

Update on Bodek-Yang Model

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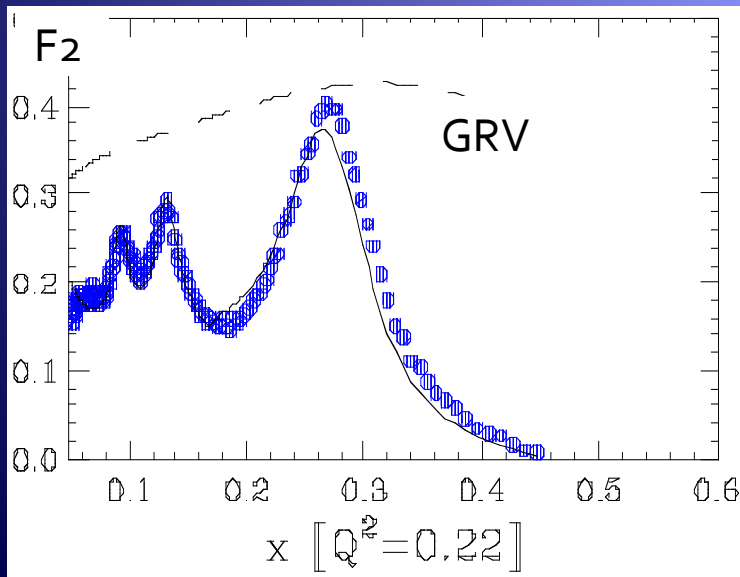
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Bodek-Yang Model

- Bodek-Yang model: describe DIS cross section in all Q^2 regions
- Challenges in e/μ -N DIS
 - High x PDFs at low Q^2
 - Resonance region overlapped with a DIS contribution
 - Hard to extrapolate DIS contribution to low Q^2 region from high Q^2 data due to non-perturbative QCD effects



- A model in terms of quark-parton model (easy to convert e/μ scattering to ν scattering)
- ✓ Understanding of high x PDFs at low Q^2 ? wealthy SLAC, JLAB data.
- ✓ Understanding of resonance scattering in terms of quark-parton model? (duality works, many studies by JLAB)

Lessons from previous QCD studies

- **NLO & NNLO analyses** with DIS data: PRL 82, 2467 (1999), Eur. Phys. J. C13, 241 (2000) by Bodek and Yang
 - Kinematic higher twist (target mass) effects are large
 - Resonance region can be well described (duality works).
 - Most of dynamic higher twist corrections (in NLO analysis) are similar to missing NNLO higher order terms.

- **NNLO pQCD+TM with NNLO PDFs can describe the non-perturbative QCD effects at low Q^2**
- **Thus, we reverse the approach to build the model:**
 - Use effective LO PDFs with a new scaling variable, ξ_w to absorb target mass, higher twist, missing QCD higher order

$$x_{Bj} = \frac{Q^2}{2Mn}$$



$$\xi_w = \frac{Q^2 + B}{\{Mv[1 + \sqrt{(1+Q^2/v^2)}] + A\}}$$

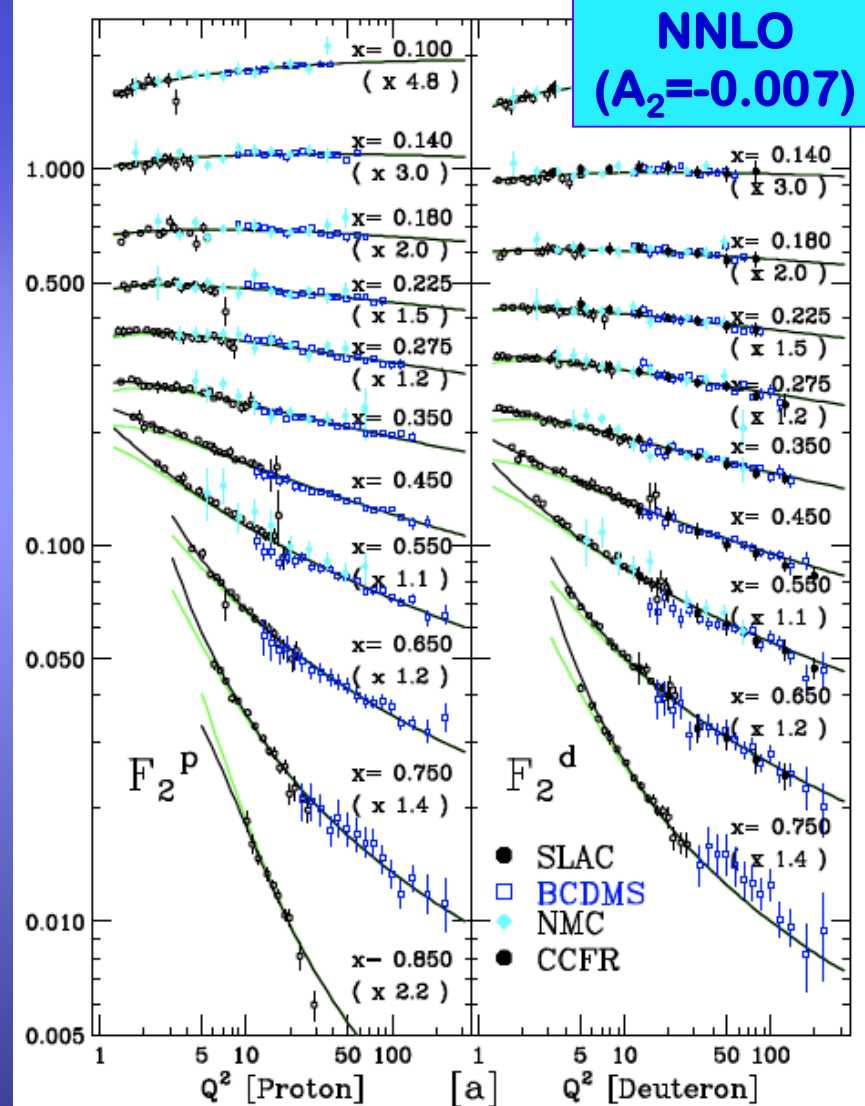
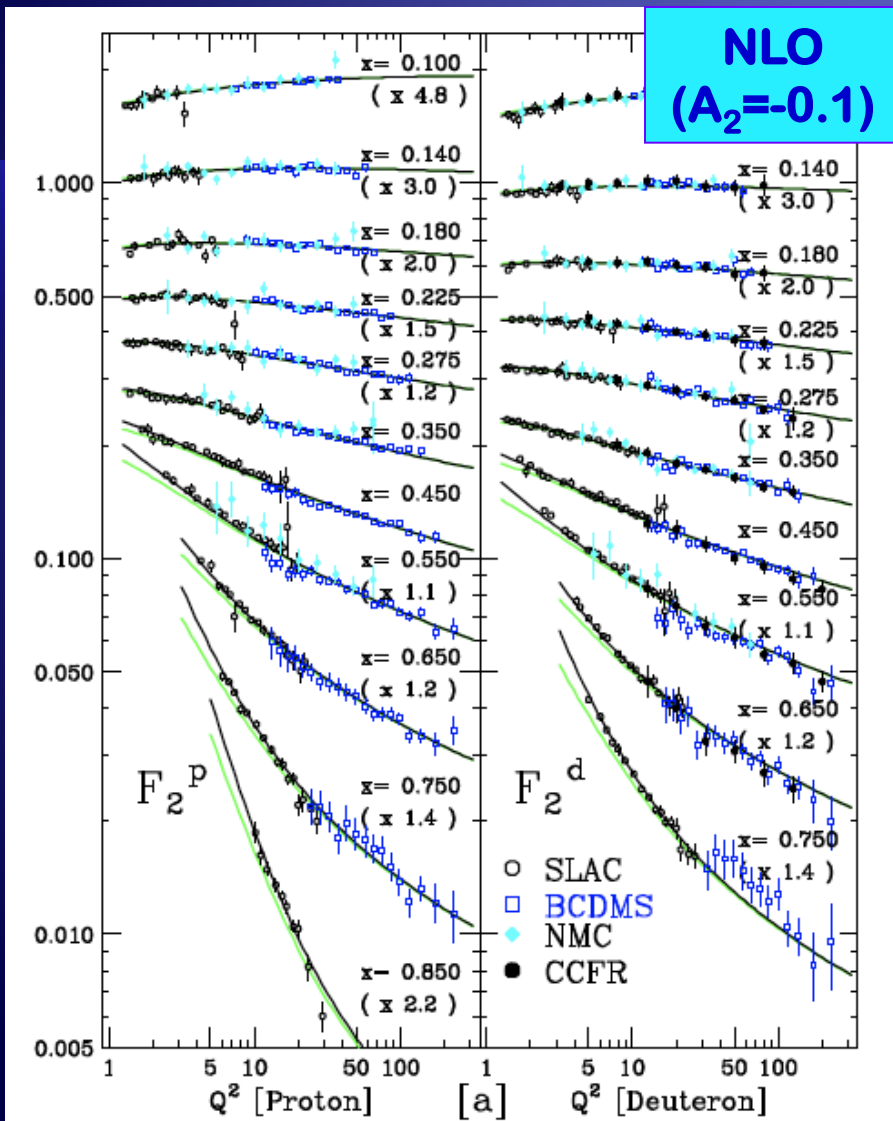
$$F_2(x, Q^2) \rightarrow \frac{Q^2}{Q^2 + C} F_2(x_w, Q^2)$$

