

# Investigating Lorentz Invariance Violation with DUNE and P20

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with  
N.R. Khan Chowdhury & N.Fiza

# Plan of the talk

- Basic theory and motivation
- Impact of LIV on probability
- $\Delta\chi^2$  correlations and various degeneracies
- Improving constraints with DUNE+P20
- Summary

# Lorentz Invariance Violation (LIV)

Kostelecky et al. (2012), Mavromatos et al.

- Introduce Lorentz violation effectively in form of the Standard model extension (SME)

- In the SME the neutrino sector is described by

$$\mathcal{L} = \frac{1}{2} \bar{\Psi} (i\gamma^\alpha \partial_\alpha - M + \mathcal{Q}) \Psi$$

Lorentz violating operator

with  $\Psi = (\nu_e, \nu_\mu, \nu_\tau, \nu_e^C, \nu_\mu^C, \nu_\tau^C)^T$

- Lorentz violating part can be decomposed into:

$$\mathcal{L}_{\text{LIV}} \supset -\frac{1}{2} \left[ a_{\alpha\beta}^\mu \bar{\psi}_\alpha \gamma_\mu \psi_\beta + b_{\alpha\beta}^\mu \bar{\psi}_\alpha \gamma_5 \gamma_\mu \psi_\beta \right]$$

- The observable effect on LH neutrinos are controlled by  $(a_L)_{\alpha\beta}^\mu = (a + b)_{\alpha\beta}^\mu$

# LIV: Theory background

Kostelecky et al. (2012), Mavromatos et al.

- $H = H_{\text{vac}} + H_{\text{mat}} + H_{\text{LIV}},$

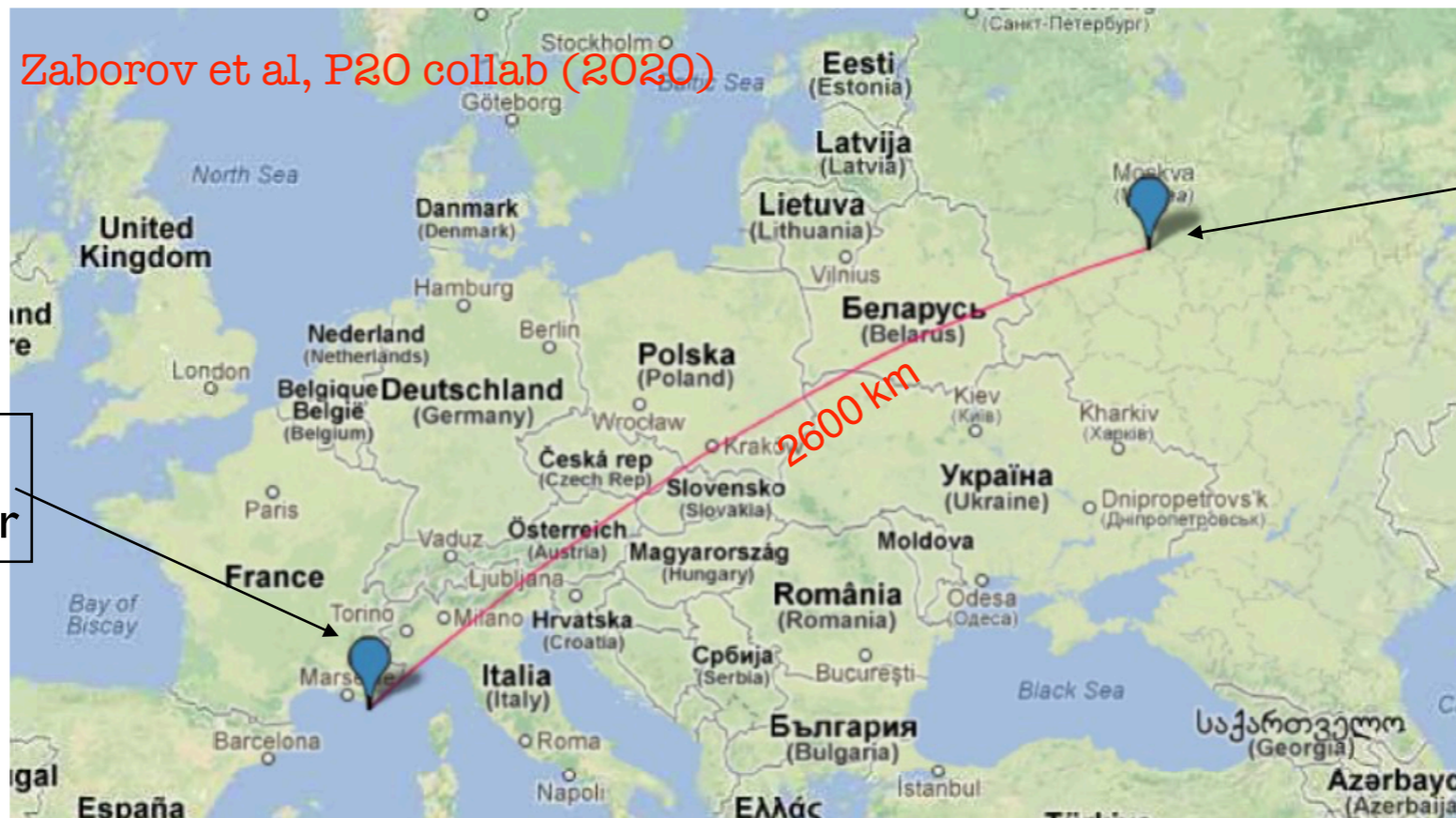
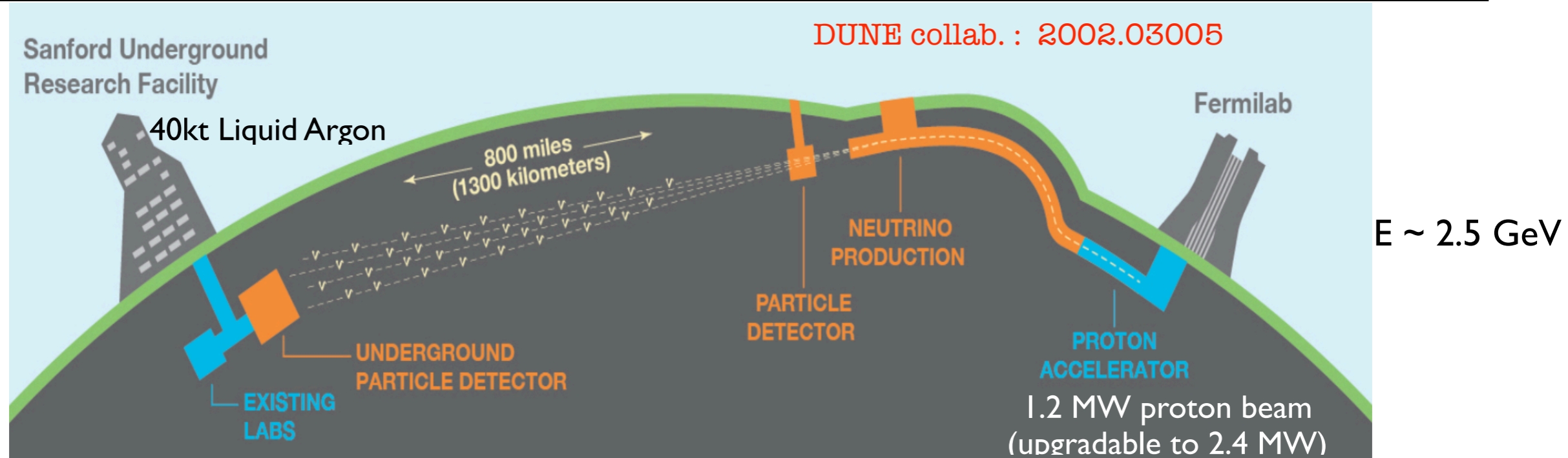
- $H_{\text{vac}} = \frac{1}{2E} U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger, \quad H_{\text{mat}} = A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

- $H_{\text{LIV}} = \begin{pmatrix} a_{ee} & a_{e\mu} & a_{e\tau} \\ a_{e\mu}^* & a_{\mu\mu} & a_{\mu\tau} \\ a_{e\tau}^* & a_{\mu\tau}^* & a_{\tau\tau} \end{pmatrix} \quad (A = 2\sqrt{2}G_F N_e E)$

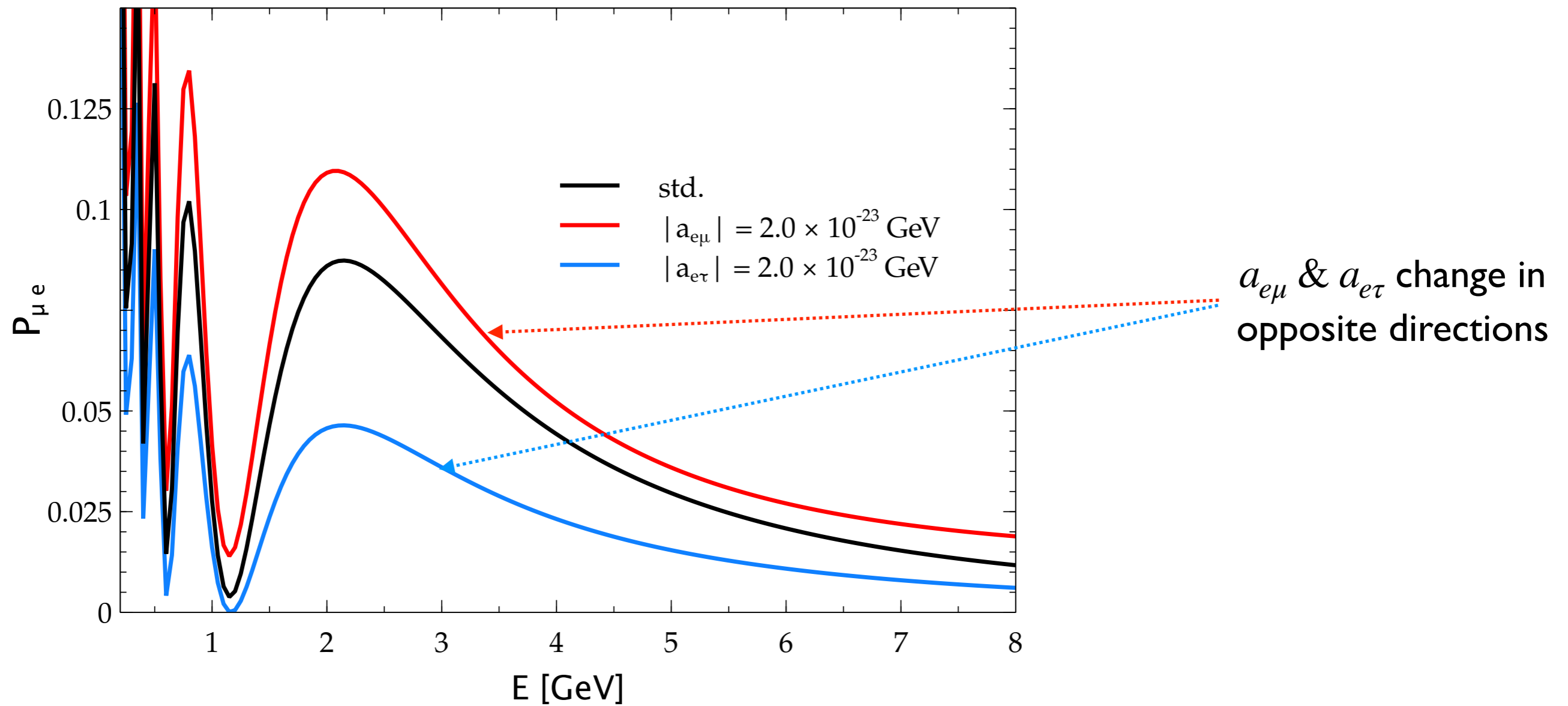
CPTV/ LIV are also probed in collider sectors: neutral meson mixing,  $t\bar{t}$  production....



