Two-Nucleon Emission in Quasielastic Neutrino and Electron Scattering induced by short-range correlations

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Contents

Introduction

- The emission of two particles requires interaction mechanisms with a pair of nucleons (by procesess like MEC, SRC, FSI...)
- In this paper we present predictions for the inclusive cross section by neutrinos and electrons including the effects of SRC and MEC separately.
- We'll apply an extended parametrized superscaling analysis with effective mass (SuSAM*) based on the RFM theory.
- SRC phenomenological model is based in two premises:
  - The tail of the scaling function is dominated by the high-momentum components of the nuclear wave function.
  - We follow the Bethe Goldstone formalism in nuclear matter.

Averaged QE hadronic tensor

The 1p1h response functions are

$$R_K^{QE}(q,\omega) = \frac{V}{(2\pi)^3}\int d^3h \frac{(m_N^*)^2}{EE'}\delta(E'-E-\omega) \times\theta(p'-k_F)\theta(k_F-h)2U_K. \quad (1)$$

where de U_K are the single nucleon responses in 1p1h excitation

$$U_L = w^{00}, \qquad U_T = w^{11} + w^{22}, \quad (2)$$

corresponding to the single nucleon tensor

$$w^{\mu\nu} = \frac{1}{2}\sum_{ss'} j^{\mu}_{OB}(\mathbf{p}',\mathbf{h})^*_{s's}\, j^{\nu}_{OB}(\mathbf{p}',\mathbf{h})_{s's}, \quad (3)$$

and

$$j^{\mu}_{OB}(\mathbf{p}',\mathbf{h})_{s's} = \overline{u}(\mathbf{p}')_{s'}\left[F_1\gamma^{\mu} + i\frac{F_2}{2m_N}\sigma^{\mu\nu}Q_{\nu}\right]u(\mathbf{h})_s, \quad (4)$$

Averaged QE hadronic Tensor

We make a change of variable in the integral (1) so we reduce it to an integral over the initial nucleon energy,

$$R_K^{QE}(q,\omega) = \frac{V}{(2\pi)^3}\frac{2\pi m_N^{*3}}{q}\int_{\epsilon_0}^{\infty} d\epsilon\, n(\epsilon)\, 2U_K(\epsilon,q,\omega), \tag{5}$$

where $\epsilon = E/m_N^*$. We've introduced the energy distribution of the Fermi gas, $n(\epsilon) = \theta(\epsilon_F - \epsilon)$ and

$$\epsilon_0 = \text{Max}\left\{\kappa\sqrt{1+\frac{1}{\tau}} - \lambda, \epsilon_F - 2\lambda\right\}, \tag{6}$$

with the dimensionless variables,

$$\lambda = \omega/2m_N^*, \quad \kappa = q/2m_N^*, \quad \tau = \kappa^2 - \lambda^2. \tag{7}$$

Averaged QE hadronic tensor

We define a mean value of the single-nucleon response by averaging with the energy distribution $n(\epsilon)$,

$$\overline{U}_K(q,\omega) = \frac{\int_{\epsilon_0}^{\infty} d\epsilon\, n(\epsilon) U_K(\epsilon,q,\omega)}{\int_{\epsilon_0}^{\infty} d\epsilon\, n(\epsilon)}. \tag{8}$$

We rewrite the QE nuclear response as,

$$R_K^{QE}(q,\omega) = \frac{V}{(2\pi)^3} \frac{2\pi m_N^{*3}}{q} 2\overline{U}_K \int_{\epsilon_0}^{\infty} d\epsilon\, n(\epsilon). \tag{9}$$

Averaged QE hadronic tensor

Using the energy distribution of the RFG, $n(\epsilon)$, we define the *Scaling function* as,

$$\frac{4}{3}(\epsilon_F - 1) f^*(\psi^*) = \int_{\epsilon_0}^{\infty} n(\epsilon) d\epsilon, \tag{10}$$

and the scaling variable,

$$\psi^* = \sqrt{\frac{\epsilon_0 - 1}{\epsilon_F - 1}} \mathrm{sgn}(\lambda - \tau). \tag{11}$$

