Flavour non-renormalisation theorems for the SMEFT

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Based on 2210.09316 with Camila Machado, Dave Sutherland

SMEFT for BSM physics

Effective theory parameterising effects of heavy new physics respecting the full SM gauge group, and containing a Higgs doublet

$$\mathcal{L}_{\rm NP} = \frac{1}{\Lambda^2}\sum_i C_i^{(6)}\mathcal{O}_i^{(6)} + \frac{1}{\Lambda^4}\sum_i C_i^{(8)}\mathcal{O}_i^{(8)} + \dots$$

ADVANTAGES

Can reproduce effects of heavy new physics at low energies

Model independent

Language to interpret experimental results

Can connect scales via anomalous dimension matrix

(Alonso), Jenkins, Manohar, Trott
1308.2627, 1310.4838, 1312.2014

CHALLENGES

Too many parameters to deal with (2499 at dimension 6)

Sometimes opaque connection between operators and observables

Categorising operators

A way to narrow down the problem is to identify *categories of important operators*

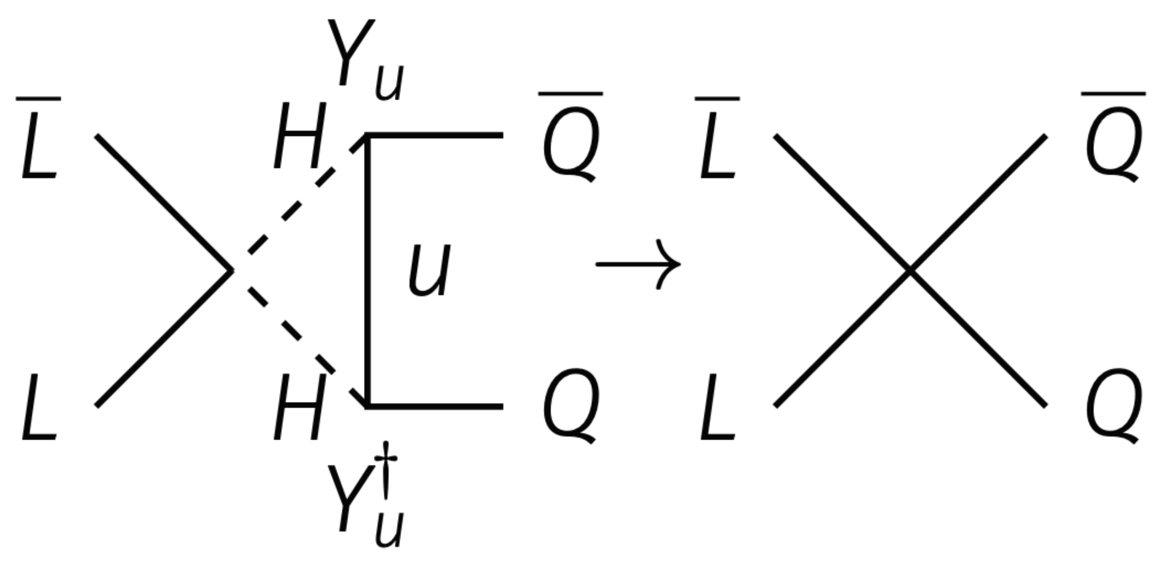
e.g.

- Operators that contribute (at tree or loop level) to a class of observables
- Operators invariant under CP or flavour symmetries
- Operators that are created at tree level by simple/motivated UV models

Running and categorisation

Categories of operators may not be conserved over scales

e.g. “flavourless” operators can produce FCNCs after running down



And “tree level” categorisations may prove less useful if scales are very different

e.g. $(g-2)_\mu$: loop-generated dipole operators can have significant effects

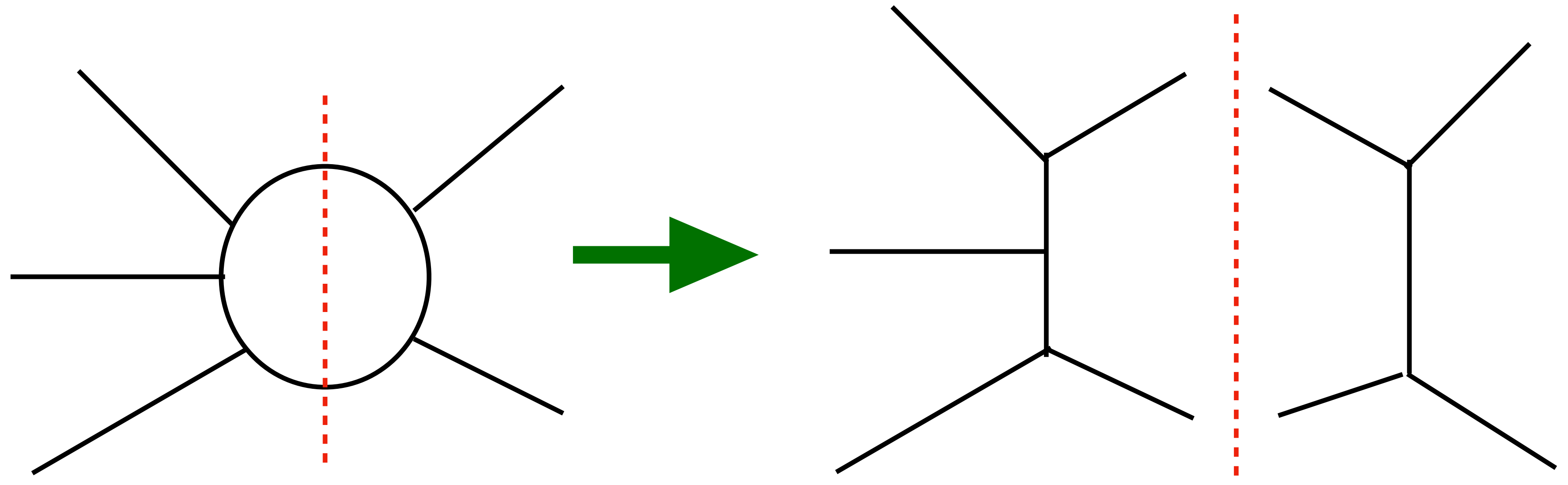
Overall, want to find categories that remain distinct over scales
i.e. they do not mix into each other under renormalisation group flow

Then can study subsets independently

Anomalous dimensions via tree amplitudes

Cutkosky's rule: 2-cuts isolate the discontinuities of the amplitude

$\Longrightarrow$ <u>can deduce divergences</u>



internal lines go onshell

Cutkosky, J. Math. Phys 1, 429 (1960)

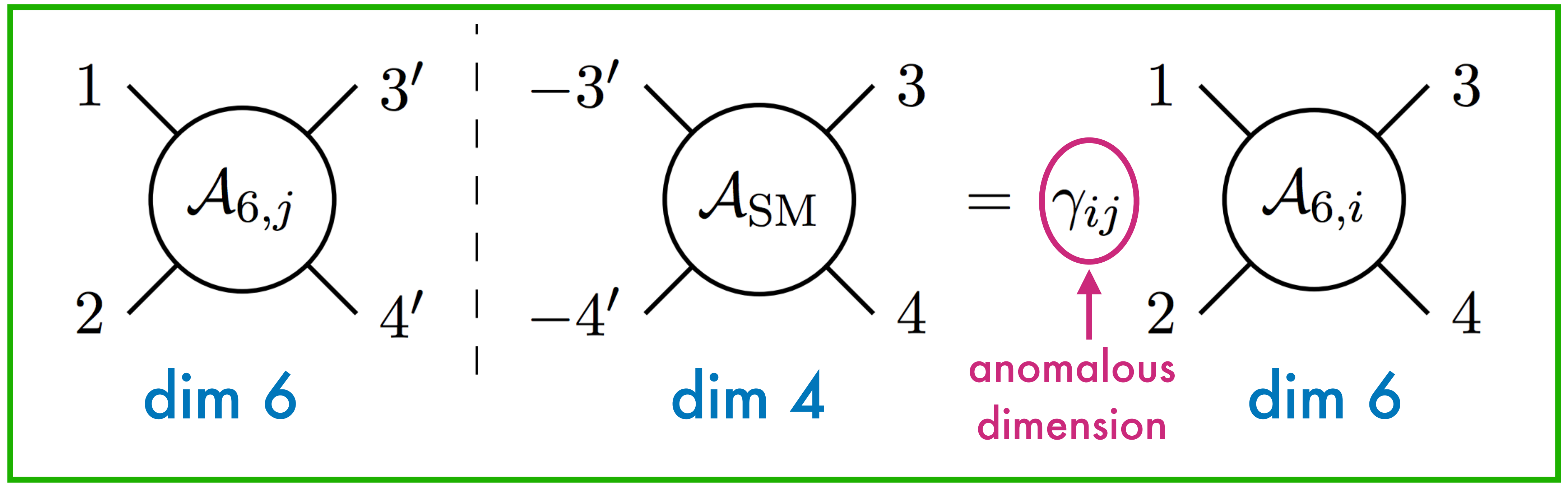
If there are no IR divergences, then 2-cuts isolate only the *UV divergent piece*

This is always the case for operator mixing* in the SMEFT at dim 6

*but not always for self-renormalisation

$\Longrightarrow$ schematically:

Caron-Huot, Wilhelm 1607.06448
Jiang, Ma, Shu, 2005.10261
Baratella, Fernandez, Pomarol, 2005.07129
Elias Miró, Ingoldby, Riembau, 2005.06983

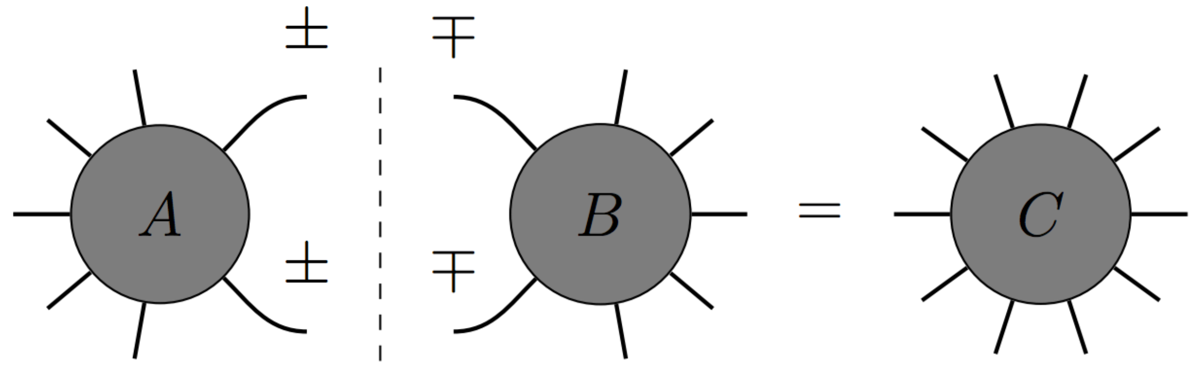


All momenta are defined *ingoing* $\Longrightarrow$ lines on either side of the cut have opposite momenta and helicity

If we know the properties of the dim 6 and dim 4 amplitudes on the LHS, we can understand which amplitudes can be produced on the RHS

