{pr}, Q_{eB}^{pr}, Q_{ed}^{prst}, Q_{eH}^{pr}, Q_{eW}^{pr}, $Q_{\ell d}^{prst}$, $Q_{\ell e}^{prst}$, $Q_{\ell equ}^{(1)prst}$, $Q_{\ell equ}^{(3)prst}$, $Q_{\ell q}^{(1)prst}$, $Q_{\ell q}^{(3)prst}$, $Q_{qd}^{(1)prst}$, $Q_{qd}^{(8)prst}$, Q_{qe}^{prst}, Q_{ye}^{pr}

Reduction Identity:

$$\begin{aligned}
R_{\ell qde}^{prst} = &-2Q_{\ell edq}^{ptsr} + \frac{1}{16\pi^2}\Big(\frac{1}{6}\overline{y_e^{pt}}\, y_d^{uv} Q_{qd}^{(1)ursv} + \frac{1}{4} g_Y y_d^{rs} Q_{eB}^{pt} \\
&+ \frac{3}{4} g_Y \overline{y_e^{pt}}\, \overline{Q_{dB}^{rs}} + Q_{eH}^{pt}\Big(6\overline{y_d^{uv}}\, y_d^{rv} y_d^{us} - 3\lambda y_d^{rs}\Big) \\
&+ Q_{\ell equ}^{(1)ptuv}\Big(\frac{3}{4} y_d^{us} y_u^{rv} + 3 y_d^{rs} y_u^{uv}\Big) + \overline{y_e^{pt}}\, y_d^{uv} Q_{qd}^{(8)ursv} \\
&+ \frac{3}{2}\overline{y_e^{uv}}\, y_d^{rs} Q_{\ell e}^{puvt} + 2\overline{y_e^{pu}}\, \overline{y_e^{vt}}\, y_e^{vu} \overline{Q_{dH}^{rs}} - \frac{1}{16} y_d^{us} y_u^{rv} Q_{\ell equ}^{(3)ptuv} \\
&- \frac{1}{4} g_L \overline{y_e^{pt}}\, \overline{Q_{dW}^{rs}} - \frac{1}{4}\overline{y_e^{ut}}\, y_d^{vs} Q_{\ell q}^{(1)puvr} - \frac{1}{4}\overline{y_e^{ut}}\, y_d^{vs} Q_{\ell q}^{(3)puvr} \\
&- \frac{1}{2}\overline{y_e^{ut}}\, y_d^{rv} Q_{\ell d}^{pusv} - \frac{1}{2}\overline{y_e^{pu}}\, y_d^{vs} Q_{qe}^{vrut} - \frac{3}{4} g_L y_d^{rs} Q_{eW}^{pt} \\
&- \overline{y_e^{pt}}\, \overline{y_u^{uv}}\, \overline{Q_{quqd}^{(1)uvrs}} - \lambda \overline{y_e^{pt}}\, \overline{Q_{dH}^{rs}} - \mu^2 \overline{y_e^{pt}}\, \overline{Q_{yd}^{rs}} \\
&- \overline{y_e^{pu}}\, y_d^{rv} Q_{ed}^{utsv} - \overline{y_e^{pt}}\, y_e^{uv} Q_{\ell edq}^{uvsr} - 3\overline{y_d^{uv}}\, y_d^{rs} Q_{\ell edq}^{ptvu} \\
&- 3\mu^2 y_d^{rs} Q_{ye}^{pt}\Big)
\end{aligned}$$

› TeX

Outlook

- (Automatic) EFT matching is crucial to BSM phenomenology
- Functional matching is ideal for automated matching
- One must carefully account for evanescent operators in computations
- **Matchete** is a public code for EFT matching. It already greatly simplifies the matching task and many more features are planned!

https://gitlab.com/matchete/matchete

Backup

Expansion by regions: an example

Find the result of a multi-scale integral as a series in $m^2/M^2 \ll 1$:

$$I = \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \frac{1}{\ell^2 - m^2} \frac{1}{\ell^2 - M^2} = \frac{i}{16\pi^2} \left(\frac{1}{\epsilon} + 1 + \log \frac{\bar{\mu}^2}{M^2} + \frac{m^2}{M^2} \log \frac{m^2}{M^2} \right)$$

$\ell^2 \sim M^2$

$$I_h = \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \frac{1}{\ell^2} \left(1 + \frac{m^2}{\ell^2} + \ldots \right) \frac{1}{\ell^2 - M^2} = \frac{i}{16\pi^2} \frac{m^2 + M^2}{M^2} \left(\frac{1}{\epsilon} + 1 + \log \frac{\bar{\mu}^2}{M^2} \right)$$

$\ell^2 \sim m^2$

$$I_s = \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \frac{1}{\ell^2 - m^2} \frac{-1}{M^2} \left(1 - \frac{\ell^2}{M^2} + \ldots \right) = \frac{-i}{16\pi^2} \frac{m^2}{M^2} \left(\frac{1}{\epsilon} + 1 + \log \frac{\bar{\mu}^2}{m^2} \right)$$

