Achilles: The BSM Pipeline

Joshua Isaacson

In Collaboration with: S. Antu, R. Farnsworth, S. Höche, W. Jay, D. Lopez Gutierrez, A. Lovato, P.A.N. Machado, L. Pickering, N. Rocco, S. Wang

PITT PACC Workshop: Nu Tools for BSM at Neutrino Beam Facilites

16 December 2022

Achilles: A CHIcago Land Lepton Event Simulator

Project Goals:

- Theory driven
- Develop modular neutrino event generator
- Provide means for easy extension by end users
- Provide automated BSM calculations for neutrino experiments
- Evaluate theory uncertainties

Measuring Oscillation Parameters

Experimental Events

$$\frac{N_{FD}}{N_{ND}}(E_{\text{reco}}) \propto \frac{\int dE_\nu \frac{d\phi_\alpha^{\text{FD}}}{dE_\nu} \; P(\nu_\alpha \to \nu_\beta; E_\nu) \;\; \sigma_\beta(E_\nu) \;\; \mathcal{M}_\alpha^{\text{FD}}(E_\nu, E_{\text{reco}})}{\int dE_\nu \frac{d\phi_\alpha^{\text{ND}}}{dE_\nu} \; \sigma_\alpha(E_\nu) \;\; \mathcal{M}_\alpha^{\text{ND}}(E_\nu, E_{\text{reco}})}$$

Measuring Oscillation Parameters

Experimental Events

$$\frac{N_{FD}}{N_{ND}}(E_{\text{reco}}) \propto \frac{\int dE_\nu \frac{d\phi_\alpha^{\text{FD}}}{dE_\nu} \; P(\nu_\alpha \to \nu_\beta; E_\nu) \;\; \sigma_\beta(E_\nu) \;\; \mathcal{M}_\alpha^{\text{FD}}(E_\nu, E_{\text{reco}})}{\int dE_\nu \frac{d\phi_\alpha^{\text{ND}}}{dE_\nu} \; \sigma_\alpha(E_\nu) \;\; \mathcal{M}_\alpha^{\text{ND}}(E_\nu, E_{\text{reco}})}$$

- Number of events in the near/far detector

Measuring Oscillation Parameters

Experimental Events

$$\frac{N_{FD}}{N_{ND}}(E_{\rm reco}) \propto \frac{\int dE_\nu \frac{d\phi_\alpha^{\rm FD}}{dE_\nu} P(\nu_\alpha \to \nu_\beta; E_\nu)\ \sigma_\beta(E_\nu)\ \mathcal{M}_\alpha^{\rm FD}(E_\nu, E_{\rm reco})}{\int dE_\nu \frac{d\phi_\alpha^{\rm ND}}{dE_\nu}\ \sigma_\alpha(E_\nu)\ \mathcal{M}_\alpha^{\rm ND}(E_\nu, E_{\rm reco})}$$

- Number of events in the near/far detector
- Probability to oscillate from α to β

Measuring Oscillation Parameters

Experimental Events

$$\frac{N_{FD}}{N_{ND}}(E_{\rm reco}) \propto \frac{\int dE_\nu \frac{d\phi_\alpha^{\rm FD}}{dE_\nu} P(\nu_\alpha \to \nu_\beta; E_\nu)\, \sigma_\beta(E_\nu)\, \mathcal{M}_\alpha^{\rm FD}(E_\nu, E_{\rm reco})}{\int dE_\nu \frac{d\phi_\alpha^{\rm ND}}{dE_\nu} \sigma_\alpha(E_\nu)\, \mathcal{M}_\alpha^{\rm ND}(E_\nu, E_{\rm reco})}$$

- Number of events in the near/far detector
- Probability to oscillate from α to β
- Neutrino-nucleus interaction cross section

Measuring Oscillation Parameters

Experimental Events

$$\frac{N_{FD}}{N_{ND}}(E_{\text{reco}}) \propto \frac{\int dE_\nu \frac{d\phi_\alpha^{\text{FD}}}{dE_\nu} P(\nu_\alpha \to \nu_\beta; E_\nu)\, \sigma_\beta(E_\nu)\, \mathcal{M}_\alpha^{\text{FD}}(E_\nu, E_{\text{reco}})}{\int dE_\nu \frac{d\phi_\alpha^{\text{ND}}}{dE_\nu} \sigma_\alpha(E_\nu)\, \mathcal{M}_\alpha^{\text{ND}}(E_\nu, E_{\text{reco}})}$$

- Number of events in the near/far detector
- Probability to oscillate from α to β
- Neutrino-nucleus interaction cross section
- Migration matrix. Depends on topology of detected event (i.e. number of protons, etc.)

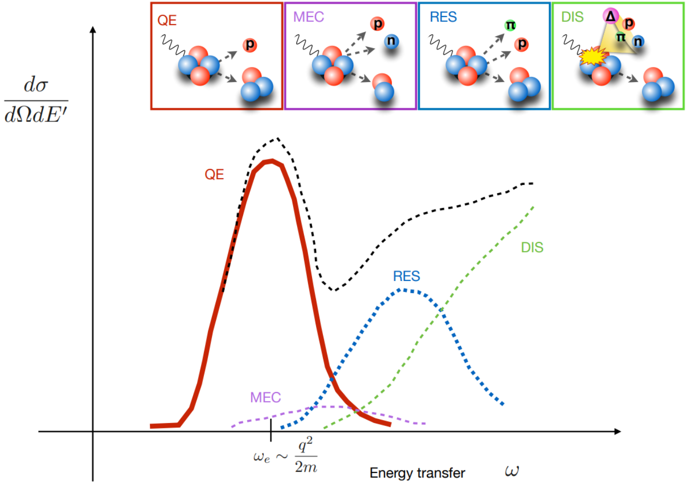
Measuring Oscillation Parameters

Experimental Events

$$\frac{N_{FD}}{N_{ND}}(E_{\text{reco}}) \propto \frac{\int dE_\nu \frac{d\phi_\alpha^{\text{FD}}}{dE_\nu} P(\nu_\alpha \to \nu_\beta; E_\nu)\, \sigma_\beta(E_\nu)\, \mathcal{M}_\alpha^{\text{FD}}(E_\nu, E_{\text{reco}})}{\int dE_\nu \frac{d\phi_\alpha^{\text{ND}}}{dE_\nu} \sigma_\alpha(E_\nu)\, \mathcal{M}_\alpha^{\text{ND}}(E_\nu, E_{\text{reco}})}$$

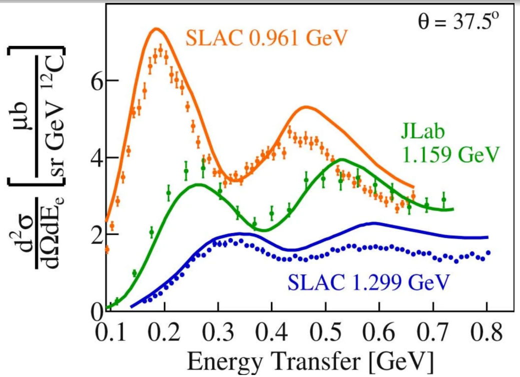
- Number of events in the near/far detector
- Probability to oscillate from α to β
- Neutrino-nucleus interaction cross section
- Migration matrix. Depends on topology of detected event (i.e. number of protons, etc.)
- Ratio helps control systematics, but cross sections do not cancel out from ratio $\to$ requires theory predictions

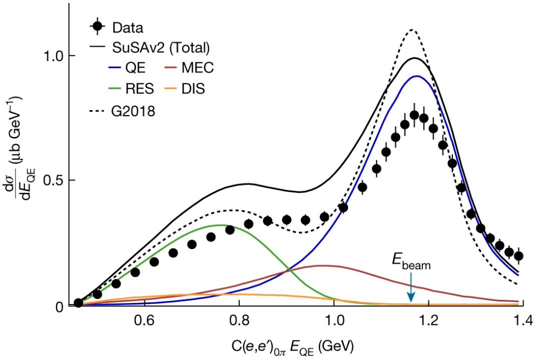
Lepton-Nucleus reaction processes



Credit: Noemi Rocco

Why a new generator? (CLAS / e4v)





- Differential distributions independent of nuclear physics work ok
- Reconstruction of the neutrino energy needs significant work

[Nature 599, 565–570 (2021)]

Why a new generator? (NOvA)

Source of Uncertainty	ν_e **signal** (%)	**Total beam background** (%)
Cross-section and FSI	7.7	8.6
Normalization	3.5	3.4
Calibration	3.2	4.3
Detector response	0.67	2.8
Neutrino flux	0.63	0.43
ν_e extrapolation	0.36	1.2
Total systematic uncertainty	9.2	11
Statistical uncertainty	15	22
Total uncertainty	18	25

[M. A. Acero, et al. NOvA collaboration, Phys. Rev. D 98, 032012]

- Cross section uncertainty one of dominant uncertainties
- NOvA systematics and statistical uncertainty equal
- DUNE and HyperK will have significantly more events

Why a new generator?