Helicity and non-renormalisation

Label amplitudes by number of legs n and total helicity $\sum h$

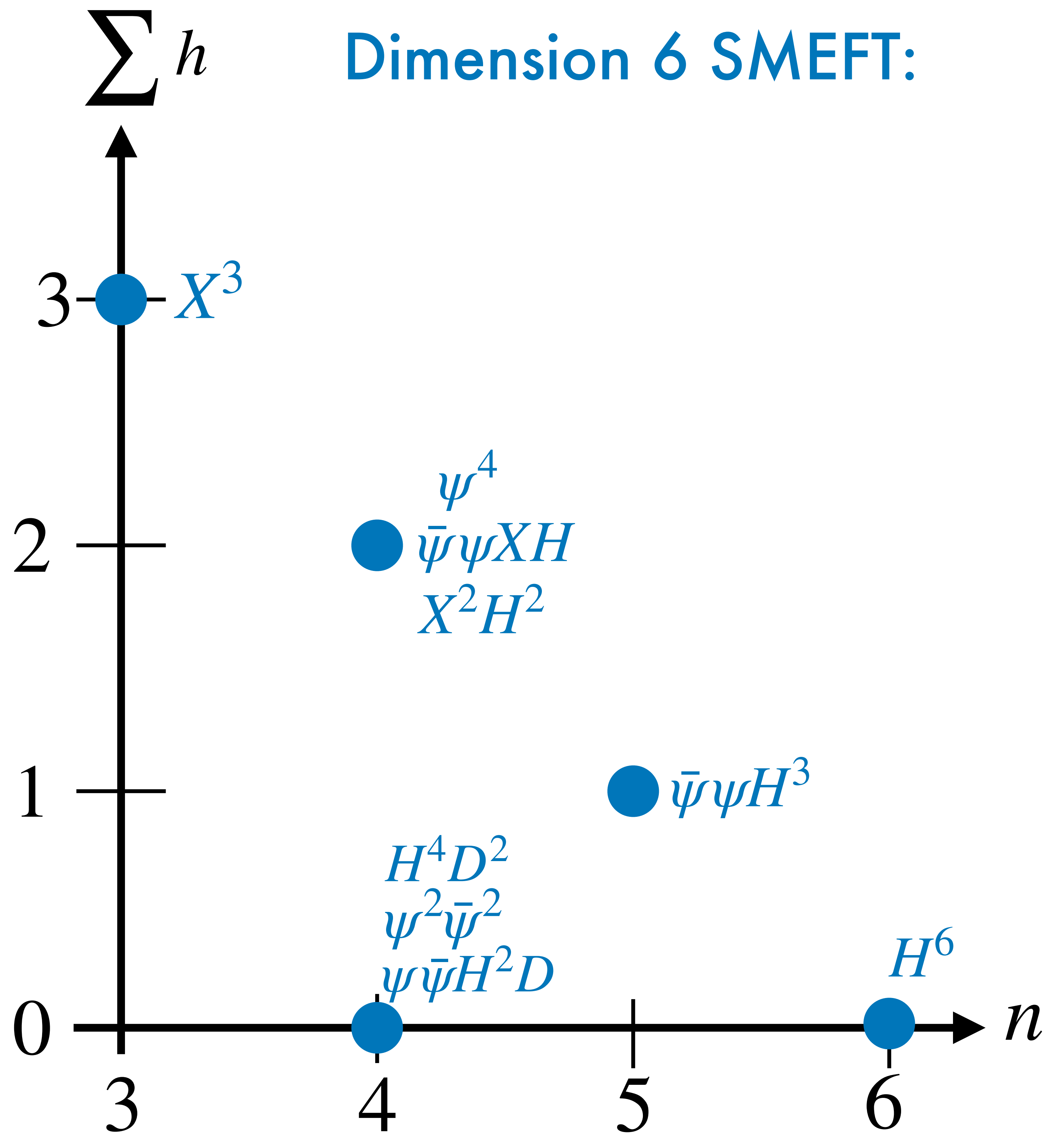


$$n_C = n_A + n_B - 4 \quad \text{and} \quad \sum h_C = \sum h_A + \sum h_B$$

$X = B_{\mu\nu}, G^A_{\mu\nu}, W^I_{\mu\nu}$

$\psi = Q, u, d, L, e$

Dimension 6 SMEFT:



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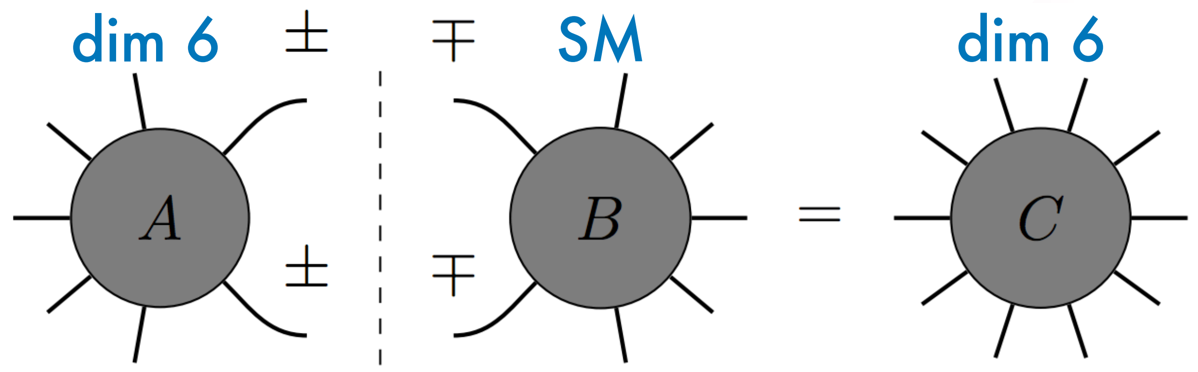

$$n_C = n_A + n_B - 4 \quad \text{and} \quad \sum h_C = \sum h_A + \sum h_B$$

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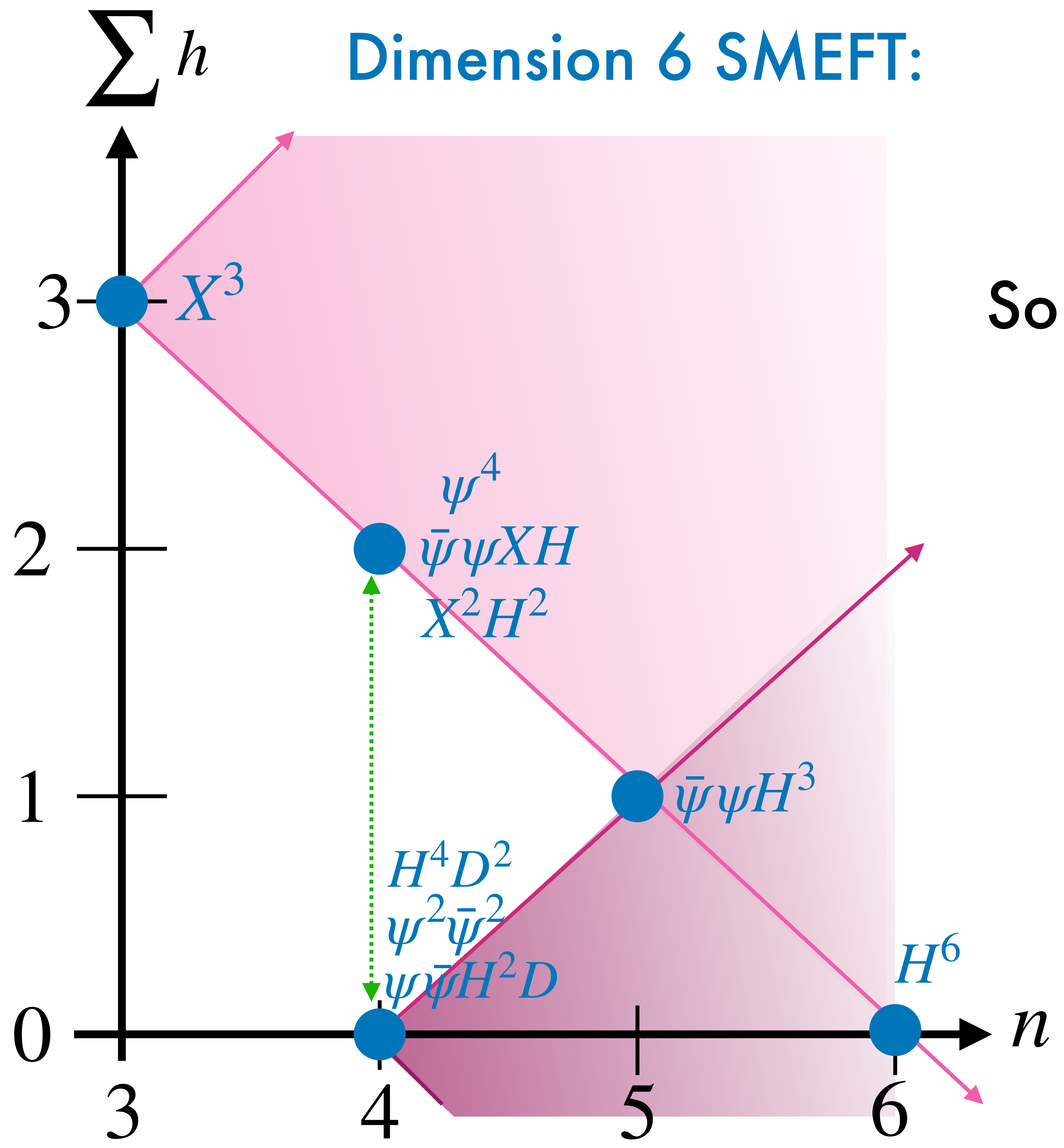


All SM amplitudes (*) lie in the cone defined by

$$\boxed{\left|\sum h\right| \leq n - 4}$$

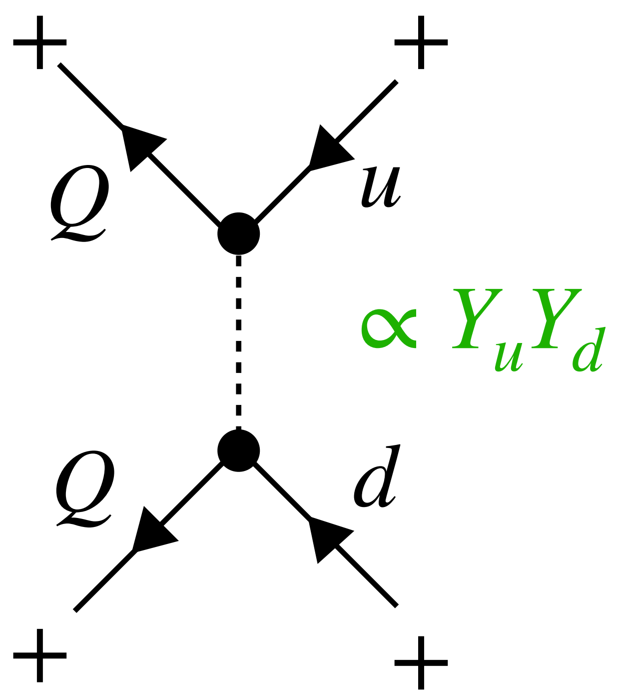
So from any operator, can only run into operators on or within the cone at one loop

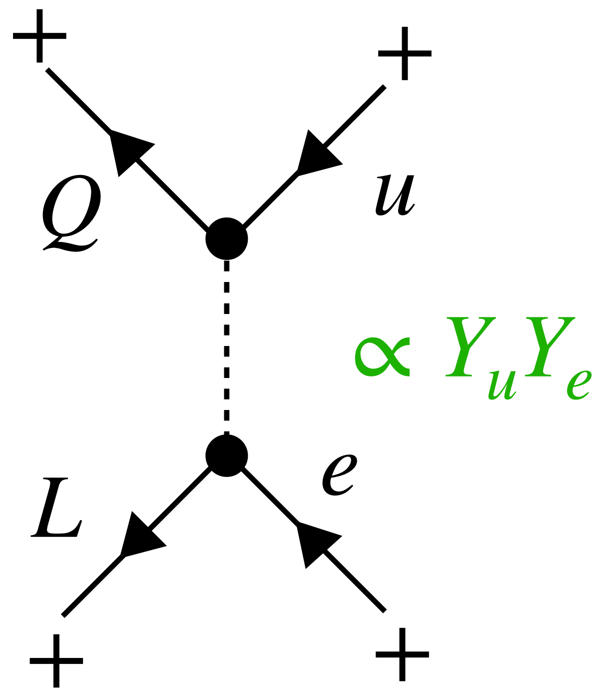
Alonso, Jenkins, Manohar 1409.0868
Cheung, Shen 1505.01844