In dimensional regularization, integrals equal the sum of their *hard* and *soft* parts

Beneke, Smirnov [hep-ph/9711391]; Jantzen [1111.2589]

$$I = I_h + I_s$$

Matchete demonstration (SM implementation)

Gauge Groups

```
DefineGaugeGroup[SU3c, SU@3, gs, G,
    FundAlphabet → CharacterRange["a", "f"],
    AdjAlphabet → CharacterRange["A", "F"]]
DefineGaugeGroup[SU2L, SU@2, gL, W,
    FundAlphabet → CharacterRange["i", "n"],
    AdjAlphabet → CharacterRange["I", "N"]]
DefineGaugeGroup[U1Y, U@1, gY, B]
```

Generation index

```
DefineFlavorIndex[Flavor, 3,
    IndexAlphabet → {"p", "r", "s", "t", "u", "v"}]
```

Fermions

```
DefineField[q, Fermion,
    Indices → {SU3c@fund, SU2L@fund, Flavor},
    Charges → {U1Y[1/6]},
    Chiral → LeftHanded,
    Mass → 0]
DefineField[u, Fermion,
    Indices → {SU3c@fund, Flavor},
    Charges → {U1Y[2/3]},
    Chiral → RightHanded,
    Mass → 0]
DefineField[d, Fermion,
    Indices → {SU3c@fund, Flavor},
    Charges → {U1Y[-1/3]},
    Chiral → RightHanded,
    Mass → 0]
```

```
DefineField[l, Fermion,
    Indices → {SU2L@fund, Flavor},
    Charges → {U1Y[-1/2]},
    Chiral → LeftHanded,
    Mass → 0]
DefineField[e, Fermion,
    Indices → {Flavor},
    Charges → {U1Y[-1]},
    Chiral → RightHanded,
    Mass → 0]
```

Higgs

```
DefineField[H, Scalar,
    Indices → {SU2L@fund},
    Charges → {U1Y[1/2]},
    Mass → 0]
```

Couplings

```
DefineCoupling[λ, SelfConjugate → True]
DefineCoupling[μ, SelfConjugate → True,
  EFTorder → 1];
DefineCoupling[Ye,
    Indices → {Flavor, Flavor}]
DefineCoupling[Yu,
    Indices → {Flavor, Flavor}]
DefineCoupling[Yd,
    Indices → {Flavor, Flavor}]
```

Matchete demonstration (SM implementation)

Lagrangian

```
ℒSM = FreeLag[] +
    -μ[]^2 Bar@H[i] × H[i] -
    λ[]/2 Bar@H[i] × H[i] × Bar@H[j] × H[j] +
    PlusHc[
        -Yu[p, r] × CG[eps@SU2L, {i, j}] ×
       Bar@H@i × Bar@q[a, j, p] ** u[a, r]
        - Yd[p, r] × H@i × Bar@q[a, i, p] ** d[a, r]
        - Ye[p, r] × H@i × Bar@l[i, p] ** e[r]
    ] // RelabelIndices;
```

```
ℒSM // NiceForm
```

Form=

$$
\begin{aligned}
&-\frac{1}{4} B^{\mu\nu 2} - \frac{1}{4} G^{\mu\nu A 2} - \frac{1}{4} W^{\mu\nu I 2} + D_\mu \bar{H}_i \, D_\mu H^i - \\
&\mu^2 \bar{H}_i H^i + i \left(\bar{d}^p_a \cdot \gamma_\mu P_R \cdot D_\mu d^{ap}\right) + i \left(\bar{e}^p \cdot \gamma_\mu P_R \cdot D_\mu e^p\right) + \\
&i \left(\bar{l}^p_i \cdot \gamma_\mu P_L \cdot D_\mu l^{ip}\right) + i \left(\bar{q}^p_{ai} \cdot \gamma_\mu P_L \cdot D_\mu q^{aip}\right) + \\
&i \left(\bar{u}^p_a \cdot \gamma_\mu P_R \cdot D_\mu u^{ap}\right) - \frac{1}{2} \lambda \bar{H}_i \bar{H}_j H^i H^j - \\
&\overline{Yd}^{pr} \bar{H}_i \left(\bar{d}^r_a \cdot P_L \cdot q^{aip}\right) - \overline{Ye}^{pr} \bar{H}_i \left(\bar{e}^r \cdot P_L \cdot l^{ip}\right) - \\
&Ye^{pr} H^i \left(\bar{l}^p_i \cdot P_R \cdot e^r\right) - Yd^{pr} H^i \left(\bar{q}^p_{ai} \cdot P_R \cdot d^{ar}\right) - \\
&Yu^{pr} \bar{H}_i \left(\bar{q}^p_{aj} \cdot P_R \cdot u^{ar}\right) \varepsilon^{ij} - \overline{Yu}^{pr} H^i \left(\bar{u}^r_a \cdot P_L \cdot q^{ajp}\right) \bar{\varepsilon}_{ij}
\end{aligned}
$$