Oscillation Measurements

- Only measure events and not fluxes directly
- Fit oscillation parameters by taking ratio of number of events in E_{reco} bins
- Cross sections do not exactly cancel in ratio, thus they are crucial
- Requires fully differential predictions (Migration matrices):
  - Requires fully-exclusive predictions (*i.e* keep track of all particles in event simulation)
- DUNE and HyperK require precision on the cross sections of about 1%

Other Measurements

- The SBN program, and both DUNE and HyperK near detectors are general purpose
- Leverage them for BSM searches
- Requires both SM and BSM fully differential predictions

Separating Primary Interaction and Cascade

General Lepton-Nucleus Scattering Cross Section

$$d\sigma = \left(\frac{1}{|v_A - v_\ell|}\frac{1}{4E_A^{\text{in}}E_\ell^{\text{in}}}\right)|\mathcal{M}|^2 \prod_f \frac{d^3p_f}{(2\pi)^3}(2\pi)^4\delta^4\left(k_A + k_\ell - \sum_f p_f\right)$$

[**JI**, et. al. 2205.06378]

Separating Primary Interaction and Cascade

General Lepton-Nucleus Scattering Cross Section

$$\mathrm{d}\sigma = \left(\frac{1}{|v_A - v_\ell|}\frac{1}{4E_A^{\text{in}}E_\ell^{\text{in}}}\right)|\mathcal{M}|^2 \prod_f \frac{\mathrm{d}^3 p_f}{(2\pi)^3}(2\pi)^4\delta^4\left(k_A + k_\ell - \sum_f p_f\right)$$

Matrix Element Schematically

$$|\mathcal{M}\left(\{k\}\to\{p\}\right)|^2 = \left|\sum_{p'} \mathcal{V}(\{k\}\to\{p'\}) \times \mathcal{P}(\{p'\}\to\{p\})\right|^2$$

[JI, et. al. 2205.06378]

Separating Primary Interaction and Cascade

General Lepton-Nucleus Scattering Cross Section

$$d\sigma = \left(\frac{1}{|v_A - v_\ell|}\frac{1}{4E_A^{\text{in}}E_\ell^{\text{in}}}\right)|\mathcal{M}|^2 \prod_f \frac{d^3p_f}{(2\pi)^3}(2\pi)^4\delta^4\left(k_A + k_\ell - \sum_f p_f\right)$$

Matrix Element Schematically

$$|\mathcal{M}(\{k\} \to \{p\})|^2 = \left|\sum_{p'} \mathcal{V}(\{k\} \to \{p'\}) \times \mathcal{P}(\{p'\} \to \{p\})\right|^2$$

- Primary interaction

[JI, et. al. 2205.06378]

Separating Primary Interaction and Cascade

General Lepton-Nucleus Scattering Cross Section

$$d\sigma = \left(\frac{1}{|v_A - v_\ell|}\frac{1}{4E_A^{\text{in}}E_\ell^{\text{in}}}\right)|\mathcal{M}|^2 \prod_f \frac{d^3p_f}{(2\pi)^3}(2\pi)^4\delta^4\left(k_A + k_\ell - \sum_f p_f\right)$$

Matrix Element Schematically

$$|\mathcal{M}(\{k\} \to \{p\})|^2 = \left|\sum_{p'} \mathcal{V}(\{k\} \to \{p'\}) \times \mathcal{P}(\{p'\} \to \{p\})\right|^2$$

- Primary interaction
- Evolution out of nucleus

[JI, et. al. 2205.06378]

Separating Primary Interaction and Cascade

General Lepton-Nucleus Scattering Cross Section

$$\mathrm{d}\sigma = \left(\frac{1}{|v_A - v_\ell|}\frac{1}{4E_A^{\mathrm{in}}E_\ell^{\mathrm{in}}}\right)|\mathcal{M}|^2\prod_f \frac{\mathrm{d}^3 p_f}{(2\pi)^3}(2\pi)^4\delta^4\left(k_A + k_\ell - \sum_f p_f\right)$$

Matrix Element Schematically Approximation

$$|\mathcal{M}(\{k\} \to \{p\})|^2 = \sum_{p'} |\mathcal{V}(\{k\} \to \{p'\})|^2 \times |\mathcal{P}(\{p'\} \to \{p\})|^2$$

- Primary interaction
- Evolution out of nucleus
- Approximate as incoherent product of primary interaction and cascade

[**JI**, et. al. 2205.06378]

Aside: Relationship to LHC event generation

Neutrino Event Generation		LHC Event Generation
Primary neutrino interaction	$\longleftrightarrow$	Hard interaction
Intranuclear cascade	$\longleftrightarrow$	Parton shower

Intranuclear Cascade:

$$\exp\left\{-i\sum_{j=2}^{A}\int_0^t d\tau\,\Gamma_{k_i}(|\mathbf{r}_1+\mathbf{v}\tau-\mathbf{r}_j|)\right\}$$

Parton Shower:

$$\exp\left\{-i\sum_{i=1}^{k}\int \mathrm{d}^4x_i\; j_a^{\mu}(x_i)A_{\mu}^{a}(x_i)\right\}$$

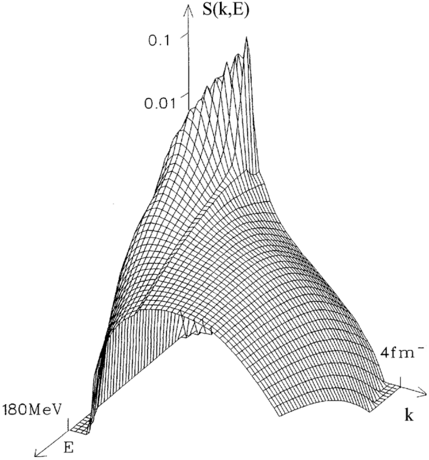
- Neutrino event generators can benefit from history of the LHC event generators push for precision

Factorization

- For Quasielastic scattering, factorize primary interaction as: $|\Psi_f\rangle = |p\rangle \otimes |\Psi_f^{A-1}\rangle$
- Initial state given via spectral function (probability distribution of removing a bound nucleon):

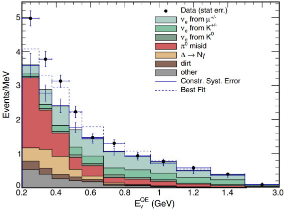
$$S_h(\mathbf{k}_h, E') = \sum_{f_{A-1}} |\langle\Psi_0|k\rangle \otimes |\Psi_f^{A-1}\rangle|^2 \delta(E' + E_0^A - E_f^{A-1})$$

- Spectral function based on correlated basis function theory [Phys. A 579, 493 (1994)]
- All but DIS implemented in this formalism [Phys.Rev.C 100 (2019) 4,045503] just need to interface with Achilles.
- Achilles provides general purpose interface to allow for other nuclear models