So finally we can factorize the nuclear response as,

$$R_K^{QE} = \frac{\epsilon_F - 1}{m_N^* \eta_F^3 \kappa} (Z \overline{U}_K^p + N \overline{U}_K^n) f^*(\psi^*), \quad \eta_F = k_F / m_N^* \tag{12}$$

Averaged QE hadronic tensor in $SuSAM^*$

- In RFG, the scaling function is zero for $1 < |\psi^*|$ for all nuclei.
- In a real nucleus the momentum is not limited by k_F since nucleons can have higher momentum.
- In $SuSAM^*$, we extend (12) by replacing $f^*(\psi^*)$ and use a phenomenological scaling function obtained from experimental data,

$$f^*_{QE} = \frac{\left(\frac{d\sigma}{d\Omega d\omega}\right)_{exp} - \left(\frac{d\sigma}{d\Omega d\omega}\right)_{MEC}}{\sigma_M(v_L r_L + v_T r_T)}, \tag{13}$$

where,

$$r_K = \frac{\epsilon_F - 1}{m^*_N \eta_F^3 \kappa}(Z\overline{U}^p_K + N\overline{U}^n_K). \tag{14}$$

Averaged 2p2h hadronic tensor

The 2p2h inclusive hadronic tensor is computed by integrating over all the 2p2h exitations,

$$
\begin{aligned}
W^{\mu\nu}_{2p2h}(q,\omega) &= \frac{V}{(2\pi)^9}\int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \frac{(m_N^*)^4}{E_1 E_2 E'_1 E'_2} \\
&\quad \times w^{\mu\nu}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2) \\
&\quad \times\theta(p'_1 - k_F^N)\theta(k_F^N - h_1)\theta(p'_2 - k_F^{N'})\theta(k_F^{N'} - h_2) \\
&\quad \times\delta(E'_1 + E'_2 - E_1 - E_2 - \omega) \\
&\quad \times\delta(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{q} - \mathbf{h}_1 - \mathbf{h}_2), \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
w^{\mu\nu}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2) &= \\
&= \frac{1}{4}\sum_{s_1 s_2 s'_1 s'_2}\sum_{t_1 t_2 t'_1 t'_2} j^\mu(1',2',1,2)^*_A\, j^\nu(1',2',1,2)_A\,. \qquad (16)
\end{aligned}
$$

where the two-body current function is antisymetrized

$$
j^\mu(1',2',1,2)_A \equiv j^\mu(1',2',1,2) - j^\mu(1',2',2,1)\,. \qquad (17)
$$

Averaged 2p2h hadronic tensor

Taking the same formalism for 2p2h excitations,

$$\overline{w}^{\mu\nu}_{NN'}(q,\omega) \equiv \frac{W^{\mu\nu}_{NN'}(q,\omega)}{\frac{V}{(2\pi)^9} F_{NN'}(q,\omega)} \quad N,N' = p,n \tag{18}$$

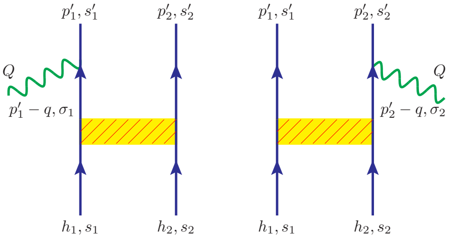
where we define the phase-space

$$\begin{aligned} F_{NN'}(q,\omega) &= \int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \frac{(m^*_N)^4}{E_1 E_2 E'_1 E'_2} \\ &\times\theta(p'_1 - k_F^N)\theta(k_F^N - h_1)\theta(p'_2 - k_F^{N'})\theta(k_F^{N'} - h_2) \\ &\times\delta(E'_1 + E'_2 - E_1 - E_2 - \omega) \\ &\times\delta(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{q} - \mathbf{h}_1 - \mathbf{h}_2), \end{aligned} \tag{19}$$

We get an exact factorization of the 2p2h hadronic tensor,

$$W^{\mu\nu}_{NN'}(q,\omega) = \frac{V}{(2\pi)^9} F_{NN'}(q,\omega) \overline{w}^{\mu\nu}_{NN'}(q,\omega). \tag{20}$$

