Revisiting "target mass corrections" in lepton-nucleus DIS

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[1] w/ Muzakka, Leger, Olness, Schienbein, et al (nCTEQ Collaboration) [2301.07715]

Thank you for the invitation!

Today's talk

Brief highlights from a "new" ☺ review on **T**arget **M**ass **C**orrections (TMCs) (more in a bit!) in deep-inelastic scattering (DIS) off **nuclear targets**

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Target mass corrections in lepton-nucleus DIS: theory and applications to nuclear PDFs

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Abstract: Motivated by the wide range of kinematics covered by current and planned deep-inelastic scattering (DIS) facilities, we revisit the formalism, practical implementation, and numerical impact of target mass corrections (TMCs) for DIS on unpolarized nuclear targets. An important aspect is that we only use nuclear and later partonic degrees of freedom, carefully avoiding a picture of the nucleus in terms of nucleons. After establishing that formulae used for individual nucleon targets (p, n), derived in the Operator Product Expansion (OPE) formalism, are indeed applicable to nuclear targets, we rewrite expressions for nuclear TMCs in terms of re-scaled (or averaged) kinematic variables. As a consequence, we find a representation for nuclear TMCs that is approximately independent of the nuclear target. We go on to construct a single-parameter fit for all nuclear targets that is in good numerical agreement with full computations of TMCs. We discuss in detail qualitative and quantitative differences between nuclear TMCs built in the OPE and the parton model formalisms, as well as give numerical predictions for current and future facilities.

Keywords: DIS, Structure Functions, Target Mass Corrections, OPE, nuclear PDFs

Contents

w/ Muzakka, Leger, Olness, Schienbein, et al (nCTEQ Collaboration) [2301.07715]

What did we do?

- Starting from OPE, derived TMCs for arbitrary A for $F_1, \ldots, F_6$ ☺

  (lenthy appendix to avoid ambiguities in conventions!) sketched in next few slides!

- ran numbers for JLAB, EIC, LBNF/DUNE ☺

- discovered some interesting physics along the way ☺ – if time allows

- found nTMCs formulae that are A-independent ☹ – no time!

  (sanity check!)

- found A-independent, 2-parameter fit for leading nTMCs ☹ – no time!

  (easy to implement in numerical codes!)

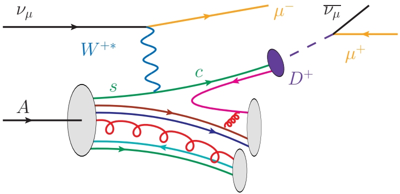
- Established correspondence between TMCs in OPE and TMCs in ACOT/parton model ☹ – no time! (nice intuition!)

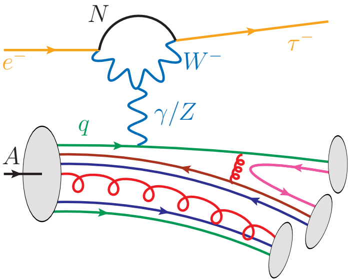
the big picture and motivation

Several νDIS and $e^{\pm}$DIS programs collecting data now:
- Fermilab
- JLab
- CERN

with more planned for $\gtrsim$'20s:
- BNL
- LBNF
- CERN

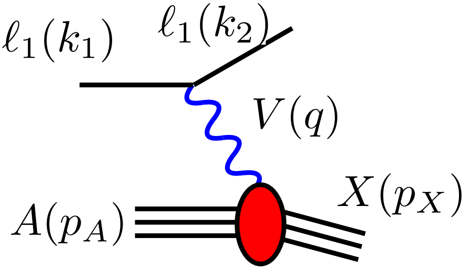
and with various agendas:
- precision ν oscillations
- precision hadronic structure
- QCD at the extremes
- search for more new physics





$\Longrightarrow$ **precise data demands precise predictions from theory**

Formally, inclusive DIS of $\ell \in \{\ell^\pm, \nu, \overline{\nu}\}$ off protons can be described by the collinear factorization theorem (**CFT**) Collins, Soper ('87); Collins ('11)



$$d\sigma(\ell_1 p \to \ell_2 X) = \underbrace{\sum_{i,X_n}}_{inclusive} \underbrace{\Delta_{ii'}}_{\text{shower/RGE}} \otimes \underbrace{f_i}_{\text{PDF}} \otimes \underbrace{d\hat{\sigma}_{i'\ell\to X_n}}_{\text{hard scattering}} + \mathcal{O}\left(\frac{\Lambda_{\rm NP}^{2+k}}{Q^{2+k}}\right)$$

target mass corrections (**TMCs**) are $\mathcal{O}(x_A^2 M_A^2/Q^2)$ "power corrections"