(*) exceptions:

$$\boxed{\begin{gathered}\left|\sum h\right| = 2 \\ n = 4\end{gathered}}$$

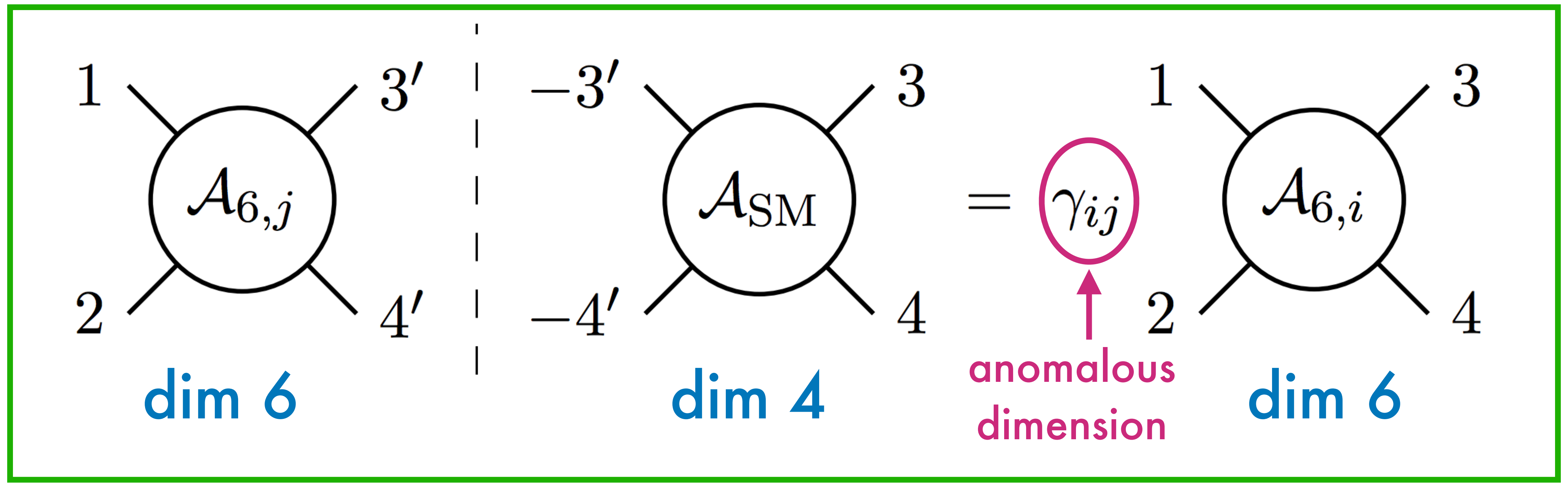
Always suppressed by a small Yukawa





Going further: gauge and flavour

We have non-renormalisation theorems based on helicity, i.e. the <u>kinematical</u> part of the amplitudes



<u>Amplitudes factorise:</u>

(kinematics) $\times$ (gauge) $\times$ (flavour)

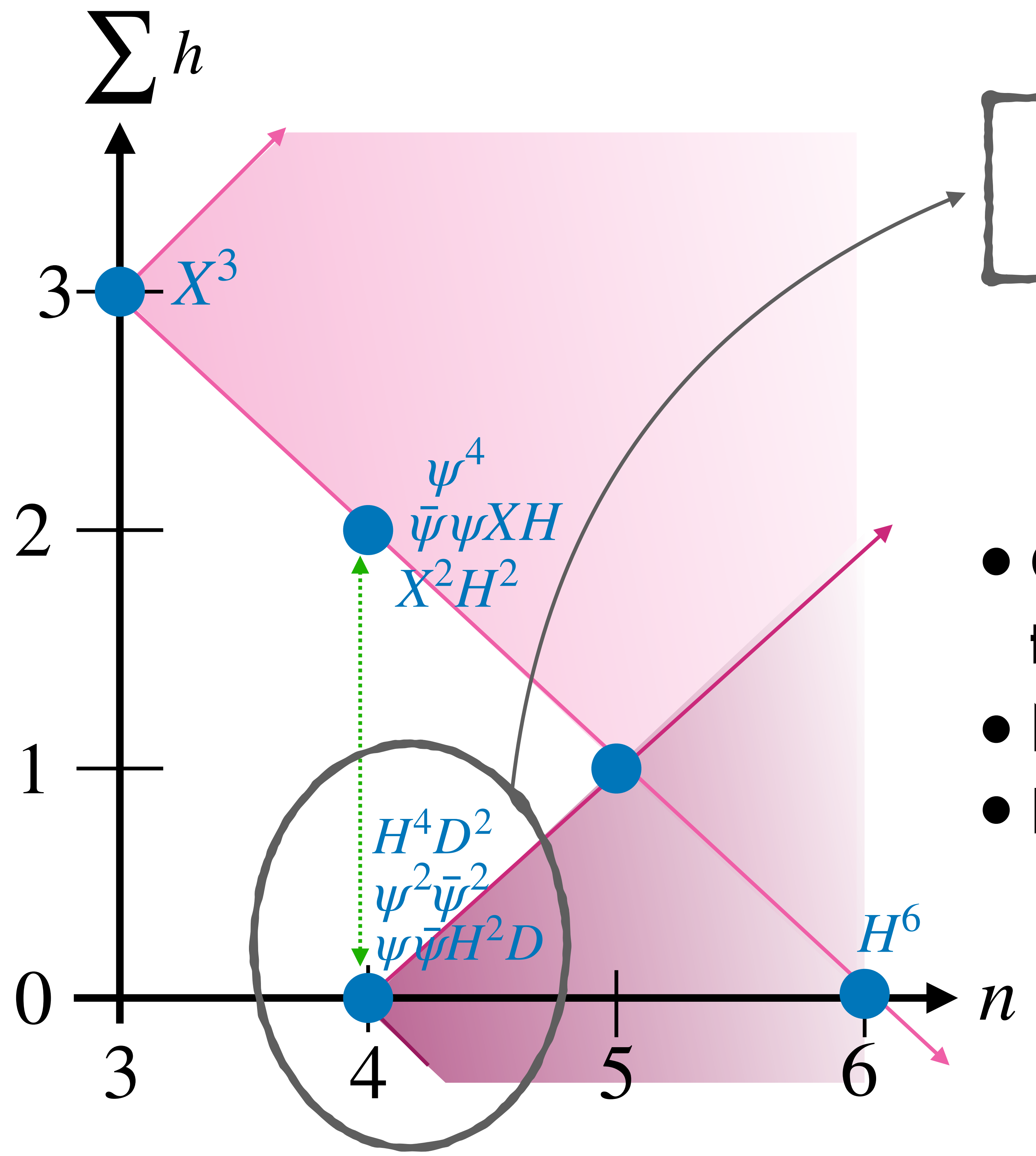
$\Longrightarrow$

<u>Anomalous dimension factorises:</u>

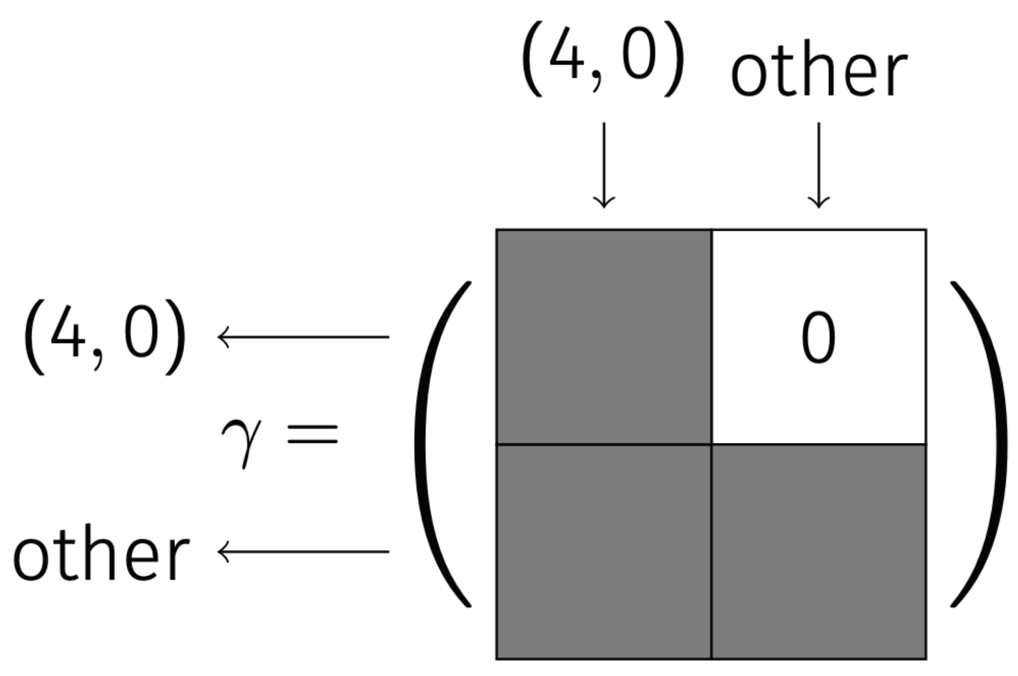
(kinematics) $\times$ (gauge) $\times$ (flavour)

Can we find good categories for the gauge and flavour parts of the operators, that are conserved under running?

The (4,0) block



All dim 6 operators with $\left(n, \sum h\right) = (4,0)$



Good place to focus, since:

- Contains most of the operators that can be generated at tree level in weakly coupled UV completions

Einhorn, Wudka 1307.0478
Craig, Jiang, Li, Sutherland 2001.00017

- Nothing runs *into* it (apart from $\propto Y_u Y_d$, $Y_u Y_e$)
- It contains most (1460) of the parameters

(N.B. We neglect O_{Hud} and O_{ledq}, which are also disconnected from the rest of the block by small Yukawas)

<u>What's in the block</u>

H^4D^2
4 Higgs operators
O_{HD} $O_{H\Box}$

$\psi\bar\psi H^2 D$
2 Higgs, 2 fermion operators
e.g. O_{Hu}, $O_{Hl}^{(1,3)}$ etc

$\psi^2\bar\psi^2$
4 fermion operators
(All except $O_{lequ}^{(1,3)}$ and $O_{quqd}^{(1,8)}$)

Flavour decomposition: irreps

Most operators have flavour matrices as Wilson coefficients

Can decompose these general matrices in any basis that is convenient

Natural choice: irreps of SM flavour group

$$\boxed{SU(3)^5 = SU(3)_Q \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e}$$