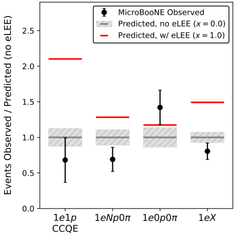


[Rev. Mod. Phys. 80, 189 (2008)]

BSM Motivation: MiniBooNE and MicroBooNE

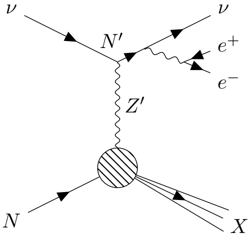


[PRL 121, 221801]



[arXiv:2110.14054]

- High intensity beams enable probes of weakly coupled BSM
- Probe different mass region than LHC
- MiniBooNE sees excess of events (MicroBooNE does not for single electrons)
- Other event generators cannot properly simulate these processes (requires properly handing spin correlations)



Using Currents

Hadronic tensor ($W^{\mu\nu}$) given by most general Lorentz structure

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \frac{\hat{p}_a^\mu \hat{p}_a^\nu}{p_a \cdot q} W_2 - i\epsilon^{\mu\nu\alpha\beta} \frac{q_\alpha p_{a\beta}}{2p_a \cdot q} W_3$$

Extending to BSM becomes complex to track all interferences:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_n} = \sum_{i,j} L_{\mu\nu}^{(ij)} W^{(ij)\mu\nu} = L_{\mu\nu}^{(\gamma\gamma)} W^{(\gamma\gamma)\mu\nu} + L_{\mu\nu}^{(\gamma Z)} W^{(\gamma Z)\mu\nu} + L_{\mu\nu}^{(Z\gamma)} W^{(Z\gamma)\mu\nu} + L_{\mu\nu}^{(ZZ)} W^{(ZZ)\mu\nu} + \cdots$$

Approaches such as the spectral function formalism can directly calculate currents:

$$W^\mu = \langle \psi_f^A | \mathcal{J}^\mu | \psi_0^A \rangle \to \sum_{p_a} \left[\langle \psi_f^{A-1} | \otimes \langle p_a | \right] | \psi_0^A \rangle \langle p_a + q | \sum_i \mathcal{J}_i^\mu | p_a \rangle,$$

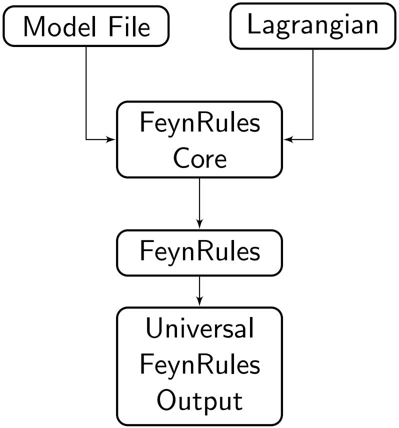
This enables the automatic handling of interferences

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_n} = \left| \sum_i L_\mu^{(i)} W^{(i)\mu} \right|^2$$

[**JI**, et. al. Phys. Rev. D 105 (2022) 9, 096006]

FeynRules

- *Mathematica* Program
- Takes model file and Lagrangian as input
- Calculates the Feynman rules
- Outputs in Universal FeynRules Output (UFO) format



[arXiv:0806.4194, arXiv:1310.1921]

Handling Form Factors

Nuclear one-body nucleon current operators:

$$\mathcal{J}^\mu = \left(\mathcal{J}_V^\mu + \mathcal{J}_A^\mu\right)$$
$$\mathcal{J}_V^\mu = \gamma^\mu \mathcal{F}_1^a + i\sigma^{\mu\nu} q_\nu \frac{\mathcal{F}_2^a}{2M}$$
$$\mathcal{J}_A^\mu = -\gamma^\mu \gamma_5 \mathcal{F}_A^a - q^\mu \gamma_5 \frac{\mathcal{F}_P^a}{M}$$

Coherent Form Factors (spin-0 nucleus):

$$\mathcal{J}^\mu = (p_{\text{in}} + p_{\text{out}})^\mu \mathcal{F}_{\text{coh}}$$

Standard Model Form Factors:

$$\mathcal{F}_i^{\gamma(p,n)} = F_i^{p,n}, \qquad \mathcal{F}_A^\gamma = 0$$
$$\mathcal{F}_i^{W(p,n)} = F_i^p - F_i^n, \qquad \mathcal{F}_A^W = F_A$$
$$\mathcal{F}_i^{Z(p)} = \left(\frac{1}{2} - 2\sin^2\theta_W\right) F_i^p - \frac{1}{2} F_i^n,$$
$$\mathcal{F}_i^{Z(n)} = \left(\frac{1}{2} - 2\sin^2\theta_W\right) F_i^n - \frac{1}{2} F_i^p$$
$$\mathcal{F}_A^{Z(p)} = \frac{1}{2} F_A, \qquad \mathcal{F}_A^{Z(n)} = -\frac{1}{2} F_A$$

[**JI**, et. al. Phys. Rev. D 105 (2022) 9, 096006]

Handling Form Factors

Nuclear one-body nucleon current operators:

$$\mathcal{J}^\mu = \left(\mathcal{J}_V^\mu + \mathcal{J}_A^\mu\right)$$
$$\mathcal{J}_V^\mu = \gamma^\mu \mathcal{F}_1^a + i\sigma^{\mu\nu} q_\nu \frac{\mathcal{F}_2^a}{2M}$$
$$\mathcal{J}_A^\mu = -\gamma^\mu \gamma_5 \mathcal{F}_A^a - q^\mu \gamma_5 \frac{\mathcal{F}_P^a}{M}$$

Coherent Form Factors (spin-0 nucleus):

$$\mathcal{J}^\mu = (p_{\text{in}} + p_{\text{out}})^\mu \mathcal{F}_{\text{coh}}$$

Straight-forward to extend to BSM

Standard Model Form Factors:

$$\mathcal{F}_i^{\gamma(p,n)} = F_i^{p,n}, \qquad \mathcal{F}_A^\gamma = 0$$
$$\mathcal{F}_i^{W(p,n)} = F_i^p - F_i^n, \qquad \mathcal{F}_A^W = F_A$$
$$\mathcal{F}_i^{Z(p)} = \left(\frac{1}{2} - 2\sin^2\theta_W\right) F_i^p - \frac{1}{2} F_i^n,$$
$$\mathcal{F}_i^{Z(n)} = \left(\frac{1}{2} - 2\sin^2\theta_W\right) F_i^n - \frac{1}{2} F_i^p$$
$$\mathcal{F}_A^{Z(p)} = \frac{1}{2} F_A, \qquad \mathcal{F}_A^{Z(n)} = -\frac{1}{2} F_A$$

[**JI**, et. al. Phys. Rev. D 105 (2022) 9, 096006]

Recursive Matrix Element Generation

$$\mathcal{J}_\alpha(\pi) = P_\alpha(\pi) \sum_{\mathcal{V}_\alpha^{\alpha_1,\alpha_2}} \sum_{\mathcal{P}_2(\pi)} \mathcal{S}(\pi_1,\pi_2) V_\alpha^{\alpha_1,\alpha_2}(\pi_1,\pi_2) \mathcal{J}_{\alpha_1}(\pi_1) \mathcal{J}_{\alpha_2}(\pi_2)$$