NLO vs NNLO

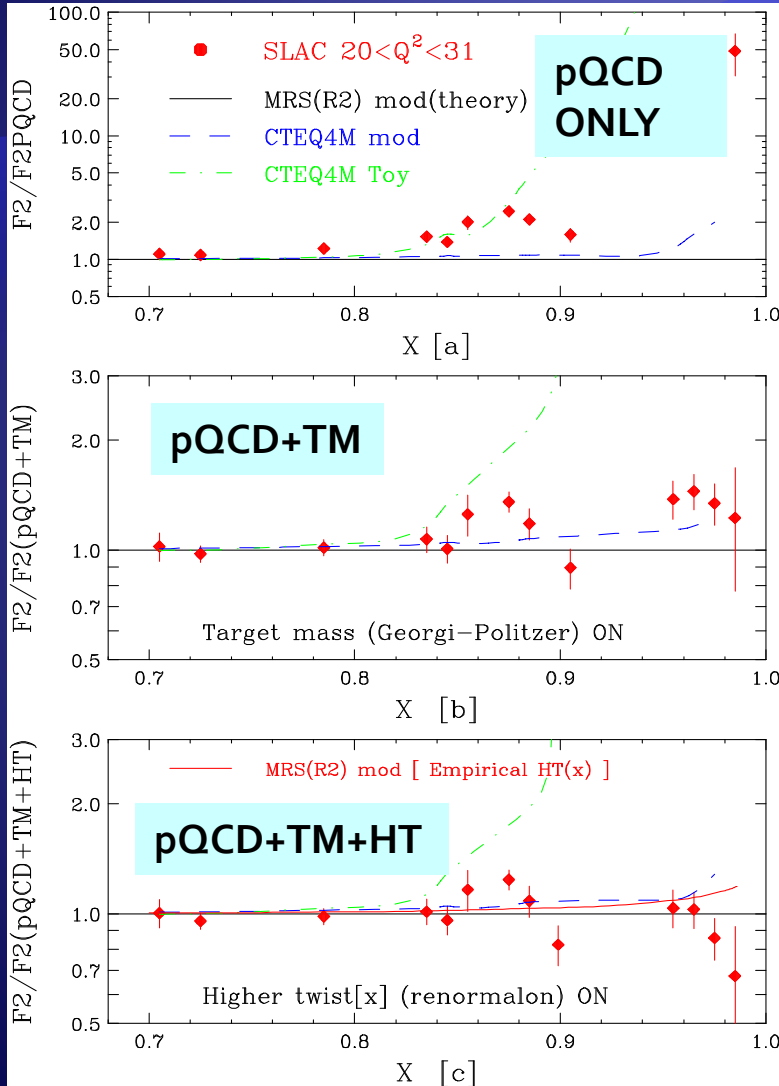
Studies of higher twist and higher order effects in NLO and NNLO QCD analysis of lepton-nucleon scattering data on F_2 and $R = \sigma_L/\sigma_T$

U.K. Yang & A. Bodek

The European Physical Journal C - Particles and Fields 13, 241–245 (2000) | [Cite this article](#)