SM fermions are in triplet irreps under their group

$SU(3)^5$ preserved by gauge interactions

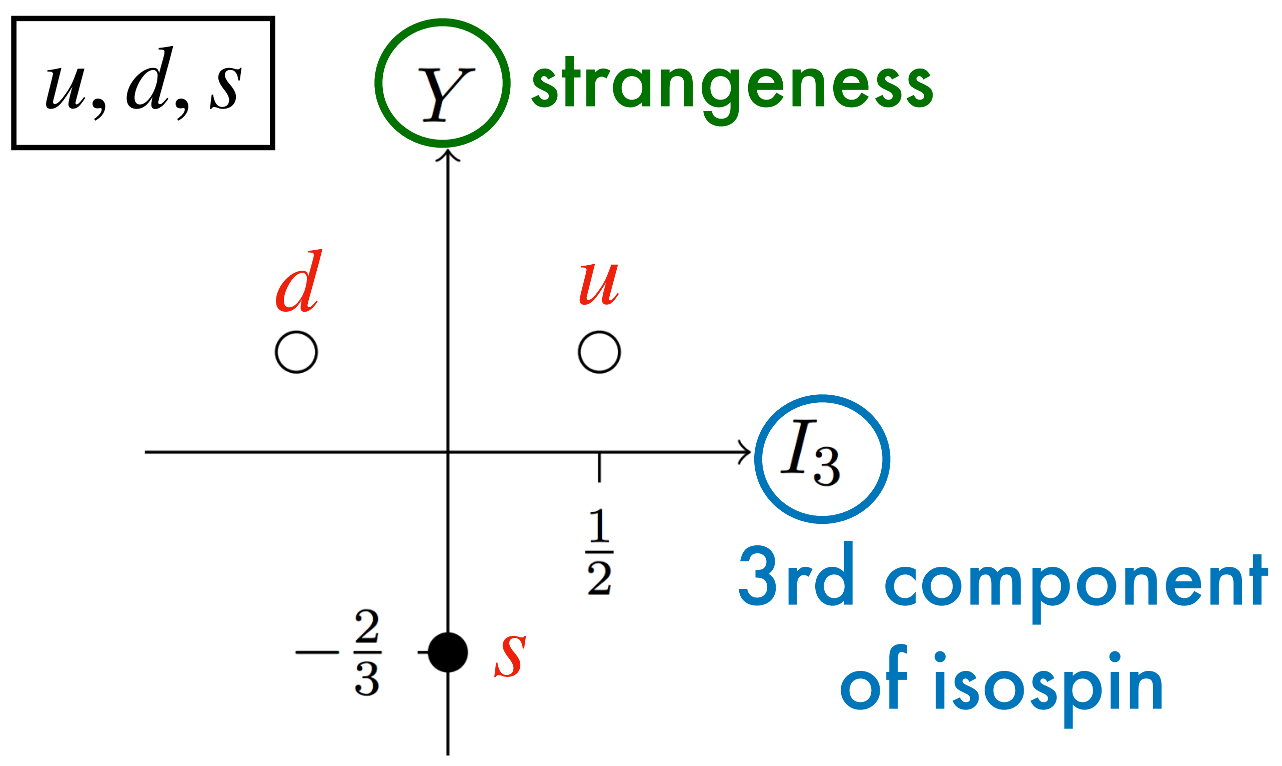
Operator type	Wilson coeff	Irrep decomposition
H^4D^2	c	$1_F \ (\forall F)$
$H^2\psi_F^2 D$	c^p_q	$3_F \otimes \bar{3}_F = 1_F \oplus 8_F$
$\psi^2\bar{\psi}^2 : (\bar{\psi}_{F_1}\psi_{F_1})(\bar{\psi}_{F_2}\psi_{F_2})$	c^{pr}_{qs}	$3_{F_1} \otimes \bar{3}_{F_1} \otimes 3_{F_2} \otimes \bar{3}_{F_2} = (1_{F_1} \otimes 1_{F_2}) \oplus (1_{F_1} \otimes 8_{F_2}) \oplus (8_{F_1} \otimes 1_{F_2}) \oplus (8_{F_1} \otimes 8_{F_2})$
$\psi_F^2\bar{\psi}_F^2$: symmetric	$c^{(pr)}_{(qs)}$	$(3_F \otimes \bar{3}_F)_{sym} \otimes (3_F \otimes \bar{3}_F)_{sym} = 1_F \oplus 8_F \oplus 27_F$
$\psi_F^2\bar{\psi}_F^2$: antisymmetric	$c^{[pr]}_{[qs]}$	$(3_F \otimes \bar{3}_F)_{antisym} \otimes (3_F \otimes \bar{3}_F)_{antisym} = 1_F \oplus 8_F$

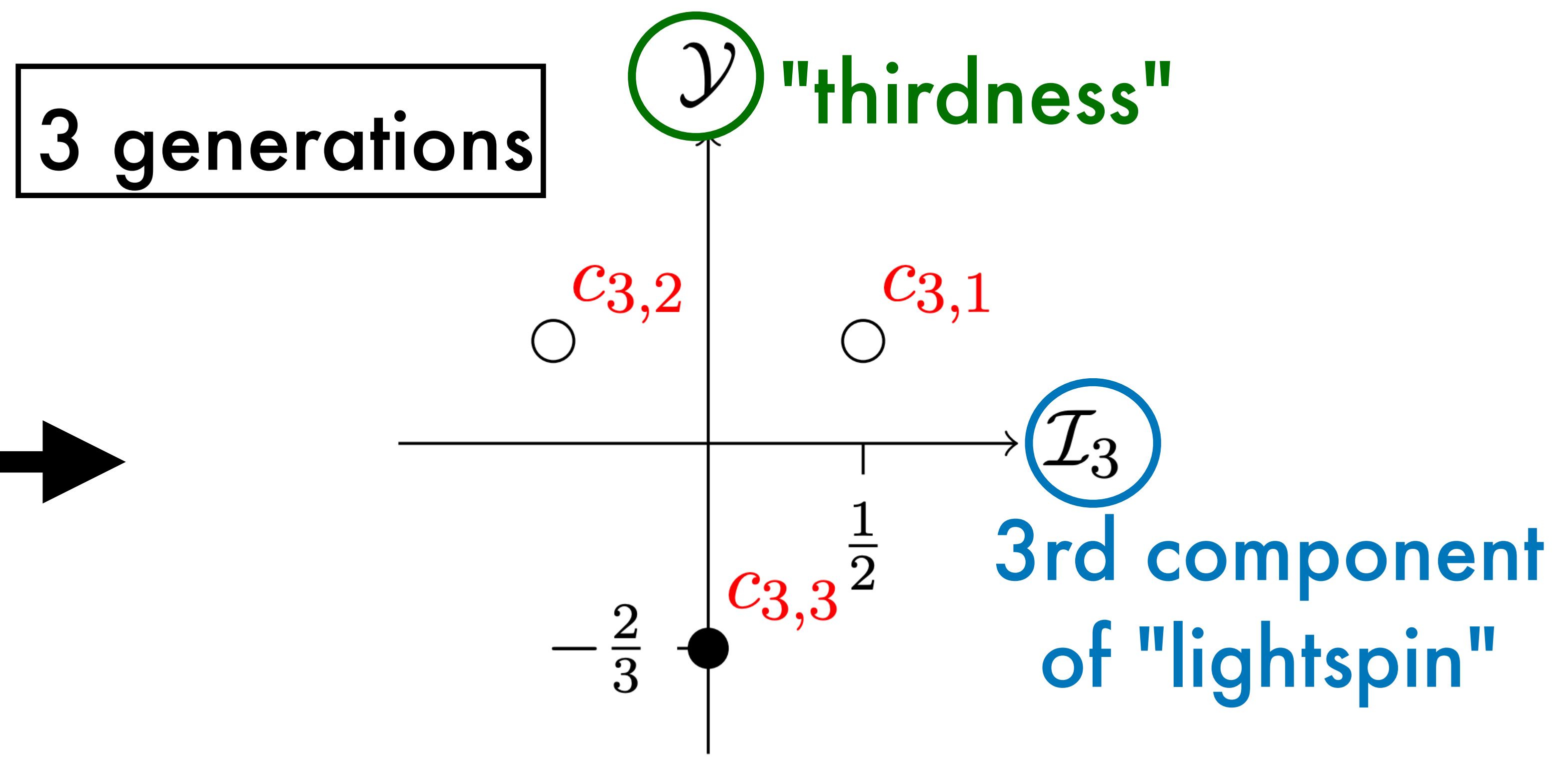
Flavour decomposition: quantum numbers

To label the components of the irreps, can use conventions developed for the $SU(3)$ of light flavours u, d, s in the 1960s

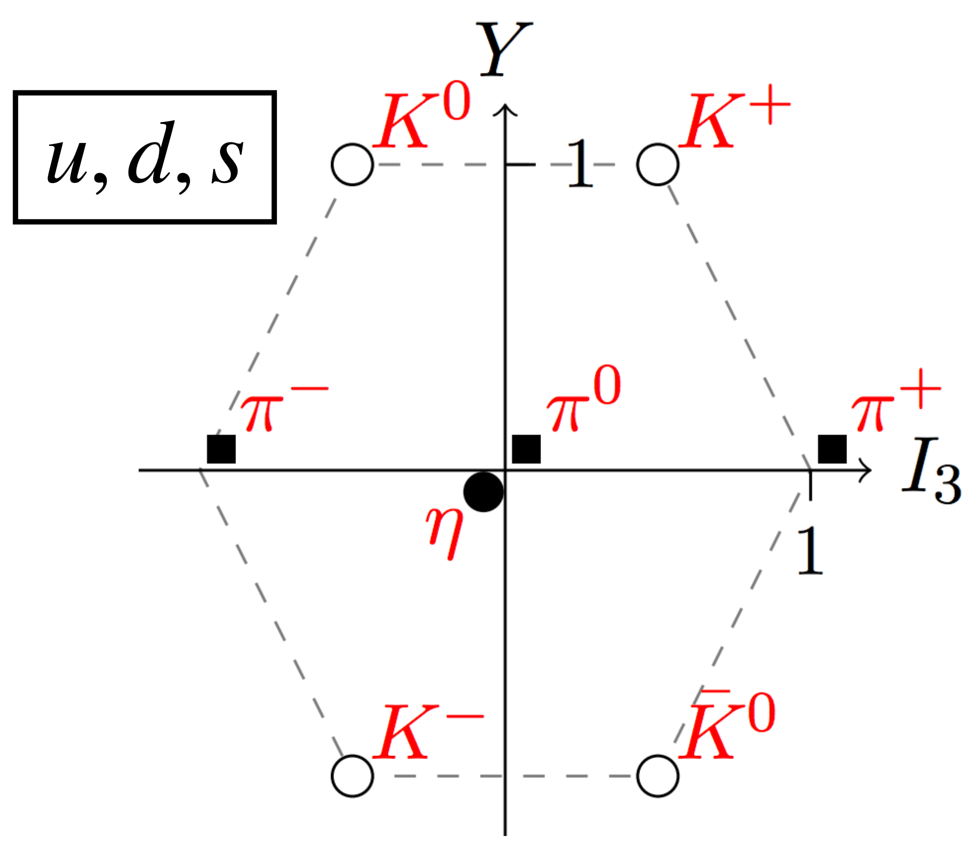
de Swart, Rev. Mod, Phys. 35 (1963) 916-939

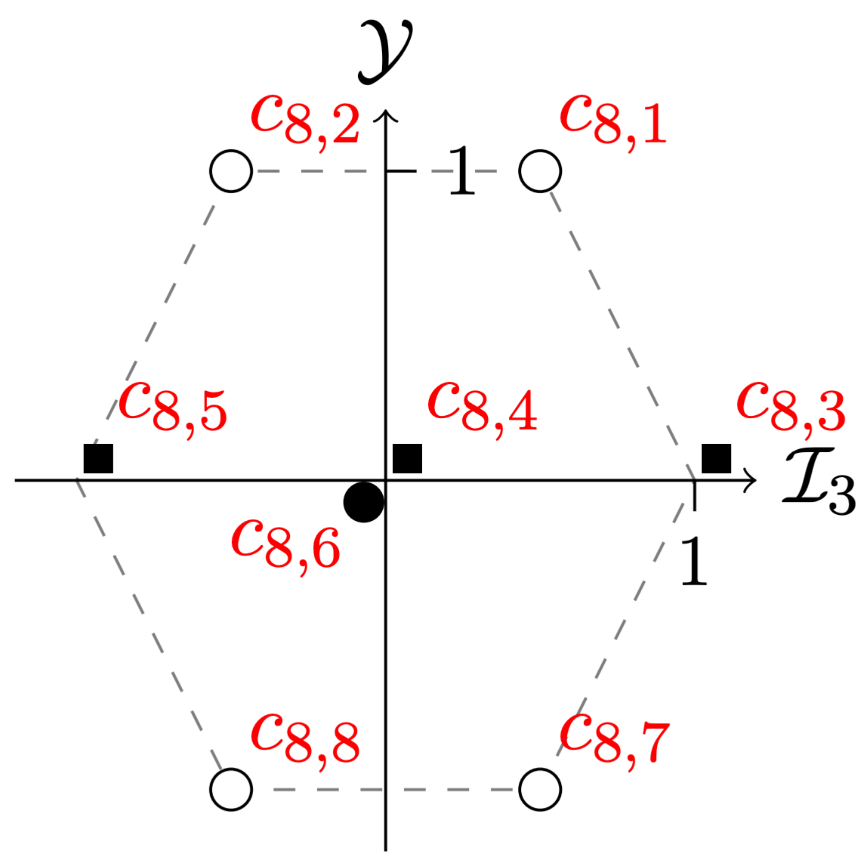
Triplet 3_F





Octet 8_F





Total $\mathcal{I}$ key:

$\bullet = 0,\ \circ = \frac{1}{2},\ \blacksquare = 1$

4 quantum numbers for each species: $\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_F$

Block-diagonalising γ via flavour decomposition

Now we have categorised the Wilson coefficients via their flavour quantum numbers
Which are conserved in the running?