Georgi, Politzer ('76,'76); Ellis, Furmanski, Petronzio ('82,'82); lots more; Kretzer, Reno ('02,'03); Schienbein, et al [0709.1775]

why look into power corrections?

nuclei (A) are not protons $(\mathcal{P})$ ☹

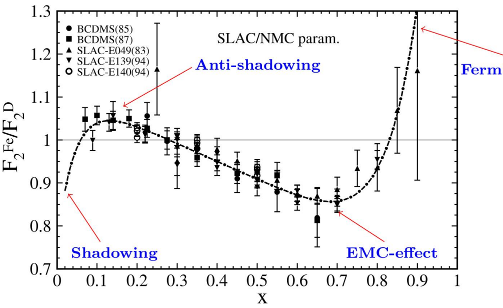
in practice, νDIS needs nuclear targets

- ν only interact through weak force: targets must be bigger ($\mathcal{O}(10)$tons), denser (Ar,Fe,Pb) $\implies$ more nuclear
- **Collinear Factorization Theorem** not proved for DIS with A (are formulae even applicable?)
- Often, $M_A^2 \gtrsim Q^2$, e.g., $M_{Fe-56} \sim 52$ GeV and $M_{Pb-208} \sim 194$ GeV

(how to reconcile this?)

- fact of life: nuclear dynamics impact hadronic structure

Plotted: $\frac{F_2^{\text{iron}}}{F_2^{\text{deuteron}}}$ for ℓ-DIS



Schienbein, et al [0710.4897]

For non-expert, QED (γ) contribution to F_2: $F_2(\xi) \approx \sum_{i\in\{q,\overline{q},g\}} Q_i^2\ \xi\ f_i^A(\xi)$, Q_i=electric charge of i

Key: **TMCs** can be incorporated in structure functions, $F_i(x, Q^2)$

Georgi, Politzer ('76,'76); Ellis, Furmanski, Petronzio ('82,'82); lots more; Kretzer, Reno ('02,'03); Schienbein, et al [0709.1775]

$$W^A_{\mu\nu} = \frac{1}{4\pi}\int d^4z\ e^{iq\cdot z}\langle A|J^\dagger_{had.\mu}(z)\ J_{had.\nu}(0)|A\rangle$$

$$= -g_{\mu\nu}\ F^A_1\ + \frac{p_{A\mu}p_{A\nu}}{Q^2}\ 2x_A\ F^A_2\ - i\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho_A q^\sigma}{Q^2}\ x_A F^A_3$$

$$+ \frac{q_\mu q_\nu}{Q^2}\ 2F^A_4\ + \frac{p_{A\mu}q_\nu + p_{A\nu}q_\mu}{Q^2}\ 2x_A\ F^A_5\ + \frac{p_{A\mu}q_\nu - p_{A\nu}q_\mu}{Q^2}\ 2x_A\ F^A_6$$

In practice, replace $F_i^{A\ \text{(No TMC)}} \to F_i^{A\ \text{(TMC)}}$ in cross sections:

$$\frac{d^2\sigma^{\rm NC}}{dx\,dy} = x(s-M^2)\frac{d^2\sigma^{\rm NC}}{dxdQ^2} = \frac{4\pi\alpha^2}{xyQ^2}\left[\frac{Y_+}{2}\ \sigma^{NC}_{\rm Red.}\right],$$

$$\sigma^{NC}_{\rm Red.} = \left(1+\frac{2y^2\varepsilon^2}{Y_+}\right)F_2^{\rm NC} \mp \frac{Y_-}{Y_+}xF_3^{\rm NC} - \frac{y^2}{Y_+}F_L^{\rm NC},$$

$$F_L = r^2F_2 - 2xF_1,\ r=\sqrt{1+4\varepsilon^2},\ \varepsilon = (xM/Q) \text{ and } Y_\pm = 1\pm(1-y)^2$$

result #1

nuclear structure functions with TMCs

Generically, structure functions with TMCs have the form:

$$F_j^{A,\mathrm{TMC}}(x_A, Q^2) \;=\; \sum_{i=1}^{6} A_j^i \underbrace{F_i^{A,(0)}(\xi_A, Q^2)}_{\text{"no" TMCs}} + B_j^i h_i^A(\xi_A, Q^2) + C_j g_2^A(\xi_A, Q^2)$$

Lots to unpack!