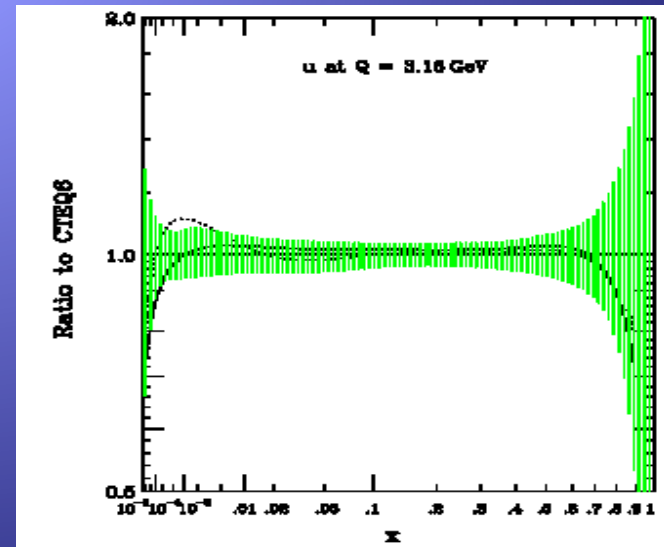
Very high x and low Q^2 data



Parton Distributions, d/u , and Higher Twist Effects at High x

U. K. Yang and A. Bodek
 Phys. Rev. Lett. **82**, 2467 – Published 22 March 1999

- Very high x and low Q^2 data is well described by the pQCD+TM+HT
- Extraction of the high x PDF is promising (1999)



Bodek-Yang Effective LO PDFs Model

1. Start with GRV LO PDF ($Q^2_{\min}=0.80$)
2. Replace x_{bj} with a new scaling, ξ_w
3. Multiply all PDFs by K factors for photo prod. limit and higher twist

$$[\sigma(\gamma) = 4\pi\alpha/Q^2 * F_2(x, Q^2)]$$

$$K_{sea} = Q^2/[Q^2+C_{sea}]$$

$$K_{val} = [1 - G_D^2(Q^2)]$$

$$* [Q^2+C_{2V}] / [Q^2+C_{1V}]$$

motivated by Adler Sum rule

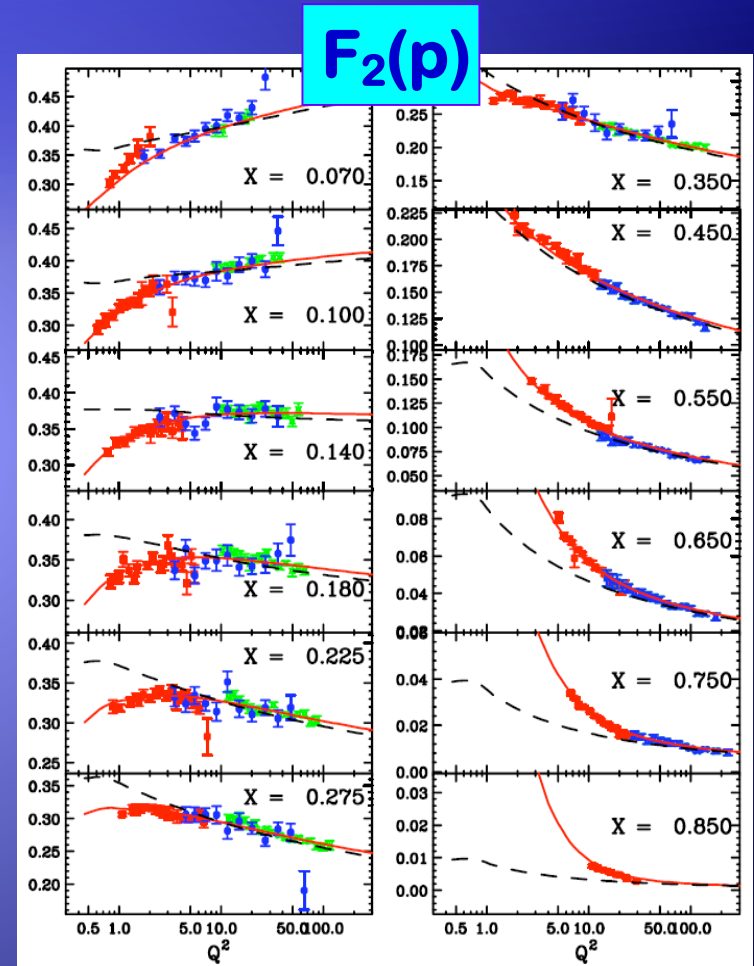
$$\text{where } G_D^2(Q^2) = 1/[1+Q^2/0.71]^2$$

4. Freeze the evolution at $Q^2 = Q^2_{\min}$

$$- F_2(x, Q^2 < 0.8) = K(Q^2) * F_2(\xi_w, Q^2=0.8)$$

5. Fit all DIS $F_2(p/D)$ data:

SLAC/BCDMS/NMC/HERA data



Nucl.Phys.B 139 (2005)