Depends on which Standard Model couplings we neglect or not

Only gauge couplings	Gauge couplings and top Yukawa	All (Gauge couplings and all Yukawas)
All flavour quantum numbers are conserved	Conserves everything but $\{d_{irrep}\}_{\{Q,u\}}$ SM flavour symm broken $SU(3)_Q \times SU(3)_u \to SU(2)_Q \times SU(2)_u \times U(1)_{Q+u}$	Only Y_{L+e} and $I_{3,L+e}$ are conserved (equivalent to two individual lepton numbers)

neglecting more parameters ⟷ neglecting fewer parameters

If we class Wilson coefficients by their flavour quantum numbers, we can trivially block-diagonalise γ

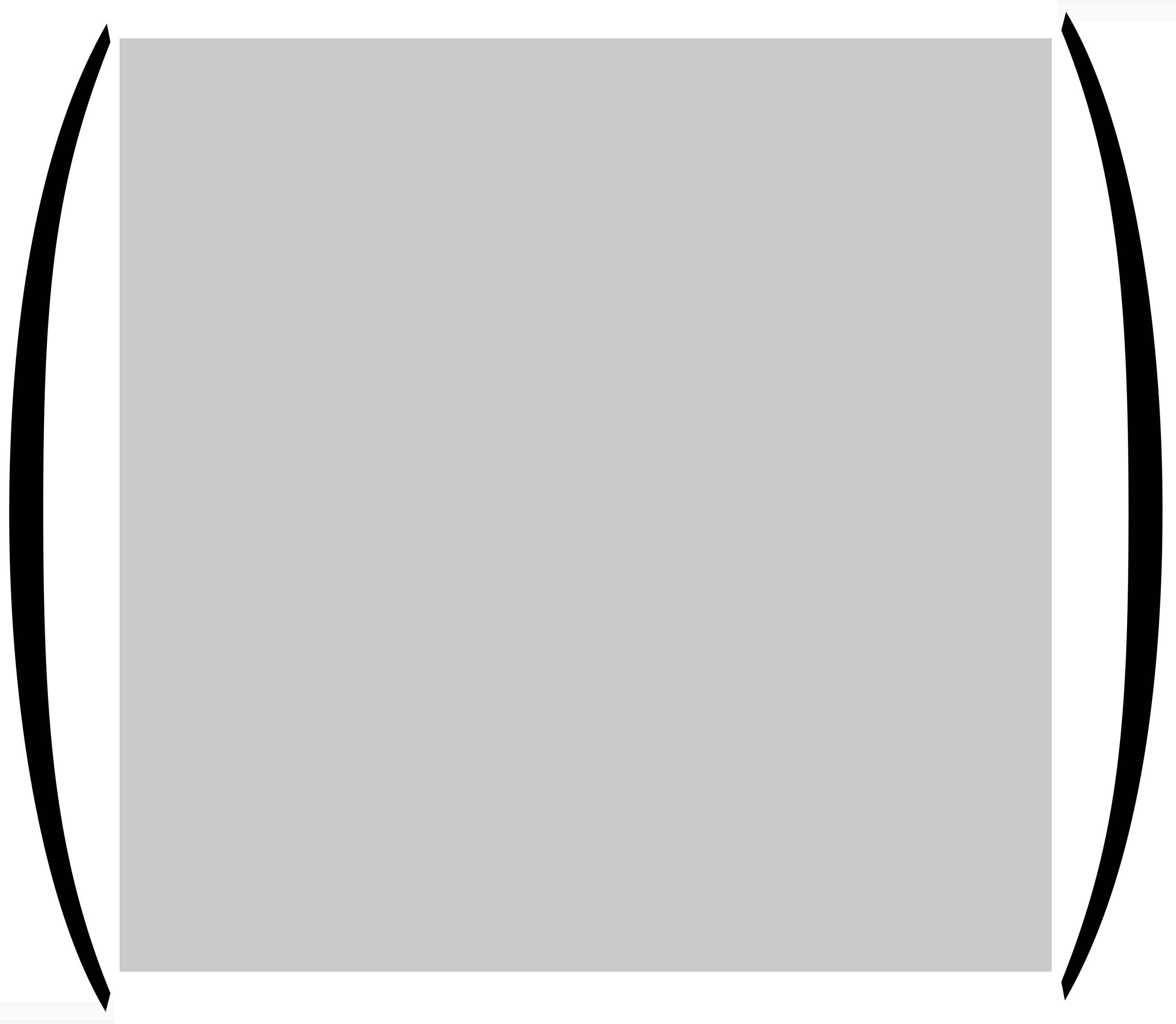
Blocks under different approximations

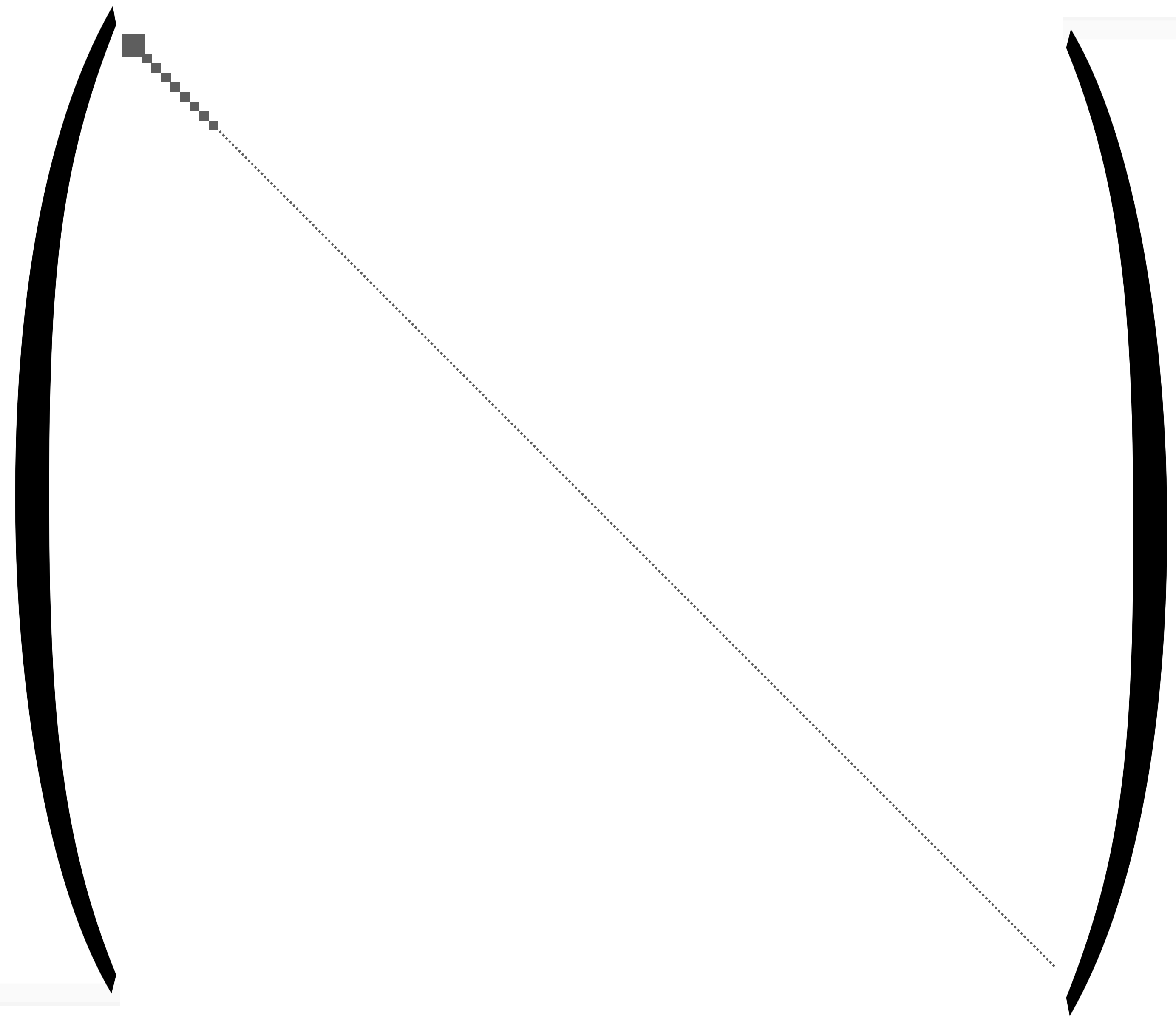
Only gauge couplings

All flavour quantum numbers conserved: $\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_{\{Q,u,d,L,e\}}$

(4,0) block
before flavour
decomposition

1460×1460





(4,0) block
after flavour
decomposition

Largest block size
34×34
(contains all the
flavour singlets)

Block size	34	13	9	6	2	1
Multiplicity	1	8	24	8	256	546

Blocks under different approximations

Gauge couplings and top Yukawa

SM flavour symm broken: $SU(3)_Q \times SU(3)_u \to SU(2)_Q \times SU(2)_u \times U(1)_{Q+u}$

Naively, expect conservation of $\mathcal{I}_Q,\ \mathcal{I}_u,\ \mathcal{I}_{3,Q},\ \mathcal{I}_{3,u},\ \mathcal{Y}_Q + \mathcal{Y}_u$

But in running within the (4,0) block, any factor of y_t comes with a $y_t^\dagger$, so in the end $\mathcal{Y}_Q$ and $\mathcal{Y}_u$ are conserved individually

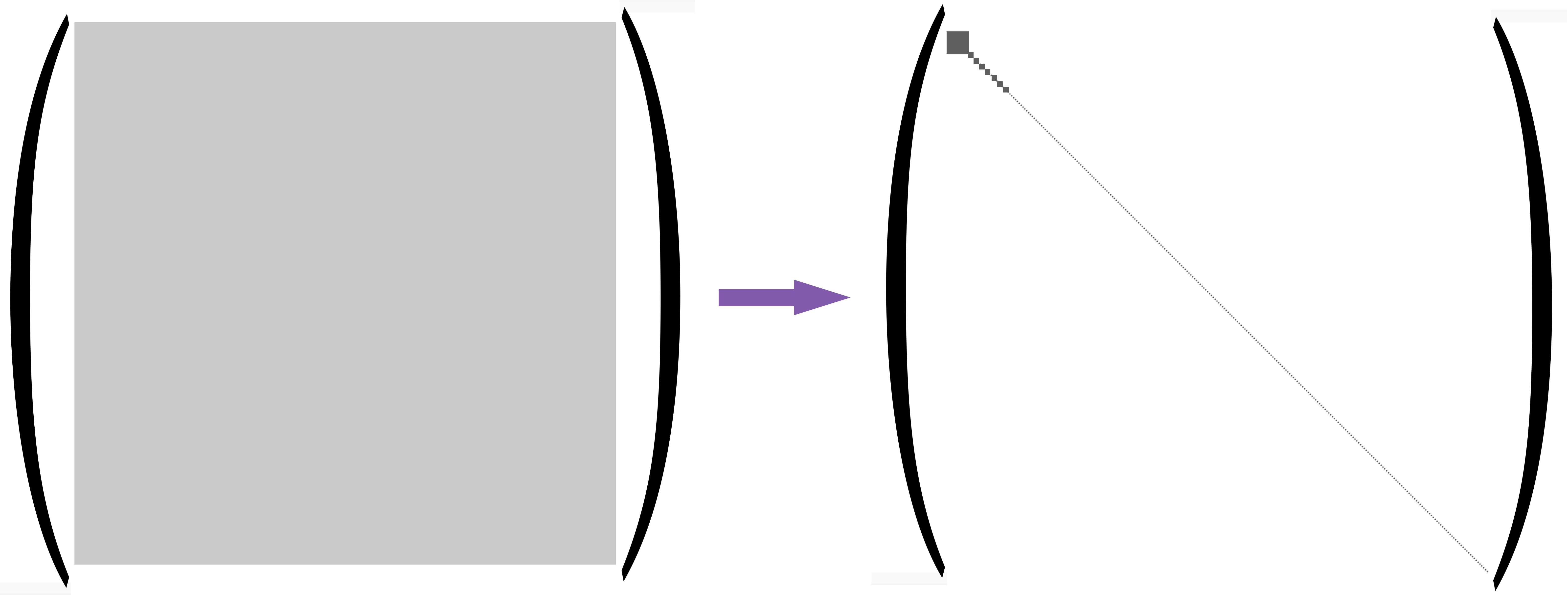
Blocks under different approximations

Gauge couplings and top Yukawa

Conserved: $\{\mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_{\{Q,u\}}$, $\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_{\{d,L,e\}}$