Short range correlations responses



(a) (b)

Let's now consider the matrix element of the OB current between a correlated pair and a two-particle state above the Fermi level,

$$\langle [\mathbf{p}'_1 \mathbf{p}'_2] | J^{\mu}_{OB}(\mathbf{q}) | [\Phi_{\mathbf{h}_1 \mathbf{h}_2}] \rangle, \tag{21}$$

Where the initial correlated state,

$$|\mathbf{h}_1, \mathbf{h}_2\rangle \rightarrow |\Phi_{\mathbf{h}_1 \mathbf{h}_2}\rangle = |\mathbf{h}_1, \mathbf{h}_2\rangle + |\Delta\Phi_{\mathbf{h}_1 \mathbf{h}_2}\rangle. \tag{22}$$

Short range correlations responses

$\Delta\Phi_{\mathbf{h}_1\mathbf{h}_2}$ is the high momentum relative wave function,

$$\langle \mathbf{p}_1\mathbf{p}_2|\Delta\Phi_{\mathbf{h}_1\mathbf{h}_2}\rangle = \delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{h}_1-\mathbf{h}_2)\Delta\varphi_{\mathbf{h}_1\mathbf{h}_2}(\mathbf{p}). \qquad (23)$$

By applying the OB current to a system of correlated nucleons,

$$\begin{aligned}\langle[\mathbf{p}'_1\mathbf{p}'_2]|J^{\mu}_{OB}(\mathbf{q})|[\Phi_{\mathbf{h}_1\mathbf{h}_2}]\rangle &= \frac{(2\pi)^3}{V^2} \\ &\delta(\mathbf{p}'_1+\mathbf{p}'_2-\mathbf{q}-\mathbf{h}_1-\mathbf{h}_2)j^{\mu}_{cor}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2). \qquad (24)\end{aligned}$$

Where we define the correlation current,

$$\begin{aligned}j^{\mu}_{cor}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2) &= \\ &(2\pi)^3 j^{\mu}_{OB}(\mathbf{p}'_1,\mathbf{p}'_1-\mathbf{q})\Delta\varphi_{\mathbf{h}_1\mathbf{h}_2}\left(\mathbf{p}'-\frac{\mathbf{q}}{2}\right) \\ &+(2\pi)^3 j^{\mu}_{OB}(\mathbf{p}'_2,\mathbf{p}'_2-\mathbf{q})\Delta\varphi_{\mathbf{h}_1\mathbf{h}_2}\left(\mathbf{p}'+\frac{\mathbf{q}}{2}\right). \qquad (25)\end{aligned}$$

with $\mathbf{p}' = \frac{1}{2}(\mathbf{p}'_1 - \mathbf{p}'_2)$ is the relative momentum of the nucleon pair.

Semiempirical 2p2h-SRC response functions

We add the correlation current (25) to the double nucleon (16), so that transform to

$$
\begin{aligned}
w^{\mu\mu}_{NN'}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2) \quad = \\
(2\pi)^6 \left| j^{\mu}_{N}(\mathbf{p}'_1,\mathbf{p}'_1-\mathbf{q})\Delta\varphi^{NN'}_{\mathbf{h}_1\mathbf{h}_2}\left(\mathbf{p}'-\frac{\mathbf{q}}{2}\right)\right|^2 \\
+(2\pi)^6 \left| j^{\mu}_{N'}(\mathbf{p}'_2,\mathbf{p}'_2-\mathbf{q})\Delta\varphi^{NN'}_{\mathbf{h}_1\mathbf{h}_2}\left(\mathbf{p}'+\frac{\mathbf{q}}{2}\right)\right|^2 \\
+2(2\pi)^6 \mathrm{Re}\left\{ j^{\mu *}_{N}(\mathbf{p}'_1,\mathbf{p}'_1-\mathbf{q})\Delta\varphi^{NN'*}_{\mathbf{h}_1\mathbf{h}_2}\left(\mathbf{p}'-\frac{\mathbf{q}}{2}\right)\right. \\
\left.\times\, j^{\mu}_{N'}(\mathbf{p}'_2,\mathbf{p}'_2-\mathbf{q})\Delta\varphi^{NN'}_{\mathbf{h}_1\mathbf{h}_2}\left(\mathbf{p}'+\frac{\mathbf{q}}{2}\right)\right\}. \qquad (26)
\end{aligned}
$$