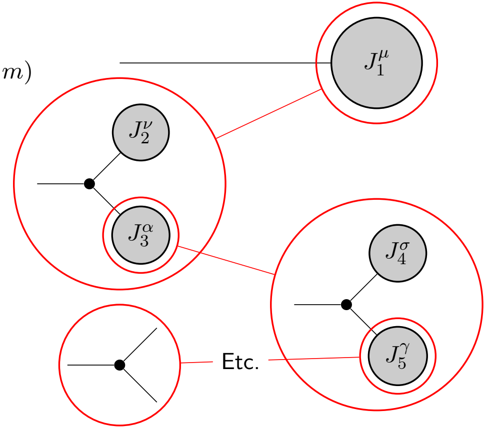
$$L_\mu^{(i)}(1,\dots,m) = \mathcal{J}_\mu^{(i)}(1,\dots,m)$$

$$L_{\mu\nu}^{(i,j)}(1,\dots,m) = \mathcal{J}_\mu^{(i)}(1,\dots,m)\mathcal{J}_\nu^{(j)\dagger}(1,\dots,m)$$

Berends-Giele Recursion

- Reuse parts of calculation
- Most efficient for high multiplicity
- Reduces computation from $\mathcal{O}(n!)$ to $\mathcal{O}(n^3)$

[Nucl. Phys. B306(1988), 759]



Phase Space Generation

$$d\Phi_n\left(a,b;1,\ldots,n\right) = \delta^{(4)}\left(p_a + p_b - \sum_{i=1}^{n} p_i\right)\left[\prod_{i=1}^{n}\frac{d^4 p_i}{(2\pi)^3}\delta\left(p_i^2 - m_i^2\right)\Theta\left(p_{i_0}\right)\right]$$

The above phase space definition does not contain the handling of initial states.

Algorithms for n-body phase space generation

- RAMBO [Comput. Phys. Commun. **40**(1986) 359]
- Multi-channel techniques [hep-ph/9405257]
- Recursive Phase Space [arXiv:0808.3674]

$2 \to 2$ Phase Space Example

Consider $l + {}^{12}C \to l' + N + X$ in the quasielastic regime.

$$\mathrm{d}\sigma \propto \; \mathrm{d}\Phi_2(a, b; 1, 2) \;\; \mathrm{d}^4 p_a \;\; \mathrm{d}^3 p_b$$

$2 \to 2$ Phase Space Example

Consider $l + {}^{12}C \to l' + N + X$ in the quasielastic regime.

$$d\sigma \propto d\Phi_2(a, b; 1, 2) \; d^4 p_a \; d^3 p_b$$

- Phase space: $d\Phi_2(a, b; 1, 2) = \frac{\lambda(s_{ab}, s_1, s_2)}{16\pi^2 2 s_{ab}} d\cos\theta_1 d\phi_1$

$2 \to 2$ Phase Space Example

Consider $l + {}^{12}C \to l' + N + X$ in the quasielastic regime.

$$\mathrm{d}\sigma \propto \mathrm{d}\Phi_2(a,b;1,2)\ \mathrm{d}^4 p_a\ \mathrm{d}^3 p_b$$

- Phase space: $\mathrm{d}\Phi_2(a,b;1,2) = \frac{\lambda(s_{ab},s_1,s_2)}{16\pi^2 2 s_{ab}} \mathrm{d}\cos\theta_1 \mathrm{d}\phi_1$
- Initial nucleon: $\mathrm{d}^4 p_a = |\vec{p}_a|^2 \mathrm{d}p_a \mathrm{d}E_r \mathrm{d}\cos\theta_a \mathrm{d}\phi_a$

$2 \to 2$ Phase Space Example

Consider $l + {}^{12}C \to l' + N + X$ in the quasielastic regime.

$$\mathrm{d}\sigma \propto \mathrm{d}\Phi_2(a,b;1,2)\ \mathrm{d}^4 p_a\ \mathrm{d}^3 p_b$$

- Phase space: $\mathrm{d}\Phi_2(a,b;1,2) = \frac{\lambda(s_{ab},s_1,s_2)}{16\pi^2 2 s_{ab}} \mathrm{d}\cos\theta_1 \mathrm{d}\phi_1$
- Initial nucleon: $\mathrm{d}^4 p_a = |\vec{p}_a|^2 \mathrm{d}p_a \mathrm{d}E_r \mathrm{d}\cos\theta_a \mathrm{d}\phi_a$
- Initial lepton (Here only monochromatic): $\mathrm{d}^3 p_b = \delta^3(p_b - p_{beam})\mathrm{d}^3 p_b$

$2 \to 2$ Phase Space Example

Consider $l + {}^{12}C \to l' + N + X$ in the quasielastic regime.

$$\mathrm{d}\sigma \propto \mathrm{d}\Phi_2(a, b; 1, 2)\ \mathrm{d}^4 p_a\ \mathrm{d}^3 p_b$$

- Phase space: $\mathrm{d}\Phi_2(a, b; 1, 2) = \frac{\lambda(s_{ab}, s_1, s_2)}{16\pi^2 2 s_{ab}} \mathrm{d}\cos\theta_1 \mathrm{d}\phi_1$
- Initial nucleon: $\mathrm{d}^4 p_a = |\vec{p}_a|^2 \mathrm{d}p_a \mathrm{d}E_r \mathrm{d}\cos\theta_a \mathrm{d}\phi_a$
- Initial lepton (Here only monochromatic): $\mathrm{d}^3 p_b = \delta^3(p_b - p_{beam}) \mathrm{d}^3 p_b$

Quasielastic Delta Function: $\delta(E_b - E_1 - E_r + m - E_2)$
Phase Space Delta Function: $\delta(E_a + E_b - E_1 - E_2)$
Define initial nucleon energy as $E_a = m - E_r$. Allows use of phase space tools developed at LHC.

Final State Interactions

Modify Primary Interaction:

- Captures rate change from FSI
- Loses all information about hadronic final state
- Primarily done using folding functions

Intranuclear Cascade:

- Unitary process (*i.e.* no rate change)
- Contains information about hadronic final state
- Primarily done via Monte Carlo methods

- **Note**: Both approaches attempt to capture effects from nuclear potential. Therefore, can only use one or the other to avoid double counting effects.
- **Note**: Intranuclear cascade is only method to provide fully exclusive final states required by experiments

[**JI**, et. al. 2205.06378]

Final State Interactions

Modify Primary Interaction:

- Captures rate change from FSI
- Loses all information about hadronic final state
- Primarily done using folding functions

Intranuclear Cascade:

- Unitary process (*i.e.* no rate change)
- Contains information about hadronic final state
- Primarily done via Monte Carlo methods

- **Note**: Both approaches attempt to capture effects from nuclear potential. Therefore, can only use one or the other to avoid double counting effects.
- **Note**: Intranuclear cascade is only method to provide fully exclusive final states required by experiments

Matrix Element Schematically Approximation

$$\left|\mathcal{M}\left(\{k\} \to \{p\}\right)\right|^2 = \sum_{p'} \left|\mathcal{V}(\{k\} \to \{p'\})\right|^2 \times \left|\mathcal{P}(\{p'\} \to \{p\})\right|^2$$

Algorithm Overview

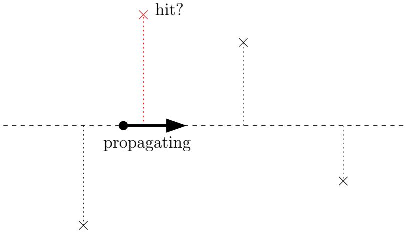
Algorithm Overview:

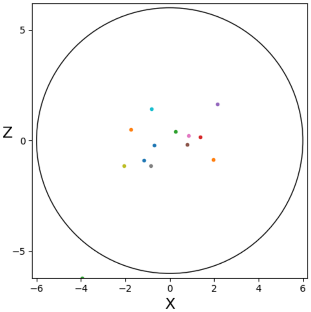
- Propagate struck nucleons
- Determine interactions based on impact parameter and cross-section
- Pauli blocking used to restrict final state phase space

