Effect of local relaxation time on drag reduction in turbulent boundary layer flow of viscoelastic fluids

Shinji Tamano[1], Hiromi Osawa[1], and Masakazu Muto[1]

[1]Nagoya Institute of Technology, Nagoya, Japan

ABSTRACT

We performed direct numerical simulations (DNSs) of a zero-pressure-gradient turbulent boundary layer flow of viscoelastic fluids with variations of solution concentration and temperature in order to investigate the effect of local variation in the relaxation time on the drag reduction. To this end, we proposed a new constitutive equation model based on the FENE-P model, c-T-FENE-P model, in which the FENE-P model was coupled with the scalar equations of solution concentration and temperature. In this study, DNSs for 6 cases of increasing and decreasing relaxation times with temperature were performed.

INTRODUCTION

As reported in recent reviews,[1] numerous DNS studies of wall-bounded turbulent flows of viscoelastic fluids can predict some experimental findings such as the steeper gradient of mean velocity profile in wall-units and more suppression of turbulence structures with the amount of the drag reduction. In the minimal channel flow, Xi & Graham[2] found that the instantaneous levels of polymer stretching and drag reduction were anticorrelated in time. Tamano et al.[3] also revealed that what occurs temporally in the minimal channel[2] was similar to what occurred spatially in boundary layer flow, and the streamwise profile of drag reduction ratio shifted downstream, as the Weissenberg number, i.e. the relaxation time became larger. However, the effect of local variation in relaxation time on the drag reduction still remains unknown in wall-bounded turbulent flows of viscoelastic fluids, since the relaxation time is constant in space and time in previous studies based on the FENE-P model. In this study, the effect of the local relaxation time on the drag reduction in wall-bounded turbulent flows is investigated by coupling the FENE-P model with equations of solution concentration and temperature.

NUMERICAL METHOD AND CONDITIONS

The non-dimensional governing equations for the incompressible viscoelastic flow are continuity and momentum equations:

$$\frac{\partial u_i}{\partial x_i} = 0, \frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1-\beta}{Re_{\theta_0}}\frac{\partial E_{ij}}{\partial x_j} + \frac{\beta}{Re_{\theta_0}}\frac{\partial^2 u_i}{\partial x_j \partial x_j}, \tag{1}$$

where u_i is the velocity component, p is pressure, x_i is a spatial coordinate, t is time, and E_{ij} is the viscoelastic stress component. In this paper, x_1 (x), x_2 (y) and x_3 (z) directions are streamwise, wall-normal and spanwise, respectively. $\beta = \eta_s/\eta_0$ is the ratio of solvent viscosity η_s to zero shear rate viscosity of solution η_0. In the present study, the inflow condition for the boundary layer is given by the Lund's method,[4] so that the computational domain is divided into the main and driver parts in which the inflow condition for the main part is obtained. In Eq. 1, the momentum-thickness Reynolds number Re_{θ_0} is defined as, $Re_{\theta_0} = \rho U_e \theta_0/\eta_0$, where U_e is the free-stream velocity, θ_0 is the momentum thickness at the inlet plane of the driver part, ρ is density. The non-dimensional FENE-P constitutive equation for conformation tensor C_{ij} for inhomogeneous polymer solutions is as follows:[4]

$$\frac{\partial C_{ij}}{\partial t} + u_k \frac{\partial C_{ij}}{\partial x_k} - \frac{\partial u_i}{\partial x_k} C_{kj} - \frac{\partial u_j}{\partial x_k} C_{ik} = -nE_{ij}. \tag{2}$$

The viscoelastic stress component is related to the conformation tensor, $E_{ij} = (fC_{ij} - \delta_{ij})/Wi$, where f is the Peterlin function $f = L^2/(L^2 - Tr(C_{ij}))$, and L represents the maximum extension of polymer. The Weissenberg number Wi is defined as, $Wi = \lambda U_e/\theta_0$, where λ is the relaxation time.

In Eq. 2, n is the number of density, i.e. the polymer concentration which is described by the following equation:[4]

$$\frac{\partial n}{\partial t} + u_j \frac{\partial n}{\partial x_j} = \frac{1}{Pe_n} \nabla^2 n, \tag{3}$$

where $Pe_n = ScRe_{\theta_0}$ is the Péclet number for concentration. In this study, the Schmidt number is set at $Sc = 1$ for the computational limitation. We called this type of FENE-P model[4] coupled with the concentration equation "c-FENE-P model". In this model, the relaxation time is constant ($\lambda = \lambda_0$), so that Wi is the same as the setting Weissenberg number $Wi_0 (= \lambda_0 Ue/\theta_0)$.