Semiempirical 2p2h-SRC response functions

We suppose that we can approximate the average of the product as the product of the averages so we'll have only the diagonal terms,

$$\overline{w}^{\mu\mu}_{NN'}(q,\omega) \simeq (2\pi)^6 \overline{\left(\left|j^{\mu}_{N}\right|^2 + \left|j^{\mu}_{N'}\right|^2\right)} \times \overline{\left|\Delta\varphi^{NN'}_{\mathbf{h}_1\mathbf{h}_2}\right|^2}, \tag{27}$$

where we use the notation,

$$\overline{\left|j^{\mu}_{N}\right|^2} \equiv \overline{\left|j^{\mu}_{N}(\mathbf{p}'_1, \mathbf{p}'_1 - \mathbf{q})\right|^2} = \overline{\left|j^{\mu}_{N}(\mathbf{p}'_2, \mathbf{p}'_2 - \mathbf{q})\right|^2} \tag{28}$$

$$\begin{aligned} \overline{\left|\Delta\varphi^{NN'}_{\mathbf{h}_1\mathbf{h}_2}\right|^2} &\equiv \overline{\left|\Delta\varphi^{NN'}_{\mathbf{h}_1\mathbf{h}_2}\left(\mathbf{p}' + \frac{\mathbf{q}}{2}\right)\right|^2} \\ &= \overline{\left|\Delta\varphi^{NN'}_{\mathbf{h}_1\mathbf{h}_2}\left(\mathbf{p}' - \frac{\mathbf{q}}{2}\right)\right|^2}. \end{aligned} \tag{29}$$

Semiempirical 2p2h-SRC response functions

We define a 2p2h coefficient dependent on the momentum transfer q:

$$\frac{c^{pn}(q)}{m_N^2 m_\pi^2} \simeq (2\pi)^6 \overline{\left(\sum_{s_1 s_2 s_1' s_2'} \left| \Delta\varphi^{pn}_{\mathbf{h}_1\mathbf{h}_2}(\mathbf{p}' + \mathbf{q}/2) \right|^2 \right)}. \qquad (30)$$

Then, for a symmetric nuclei, $N = Z$, the semi-empirical response due to the 2p2h contribution reduces to

$$R_K^{\mathrm{2p2h}} = \frac{V}{(2\pi)^9} F(q,\omega) \frac{Z + \alpha(Z-1)}{2Z-1} \frac{c^{pn}(q)}{m_N^2 m_\pi^4} (\overline{U}_K^p + \overline{U}_K^n). \qquad (31)$$

Cross section

From the response functions we can write the semiempirical formula for the inclusive cross section in the 2p2h channel induced by the OB current. In the case of electromagnetic scattering the semiempirical formula is,

$$\left(\frac{d\sigma}{d\Omega' d\epsilon'}\right)^{em}_{\text{2p2h}} = \frac{\sigma_{\text{Mott}} V}{(2\pi)^9} \frac{F(q,\omega) c^{pn}(q)}{m_N^2 m_\pi^4} \frac{Z + \alpha(Z-1)}{2Z-1} \left[v_L (\overline{U}_L^p + \overline{U}_L^n) + v_T (\overline{U}_T^p + \overline{U}_T^n) \right], \tag{32}$$

In the case of CC neutrino scattering formula extends naturally by replacing the electromagnetic single nucleon responses with the corresponding to the $n(\nu_\mu, \mu)p$ or $p(\overline{\nu}_\mu, \mu^+)n$,

$$\left(\frac{d\sigma}{d\Omega' d\epsilon'}\right)^{\nu}_{\text{2p2h}} = \frac{\sigma_0 V}{(2\pi)^9} \frac{F(q,\omega) c^{pn}(q)}{m_N^2 m_\pi^4} \frac{Z + \alpha(Z-1)}{2Z-1} \left[V_{CC}\overline{U}_{CC} + 2V_{CL}\overline{U}_{CL} + V_{LL}\overline{U}_{LL} + V_T \overline{U}_T \pm 2V_{T'}\overline{U}_{T'} \right]. \tag{33}$$