Interaction Probabilities:

$$P_{\text{cyl}}(b) = \Theta(\sigma/\pi - b^2)$$

$$P_{\text{Gau}}(b) \equiv \exp\left(-\frac{\pi b^2}{\sigma}\right)$$

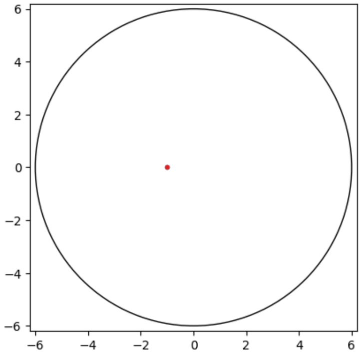




[JI, et. al. Phys. Rev. C 103(2021) 1, 015502] , [JI, et. al. 2205.06378]

Propagation with Potential

Initial Momentum: 250 MeV

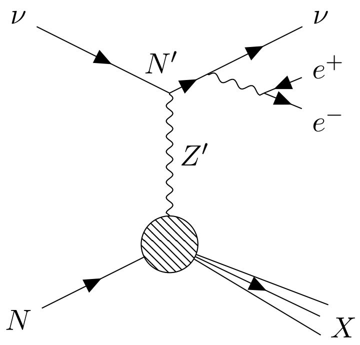


- Propagation using symplectic integrator for non-separable Hamiltonians [1609.02212]
- Energy is conserved to a high degree of precision
- Extremely stable

- Blue: Non-relativistic potential $(E = \sqrt{p^2 + m^2} + V)$
  [Phys. Rev. C. 38, 2967]
- Red: Relativistic optical potential $(E = \sqrt{p^2 + (m + S)^2} + V)$
  [Phys. Rev. C. 80, 034605]

[**JI**, et. al. 2205.06378]

Dark Neutrino



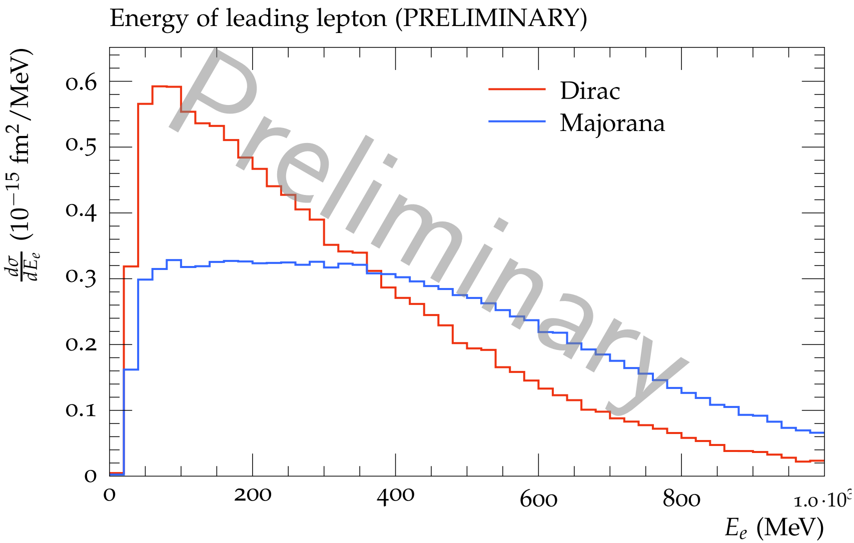
Key Notes:

- Handles both Dirac and Majorana fermions
- Results are flux-averaged over the MiniBooNE / MicroBooNE neutrino flux
- Generates full $2 \to 4$ body phase space with complete angular dependence included (Neglecting spin correlations limits you to only the Majorana case)

Parameters:

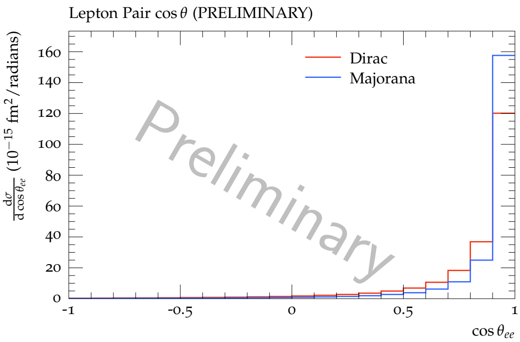
- $m_{N'} = 420$ MeV
- $m_{Z'} = 30$ MeV
- $\alpha_D = 0.25$
- $\alpha\epsilon^2 = 2 \times 10^{-10}$
- $|U_{42}^{\mu}| = 9 \times 10^{-7}$

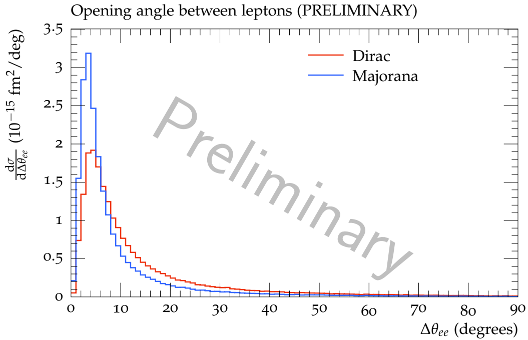
Dark Neutrino



Dark Neutrino

- No cuts applied yet
- Typical opening angle around 5-6 degrees
- MiniBooNE needs separation of about 10 degrees to distinguish 1 or 2 electrons





- Need to include background to compare to MiniBooNE data
- Simulate possible MicroBooNE limits

MicroBooNE Simulation

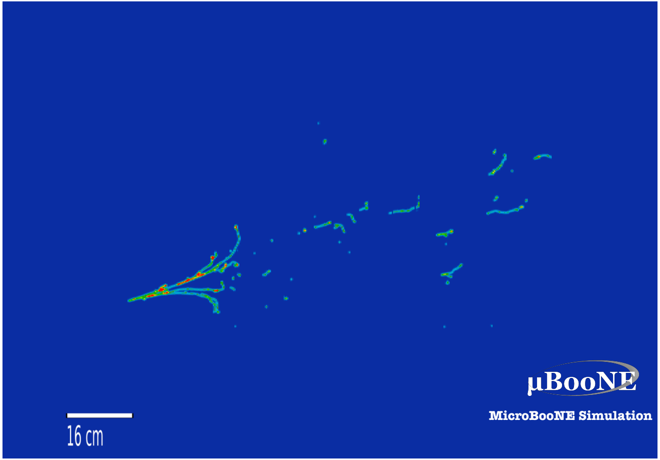
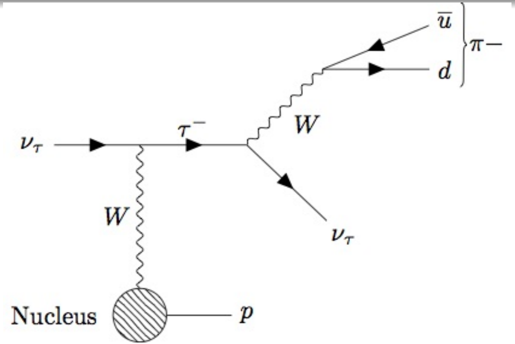
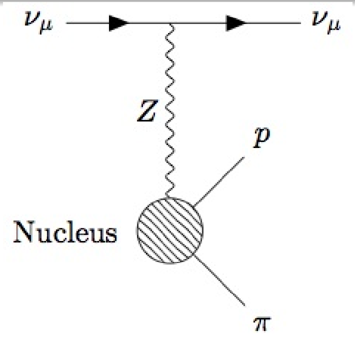


Image generated by the MicroBooNE collaboration using Achilles

- Working on implementing into MicroBooNE Pipeline
- Developing interface to LArSoft