- $x_A =$ Bjorken x
- $\xi_A(x_A) =$ Natchmann $x = 2x_A/(1+\sqrt{1+4x_A^2M_A^2/Q^2})$
- A_j^i, B_j^i, C_j are closed-form functions $\sim f\left[x_A, (x_A^2M_A^2/Q^2)\right]$
- for $i \neq j$, structure function mixing! ←this is cool
- $A_i^i \sim \mathcal{O}(1)$, while B_j^i, $C_j \sim \mathcal{O}(x_A M_A^2/Q^2)$.
- h_i and g_2 are convolutions over $F_i(y)|_{\mathrm{no-TMC}}$

Nuclear structure functions with TMCs

$$\tilde{F}_1^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A}\right)\tilde{F}_1^{A,(0)}(\xi_A) + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^2}\right)\tilde{h}_2^A(\xi_A) + \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^3}\right)\tilde{g}_2^A(\xi_A)\,,$$

$$\tilde{F}_2^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A^2}{\xi_A^2 r_A^3}\right)\tilde{F}_2^{A,(0)}(\xi_A) + \left(\frac{6M_A^2 x_A^3}{Q^2 r_A^4}\right)\tilde{h}_2^A(\xi_A) + \left(\frac{12M_A^4 x_A^4}{Q^4 r_A^5}\right)\tilde{g}_2^A(\xi_A)\,,$$

$$\tilde{F}_3^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A^2}\right)\tilde{F}_3^{A,(0)}(\xi_A) + \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_3^A(\xi_A)\,,$$

$$\begin{aligned}\tilde{F}_4^{A,\mathrm{TMC}}(x_A) = {} & \left(\frac{x_A}{\xi_A r_A}\right)\tilde{F}_4^{A,(0)}(\xi_A) - \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^2}\right)\tilde{F}_5^{A,(0)}(\xi_A) + \left(\frac{M_A^4 x_A^3}{Q^4 r_A^3}\right)\tilde{F}_2^{A,(0)}(\xi_A) \\ & + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_5^A(\xi_A) - \left(\frac{2M_A^4 x_A^4}{Q^4 r_A^4}\right)\left(2 - \xi_A^2 M_A^2/Q^2\right)\tilde{h}_2^A(\xi_A) \\ & + \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^5}\right)\left(1 - 2x_A^2 M_A^2/Q^2\right)\tilde{g}_2^A(\xi_A)\,,\end{aligned}$$

$$\begin{aligned}\tilde{F}_5^{A,\mathrm{TMC}}(x_A) = {} & \left(\frac{x_A}{\xi_A r_A^2}\right)\tilde{F}_5^{A,(0)}(\xi_A) - \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3 \xi_A}\right)\tilde{F}_2^{A,(0)}(\xi_A) \\ & + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_5^A(\xi_A) - \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^4}\right)\left(1 - x_A \xi_A M_A^2/Q^2\right)\tilde{h}_2^A(\xi_A) \\ & + \left(\frac{6M_A^4 x_A^3}{Q^4 r_A^5}\right)\tilde{g}_2^A(\xi_A)\,,\end{aligned}$$

$$\tilde{F}_6^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A^2}\right)\tilde{F}_6^{A,(0)}(\xi_A) + \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_6(\xi_A)\,.$$

result #2: some numbers

we use NLO PDFs (nCTEQ15) to build str. fns. At LO, these are

$$F_1^{\nu A} = (d + s + \bar{u} + \bar{c}), \qquad F_1^{\bar{\nu} A} = (u + c + \bar{d} + \bar{s})$$
$$F_2^{\nu A} = 2x\,(d + s + \bar{u} + \bar{c}), \qquad F_2^{\bar{\nu} A} = 2x\left(u + c + \bar{d} + \bar{s}\right)$$
$$F_3^{\nu A} = +2\,(d + s - \bar{u} - \bar{c}), \qquad F_3^{\bar{\nu} A} = -2\left(u + c - \bar{d} - \bar{s}\right)$$