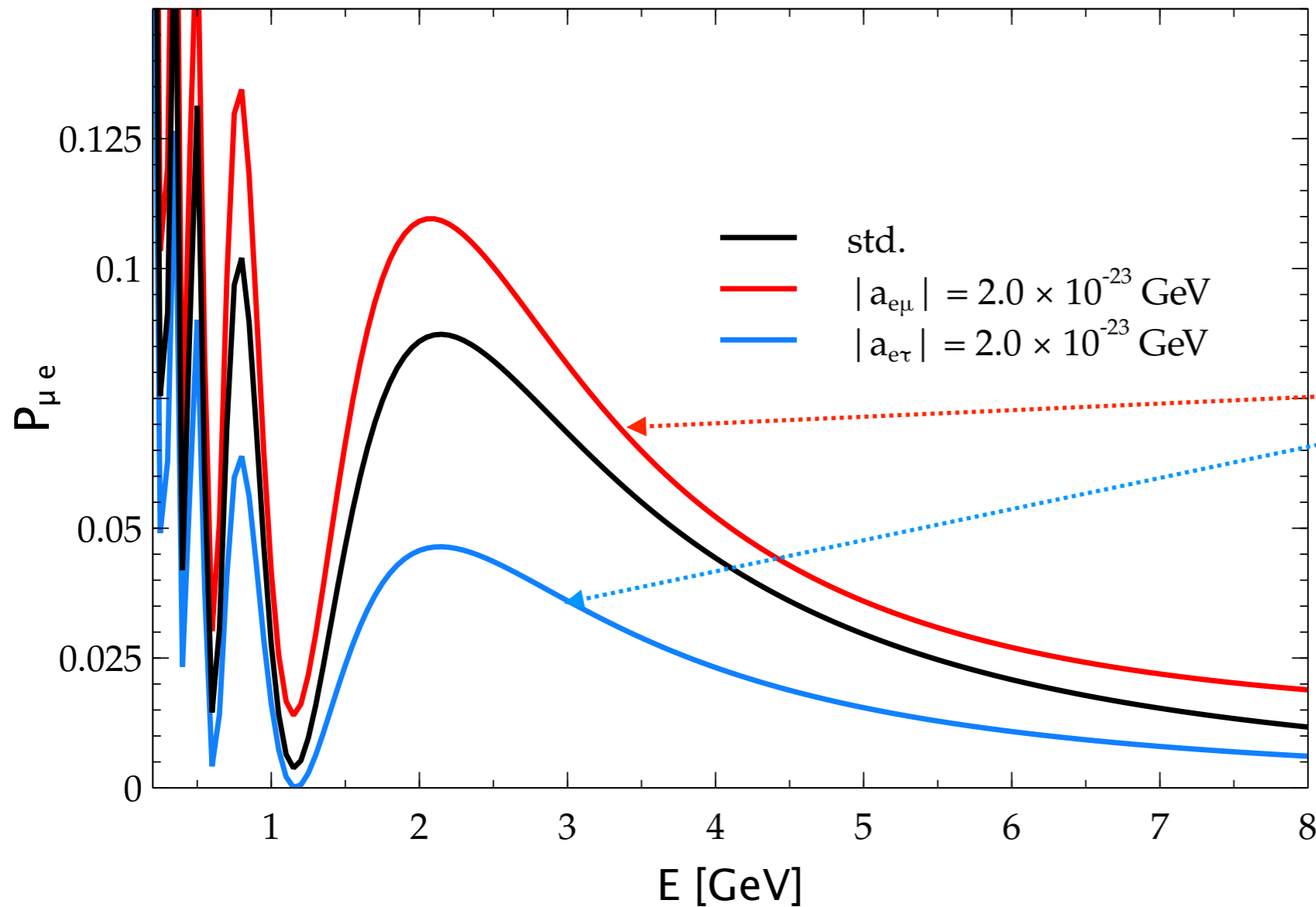
# Schematics of DUNE & P20



# Impact of LIV parameters at probability



# Impact of LIV parameters at probability



$a_{e\mu}$  &  $a_{e\tau}$  change in opposite directions

$$\delta_{13} \simeq -122.4^\circ$$

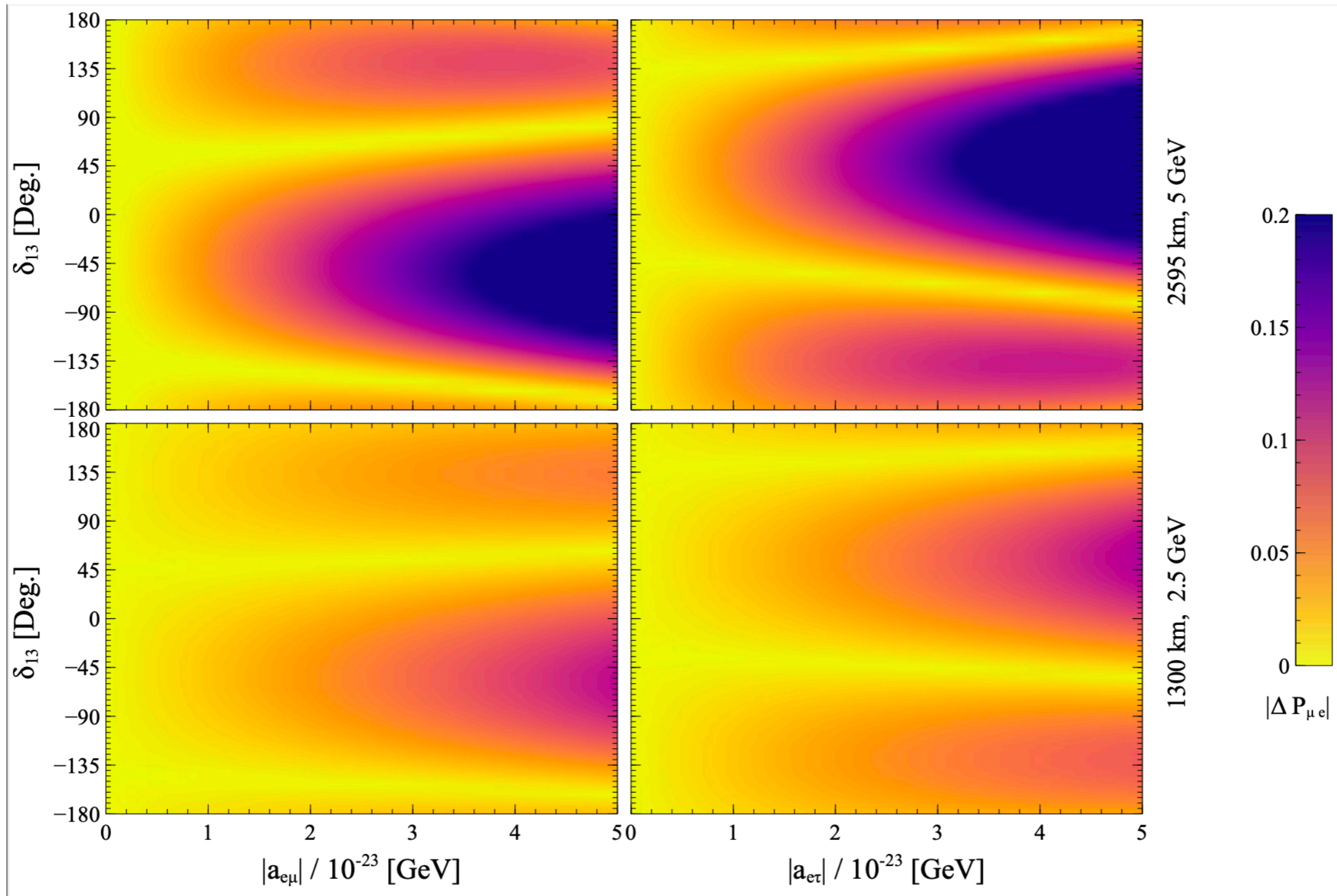
$$\Delta P = P(\text{Std.} + \text{LIV}) - P(\text{Std.})$$

$$\Delta P_{\mu e}(|a_{e\mu}|) \simeq 8|a_{e\mu}| \frac{\pi}{2} E s_{13} \sin 2\theta_{23} c_{23} \left[ -\sin \delta_{13} + \frac{2}{\pi} \frac{s_{23}^2}{c_{23}^2} \cos \delta_{13} \right], \quad +ve$$

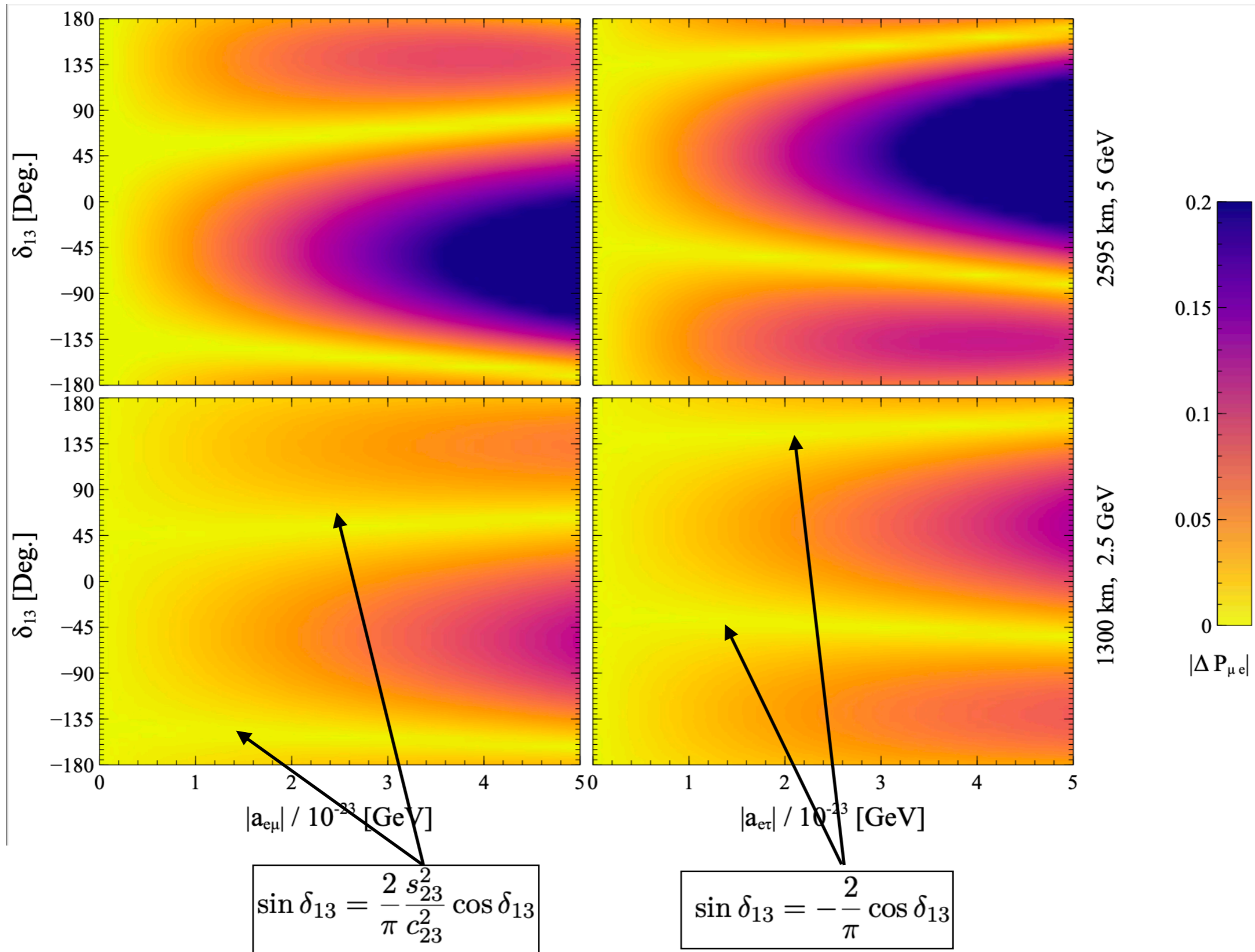
$$\Delta P_{\mu e}(|a_{e\tau}|) \simeq 8|a_{e\tau}| \frac{\pi}{2} E s_{13} \sin 2\theta_{23} s_{23} \left[ \sin \delta_{13} + \frac{2}{\pi} \cos \delta_{13} \right]. \quad -ve$$