Photo-production data

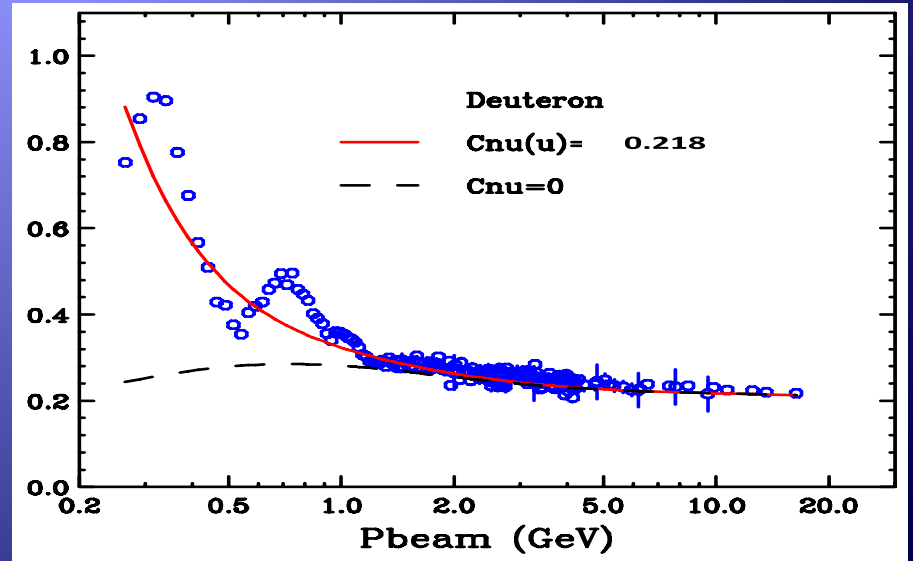
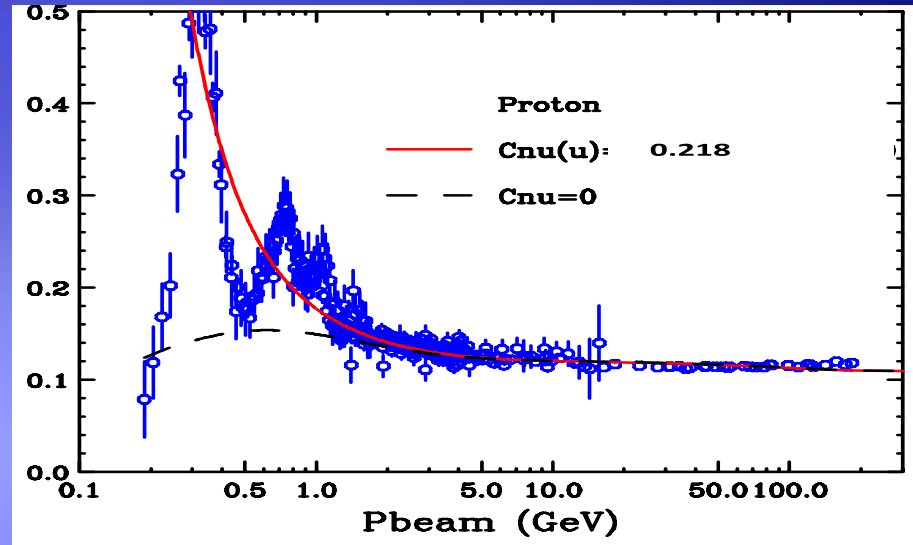
- Additional K^{LW} factor for valence quarks:

$$K_{val} = K^{LW} * [1 - G_D^2(Q^2)] * [Q^2 + C_{2V}] / [Q^2 + C_{1V}]$$

$$K^{LW} = (v^2 + C^v) / v^2$$

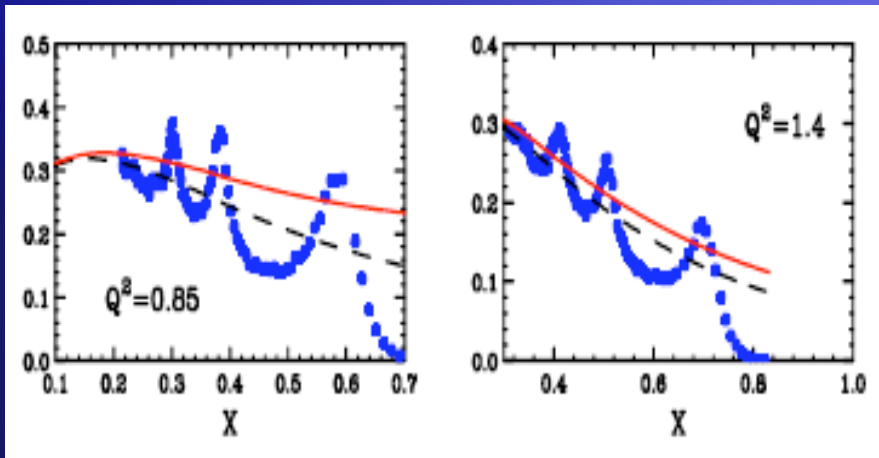
This makes a duality work all the way down to $Q^2=0$ (for charged leptons)

- Photo-production data with $v(P_{beam}) > 1$ GeV included in the fitting

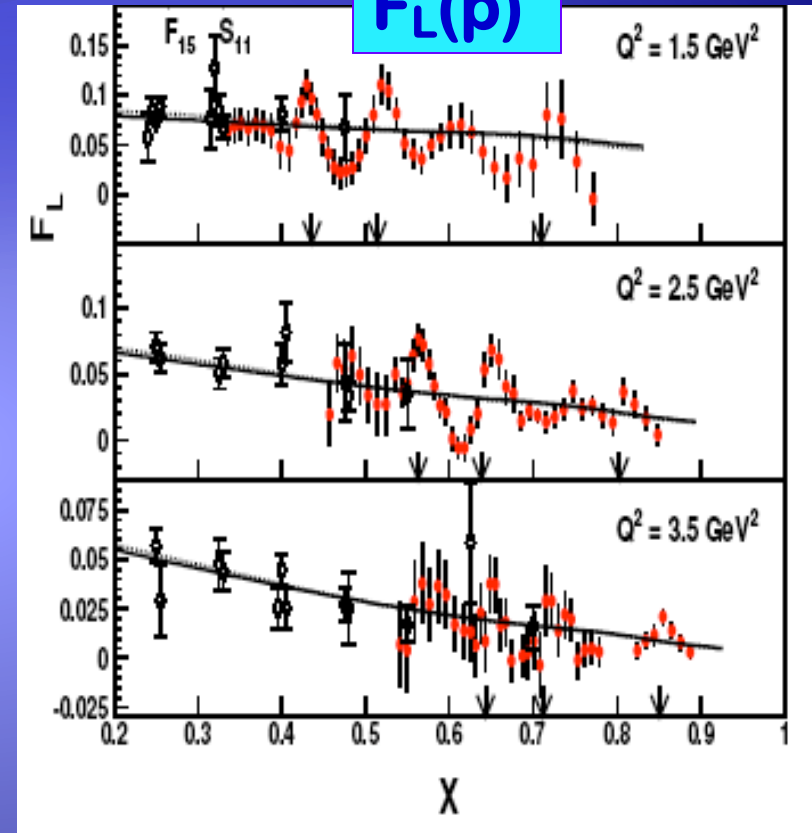


F₂ & F_L Resonance data

F₂(p)



F_L(p)