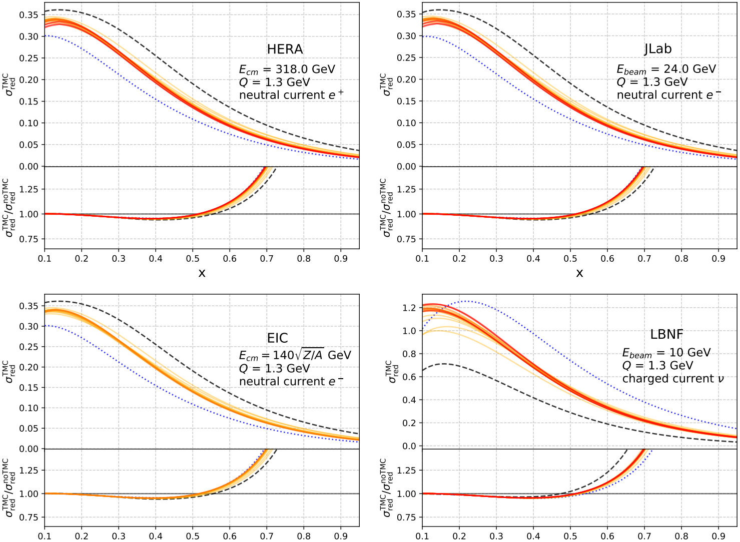
$$F_2^{l^\pm A} = x\frac{1}{9}\left[4(u+\bar{u}) + (d+\bar{d}) + 4(c+\bar{c}) + (s+\bar{s})\right]$$

for many targets

Symbol	A	Z	Symbol	A	Z	Symbol	A	Z	Symbol	A	Z
H	1	1	Be	9	4	Ca	40	20	Xe	131	54
D	2	1	C	12	6	Fe	56	26	W	184	74
^{3}He	3	2	N	14	7	Cu $_{\rm iso}$	64	32	Au	197	79
He	4	2	Ne	20	10	Kr $_{\rm iso}$	84	42	Au $_{\rm iso}$	197	98.5
Li	6	3	Al	27	13	Ag $_{\rm iso}$	108	54	Pb $_{\rm iso}$	207	103.5
Li	7	3	Ar	40	18	Sn $_{\rm iso}$	119	59.5	Pb	208	82

reduced cross sections for many nuclear targets

Plotted: (upper) reduced cross sections with nTMCs; (lower) ratio to w/o



result #3: something interesting

the operator product expansion (in a nutshell)

The OPE is a formalism for decomposing products of operators

Wilson ('69); Brandt, Preparata ('71); Christ, et al ('72)

$$\langle\text{some number of operators } \hat{\mathcal{O}}\rangle = \sum_k \underbrace{\mathcal{C}_k}_{\text{Wilson coeff.}} \times \langle\text{fewer operators } \hat{\mathcal{O}}\rangle$$

As an intermediate step, we 1. took the OPE of the hadronic current, 2. set $(M_A^2/Q^2) \to 0$, and 3. derived/recovered an important sanity check:

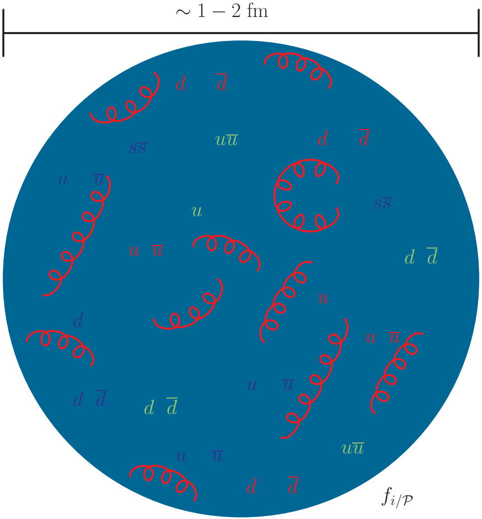
$$\tilde{F}_i^{AN}\Big|_{\text{No TMC}} = C_i^N A_{\tau=2}^N + \mathcal{O}(\tau > 2) \quad \text{for} \quad i = 1, 3-6 ,$$

$$\tilde{F}_2^{A(N-1)}\Big|_{\text{No TMC}} = C_2^N A_{\tau=2}^N + \mathcal{O}(\tau > 2)$$