Results

Dividing the cross section by the single nucleon,

$$f^*_{2p2h} \equiv \frac{\left(\frac{d\sigma}{d\Omega d\omega}\right)_{2p2h}}{\sigma_M(v_L r_L + v_T r_T)}. \quad (34)$$

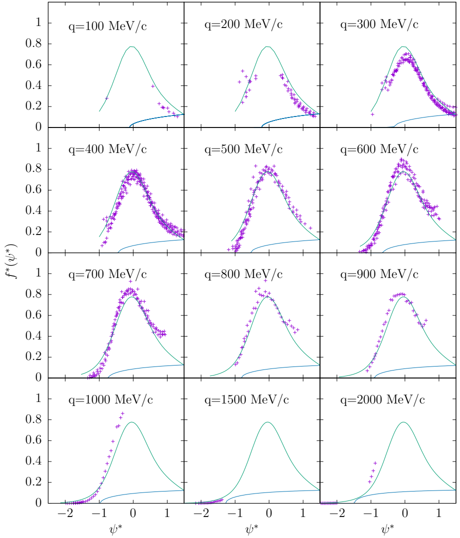
$$\Downarrow$$

$$f^*_{\mathrm{2p2h}}(q,\omega) = \frac{VF(q,\omega)}{(2\pi)^9}\frac{m^*_N \eta^3_F \kappa}{Z\xi_F}\frac{Z+\alpha(Z-1)}{2Z-1}\frac{c^{pn}(q)}{m^2_N m^4_\pi}. \quad (35)$$

Finally, the semiempirical formula will make use of the frozen aproximation for the phase-space function to compute the integral (17),

$$F_{NN'}(q,\omega) = \left(4\pi k^N_F k^{N'}_F\right)^3 \frac{m^{*2}_N}{18}\sqrt{1 - \frac{4m^{*2}_N}{(2m^*_N+\omega)^2 - q^2}}. \quad (36)$$

Results

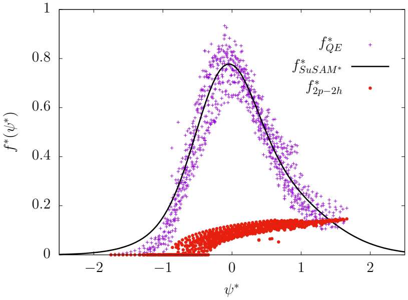


- f^*_{QE} data for several q-values are shown.
- $c^{pn}(q)$ is fitted from the tail of the scaling function $f^*_{SuSAM^*}$ for $\Psi^* > 1.5$.
- The $f^*_{SuSAM^*}$ is obtained by fitting a sum of Gaussians:

$$f^*(\psi^*) = a_3 e^{-(\psi^*-a_1)^2/(2a_2^2)}$$

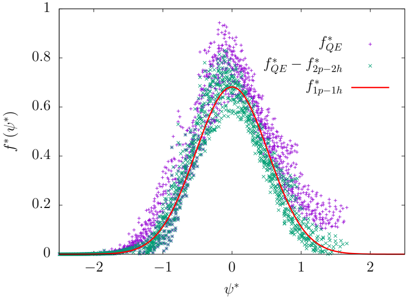
$$+b_3 e^{-(\psi^*-b_1)^2/(2b_2^2)}.$$

Results



- The total QE data are concentrated in a band for $q \leq 1000$ MeV/c.
- We compare the f^*_{2p2h} contribution for the kinematics of the total QE experimental data.
- f^*_{2p2h} points generate a band that could explain the tail of the scaling function.

