COMPUTATIONAL ROD CLIMBING AND DIPPING DEPENDENT UPON NORMAL STRESSES

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ABSTRACT

With recent advancement in accurately measuring the second normal stress difference, the rheology community becomes interested in the Weissenberg effect, i.e., rod climbing[1], which is caused and regulated by the second (N_2) as well as the first normal stress difference (N_1). In this study, computational description of the rod climbing is carried out with the Leonov viscoelastic model implemented. First, we adjust the relative magnitude of the first and the second normal stress differences in simple shear flow in terms of constitutive modification of the equations within appropriateness for mathematical well-posedness. Applying the finite element analysis combined with the level set method, we observe that the climbing height of the liquid decreases as the relative strength of the second normal stress increases and eventually the rod climbing transforms into the rod dipping. This transition is proven to occur when the absolute value of the second normal stress difference is a quarter of the first one, which coincides with the classical result derived under the assumption of the second order fluid[2]. Furthermore, it is also shown to incur reversal of rotational direction in secondary flow.

RESULTS AND DISCUSSION

To numerically express the rod climbing of the viscoelastic liquid, we employ the Leonov viscoelastic constitutive equation. Following is the mathematical representation of rheological field equations applied in this study:

$$\overset{\nabla}{\mathbf{c}} + \frac{1}{\theta}\left[b_1\left(\mathbf{c}^2 - \frac{I_1 + I_2}{3}\mathbf{c} + \boldsymbol{\delta}\right) + \frac{I_2}{3}\mathbf{c} - \boldsymbol{\delta}\right] = \mathbf{0},$$

$$\text{Mooney potential: } U = \frac{G}{2}[(1-\beta)I_1 + \beta I_2 - 3],$$

$$\boldsymbol{\tau} = 2(1-s)\left(\frac{\partial U}{\partial I_1}\mathbf{c} - \frac{\partial U}{\partial I_2}\mathbf{c}^{-1}\right) + 2s\eta\mathbf{e} = (1-s)G[(1-\beta)\mathbf{c} - \beta\mathbf{c}^{-1}] + 2s\eta\mathbf{e}. \qquad (1)$$

Here $\mathbf{c}$ is the conformation tensor expressing the recoverable strain, $\mathbf{v}$ is velocity, G is the modulus, θ is the relaxation time, $\eta = G\theta$ is the zero shear viscosity, the invariants of $\mathbf{c}$ are defined as $I_1 = \mathrm{tr}\,\mathbf{c}$ and $I_2 = \frac{1}{2}\left(I_1{}^2 - \mathrm{tr}\,\mathbf{c^2}\right)$, and $\overset{\nabla}{\mathbf{c}} \equiv \frac{d\mathbf{c}}{dt} - \nabla\mathbf{v}^T \cdot \mathbf{c} - \mathbf{c} \cdot \nabla\mathbf{v}$ is its upper convected time derivative. $\boldsymbol{\tau}$ is the stress, s is the retardation parameter and $\mathbf{e} = 1/2\,(\nabla\mathbf{v} + \nabla\mathbf{v}^T)$. b_1 and β are fitting parameters for proper description of liquid, they as well control the magnitude of the normal stress differences in shear flow and their admissible range lie between 0 and 1. Since the steady shear flow curve (shear rate vs. shear stress) shows non-monotonic behaviour in the case of $1/2 < b_1 \le 1$, its specification in $0 \le b_1 \le 1/2$ is appropriate for physical consistency as well as mathematical stability.

Simple analysis shows that the relation $-N_2/N_1 = b_1/2$ in simple shear flow holds in the asymptotic limit of vanishing shear rate when β = 0. Also, we can prove that the effect of normal stresses in the Weissenberg effect disappears when $-N_2/N_1 = 1/4$ and the rod climbing (precisely dipping) is solely determined by the centrifugal force, which exactly coincides with the classical analysis obtained for the second order fluid[2].

Rod climbing is a viscoelastic phenomenon expressed by the combination of normal stresses, centrifugal force and surface tension. Adjusting values of β as well as b_1 and thus modulating the relative amount of the first and second normal stress differences in the flow, we obtain the result of **Fig. 1** which shows simulated interfaces of rotating viscoelastic liquid according to the ratio of normal stresses on the rod surface. One can distinctly observe that the height of liquid surface near the rod decreases and eventually the climbing transforms to the dipping as the relative amount of N_2 increases.

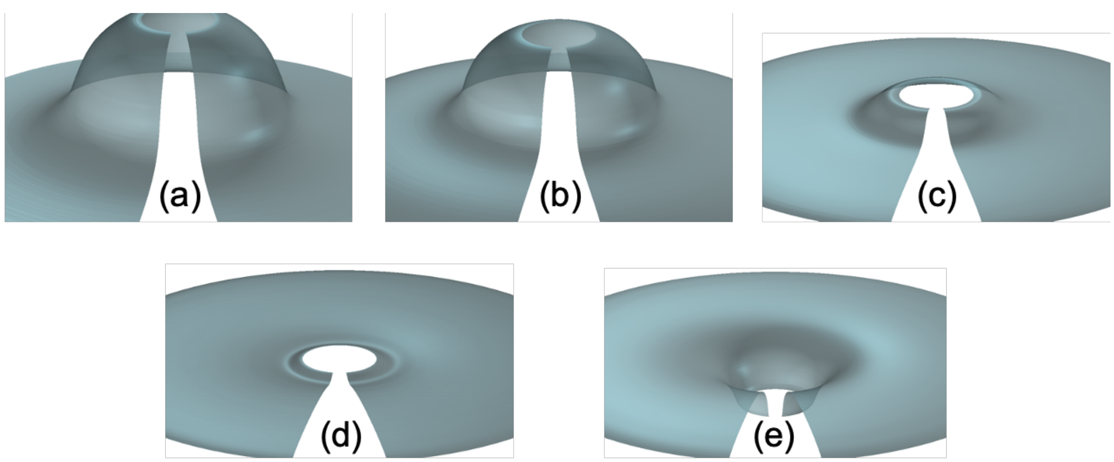

Figure 1: Shapes of gas-liquid interface at the Deborah number $De = 0.298$ for $-N_2/N_1$ equal to (a) 0; (b) 0.092; (c) 0.187; (d) 0.236; (e) 0.286

Indeed, the surface profile is shown to be determined mainly by the weak secondary flows, and at the transition of rod climbing into dipping the reversal of rotational direction in secondary flow is found to occur. The present work suggests that the rod climbing experiment cooperated with computational modelling may be utilized at least as a tool supporting the rheometry measuring normal stresses in simple shear flow.

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