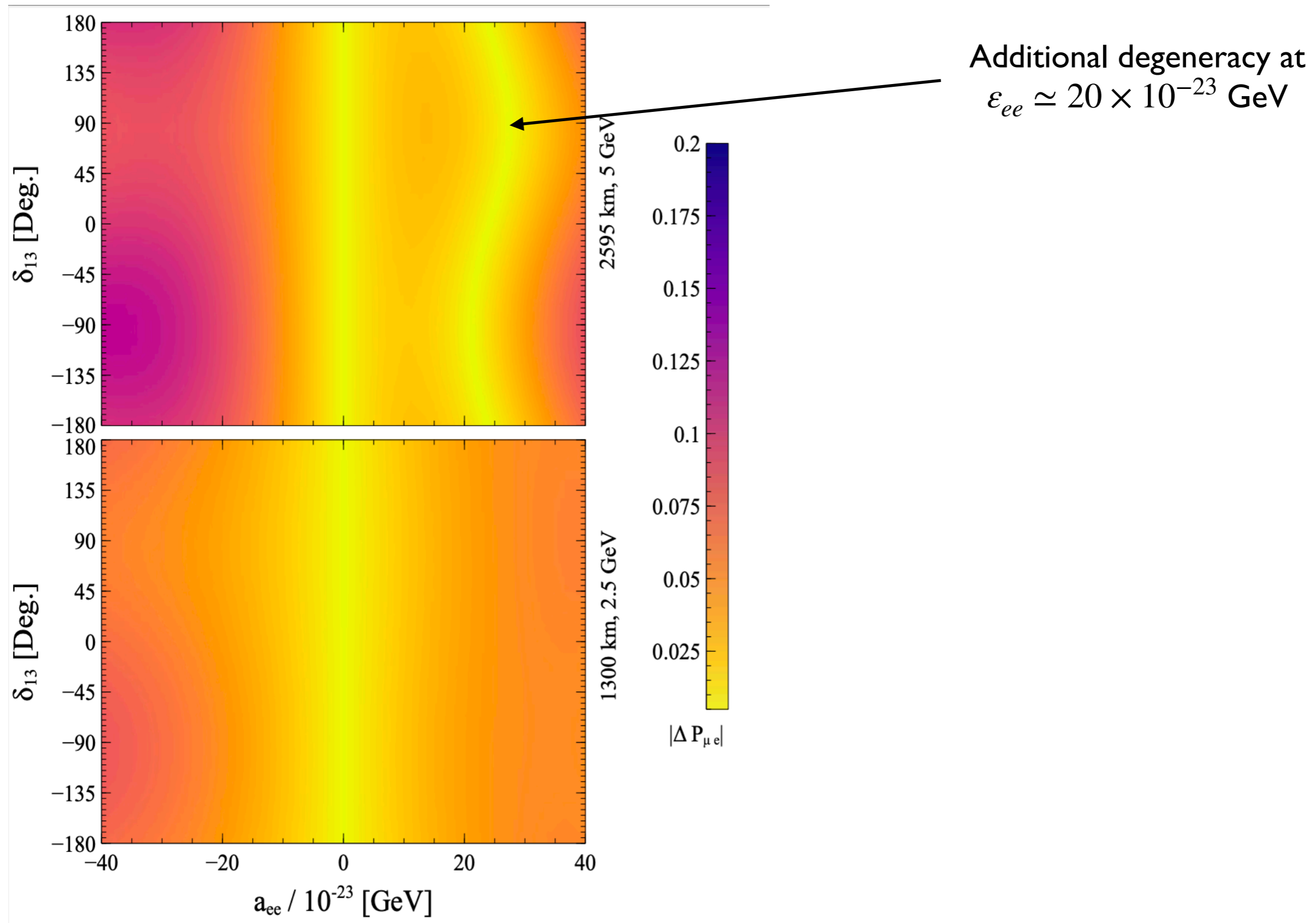
# $\Delta P_{\mu e}$ heatplots in $(\delta_{13} - a_{\alpha\beta})$ plane



# $\Delta P_{\mu e}$ heatplots in $(\delta_{13} - a_{\alpha\beta})$ plane



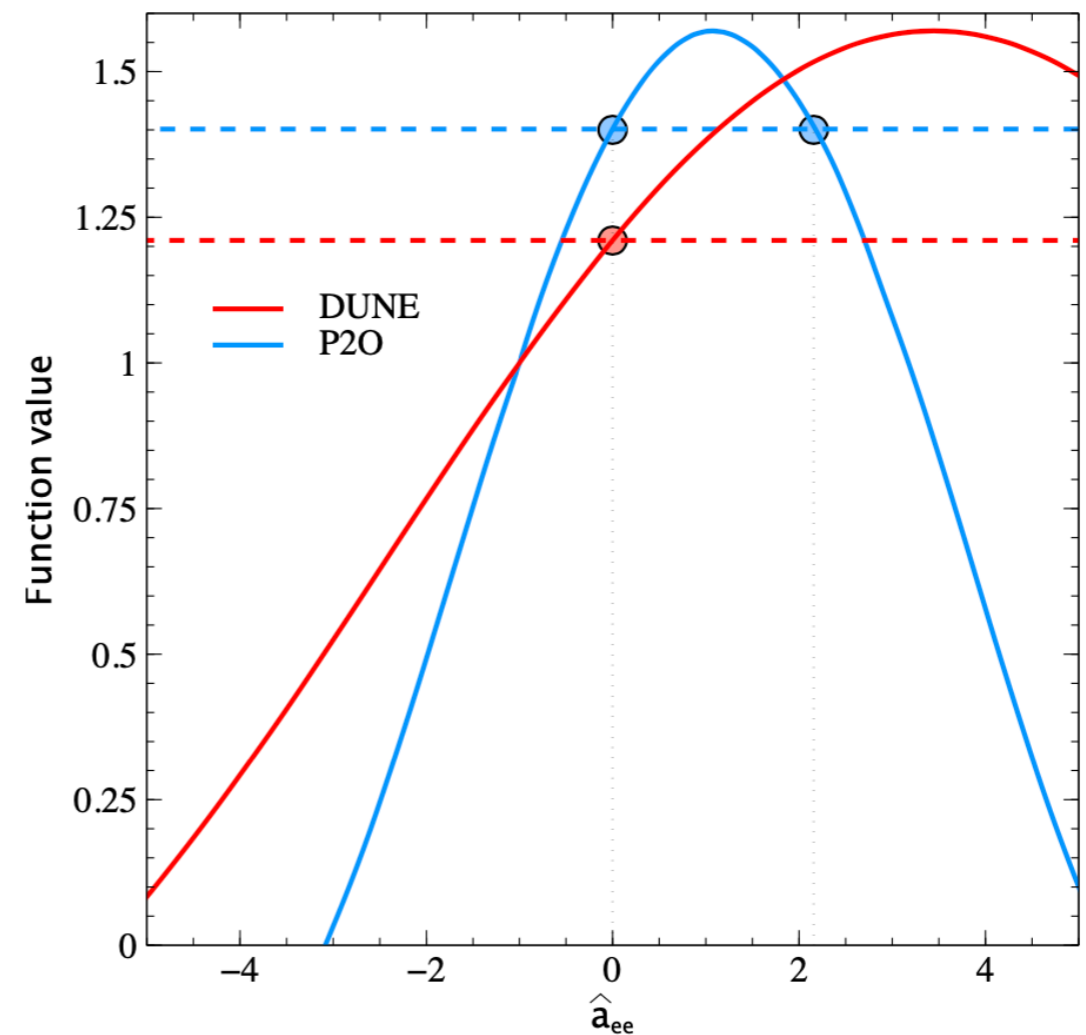
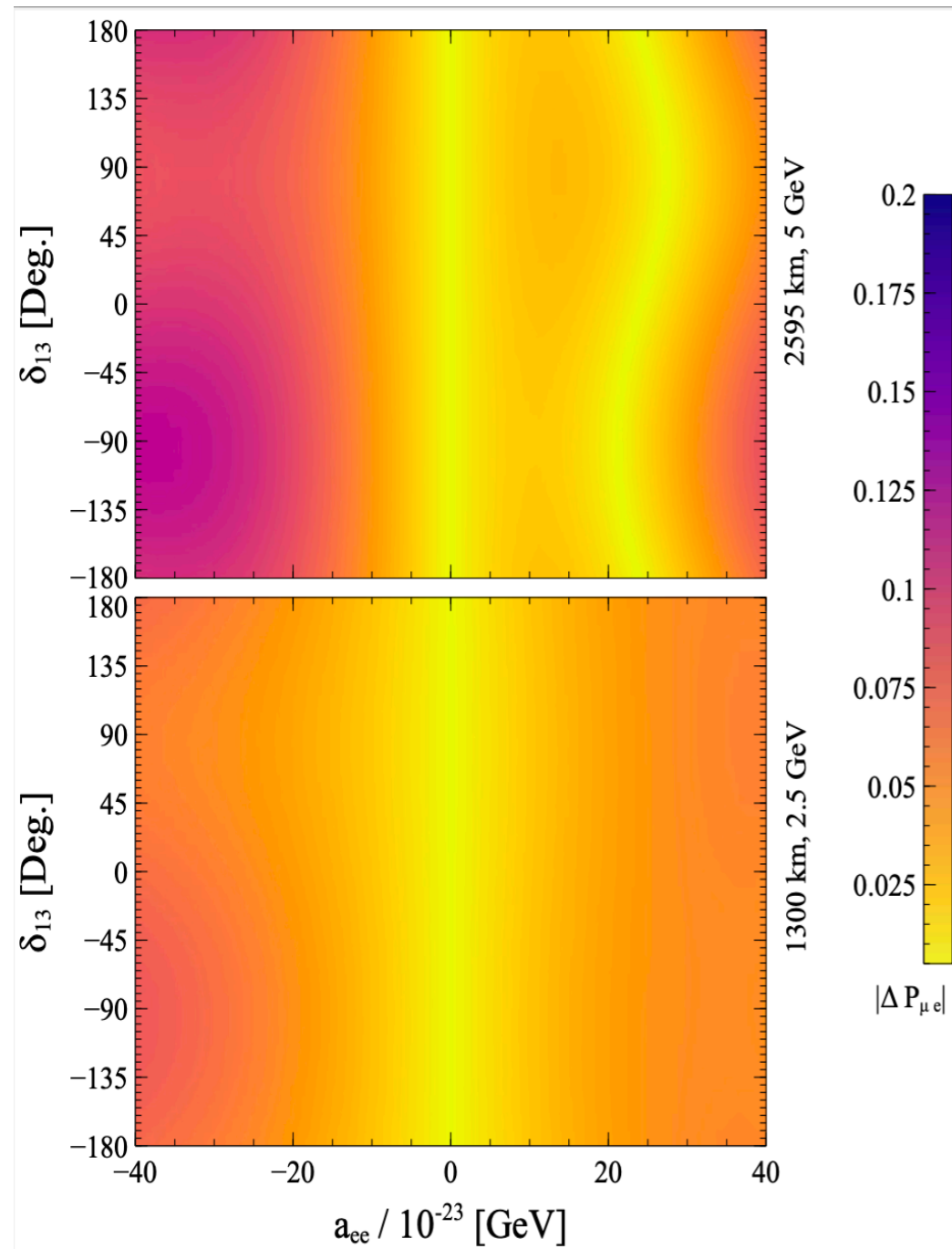
# $\Delta P_{\mu e}$ heatplots in $(\delta_{13} - a_{ee})$ plane