- Predictions for F_L are in good agreement (not included in the fit) duality works

$$F_L = F_2 \left(1 + 4M^2 x^2 / Q^2\right) \frac{R}{(1 + R)}$$

Neutrino cross sections

- Effective LO model with ξw describe all DIS and resonance F_2 data as well as photo-production data ($Q^2=0$ limit): vector contribution works well
- Neutrino Scattering:
 - Effective LO model works for xF_3 ?
 - Nuclear effects using e/μ scattering data
 - Axial vector contribution at low Q^2 ?
 - Use $R=R_{1998}$ to get $2xF_1$
 - Implement charm mass effect through ξw slow rescaling algorithm for F_2 , $2xF_1$, and xF_3

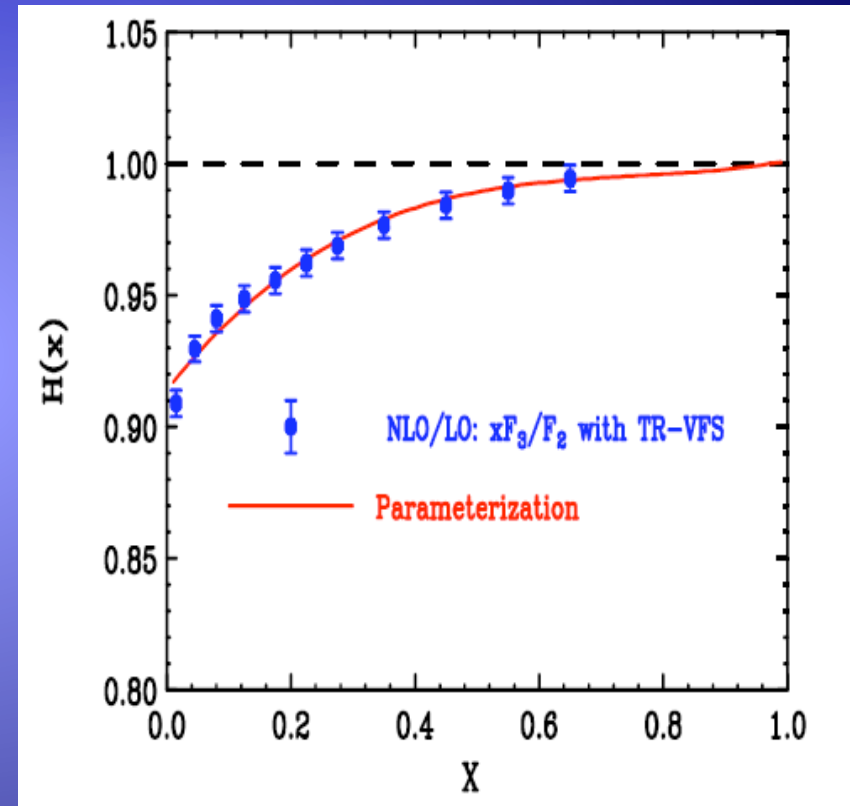
Effective LO model for xF_3 ?

- Scaling variable, ξw absorbs higher order effect for F_2 , but the higher order effects for F_2 and xF_3 are not the same
- Use NLO QCD to get double ratio

$$H(x) = \frac{xF_3(\text{NLO})}{xF_3(\text{LO})} / \frac{F_2(\text{NLO})}{F_2(\text{LO})}$$

not 1 but almost indep. of Q^2

- Enhance anti-neutrino cross section by 3%



Axial Vector Structure Functions

- At high Q^2 , vector and axial vector contribution are same, but not at low Q^2
- The non-zero PCAC component of F_2^{axial} at low Q^2 : mostly longitudinal

$$2xF_1^{axial} = 2xF_1^{vector}$$

- K factors for axial contributions (F_2 & xF_3): type II

$$K_{sea}^{vector} = \frac{Q^2}{Q^2 + C} \quad \text{or} \quad K_{sea}^{axial} = \frac{Q^2 + 0.55C_{sea}^{axial}}{Q^2 + C_{sea}^{axial}} \quad K_{val}^{axial} = \frac{Q^2 + 0.1C_{val}^{axial}}{Q^2 + C_{val}^{axial}}$$

$$\text{where } C_{sea}^{axial} = 0.75, \quad C_{val}^{axial} = 0.18$$

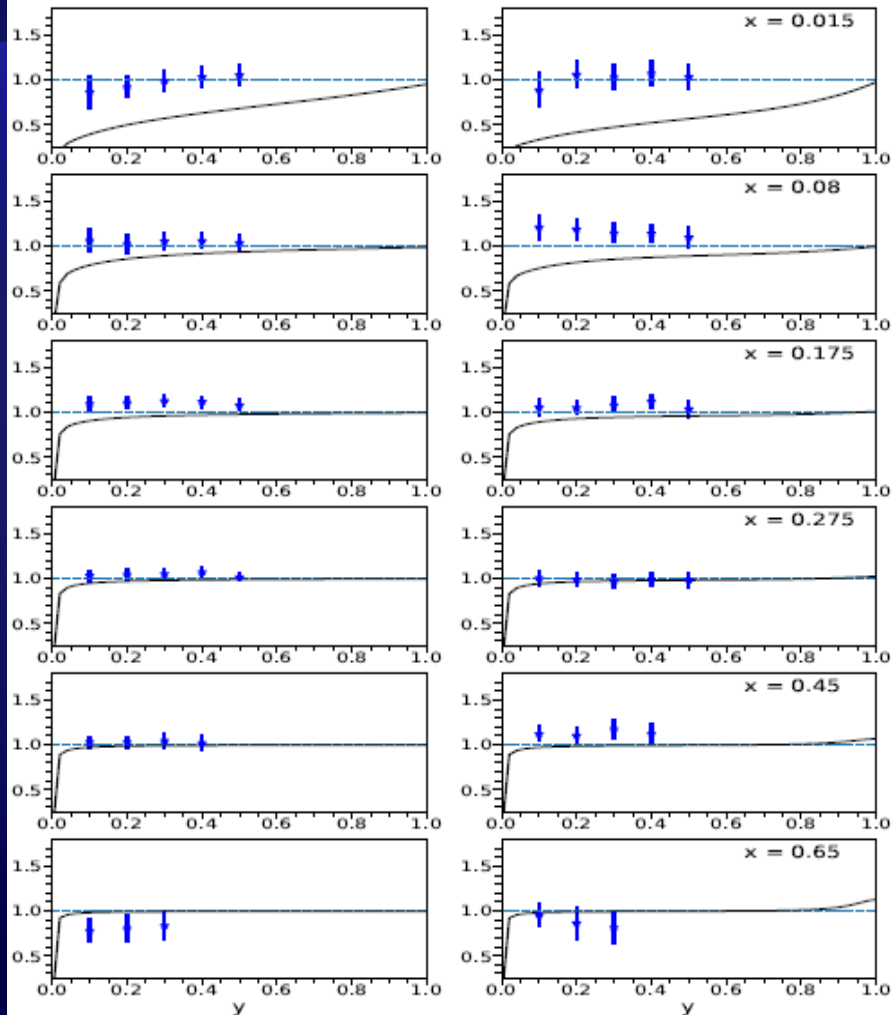
Comparison with CCFR(Fe) , CHORUS (Pb) data