(4,0) block before flavour decomposition

1460×1460



(4,0) block after flavour decomposition

Largest block size 61×61 (contains all the flavour singlets plus things with $\mathcal{I}_{\{Q,u\}} = 0$)

Block size	61	17	13	12	8	2	1
Multiplicity	1	7	8	15	8	217	498

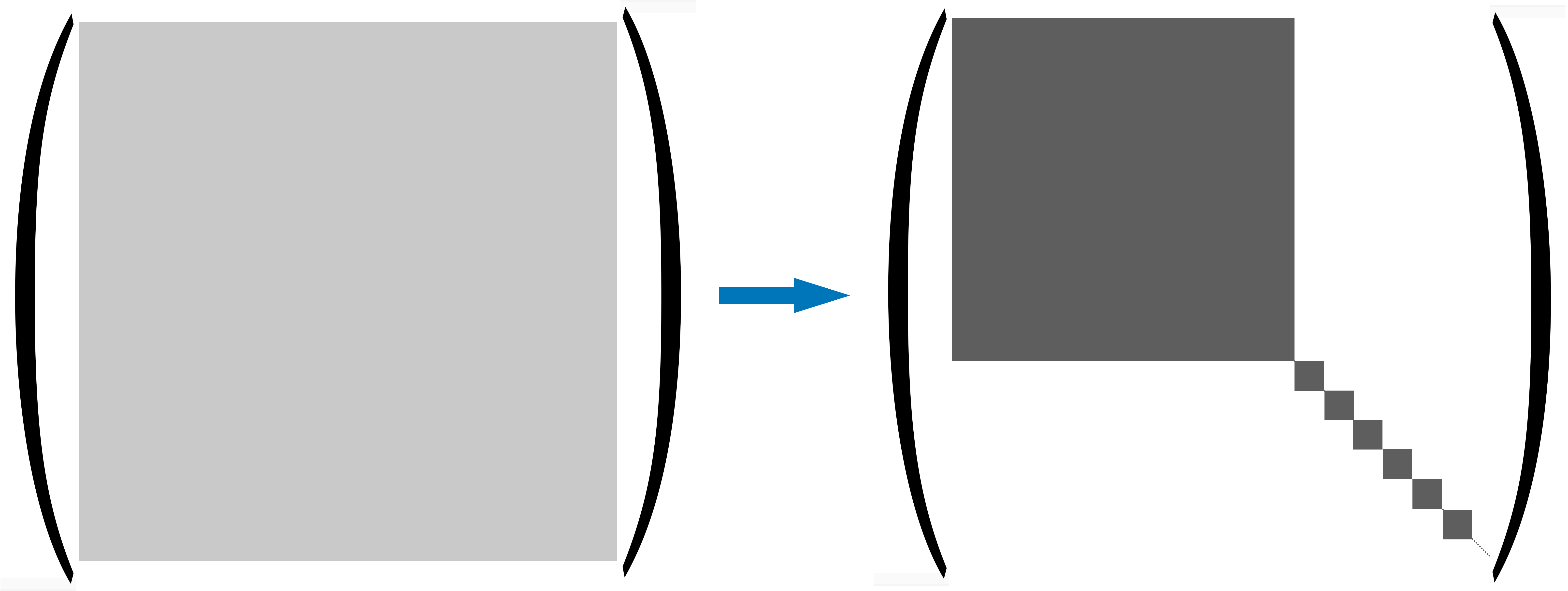
Blocks under different approximations

All (Gauge couplings and all Yukawas)

Conserved: $\mathcal{I}_{3,L} + \mathcal{I}_{3,e}, \ \mathcal{Y}_L + \mathcal{Y}_e$

(4,0) block
before flavour
decomposition

1460×1460



(4,0) block
after flavour
decomposition

Largest block size

932×932

6 4×4 blocks
6 3×3 blocks

Block size	932	81	4	3
Multiplicity	1	6	6	6

Invariant categorisations: a minimal parameter set

e.g. Assume that the flavour breaking we see in the SM is dominant

i.e. NP respects (at least) $U(2)_Q \times U(2)_u \times U(3)^3$ (and CP)

So within the (4,0) block we need the 61 parameters with

$$\mathcal{I}_{\{Q,u\}} = 0,\ d_{\{d,L,e\}} = 1$$

+ other operator coefficients $\underbrace{C_W\ C_G}_{(3,3)}$ $\underbrace{C_{tB}\ C_{tW}\ C_{tG}}_{(4,2)}$ $\underbrace{C_{HB}\ C_{HW}\ C_{HG}\ C_{HWB}}_{(4,2)}$ $\underbrace{C_{tH}}_{(5,1)}$ $\underbrace{C_H}_{(6,0)}$ = 11 parameters

$\left(n, \sum h\right) =$

72 total

agrees with Greljo, Palavric, Thomsen 2203.09561 Table 1

If we neglect y_b and smaller, this set is complete across scales

This is a consistent choice for global fits

The non-(4,0) operators do not run into the (4,0) block

Invariant categorisations: a minimal parameter set

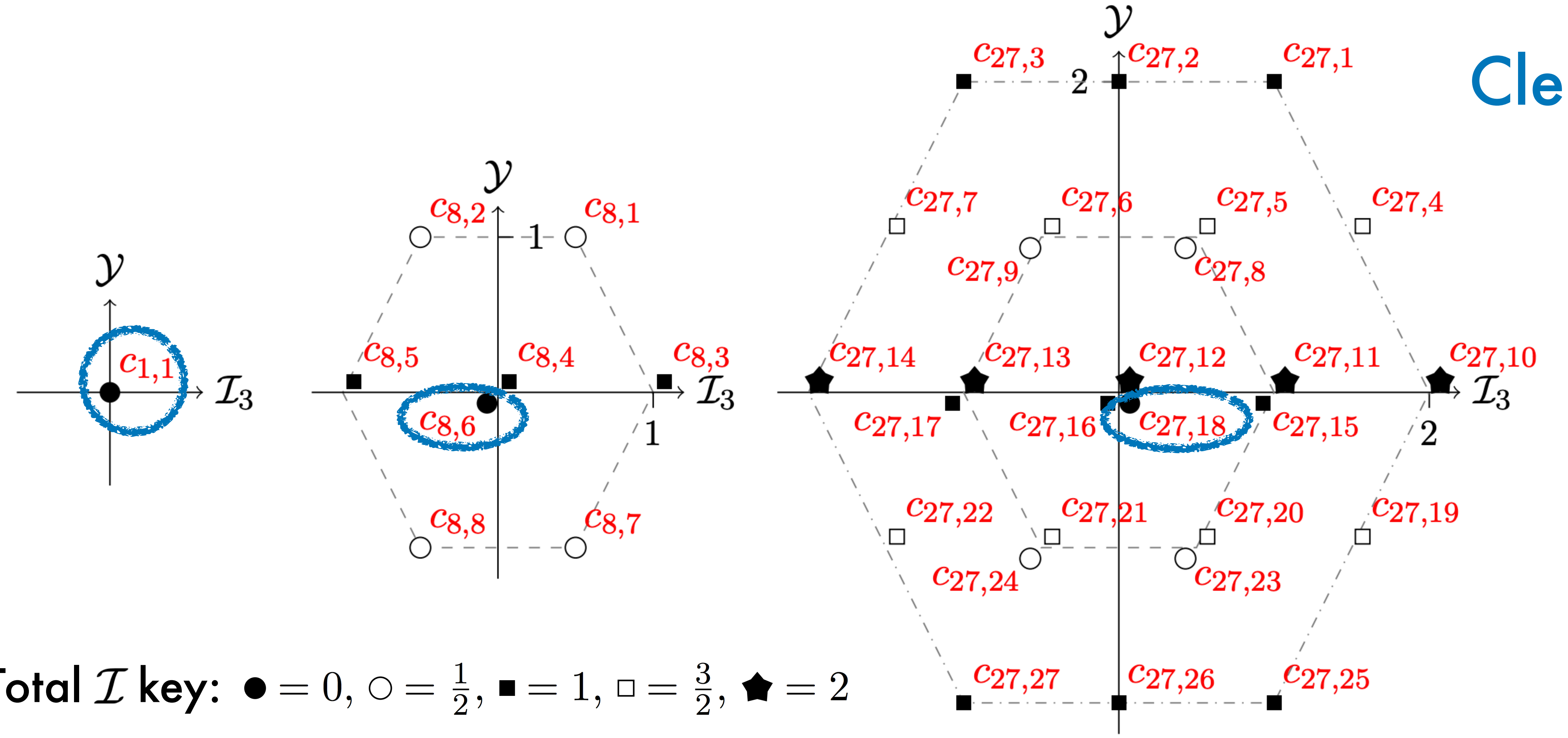
e.g. Assume that the flavour breaking we see in the SM is dominant

i.e. NP respects (at least) $U(2)_Q \times U(2)_u \times U(3)^3$

So within the (4,0) block we need the 61 parameters with

Not just counting!
Can see the explicit parameters

$$\mathcal{I}_{\{Q,u\}} = 0,\ d_{\{d,L,e\}} = 1$$



Total $\mathcal{I}$ key: $\bullet = 0$, $\circ = \frac{1}{2}$, $\blacksquare = 1$, $\square = \frac{3}{2}$, $\bigstar = 2$

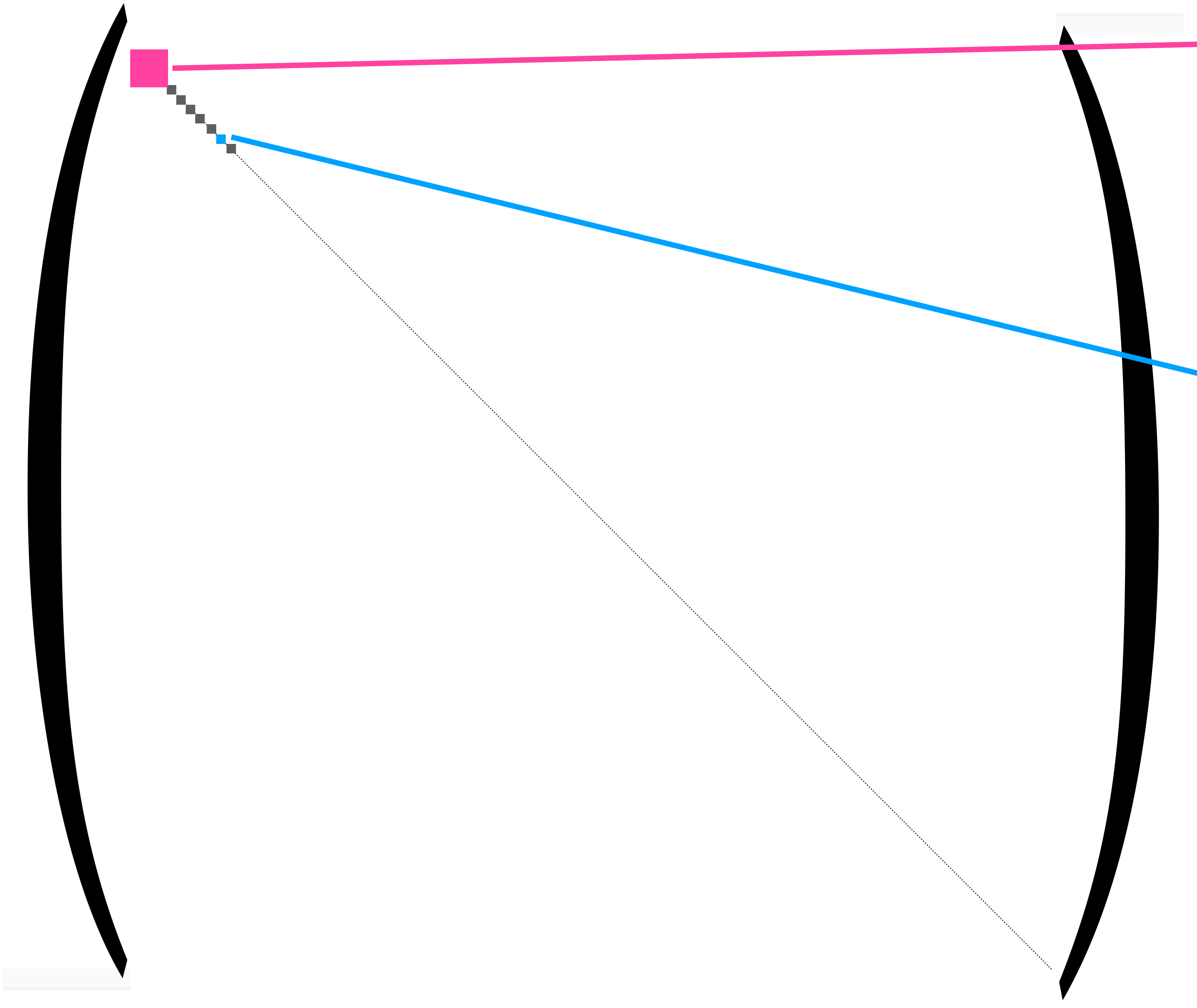
Clebsch-Gordan decompositions of the parameters:

$$c_{8,6} = \sqrt{\frac{1}{6}}\left(-c_1^1 - c_2^2 + 2c_3^3\right)$$

$$c_{27,18} = \sqrt{\frac{1}{30}}\left(c_{11}^{11} + 2c_{(12)}^{(12)} + c_{22}^{22} - 6c_{(13)}^{(13)} - 6c_{(23)}^{(23)} + 3c_{33}^{33}\right)$$