# $\Delta P_{\mu e}$ heatplots in $(\delta_{13} - a_{\alpha\beta})$ plane

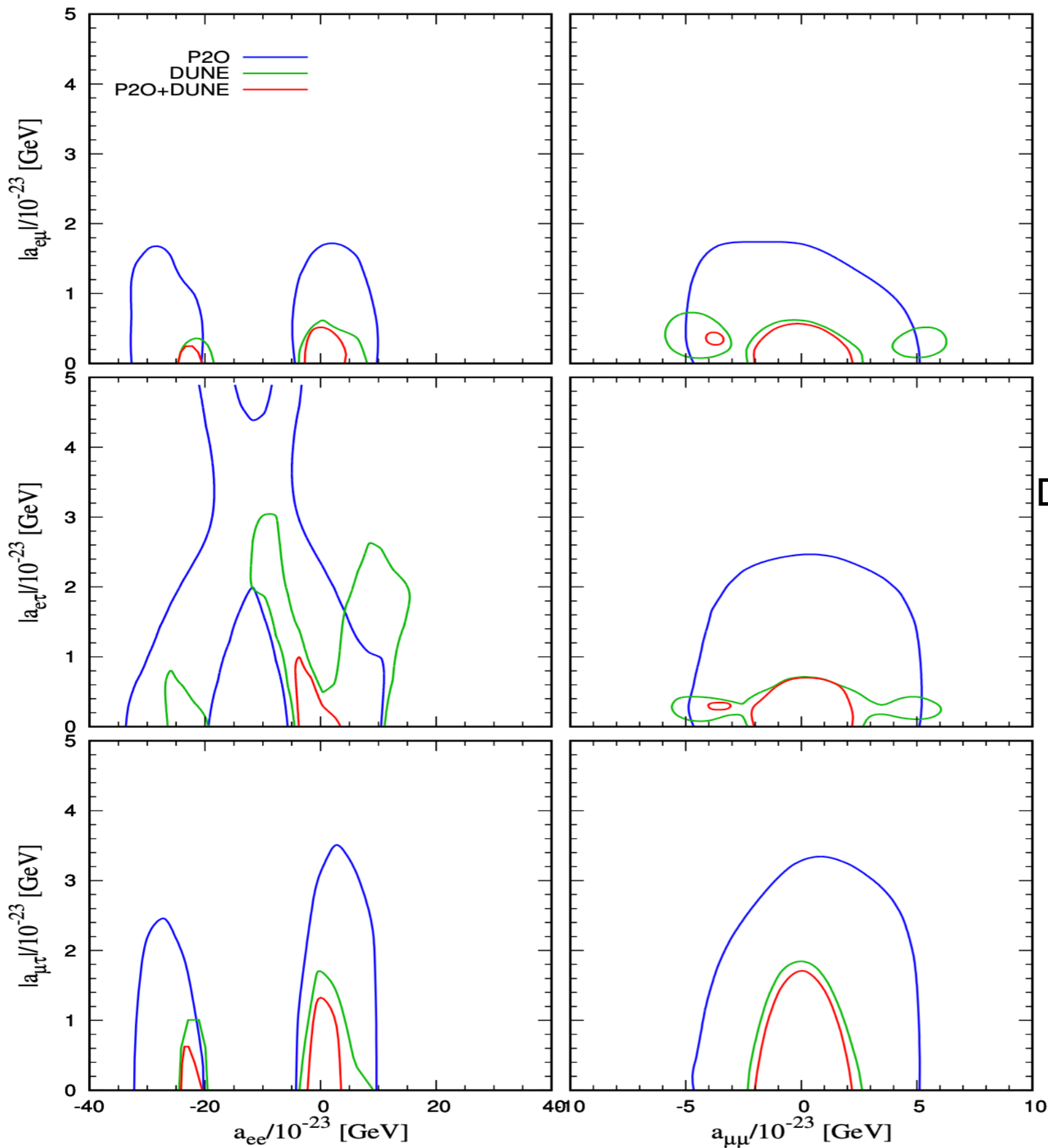
$$\hat{A} \rightarrow \hat{A}[1 + a_{ee}/\sqrt{2}G_F N_e] = 2\sqrt{2}G_F N_e E[1 + a_{ee}/\sqrt{2}G_F N_e] = \hat{A}[1 + \hat{a}_{ee}] \quad (\text{Define } \hat{a}_{ee} = a_{ee}/\sqrt{2}G_F N_e)$$

$$\Delta P_{\mu e}(a_{ee}) = 0 \implies \left[ \frac{\sin [1 - \hat{A}(1 + \hat{a}_{ee})] \Delta}{1 - \hat{A}(1 + \hat{a}_{ee})} - \frac{\sin [1 - \hat{A}] \Delta}{1 - \hat{A}} \right] = 0$$



P2O has two solutions in this range

# LIV-LIV parameter space ( $a_{\alpha\beta} - a_{\gamma\gamma}$ ) at 95% C.L.

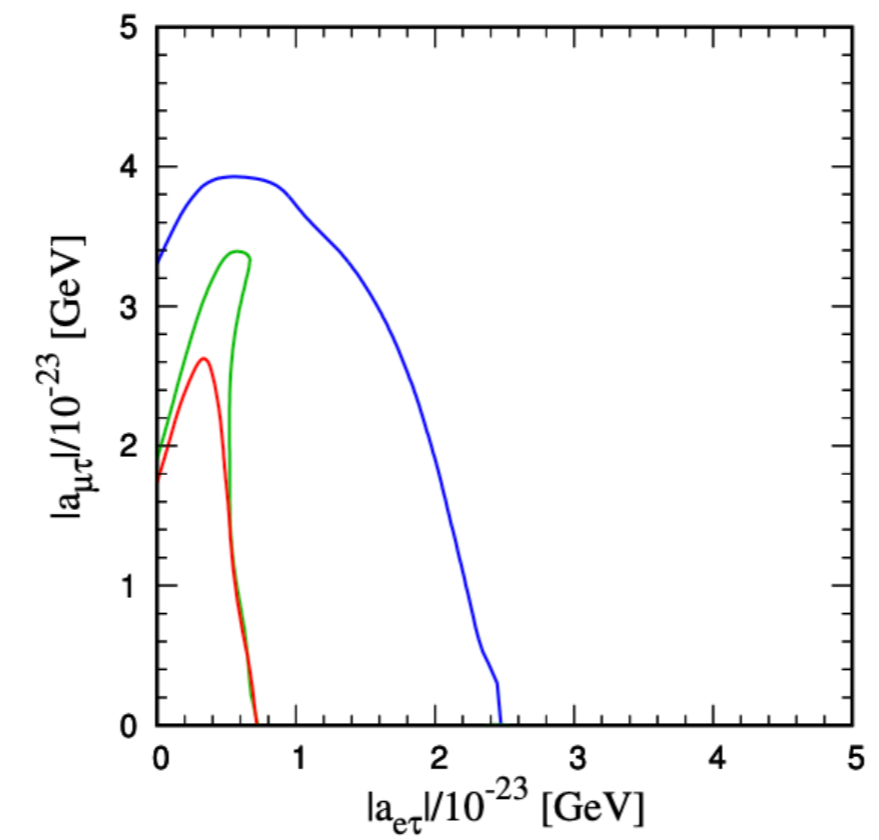
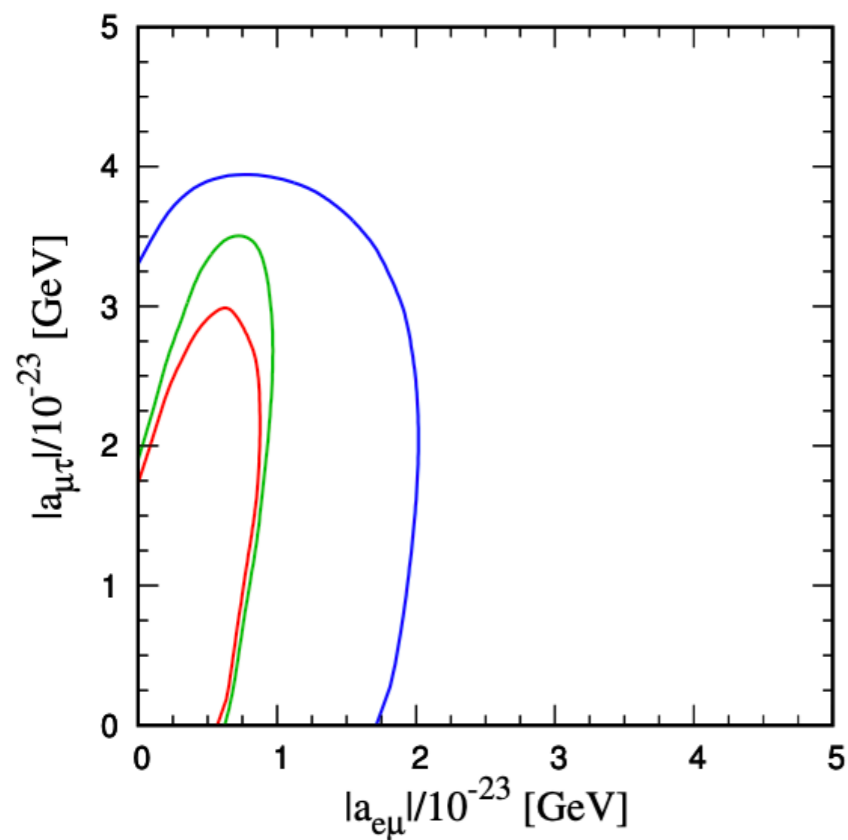
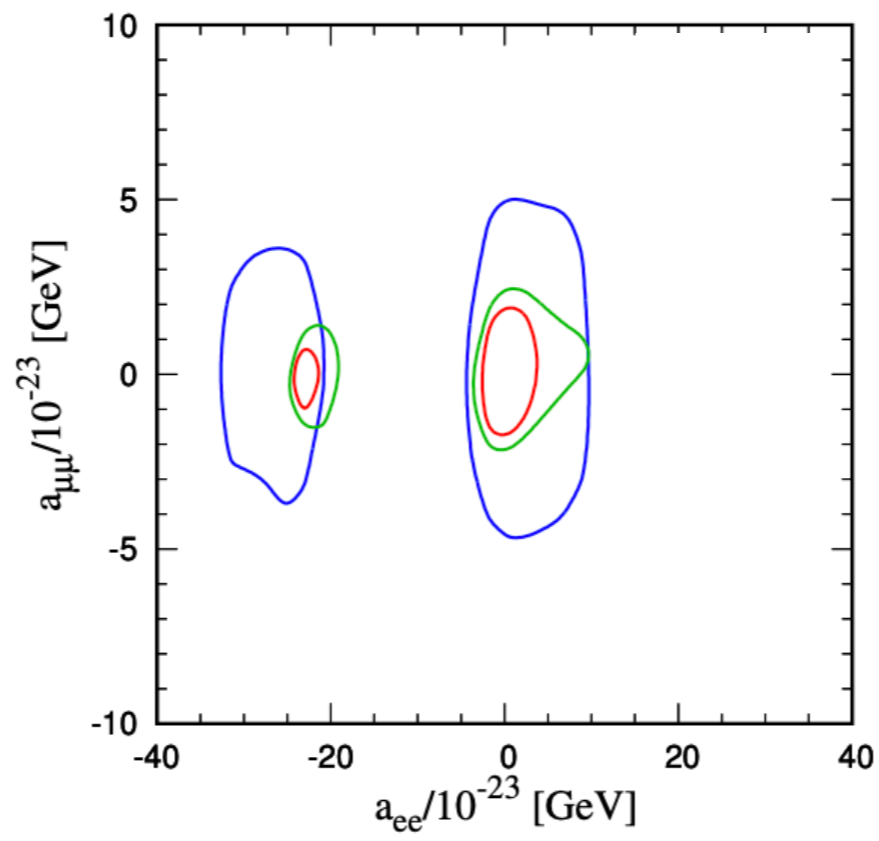
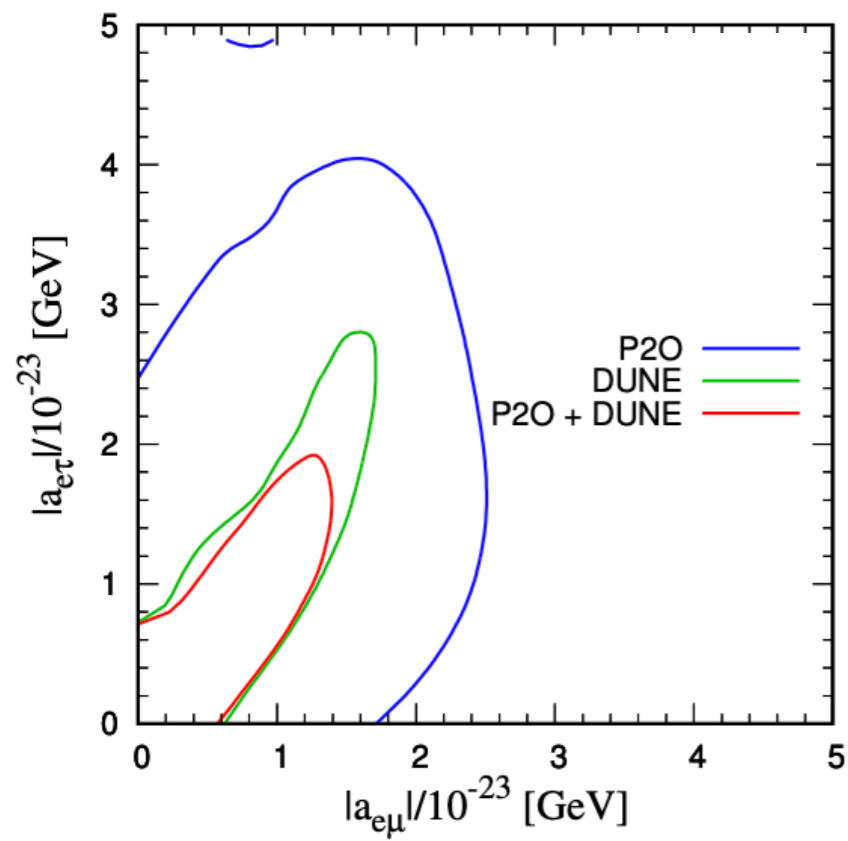