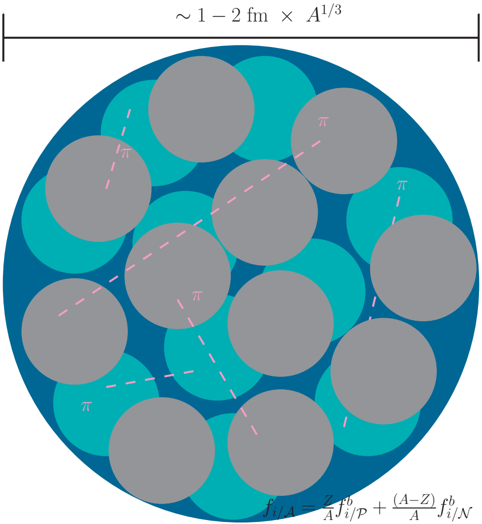
structure fns. = (short-dist. phys.) $\times$ (hadronic matrix element)

for proton, $F_i^N = C_i^N \times A^N$ + power corrections

$\Longrightarrow$ "PDFs = QCD $\times$ hadronic matrix element"



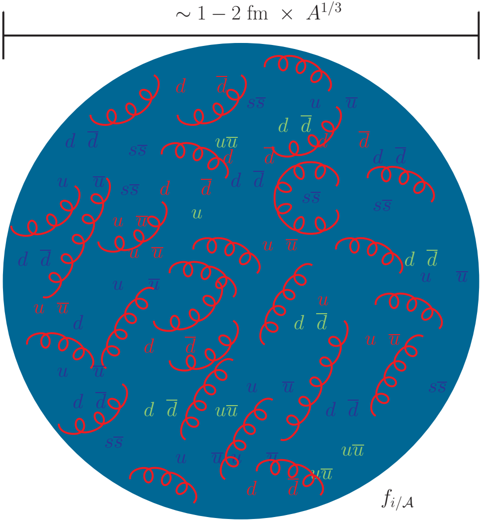
for *A*, it is common to parameterize PDF as combination of “bound” $\mathcal{P}$ and $\mathcal{N}$ PDFs



$$f_{i/\mathcal{A}} = \frac{Z}{A} f^{b}_{i/\mathcal{P}} + \frac{(A-Z)}{A} f^{b}_{i/\mathcal{N}}$$

for A, $F_i^{AN} = C_i^N \times A^N$ + power corrections

$\Longrightarrow$ "PDFs = QCD $\times$ had. ME" ("nucleon" picture not necessary)



Summary and conclusion

The nCTEQ collaboration has revisited the theory and phenomenology TMCs in DIS off nuclear targets nCTEQ Collaboration [2301.07715]

- **extended** formalism for protons to nuclei
- **pedagogical appendix** that fills in gaps in literature/texts
- **lots of phenomenology, numbers, and plots** (... so many plots - JLAB, EIC, LBNF)
- hope this work **guides future discussions**
- lots not covered (ACOT, uncertainties, $x_N > 1$, fit results), so see the paper!

Thank you!

backup

Rescaling

Interestingly, TMCs have particular kinematical dependence:

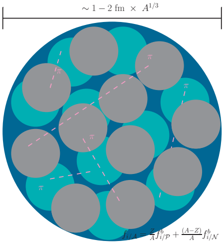
$$\frac{x_A}{\xi_A} \quad \text{or} \quad \left(\frac{x_A^2 M_A^2}{Q^2}\right)$$

Define "average (nucleon) kinematics": $M_N \equiv M_A/A$ **and** $x_N \equiv A x_A$

$$\frac{x_A}{\xi_A} = \frac{x_N}{\xi_N} \quad \text{or} \quad \left(\frac{x_A^2 M_A^2}{Q^2}\right) = \left(\frac{x_N^2 M_N^2}{Q^2}\right)$$

Consequence: TMCs for A-independent, "nucleon" str. fns. that matches intuitive picture of nuclei →