Results



- The remaining 2p2h contribution is also subtracted so Purely QE 1p1h data remains.
- New band of points is symmetrical as is the new f^*_{1p1h}:

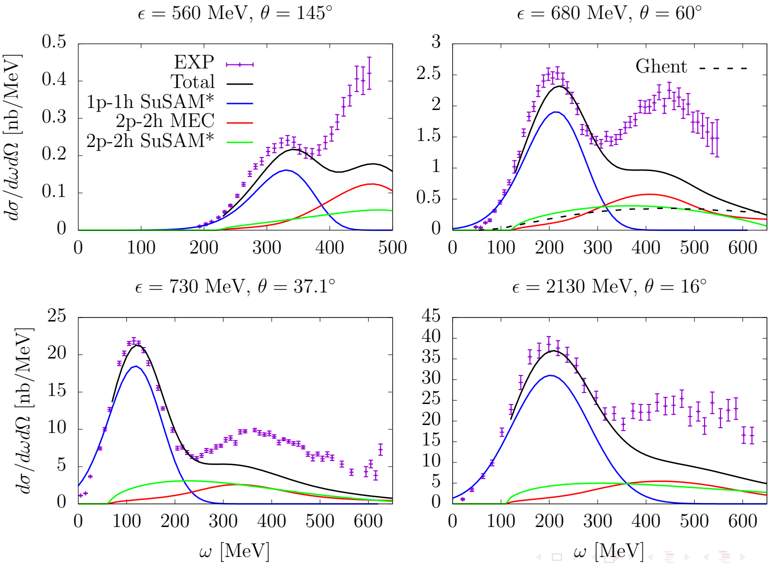
$$f^*_{1p1h}(\psi^*) = be^{-(\psi^*)^2/a^2}.$$

Results

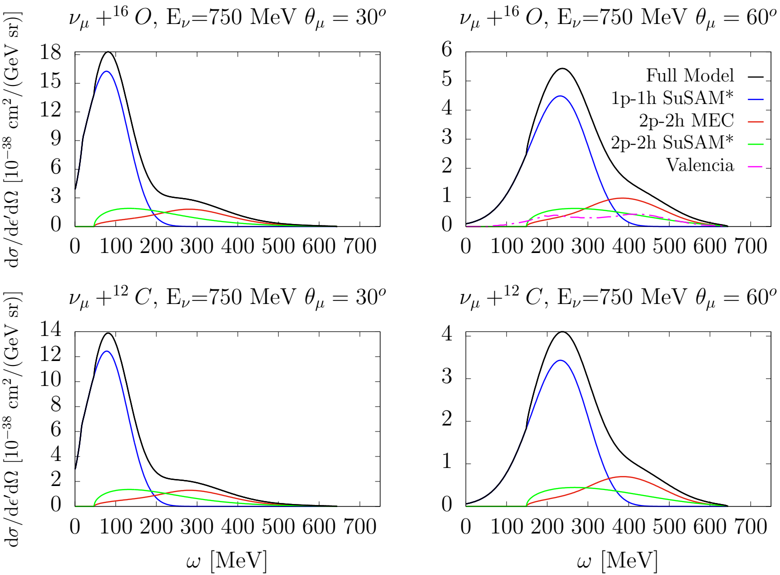
Total Cross Section

$$\frac{d\sigma}{d\Omega' d\epsilon'} = \left(\frac{d\sigma}{d\Omega' d\epsilon'}\right)_{1p1h} + \left(\frac{d\sigma}{d\Omega' d\epsilon'}\right)_{\mathrm{2p2h}} + \left(\frac{d\sigma}{d\Omega' d\epsilon'}\right)_{MEC}$$

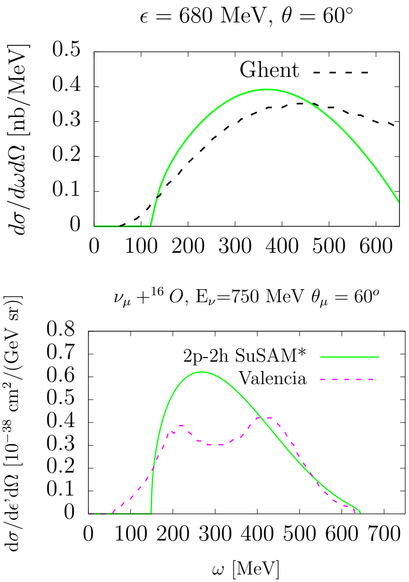
Results


ϵ = 560 MeV, θ = 145°
ϵ = 680 MeV, θ = 60°
ϵ = 730 MeV, θ = 37.1°
ϵ = 2130 MeV, θ = 16°
dσ/dωdΩ [nb/MeV]
ω [MeV]
EXP
Total
1p-1h SuSAM*
2p-2h MEC
2p-2h SuSAM*
Ghent