Point: CHORUS/BY (Type II)

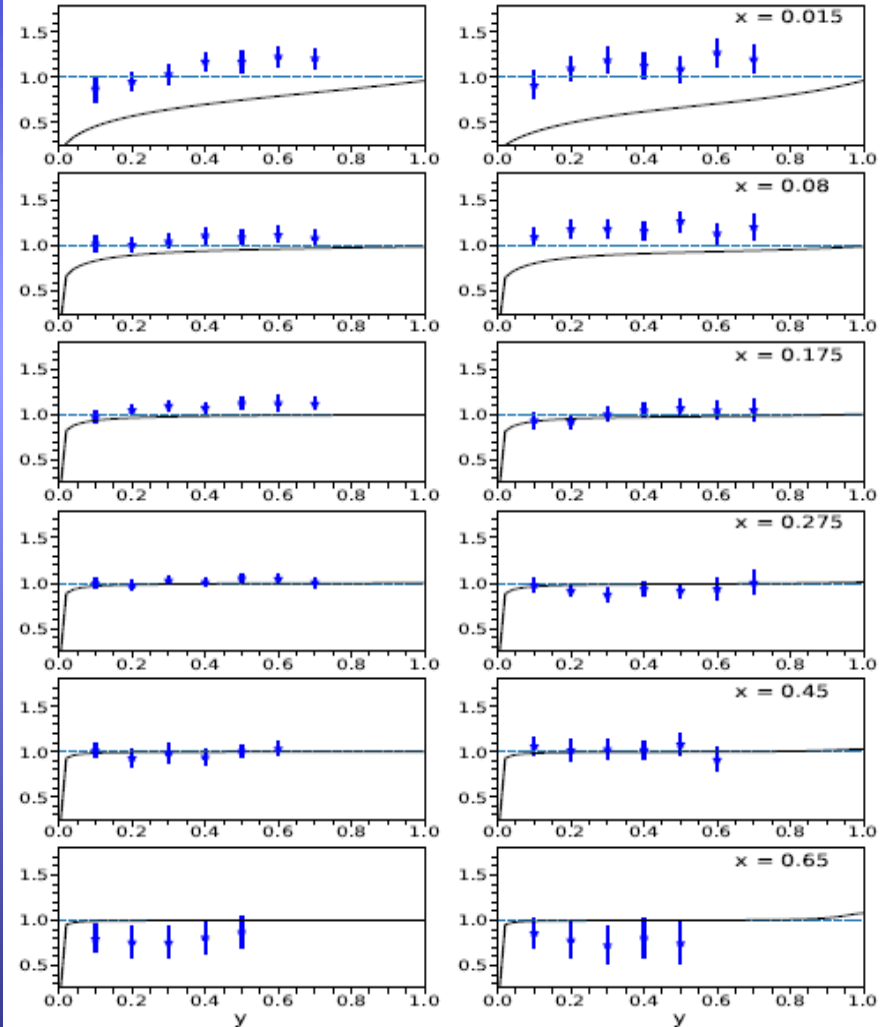
Line: Type I/Type II

Type 1 (Vector = Axial)

$E_\nu = 15.0$

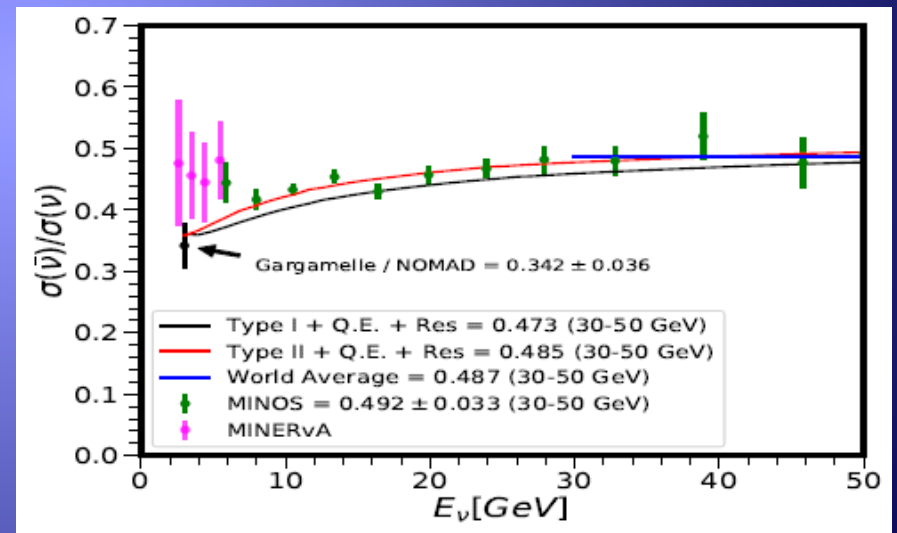
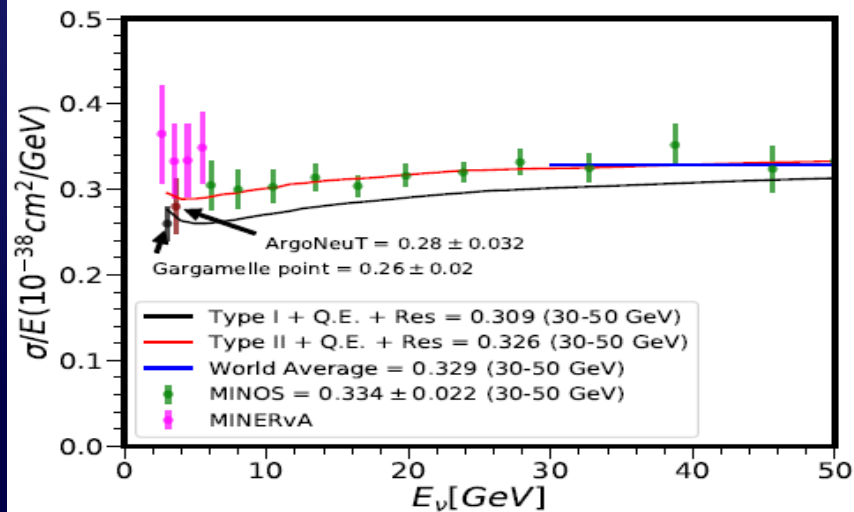
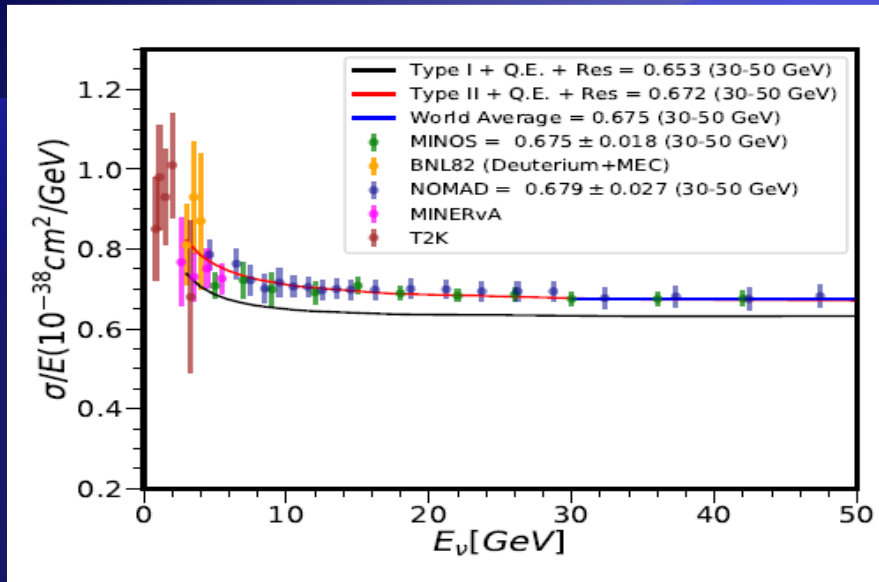