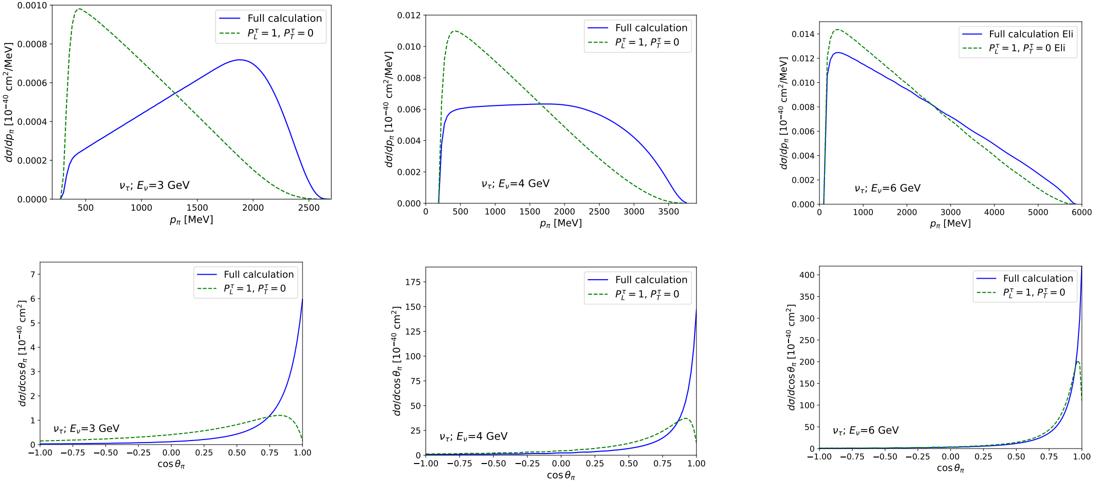
Tau Polarization





- ν_τ least understood particle (only detected a total of 14 events have been positively identified)
- DUNE ν_τ sample will be most important given reconstruction, statistics, and background rejection
- Need to separate NC background from ν_τ CC interactions [2007.00015]
- Key is to understand angular distribution of decay products
- Requires properly handling the τ polarization [1906.05656] [2202.07539]

Tau Decay to Pion Angular Distributions



Analytical calculation from: [2202.07539] only for $\nu_\tau {}^{16}O \to \tau X, \tau \to \pi\nu_\tau$

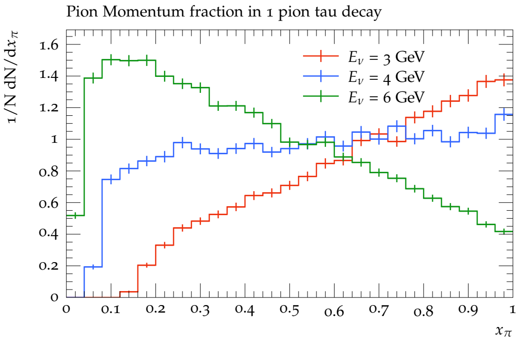
Taus in Achilles

Decay mode	Branching ratio
Leptonic	35.2%
$e^-\bar{\nu}_e\nu_\tau$	17.8%
$\mu^-\bar{\nu}_\mu\nu_\tau$	17.4%
Hadronic	64.8%
$\pi^-\pi^0\nu_\tau$	25.5%
$\pi^-\nu_\tau$	10.8%
$\pi^-\pi^0\pi^0\nu_\tau$	9.3%
$\pi^-\pi^-\pi^+\nu_\tau$	9.0%
$\pi^-\pi^-\pi^+\pi^0\nu_\tau$	4.5%
other	5.7%

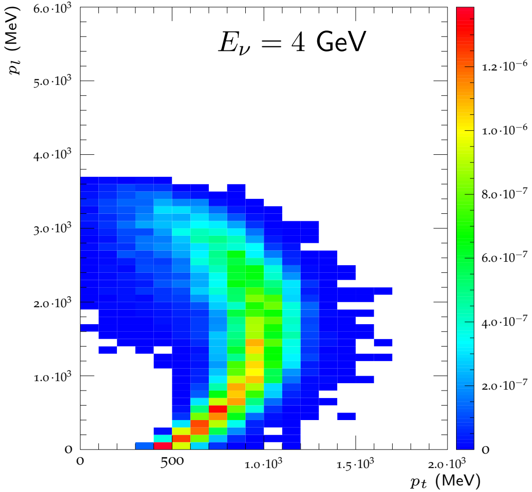
[2007.00015]

- Interface Achilles with Sherpa
- Provides spin correlated decays following [hep-ph/0110108]
- Enables the ability to have all possible decay modes
- Sherpa interface provides ability to include QED showers

Taus in Achilles: One-body

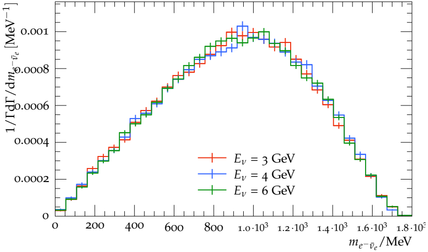


$$x_\pi = \frac{p_\pi}{p_\tau}$$

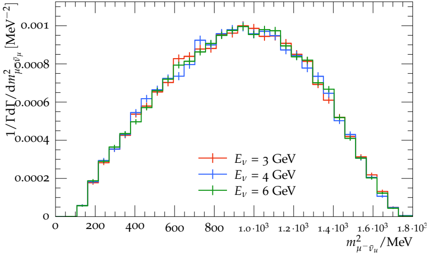


Taus in Achilles

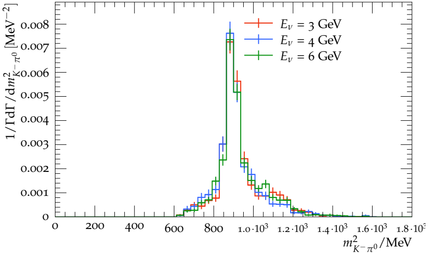
Mass distribution of the electron/neutrino for $\tau^- \to e^- \bar{\nu}_e \nu_\tau$



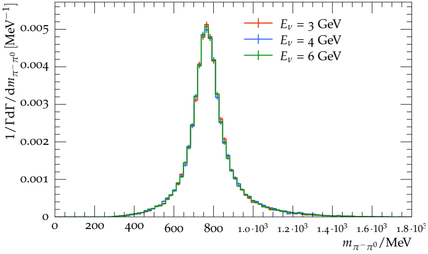
Mass distribution of the hadrons for $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$



Mass distribution of the hadrons for $\tau^{--}\pi^0\nu_\tau$

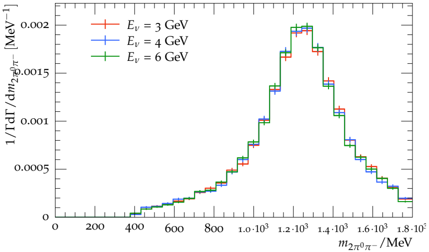


Mass distribution of the hadrons for $\tau^- \to \pi^- \pi^0 \nu_\tau$

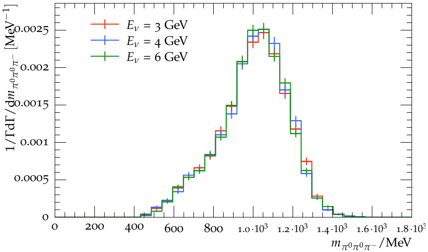


Taus in Achilles

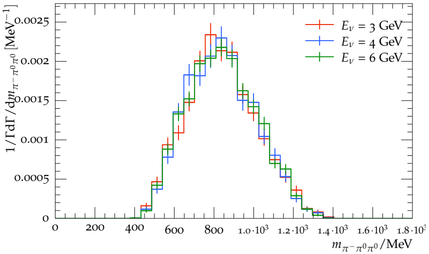
Distribution of the hadronic mass in $\tau^- \to 2\pi^0\pi^-\nu_\tau$



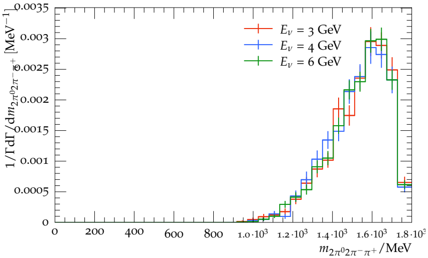
Distribution of the $\pi^0\pi^0\pi^-$ mass in $\tau^- \to 3\pi^0\pi^-\nu_\tau$