Results

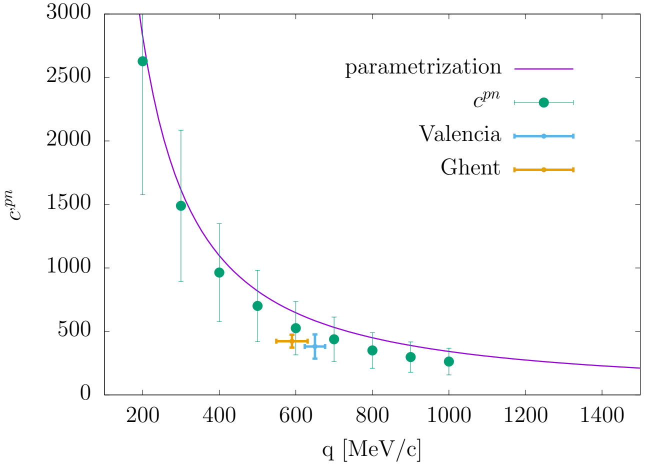

$\nu_\mu +^{16} O$, E$_\nu$=750 MeV $\theta_\mu = 30^o$
$\nu_\mu +^{16} O$, E$_\nu$=750 MeV $\theta_\mu = 60^o$
$\nu_\mu +^{12} C$, E$_\nu$=750 MeV $\theta_\mu = 30^o$
$\nu_\mu +^{12} C$, E$_\nu$=750 MeV $\theta_\mu = 60^o$
$d\sigma/d\epsilon' d\Omega$ [10^{-38} cm^2/(GeV sr)]
ω [MeV]
Full Model
1p-1h SuSAM*
2p-2h MEC
2p-2h SuSAM*
Valencia
0 100 200 300 400 500 600 700
0 3 6 9 12 15 18
0 1 2 3 4 5 6
0 2 4 6 8 10 12 14
0 1 2 3 4

Results


$\epsilon = 680$ MeV, $\theta = 60°$
$d\sigma/d\omega d\Omega$ [nb/MeV]
Ghent
0.5
0.4
0.3
0.2
0.1
0
0 100 200 300 400 500 600
$\nu_\mu +^{16} O$, E$_\nu$=750 MeV $\theta_\mu = 60^o$
dσ/dϵ'dΩ [10^{-38} cm^2/(GeV sr)]
2p-2h SuSAM*
Valencia
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 100 200 300 400 500 600 700
ω [MeV]

Results

$$c^{pn}(q) = a_0 \sqrt{\frac{m_N^* + q}{q}} \frac{m_N^*}{q} \quad (38)$$



Summary

- We've presented a new parameterization of the phenomenological scaling function which consists of a sum of a symmetric function corresponding to the single particle (1p1h) emission plus a contribution from two particle (2p2h) processes, associated to the high-energy tail of the scaling function.
- By a correlation current, we've shown that the hadronic tensor for 2p2h process can be written as the product of the 2p2h phase space, the averaged single-nucleon tensor, and a 2p2h coefficient dependent on the momentum transfer q, for pn, pp or nn pairs.
- In the independent pair aproximation, $c_{pn}(q)$, fitted to the tail of the scaling function, are related to the average high-momentum distribution of the pair.

Summary

- Even if in reality, those parameters encompass additional contributions.
- We've restored the symmetry of the 1p1h scaling function with the new parameterization and we can effectively separate the 1p1h, 2p2h contributions.
- The parameterization of the scaling function approximately captures the two-nucleon emission, including correlations and other effects, induced by the OB current and we've seen that it can be applied to neutrino calculations.
- Our model approximately reproduces the calculations of Ghent with SRC and the calculations of the Valencia model.

¡MUCHAS GRACIAS!