$\Delta\chi^2_{\min}$  after marginalization over  $\theta_{23}, \delta_{13}, \Delta m_{31}^2$  (sign, magnitude)

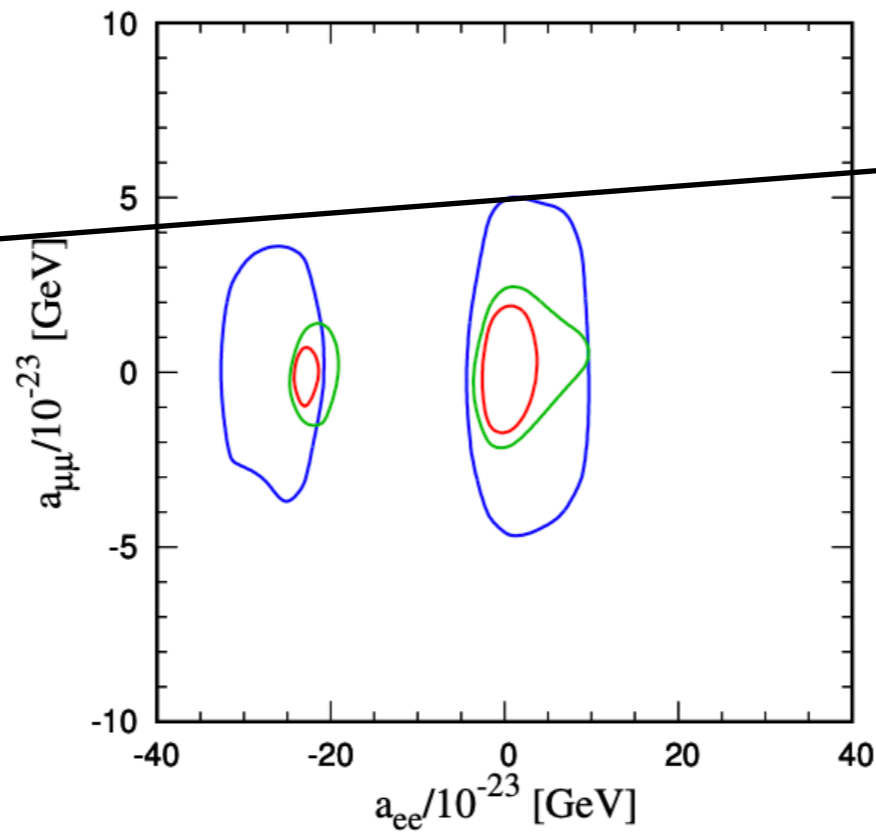
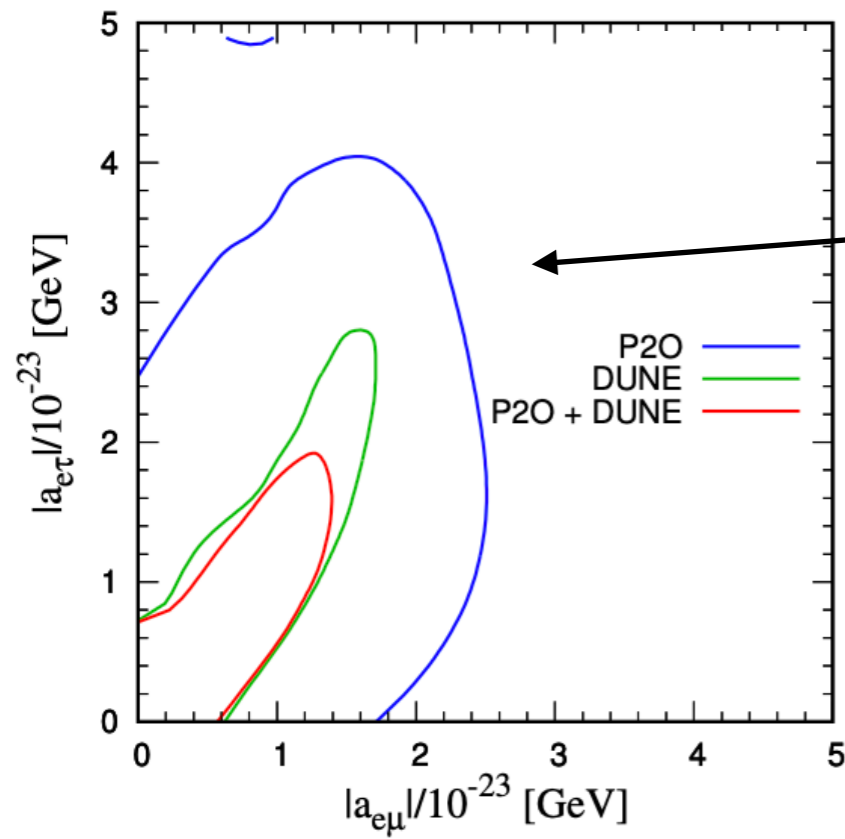
Degeneracy at  $a_{ee} \simeq -20 \times 10^{-23}$  GeV (due to mass ordering marginalization)



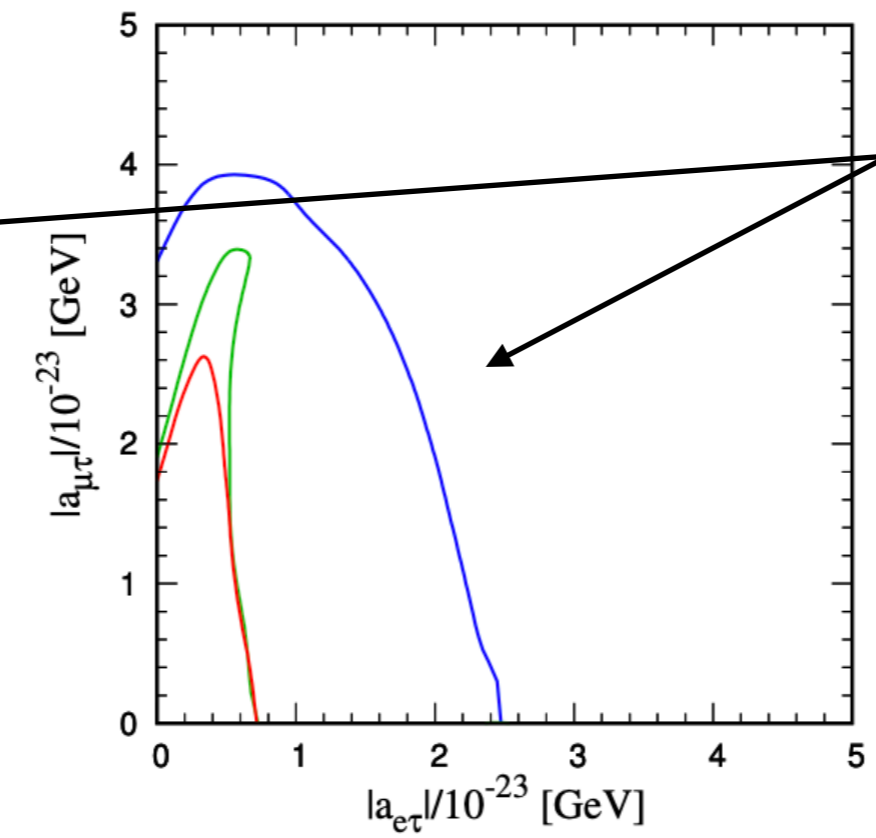
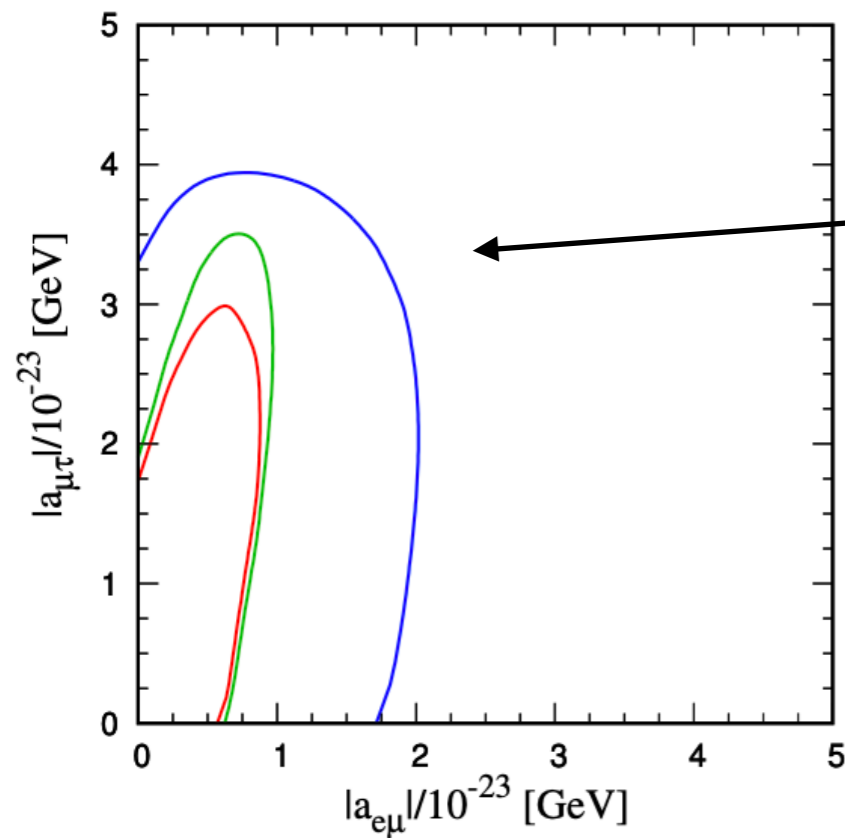
# LIV-LIV parameter space