In this study, moreover, we introduced the equation of the non-dimensional solution temperature T as follows:

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \frac{1}{Pe_T} \nabla^2 T, \tag{4}$$

where $Pe_T = PrRe_{\theta_0}$ is the Péclet number for temperature. In this study, the Prandtl number is set at $Pr = 1$ for simplicity. Here, it is assumed that the relaxation time λ is the simple function of the solution temperature, $\lambda(T) = \lambda_0 T^{\pm 0.5, \pm 1.0, \pm 2.0}$, so that the Weissenberg number is rewritten as,

$$Wi = \frac{\lambda(T) Ue}{\theta_0} = \frac{\lambda(T)}{\lambda_0} Wi_0. \tag{5}$$

We name this type of constitutive equation model "c-T-FENE-P model."

The second-order accurate finite difference scheme on a staggered grid is used. The semi-implicit time marching algorithm is used where the diffusion term in the wall-normal direction is treated implicitly with the Crank-Nicolson scheme, and the third-order Runge-Kutta scheme is used for all other terms. The setting momentum-thickness Reynolds and Weissenberg numbers are $Re_{\theta_0} = 670$ and $Wi_0 = 50$, respectively. The viscosity ratio is $\beta = 0.9$. The maximum extension of the polymer L is 100. The size of the computational domain for the present simulations is equal to $(L_x \times L_y \times L_z)$ $= (300\theta_0 \times 30\theta_0 \times 20\pi\theta_0/3)$ in the streamwise, wall-normal, and spanwise directions, respectively. The grid size is $(N_x \times N_y \times N_z) = (384 \times 64 \times 64)$. The concentration inlet boundary condition is given by the constant Gaussian profile.[4] The inflow temperature is constant ($T = T_{in}$), and the isothermal heated wall boundary condition ($T_w = 4T_{in}$) is imposed.

RESULTS

The streamwise variation in the drag reduction ratio DR is shown in Fig. 1. $DR(x)$ is defined as,

$$DR(x) = \frac{C_{f_{\text{Newtonian}}}(x) - C_{f_{\text{Viscoelastic}}}(x)}{C_{f_{\text{Newtonian}}}(x)}, \tag{6}$$

where $C_{f_{\text{Newtonian}}}(x)$ and $C_{f_{\text{Viscoelastic}}}(x)$ are the skin friction coefficients for Newtonian and viscoelastic fluids, which are functions of the streamwise position x and are evaluated at the same x. For the case of increasing relaxation time with temperature (open symbols), the DR of the c-T-FENE-P model is smaller than that of the c-FENE-P model ($\lambda = \lambda_0$), while it is larger for the case of decreasing relaxation time with temperature (closed symbols).

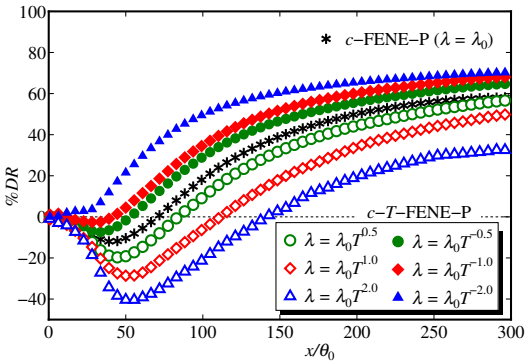

Figure 1: Streamwise variation in drag reduction ratio

CONCLUSIONS

Direct numerical simulations of the zero-pressure gradient drag-reducing turbulent boundary layer of viscoelastic fluids were performed using the proposed c-T-FENE-P model. The effect of local variation in the relaxation time on the drag reduction ratio was discussed.

ACKNOWLEDGEMENTS

This work was supported by a Grant-in-Aid for Scientific Research (JP15H03918, 20H02065) from Japan Society for Promotion of Science.

REFERENCES

1. Xi, L. Turbulent drag reduction by polymer additives: Fundamentals and recent advances, *Phys. Fluids*, **31**, No. 121302, 2019.
2. Xi, L., Graham M.D. Active and hibernating turbulence in minimal channel flow of Newtonian and polymeric fluids, *Phys. Rev. Lett.*, **104**, No. 218301, 2010.
3. Tamano, S., Graham, M.D., Morinishi, Y. Streamwise variation of turbulent dynamics in boundary layer flow of drag-reducing fluid, *J. Fluid Mech.*, **686**, 352-377, 2011.
4. Dimitropoulos, C.D., Dubief, Y., Shaqfeh, E.S.G., Moin, P. Direct numerical simulation of polymer-induced drag reduction in turbulent boundary layer flow of inhomogeneous polymer solutions, *J. Fluid Mech.*, **566**, 153-162, 2006.