$E_\nu = 25.0$



Total cross sections

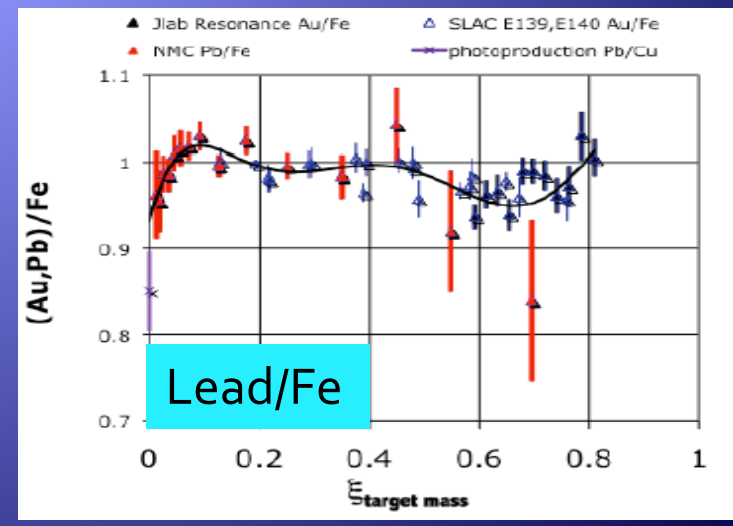
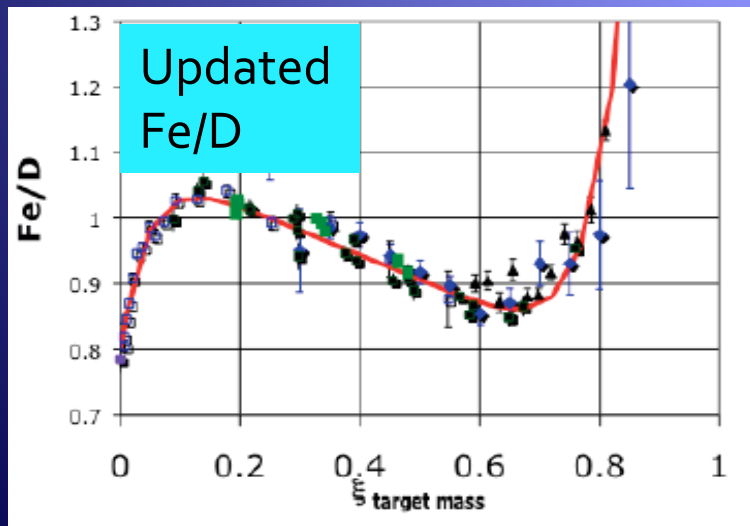
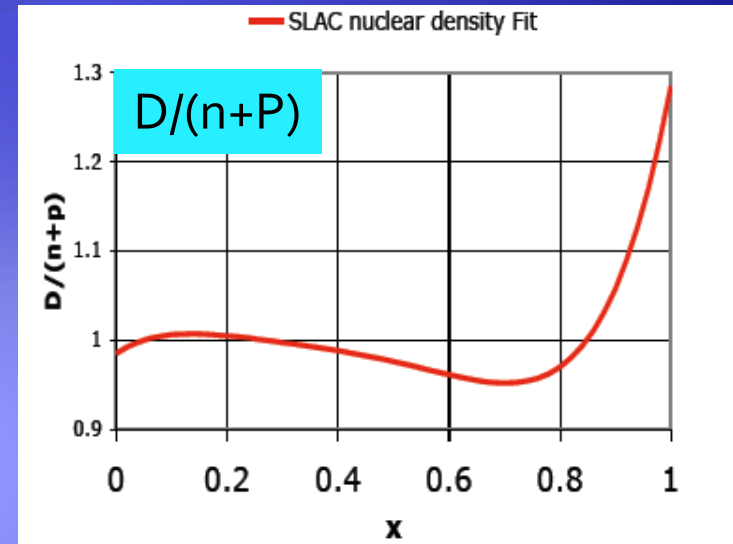
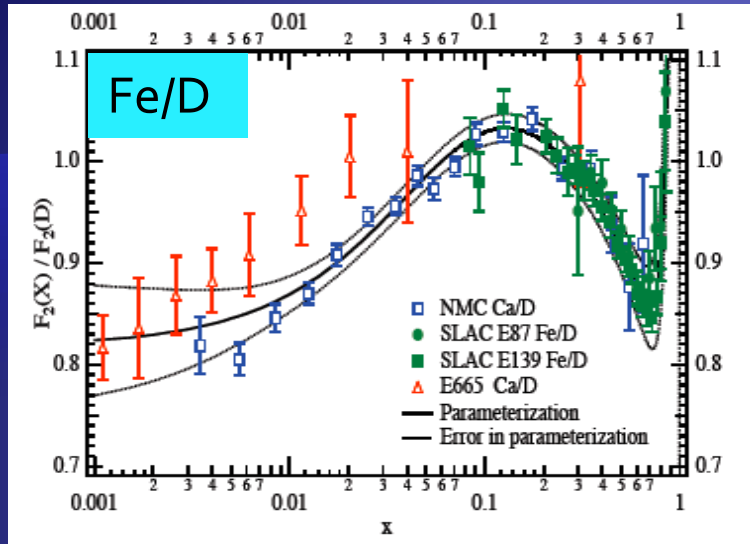
- BY(DIS, $W > 1.4$) + Q.E. + Resonance



