Lubrication force between two approaching cylinders in a Bingham fluid

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ABSTRACT

We examine the problem of two circular cylinders immersed in a Bingham fluid approaching each other at zero Reynolds number. The goal of this study is to understand how lubrication forces manifest in the small gap between the cylinders as a function of the Bingham number. We consider both (i) the case of purely normal lubrication force when the relative position vector and the velocity vector of both cylinders are aligned and (ii) the case with normal and tangential lubrication forces when this alignment is broken.

We select a large enough flow domain such that the assumption of two cylinders immersed in an infinite domain is reasonably satisfied and the spurious influence of the boundary conditions at the limit of the finite size domain is minimal in the case of small Bingham numbers. The flow domain is therefore a simple and large rectangle and the two cylinders are located close to its center. We compute the two-dimensional steady flow solution for a set of cylinder positions and cylinder velocities. The set of governing equations comprise the Stokes equations and the Bingham constitutive law, combined to Dirichlet conditions imposed on the velocity field over all boundaries of the flow domain. While the magnitude of the cylinder velocities is not a relevant parameter, we coherently set it to a value such that the corresponding Reynolds number is indeed very small and representative of Stokes flow conditions. The solution is obtained by an Augmented Lagrangian algorithm or a FISTA algorithm that both solve the Bingham model without any regularization [1]. We use a boundary fitted Finite Element discretization scheme with either P1-P2-P1DC or P1-P1isoP2-P0isoP2 elements for pressure, velocity and strain rate tensor components, respectively, combined to adaptive unstructured mesh refinement to capture the details of the flow field and the yield surfaces as accurately as possible [3]. The code is implemented on the FreeFEM++ platform [2] and is validated through comparisons with results published in the literature on other similar flow configurations [3].

We discuss the computed results in terms of flow field, yielded/unyielded regions and magnitude of the lubrication force. Given that our computed results fully resolve the flow in the thin lubrication gap, we are also in a position to compare them to approximate analytical solutions previously published in the literature [4]. The findings of this work also contribute to extend our understanding of the role of lubrication in dense suspensions of rigid particles in a yield stress fluid.

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