So, for example, as well as the full singlet piece of C_{qe}, we need $\frac{1}{\sqrt{3}}\left(-C_{qe}^{11ii} - C_{qe}^{22ii} + 2C_{qe}^{33ii}\right)$

Invariant categorisations: step by step approach



The previous slide was about this subset of broadly flavour conserving coefficients

To go further, we can also look at other subsets containing flavour-violating coefficients

Block diagonalisation ensures they are theoretically disconnected. Depending on observables they may or may not be experimentally disconnected too

More pheno uses

<u>Which</u> coefficients can be induced by running from any given coefficient (including flavour structure), or vice versa?

e.g. the lepton flavour non-universal part of the operator

$$\mathcal{L}_{\rm NP} = \frac{C}{\Lambda^2}\Big((\bar{Q}'_3\gamma^\mu Q'_3)(\bar{L}'_3\gamma_\mu L'_3) + (\bar{Q}'_3\gamma^\mu\sigma^I Q'_3)(\bar{L}'_3\gamma_\mu\sigma^I L'_3)\Big)$$

which can be responsible for LFUV in B decays

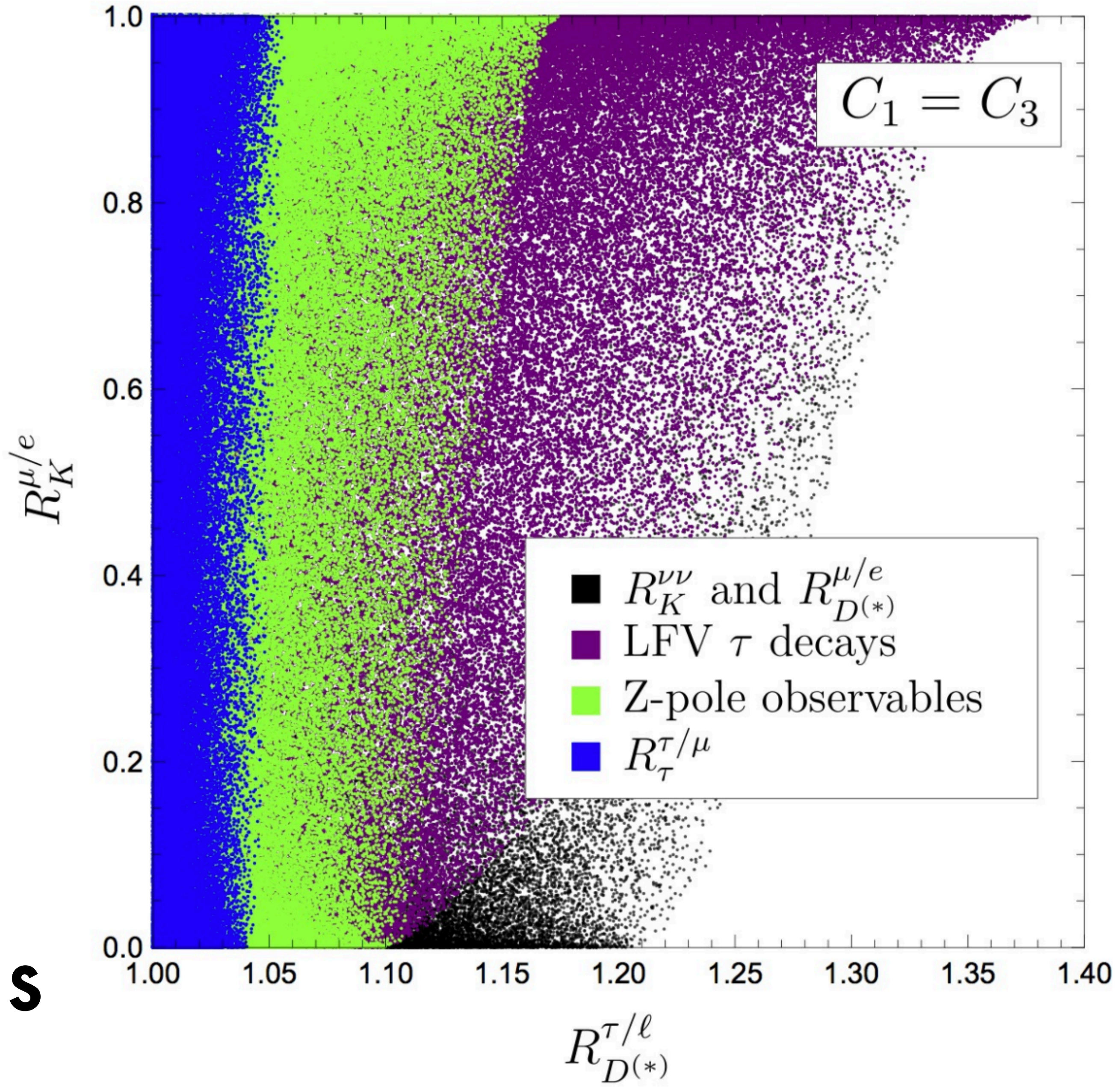
Clebsch-Gordan coefficients

$$\left.\begin{matrix} c_{1,1,8,6} \\ c_{8,6,8,6} \end{matrix}\right\} \begin{matrix} 12\times 12 \\ \text{block} \end{matrix}$$

Mixes with the $c_{8,6}$ lepton octet components of:

$$C^{(1)}_{LQ}\,(\times 2),\ C^{(3)}_{LQ}(\times 2),\ C_{Lu}\,(\times 2),\ C_{Ld},\ \underbrace{C_{LL}\,(\times 2),\ C_{Le}}_{\tau\text{ decays}},\ \underbrace{C^{(1)}_{HL},\ C^{(3)}_{HL}}_{\text{LFUV in Z couplings}}$$