# LIV-LIV parameter space

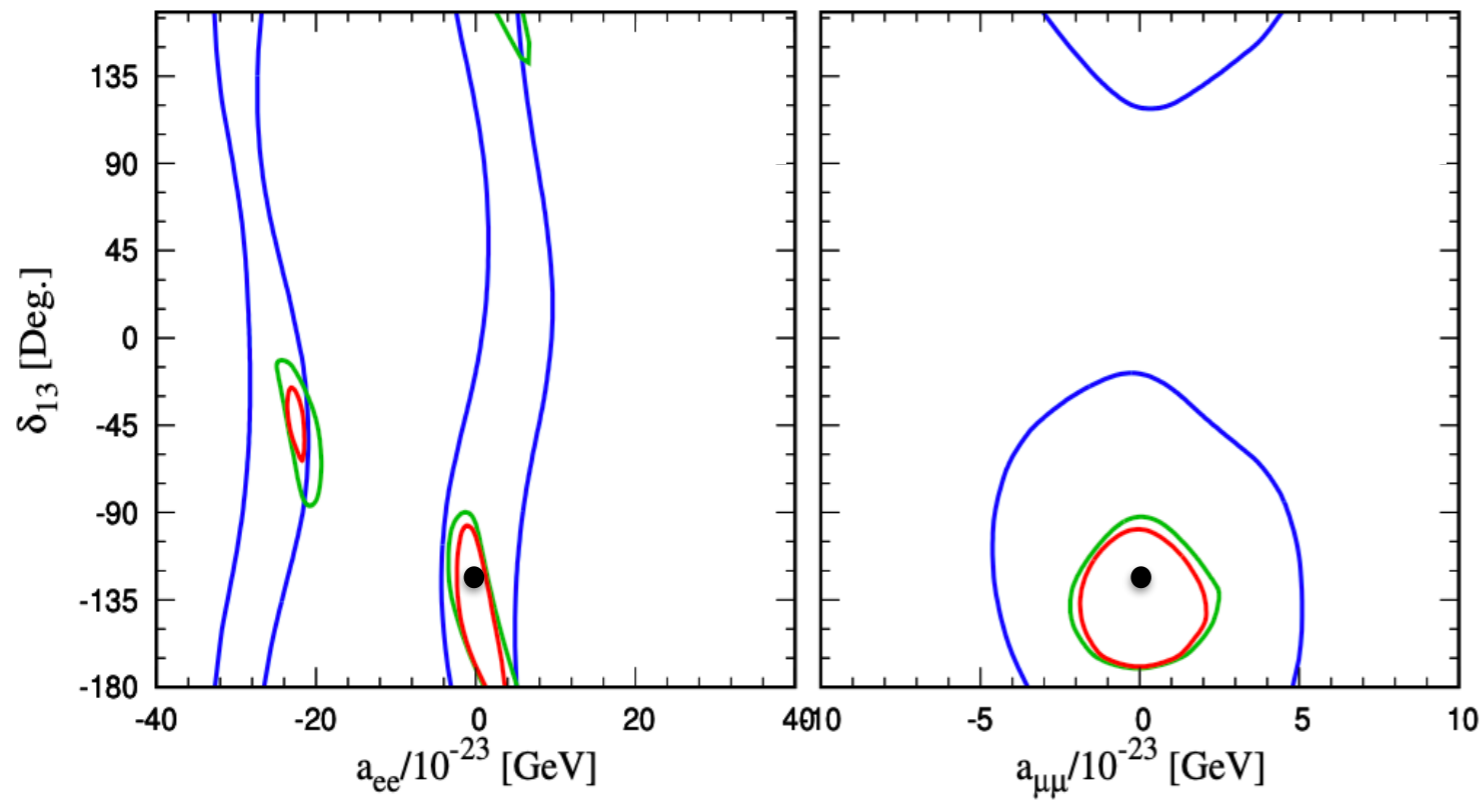


Similar impacts for  $a_{e\mu}$  &  $a_{e\tau}$



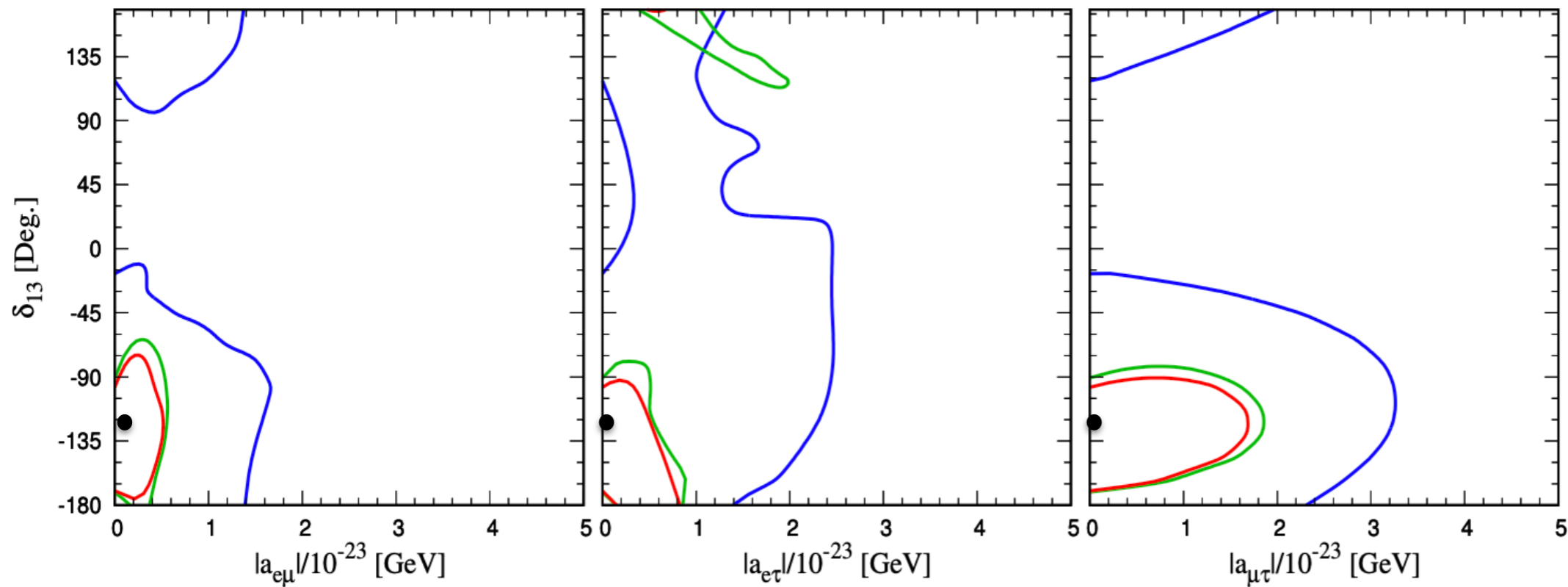
Not as effective in probing  $a_{\mu\tau}$