– same expressions as for A but replace "$_A$" with "$_N$"



some numbers

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$$F_2^{l^\pm A} = x\frac{1}{9}\left[4(u + \bar{u}) + (d + \bar{d}) + 4(c + \bar{c}) + (s + \bar{s})\right]$$

for many targets

Symbol	A	Z	Symbol	A	Z	Symbol	A	Z	Symbol	A	Z
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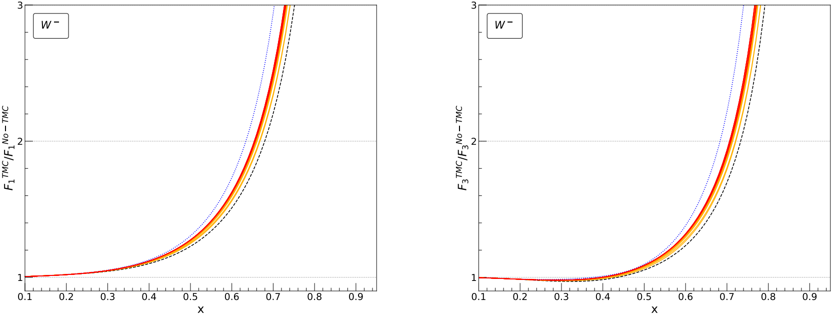
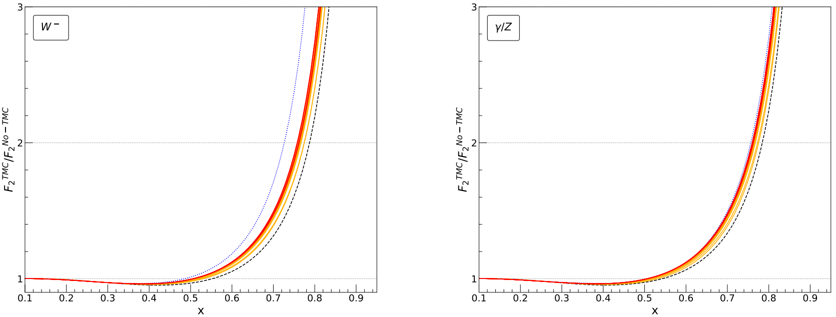
ratio of F_i^{TMC} / $F_i^{\mathrm{no\ TMC}}$

Plotted: ratio for (L) $F_1^{W^-}$ and (R) $F_3^{W^-}$ at $Q = 1.5$ GeV



Can you spot the ^{1}H and ^{2}D curves?

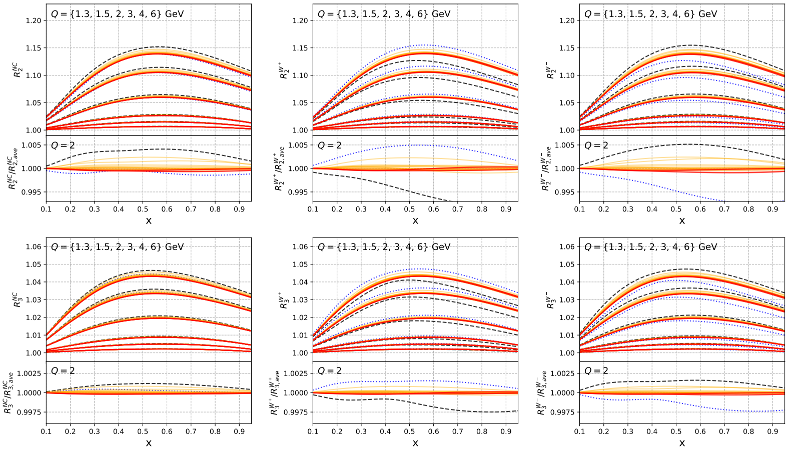
Plotted: ratio for (L) $F_2^{W^-}$ and (R) $F_2^{\gamma/Z}$ at $Q = 1.5$ GeV



Can you spot the ^{1}H and ^{2}D curves?

ratio of F_i^{TMC} / $F_i^{\mathrm{leading\ TMC}}$

Plotted: ratio for (L) $F_i^{Z/\gamma}$, (C) $F_i^{W^+}$, (R) $F_i^{W^-}$ for $i = 2$ (upper) and $i = 3$ (lower)



remarkable uniformity! (good enough to fit! ☺)

reduced cross sections

Plotted: (upper) reduced cross sections with nTMCs; (lower) ratio to w/o