Feruglio, Paradisi, Pattori, 1606.00524 & 1705.00929



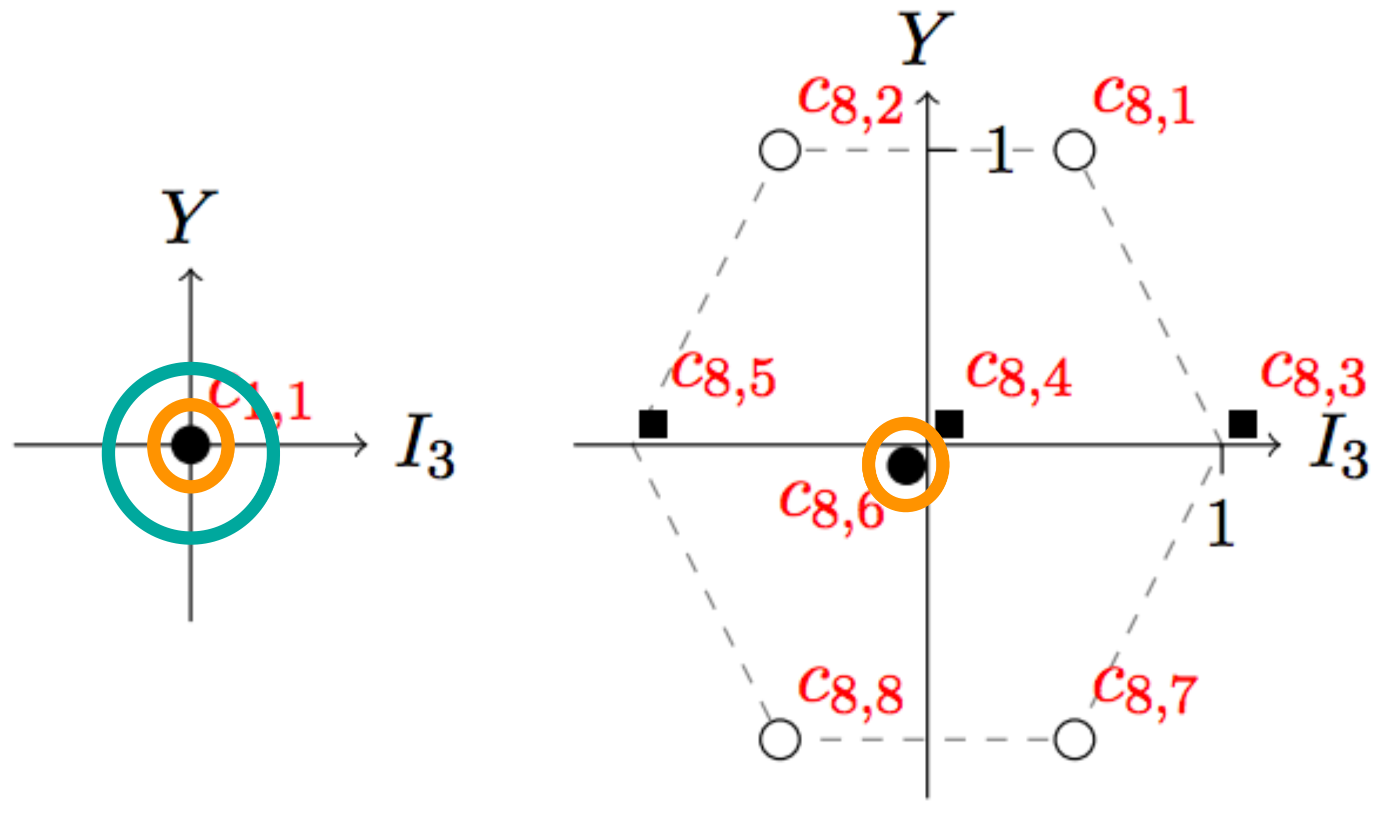
Summary and outlook

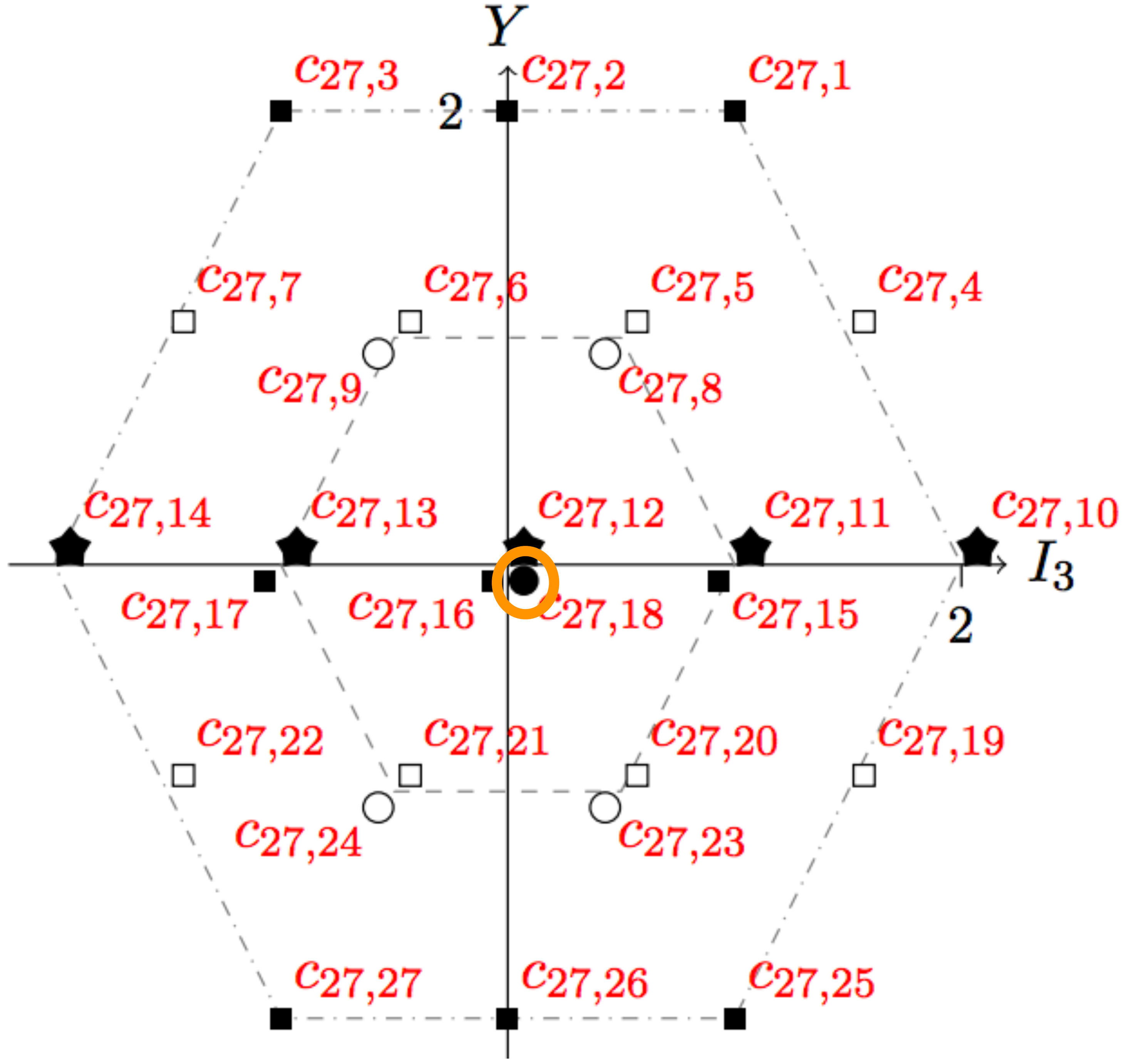
- Large non-diagonal anomalous dimension matrix complicates SMEFT phenomenology
- γ_{SMEFT} is very flavourful
- Using a symmetry-based flavour decomposition, achieve simple block diagonalisation of (4,0) operators
- Blocks allow you to understand closed subsets of parameters and narrow in on loop-level pheno

Backups…

Flavour symmetry subsets

4 quantum numbers: $\{d_{irrep}, I, I_3, Y\}$





Total I key: $\bullet = 0$, $\circ = \frac{1}{2}$, $\blacksquare = 1$, $\square = \frac{3}{2}$, ⬟ $= 2$

This is a fully general decomposition which does not restrict form of Wilson coefficients
But, since it is couched in flavour symmetry irreps,
easy to identify the subsets of coefficients that are invariant under exact flavour symmetries

e.g.

Exact $U(3)$ symmetry: just singlets

Exact $U(2)$ symmetry: just $I = 0$

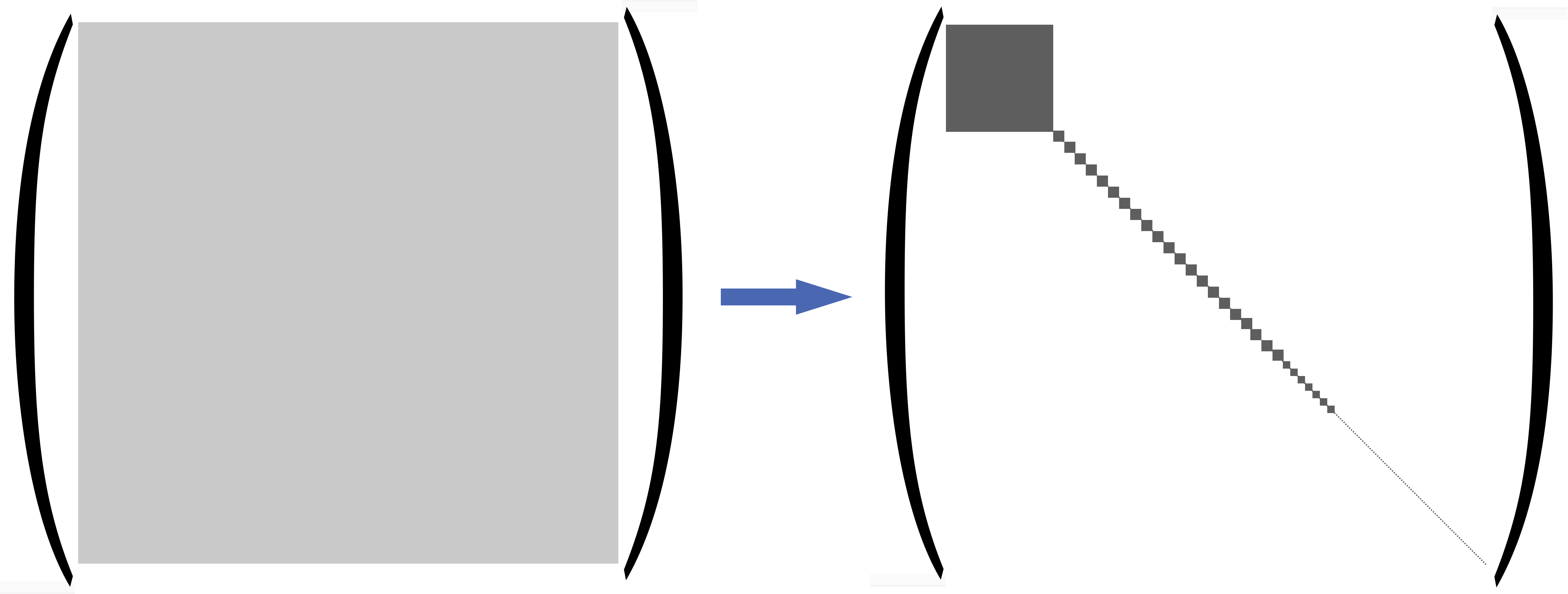
Blocks under different approximations

Gauge couplings and all 3rd generation Yukawas (full CKM)

Conserved: $\{I, I_3, Y\}_{\{u,d,L,e\}}$

(4,0) block before flavour decomposition

1460×1460



(4,0) block after flavour decomposition

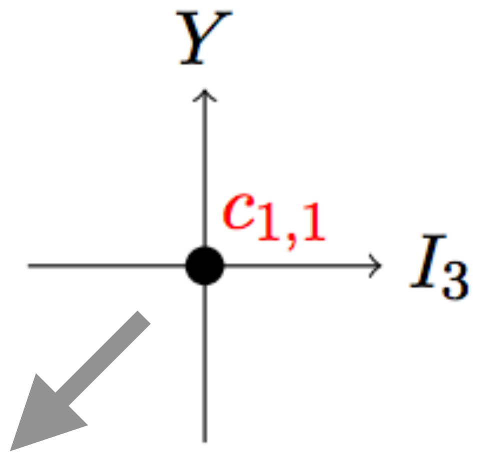
Largest block size 292×292

49 2×2 blocks

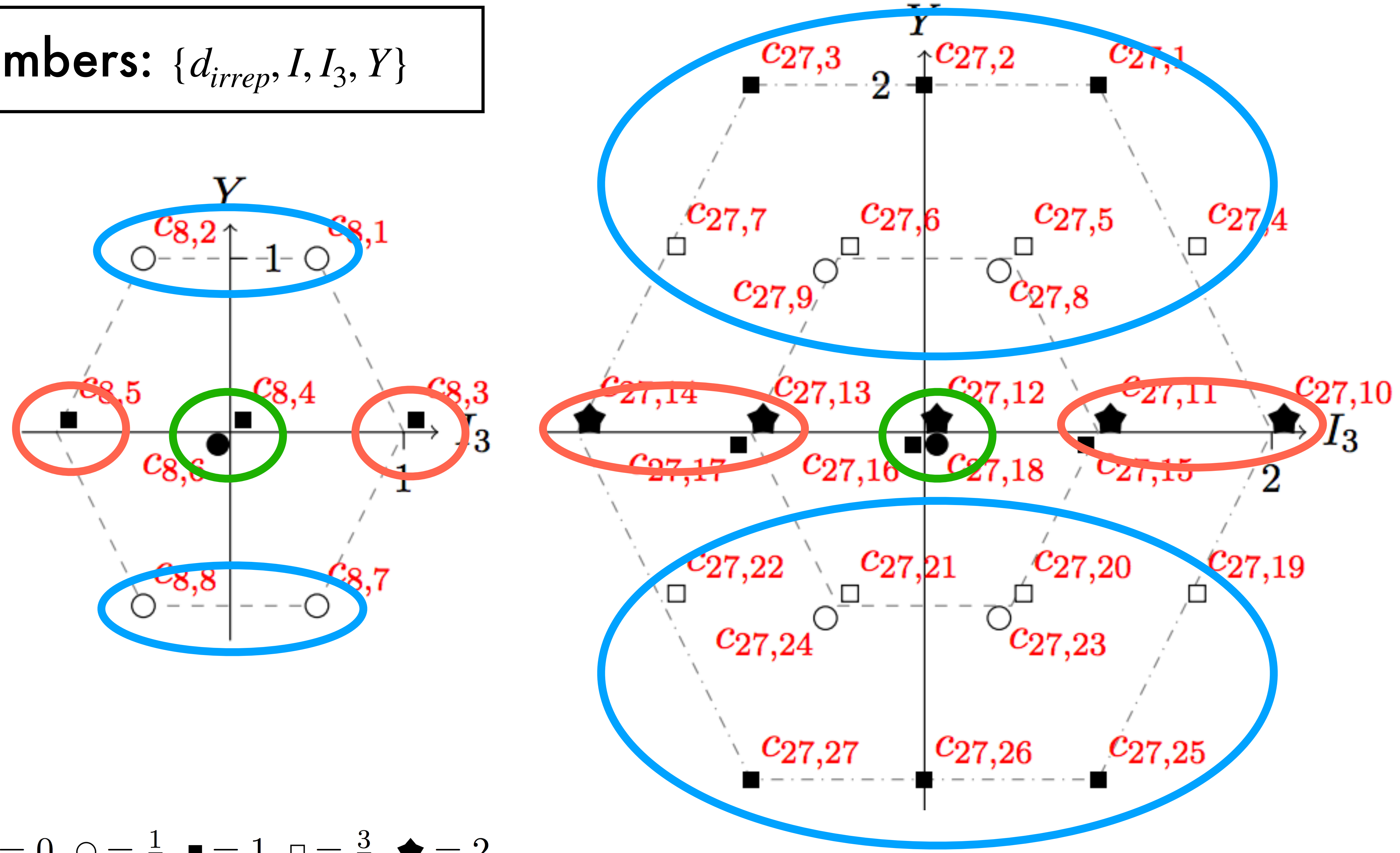
321 1×1 blocks

Flavour quantum numbers and pheno

For each flavour: $\boxed{\text{4 quantum numbers: } \{d_{irrep}, I, I_3, Y\}}$



Singlet:
flavour conserving &
flavour universal



Total I key: $\bullet = 0$, $\circ = \frac{1}{2}$, $\blacksquare = 1$, $\square = \frac{3}{2}$, $\bigstar = 2$

$d_{irrep} > 1, \ \{I_3, Y\} = 0$
Flavour conserving but non-universal

$I_3 > 0, \ Y = 0$
Flavour changing in first two generations only

$Y \neq 0$
Flavour changing involving 3rd generation

For 27-plet, the larger the values of I_3 and Y, the more flavour violating