Distribution of the $\pi^-\pi^0\pi^0$ mass for $\tau^- \to 2\pi^0 2\pi^-\pi^+\nu_\tau$

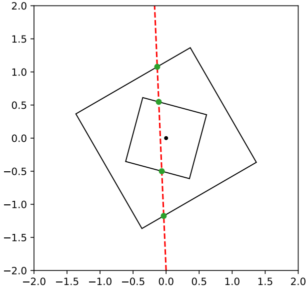


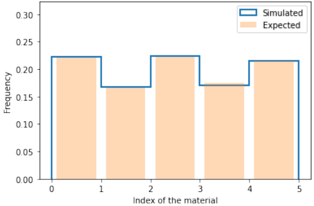
Mass distribution of the hadrons for $\tau^- \to 2\pi^0 2\pi^-\pi^+\nu_\tau$



Geometry Driver

- Work done in collaboration with SIST summer student Santanu Antu
- Able to parse GDML files
- Propagate neutrinos through detector geometry
- Future Steps:
  - Finalize interface to flux driver and cross section codes
  - Keep track of number of protons on target
  - Interface with Achilles





Conclusions

Current Status:

- Achilles aims to be a modular theory driven generator to address these needs
- BSM important for the current and next generation neutrino experiments
- Robust BSM program requires automating theory calculations
- Properly handling spin is crucial in analyses
- Fully polarized tau decays for all channels now available

Future Steps:

- Implement QED showers to handle radiative corrections
- Interface with LArSoft
- Improve BSM user interface and study more models (If you have a model and want to investigate it, come talk with Pedro or myself)

Achilles code can be found at: `https://github.com/AchillesGen/Achilles`

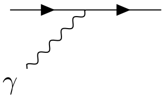
Universal FeynRules Output (UFO)

- Python output files
- Contains model-independent files and model-dependent files
- Contains all information to calculate any tree level matrix element
- Has parameter file to adjust model parameters to scan allowed regions

Example QED ($e^+e^-\gamma$ Vertex):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(iD^\mu\gamma_\mu - m\right)\psi$$

$$V_{e^+e^-\gamma} = ie\gamma^\mu =$$



[arXiv:1108.2040]

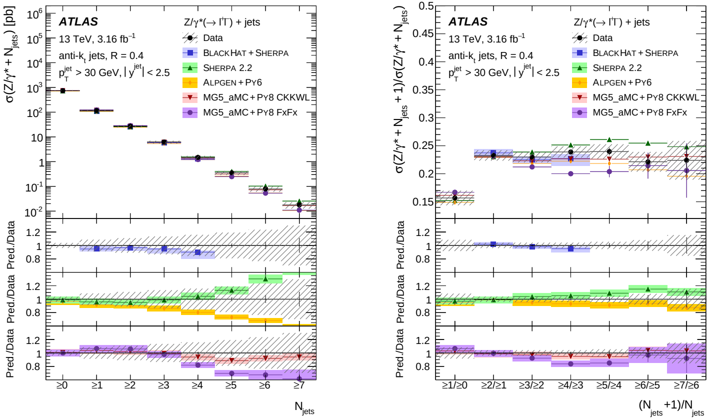
Universal FeynRules Output (UFO)

Example for photon-electron vertex

```
e__minus__ = Particle(pdg_code=11, name='e-', antiname='e+',
                      spin=2, color=1, mass=Param.ZERO,
                      width=Param.ZERO, texname='e-',
                      antitexname='e+', charge=-1,
                      GhostNumber=0, LeptonNumber=1,
                      Y=0)
V_77 = Vertex(name='V_77',
              particles=[ P.e__plus__, P.e__minus__, P.a ],
              color=[ '1' ], lorentz=[ L.FFV1 ],
              couplings={(0,0):C.GC_3})
FFV1 = Lorentz(name='FFV1', spins=[ 2, 2, 3 ],
               structure='Gamma(3,2,1)')
GC_3 = Coupling(name='GC_3', value='-(ee*complex(0,1))',
                order={'QED':1})
```

Tree Level Matrix Element Generators

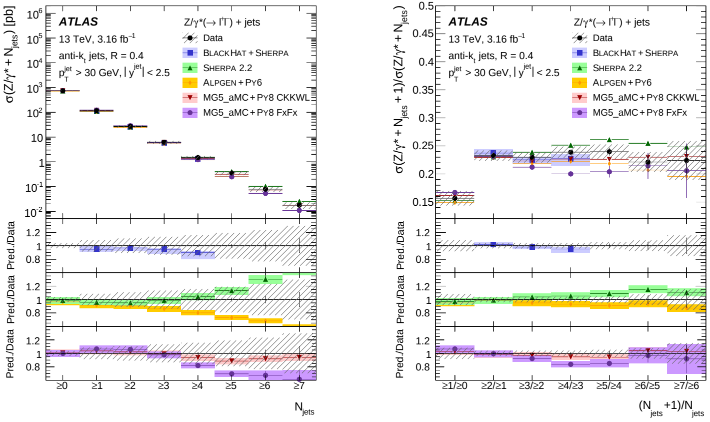
- ALPGEN [arXiv:hep-ph/0206293]
- AMEGIC [arXiv:hep-ph/0109036]
- COMIX [arXiv:0808.3674]
- CALCHEP [arXiv:1207.6082]
- HERWIG [arXiv:0803.0883]
- MADGRAPH
  [arXiv:1405.0301]
- WHIZARD [arXiv:0708.4233]
- etc.



[arXiv:1702.05725]

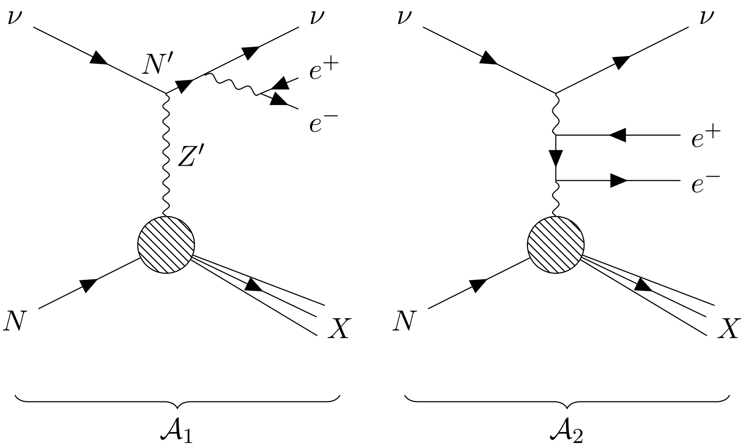
Tree Level Matrix Element Generators

- ALPGEN [arXiv:hep-ph/0206293]
- AMEGIC [arXiv:hep-ph/0109036]
- COMIX [arXiv:0808.3674]
- CALCHEP [arXiv:1207.6082]
- HERWIG [arXiv:0803.0883]
- MADGRAPH [arXiv:1405.0301]
- WHIZARD [arXiv:0708.4233]
- etc.