# LIV-SI( $\delta_{13}$ ) parameter space

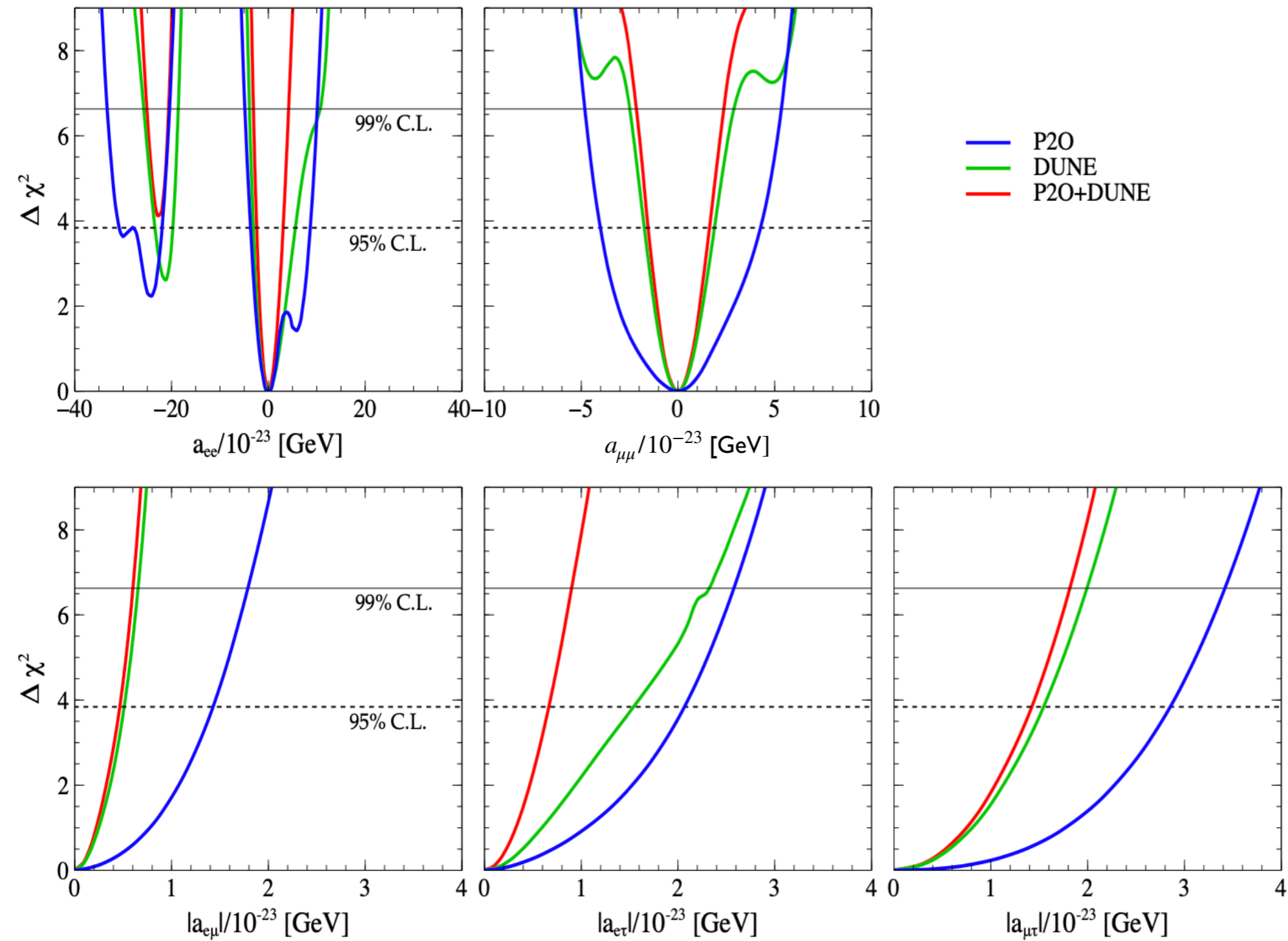


P2O — blue  
DUNE — green  
P2O+DUNE — red

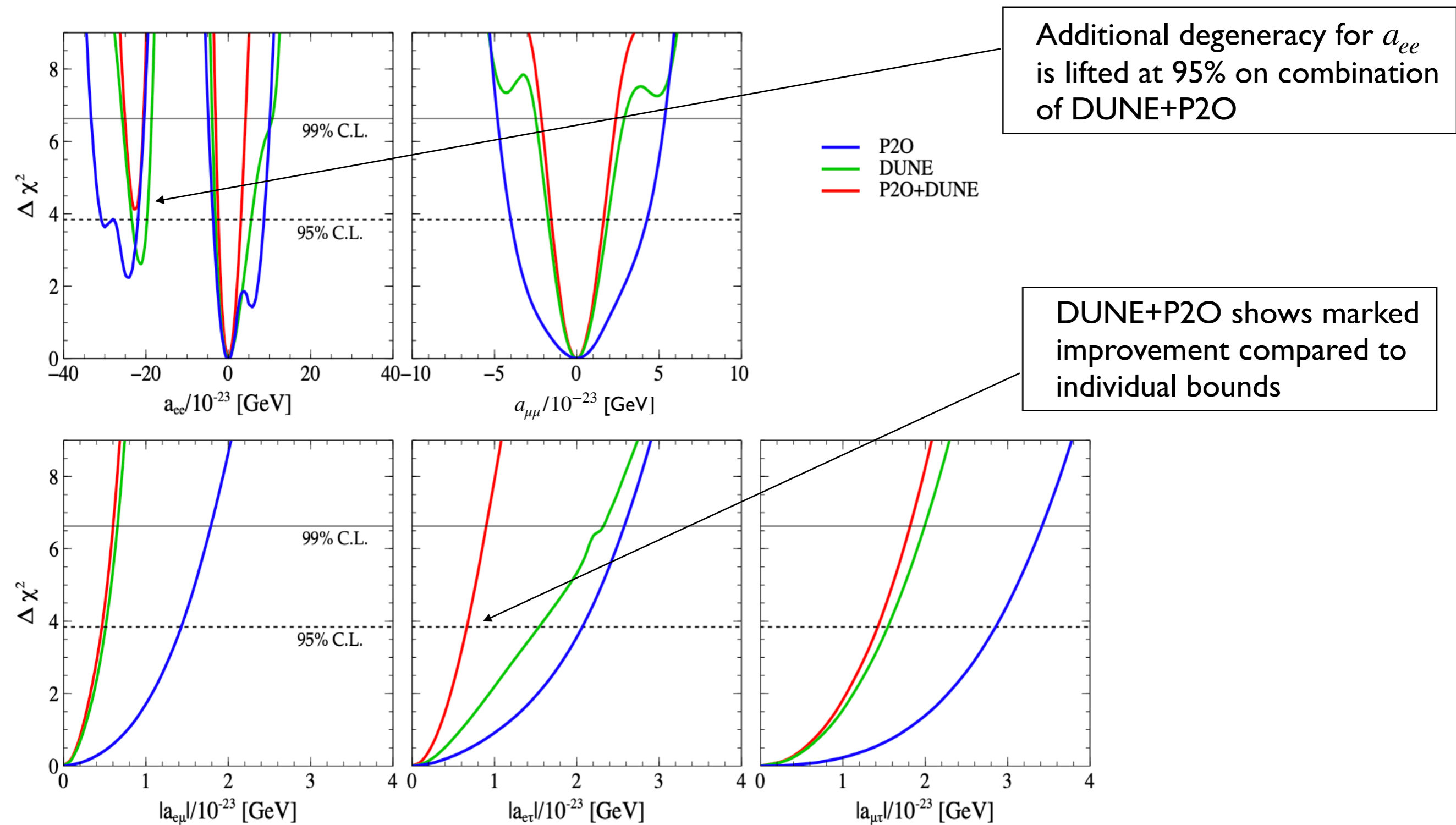
P2O has much less sensitivity to the CP phase than DUNE



# Constraints on LIV parameters



# Constraints on LIV parameters



# Constraints on LIV parameters (95% C.L.)

Parameter	Bounds from DUNE [ $10^{-23}$ GeV]	Bounds from P2O [ $10^{-23}$ GeV]	Bounds from (P2O+DUNE) [ $10^{-23}$ GeV]
$a_{ee}$	$[-24 < a_{ee} < -20]$ $\cup [-3.2 < a_{ee} < 5.6]$	$[-30.8 < a_{ee} < -21.9]$ $\cup [-3.9 < a_{ee} < 8.6]$	$-2.6 < a_{ee} < 3.3$
$a_{\mu\mu}$	$-1.9 < a_{\mu\mu} < 2.0$	$-4.0 < a_{\mu\mu} < 4.3$	$-1.6 < a_{\mu\mu} < 1.6$
$ a_{e\mu} $	0.6	1.6	0.4
$ a_{e\tau} $	1.3	2.1	0.7
$ a_{\mu\tau} $	1.5	2.9	1.3

# Constraints on LIV parameters (95% C.L.)

Parameter	Bounds from DUNE [ $10^{-23}$ GeV]	Bounds from P2O [ $10^{-23}$ GeV]	Bounds from (P2O+DUNE) [ $10^{-23}$ GeV]
$a_{ee}$	$[-24 < a_{ee} < -20]$ $\cup [-3.2 < a_{ee} < 5.6]$	$[-30.8 < a_{ee} < -21.9]$ $\cup [-3.9 < a_{ee} < 8.6]$	$-2.6 < a_{ee} < 3.3$
$a_{\mu\mu}$	$-1.9 < a_{\mu\mu} < 2.0$	$-4.0 < a_{\mu\mu} < 4.3$	$-1.6 < a_{\mu\mu} < 1.6$
$ a_{e\mu} $	0.6	1.6	0.4
$ a_{e\tau} $	1.3	2.1	0.7
$ a_{\mu\tau} $	1.5	2.9	1.3

SK:

$$|a_{e\mu}| \lesssim 3.2 \times 10^{-23} \text{ GeV};$$

$$|a_{e\tau}| \lesssim 5 \times 10^{-23} \text{ GeV}.$$

IceCube:

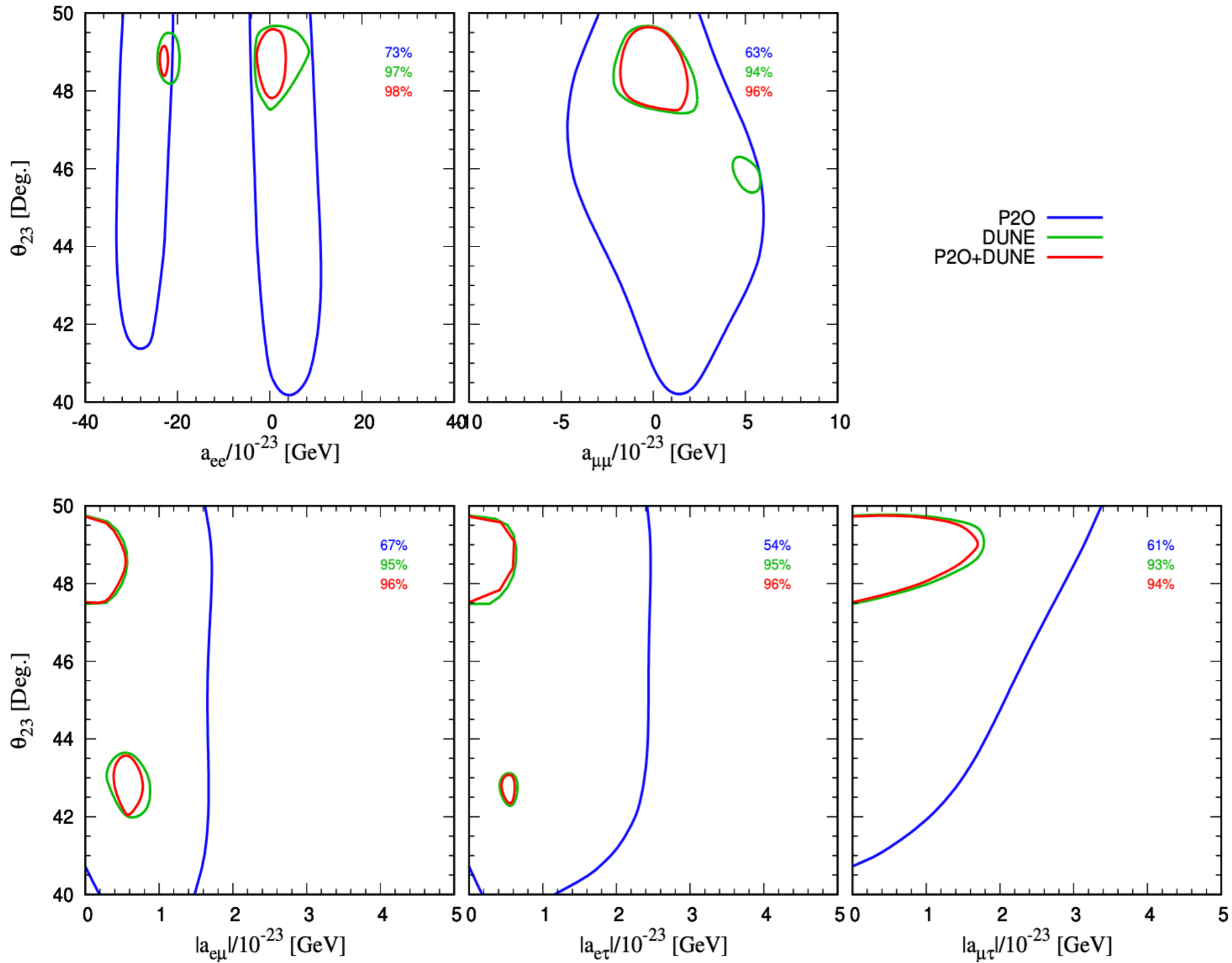
$$|a_{\mu\tau}| \lesssim 0.41 \times 10^{-23} \text{ GeV}.$$

# Summary

- LIV in  $ee, e\mu, e\tau$  sector can be efficiently explored in LBL expts.
- Interesting degeneracies at different L,E: can be understood from probability analysis
- DUNE & P20 can help lift hard-to-remove degeneracies for LIV in  $ee$  sector
- Improvement of bounds by factors of 7-8 on  $a_{e\mu}, a_{e\tau}$



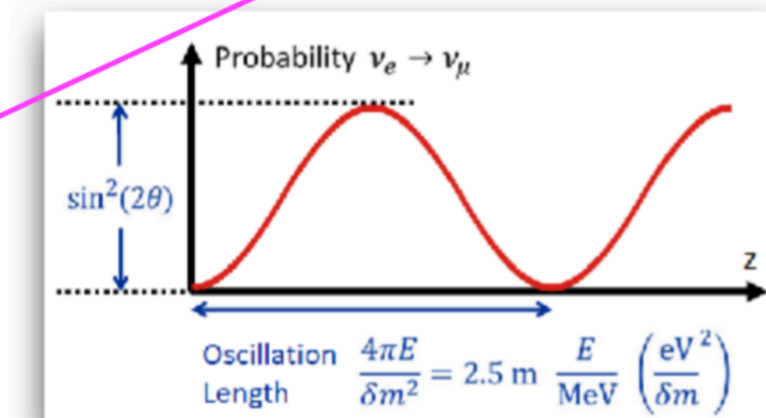
# LIV-SI( $\theta_{23}$ ) parameter space



# Backup

- Each  $\nu_k$  propagates as  $e^{-iE_k L}$  with  $E_k \simeq \sqrt{|\vec{p}|^2 + m_k^2} \simeq E + \frac{m_k^2}{2E}$  where  $E \simeq |\vec{p}|$
- Oscillation arises due to the difference  $\Delta m_{kj}^2 = m_k^2 - m_j^2$

- $P(\nu_\mu \rightarrow \nu_e) = |\langle \nu_e | \nu_\mu(L) \rangle|^2$   
 $= \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)$



$$\simeq 1.27 \times \frac{\Delta m^2 [\text{eV}^2] \cdot L [\text{km}]}{E [\text{GeV}]} \simeq \pi/2$$

(For maximal oscillation)

$$\frac{L [\text{km}]}{E [\text{GeV}]} \sim (\Delta m^2 [\text{eV}^2])^{-1}$$

Main Principle for Long-Baseline expts.

