HYSTERESES IN ONE-DIMENSIONAL COMPRESSION OF A POROELASTIC HYDROGEL

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ABSTRACT

We investigate theoretically the one-dimensional compression of a hydrogel layer by a uniform fluid flow normal to the gel surface. The flow is driven by a pressure drop across the gel layer, which is modelled as a poroelastic medium. Since the pressure simultaneously drives the Darcy flow through the pores and compresses the gel, the flux-pressure relationship is in general non-monotonic. Most interestingly, we discover hysteresis when the pressure drop is controlled, which are also confirmed by transient numerical simulations. The hysteresis stems from the interplay between the gel compression at the upstream interface and that in the bulk of the gel, and would not be predicted by models that ignore the interfacial compression. Finally, we suggest experimental setups and conditions to seek such hystereses in real gels.

METHODS

Consider the one-dimensional (1D) compression of a layer of hydrogel by a normal ow of the solvent (Fig. 1). The gel layer extends to infinity in y-direction and the uniform flow with the flux Q in x-direction is driven by the prescribed pressure drop across the layer. The downstream surface of the gel layer is fixed in space but allows passage of the solvent, as if by a stiff and highly permeable mesh. The upstream gel surface is displaced by the flow as the layer compacts. The layer of hydrogel is modelled as a poroelastic medium, with volume fractions ϕ_f and $\phi_s = 1-\phi_f$ for the fluid and solid components. We assume Darcy flow inside the gel layer and adopt the same constitutive relation as Hewitt *et al.*[1], which is close to the linear elastic behavior for small strains but diverges as the strain approach to a prescribed maximum value.

In our study, we compared the influence of various permeability models and entry boundary conditions of the layer of hydrogel. Specifically, we investigated the difference between the Kozeny-Carmen model and our own permeability model with a constant drag coefficient. For boundary conditions, we examined two options: the stress-free boundary condition used by Hewitt *et al*[1] and our entry condition based on interfacial permeability η.[2,3] The following results will be in dimensionless variables that have been properly scaled.

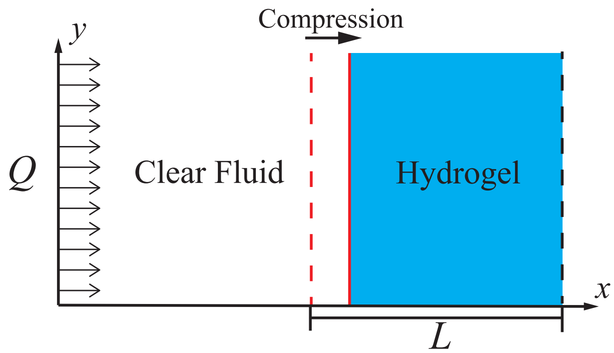

Figure 1: A diagram showing the 1-D compression of a layer of hydrogel. The dashed red line denotes the initial interface between the clear fluid and the hydrogel, and the solid red line the displaced interface under compression. The downstream surface of the gel layer is constrained by a fixed permeable plate (the black dash line), and has zero displacement.

RESULTS

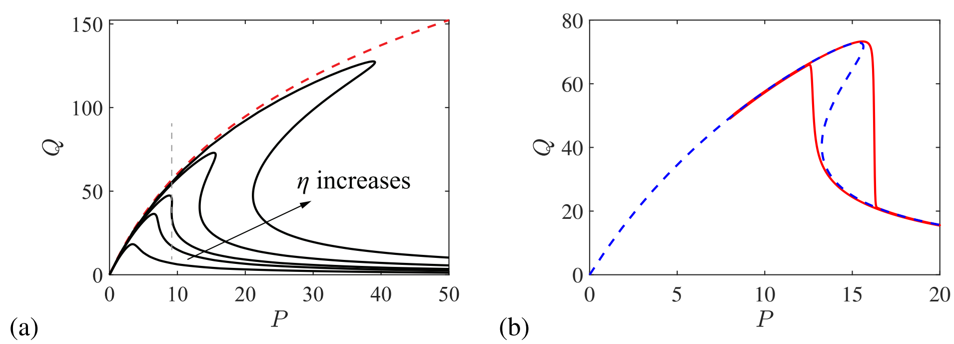

Figure 2: (a) The flux-pressure $Q \sim P$ curve with different $\eta = 1, 2, 2.6, 4, 7$. The gray dashed line marks the vertical segment of the curve at the threshold $\eta \approx 2.6$, and the red dashed line represents the curve at the limit of $\eta \to \infty$; (b) Hysteresis observed by increasing P gradually from $P = 8.2$ to 20, and then decreasing it back to 8.2, at a constant rate $dP/dt = \pm 25$. The red solid lines represent the transient solution whereas the blue dashed lines the steady-state solution corresponding to the $\eta = 4$ curve of (a).

We will present two types of results, steady-state $Q(P)$ relationships that exhibit hystereses, and transient simulations to confirm the hysteretic behaviors expected in an experimental setup. Numerical method is shown in our previous study[4]. Here we present predictions of our entry boundary condition and permeability model. The interfacial permeability η turns out to be a key parameter for the flux-pressure relation. Fig. 2(a) depicts the steady-state flow rate Q as a function of the pressure drop P for a range of η values. Two distinct behavior can be identified. For small η, Q initially increases with P, reaches a maximum, and then declines as P increases further. In this regime, there is a single Q value for each P, and no hysteresis exists. Above a threshold $\eta = 2.6$, the curve overturns as a breaking wave to produce three Q for an intermediate range of the P. Now hysteresis is expected if one varies P in small increments and monitors the reaction of Q. With greater η values, the achievable flow rates are higher, and the curve extends toward the upper-right of the plot, with greater jumps expected for the hysteresis.

The hysteretic behavior, exemplified by η = 4 in Fig. 2(a), is more intricate. We note that the transient solution largely confirms the expectations from the analytical steady solution, with hysteretic jumps when P crosses thresholds while increasing and decreasing. we can rationalize the hysteresis from the dynamics of $Q(P)$ which clearly indicates two distinct states; let us call them "relaxed" and "compressed" for brevity. The hysteresis happens for pressure between two thresholds, $13 \leq P \leq 16.2$. As P increases from below the lower threshold $P_L = 13$, The flux keeps increasing with pressure. The interfacial compression remains small and the upstream portion of the gel layer stays "relaxed", until P reaches the upper threshold of $P_U = 16.2$, when the upstream portion of the gel suddenly becomes "compressed", accompanied by a drop in the fluid velocity. A similar hysteresis occurs when P decreases from above the upper threshold. Note that the hysteretic behaviors can be observed from the flux as well as the thickness of the layer and interfacial volume fraction. We have explained the observed behaviors by analyzing the total pressure drop into two components: interfacial and bulk. Additionally, we discussed the condition of hysteresis in relation to various permeability models and entry conditions.

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