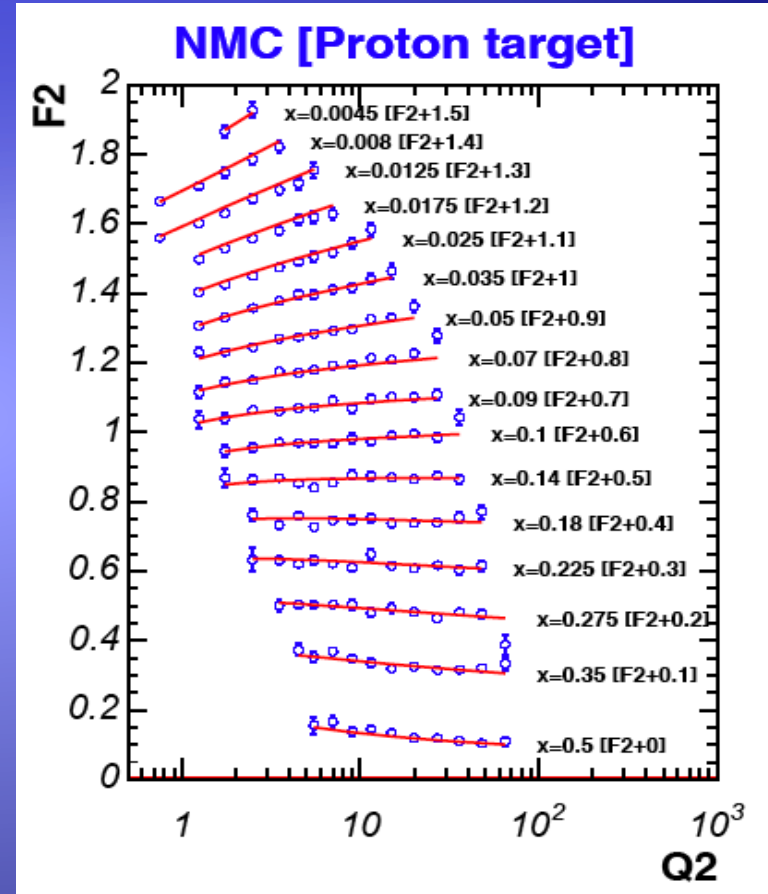
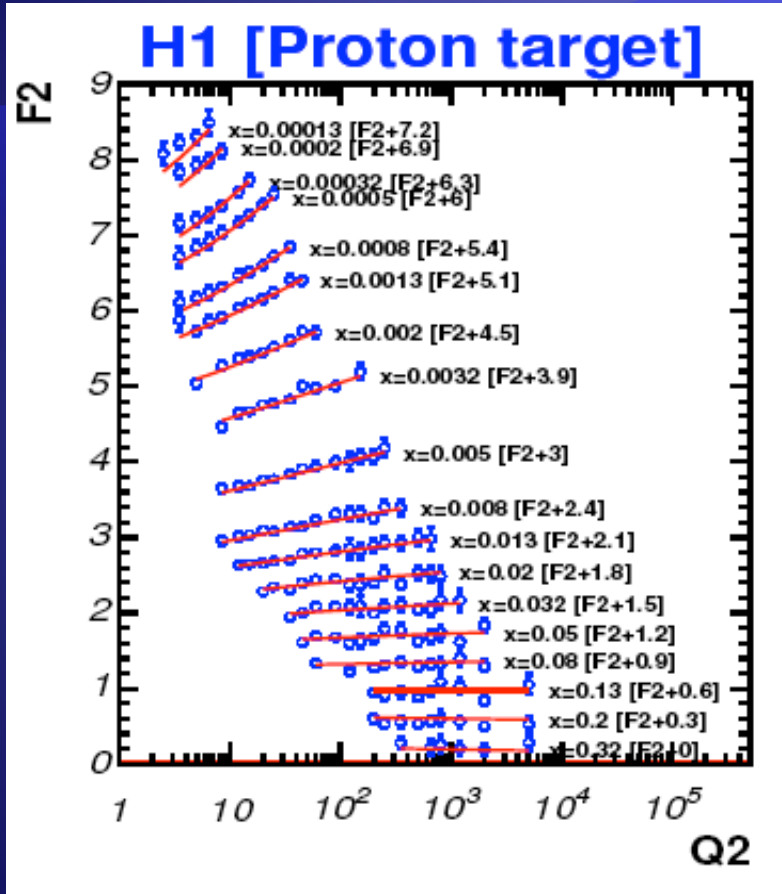
Summary & Discussions

- BY Effective LO model with ξw describe all e/ μ DIS and resonance data as well as photo-production data (down to $Q^2=0$): provide a good reference for vector SF for neutrino cross section
- BY(DIS) type II model (axial=PCAC) provide a good reference for both neutrino and anti-neutrino cross sections ($W>1.8$). Low energy neutrino experiments can normalize their data to our model to extract their flux
- Model also works well down to $W=1.4$ GeV on average (duality), thus providing overlap with resonance models

Nuclear Effects: use e/ μ data



Low x HERA and NMC data



➤ Fit works at low x