# Leptonic CP violation?

Table: de Salas, Forero, Gariazzo, Martinez-Mirave, Mena, Ternes, Tortola, Valle: 2006.11237

Is the CP phase non-zero?

could help explain baryon asymmetry

Oscillation parameter	Best fit value	$3\sigma$ range
$\theta_{12}/^\circ$	34.3	[31.4, 37.4]
$\theta_{23}/^\circ$	48.8	[41.6, 51.3]
$\theta_{13}/^\circ$	8.6	[8.2, 8.9]
$\delta_{13}/\pi$	-0.8	$[-1, 0] \cup [0.8, 1]$
$\Delta m_{21}^2/10^{-5} \text{ eV}^2$	7.5	[6.9, 8.1]
$\Delta m_{31}^2/10^{-3} \text{ eV}^2$	2.6	[2.5, 2.7]

$$\begin{aligned}
 P_{\mu e} = & \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2(1-A)\Delta}{1-A} \\
 & + \alpha^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2 A\Delta}{A} \\
 & + \alpha \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin(1-A)\Delta}{1-A} \frac{\sin A\Delta}{A} \cos(\delta + \Delta)
 \end{aligned}$$

where,

$$A = \frac{2\sqrt{2}EG_F n_E}{\Delta m_{31}^2}$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}$$

$$\alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$$

# Backup

$$\Delta P_{\mu e}(a_{ee}) \simeq 4s_{13}^2 c_{13}^2 s_{23}^2 \left\{ \frac{\sin^2 [1 - \hat{A}(1 + a_{ee}/\sqrt{2}G_F N_e)] \Delta}{[1 - \hat{A}(1 + a_{ee}/\sqrt{2}G_F N_e)]^2} - \frac{\sin^2 [1 - \hat{A}] \Delta}{[1 - \hat{A}]^2} \right\} + \cos \delta_{13}\text{-term.}$$

$$\underbrace{\left[ \frac{\sin [1 - \hat{A}(1 + \hat{a}_{ee})] \Delta}{1 - \hat{A}(1 + \hat{a}_{ee})} - \frac{\sin [1 - \hat{A}] \Delta}{1 - \hat{A}} \right]}_{I_-} \times \underbrace{\left[ \frac{\sin [1 - \hat{A}(1 + \hat{a}_{ee})] \Delta}{1 - \hat{A}(1 + \hat{a}_{ee})} + \frac{\sin [1 - \hat{A}] \Delta}{1 - \hat{A}} \right]}_{I_+} = 0,$$

$$\Delta \chi^2(a_{ee}, c) \sim \Delta P_{\mu e}(a_{ee})$$

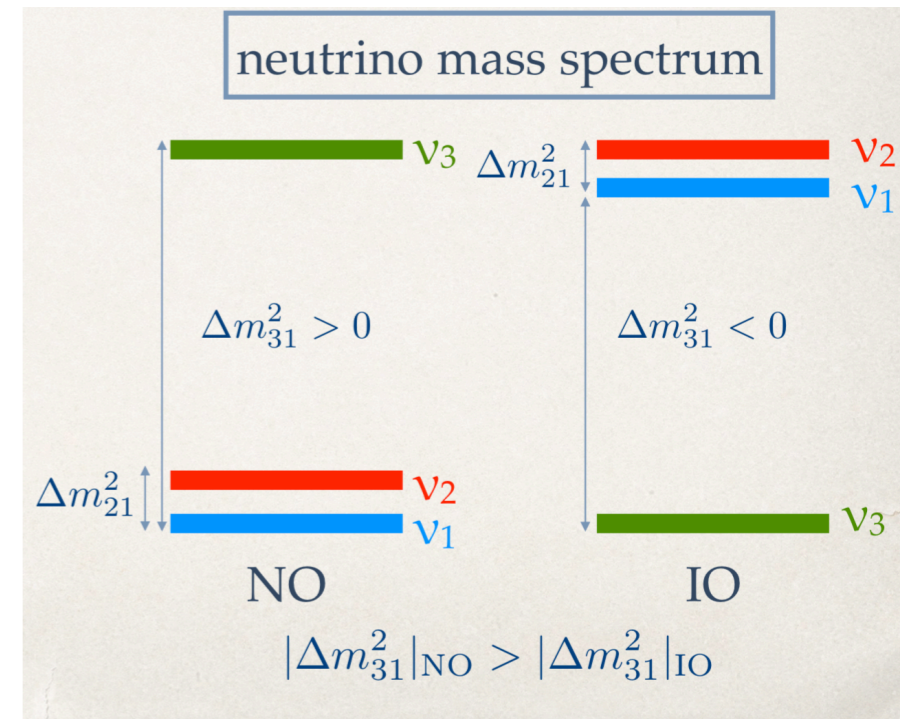
$$\sim \underbrace{\left[ \frac{\sin [1 - \hat{A}(1 + \hat{a}_{ee})] \Delta}{1 - \hat{A}(1 + \hat{a}_{ee})} - \frac{\sin [1 - \hat{A}] \Delta}{1 - \hat{A}} \right]}_{I_-} \times \underbrace{\left[ \frac{\sin [1 - \hat{A}(1 + \hat{a}_{ee})] \Delta}{1 - \hat{A}(1 + \hat{a}_{ee})} + \frac{\sin [1 - \hat{A}] \Delta}{1 - \hat{A}} \right]}_{I_+}.$$

$$\sim \underbrace{\left[ \frac{\sin [1 + \hat{A}(1 + \hat{a}_{ee})] \Delta}{1 + \hat{A}(1 + \hat{a}_{ee})} + \frac{\sin [1 - \hat{A}] \Delta}{1 - \hat{A}} \right]}_{I_-} \times \underbrace{\left[ \frac{\sin [1 + \hat{A}(1 + \hat{a}_{ee})] \Delta}{1 + \hat{A}(1 + \hat{a}_{ee})} - \frac{\sin [1 - \hat{A}] \Delta}{1 - \hat{A}} \right]}_{I_+}.$$

# Mass ordering ambiguity

**Table:** de Salas, Forero, Gariazzo, Martinez-Mirave, Mena, Ternes, Tortola, Valle: 2006.11237

Oscillation parameter	Best fit value	$3\sigma$ range
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$\Delta m_{31}^2/10^{-3} \text{ eV}^2$	2.6	[2.5, 2.7]



What is the sign of  $\Delta m_{31}^2$  ?

$$\begin{aligned}
 P_{\mu e} = & \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2(1-A)\Delta}{1-A} \\
 & + \alpha^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2 A\Delta}{A} \\
 & + \alpha \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin(1-A)\Delta}{1-A} \frac{\sin A\Delta}{A} \cos(\delta + \Delta)
 \end{aligned}$$

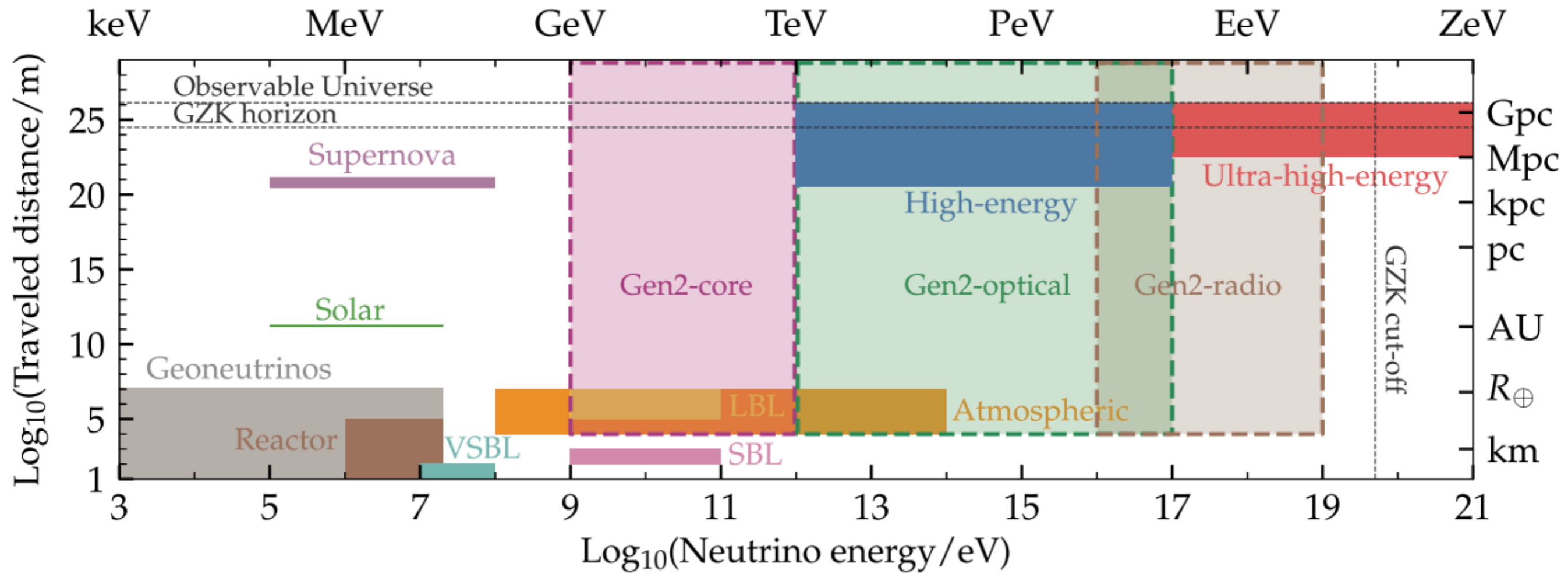
where,

$$A = \frac{2\sqrt{2}EG_F n_E}{\Delta m_{31}^2}$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}$$

$$\alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$$

# Icecube-Gen2





# Chisquare

$$\Delta\chi^2(p^{\text{true}}) = \text{Min}_{p^{\text{test}}, \eta} \left[ 2 \sum_k \sum_j \sum_i^{\text{mode channel bin}} \left\{ N_{ijk}^{\text{test}}(p^{\text{test}}; \eta) - N_{ijk}^{\text{true}}(p^{\text{true}}) \right. \right. \\ \left. \left. + N_{ijk}^{\text{true}}(p^{\text{true}}) \ln \frac{N_{ijk}^{\text{true}}(p^{\text{true}})}{N_{ijk}^{\text{test}}(p^{\text{test}}; \eta)} \right\} + \sum_l \frac{(p_l^{\text{true}} - p_l^{\text{test}})^2}{\sigma_{p_l}^2} + \sum_m \frac{\eta_m^2}{\sigma_{\eta_m}^2} \right].$$