[arXiv:1702.05725]

Multi-channel Integration



- Both diagrams contribute to cross section
- They have different pole structures
- Need method to sample these structures efficiently (i.e. $|\mathcal{A}_1 + \mathcal{A}_2|^2$)

Multi-channel Integration and VEGAS

Multi-channel Integration

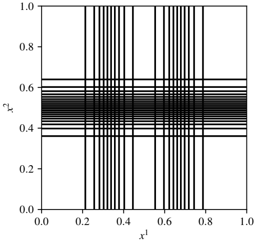
- Generate PS efficiently for $|\mathcal{A}_1|^2$ or $|\mathcal{A}_2|^2$
- Do not know how to efficiently sample $2Re(\mathcal{A}_1\mathcal{A}_2^\dagger)$
- Define channels: C_1 and C_2
- Generate events according to distributions g_i for channel i

$$\int d\vec{x} f(\vec{x}) = \sum_i \alpha_i \int d\vec{x} g_i(\vec{x}) \frac{f(\vec{x})}{g_i(\vec{x})}$$

- Optimize α_i to minimize variance

VEGAS

- Adaptive importance sampling
- Use this to get interference terms more accurately



VEGAS grid for $\int_0^1 d^4x \left(e^{-100(\vec{x}-\vec{r_1})^2} + e^{-100(\vec{x}-\vec{r_2})^2}\right)$

[J.Comput.Phys. 27 (1978) 291, 2009.05112]

Recursive Phase Space Decomposition

Phase space can be decomposed as:

$$\mathrm{d}\Phi_n(a,b;1,\ldots,n) = \mathrm{d}\Phi_{n-m+1}(a,b;m+1,\ldots,n)\frac{\mathrm{d}s_\pi}{2\pi}\mathrm{d}\Phi_m(\pi;1,\ldots,m)$$

Iterate until only $1 \to 2$ phase spaces remain.
Basic building blocks:

$$S_\pi^{\rho,\pi\setminus\rho} = \frac{\lambda(s_\pi, s_\rho, s_{\pi\setminus\rho})}{16\pi^2\, 2\, s_\pi}\,\mathrm{d}\cos\theta_\rho\,\mathrm{d}\phi_\rho$$

$$T_{\alpha,b}^{\pi,\overline{\alpha b\pi}} = \frac{\lambda(s_{\alpha b}, s_\pi, s_{\overline{\alpha b \pi}})}{16\pi^2\, 2 s_{\alpha b}}\,\mathrm{d}\cos\theta_\pi\,\mathrm{d}\phi_\pi$$

Momentum conservation: $(2\pi)^4\mathrm{d}^4 p_{\overline{\alpha b}}\delta^{(4)}(p_\alpha + p_b - p_{\overline{\alpha b}})$

Results

Processes Considered:

- Electron-Carbon Scattering
- Neutrino-Carbon Scattering
- Dirac/Majorana Dark neutrino [1807.09877]

Experimental Setup:

- Target Nucleus: Carbon (Argon for Dark Neutrino)
- Electron: 961 MeV and 1299 MeV
- Neutrino: 1000 MeV
- Validating beam fluxes

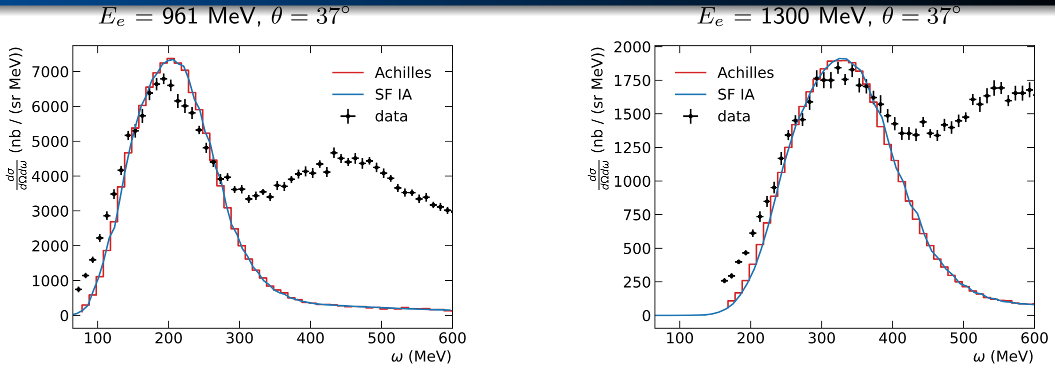
NOTE: All processes are fully differential

Parameters:

- Only quasielastic scattering (coherent for Dark Neutrino) is included and no FSI
- EM Form Factors: Kelly [PRC 70, 068202 (2004)]
- Coherent Form Factor: Lovato [1305.6959]
- Axial Form Factor:
  - Dipole
  - $M_A = 1.0$ GeV
  - $g_A = 1.2694$
- $\alpha = 1/137$
- $G_F = 1.16637 \times 10^{-5}$
- $M_Z = 91.1876$ GeV

[**JI**, et. al. Phys. Rev. D 105 (2022) 9, 096006]

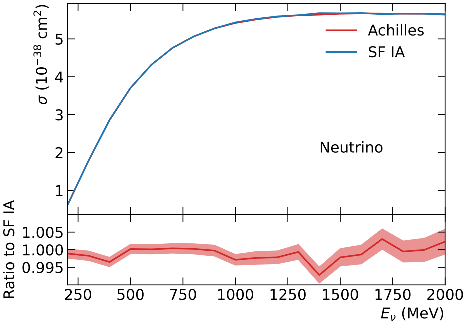
Electron Scattering

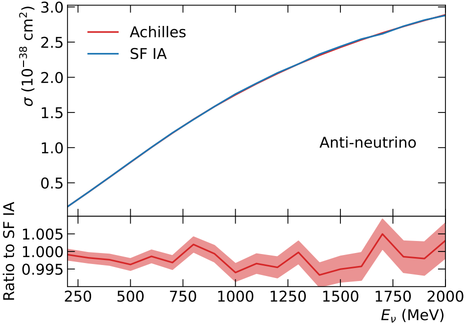


Observable: Double differential cross section with respect to outgoing electron angle and energy transfer from electron to nuclear system ($\omega = E^e_{\rm in} - E^e_{\rm out}$)

[**JI**, et. al. Phys. Rev. D 105 (2022) 9, 096006]

Neutrino Total Cross Section

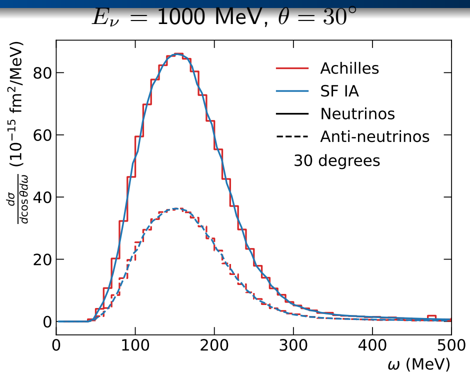


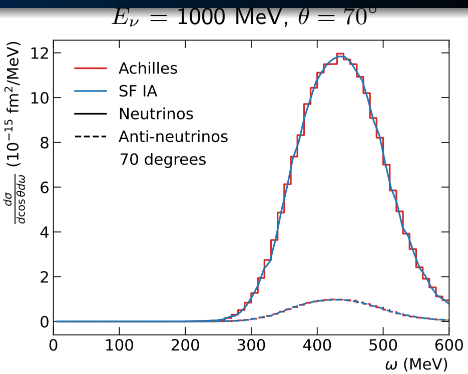


Observable: Total neutrino-Carbon cross section versus neutrino energy

[**JI**, et. al. Phys. Rev. D 105 (2022) 9, 096006]

Neutrino Differential Cross Section

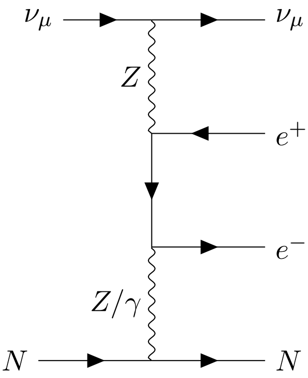


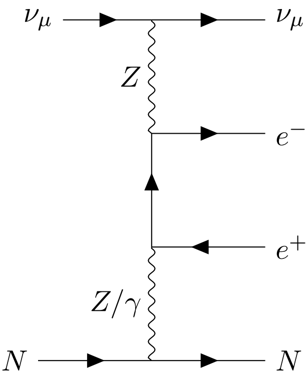


Observable: Double differential cross section with respect to outgoing electron angle and energy transfer from neutrino to nuclear system ($\omega = E_\nu - E_e$)

[**JI**, et. al. Phys. Rev. D 105 (2022) 9, 096006]

Neutrino Tridents





Neutrino Tridents