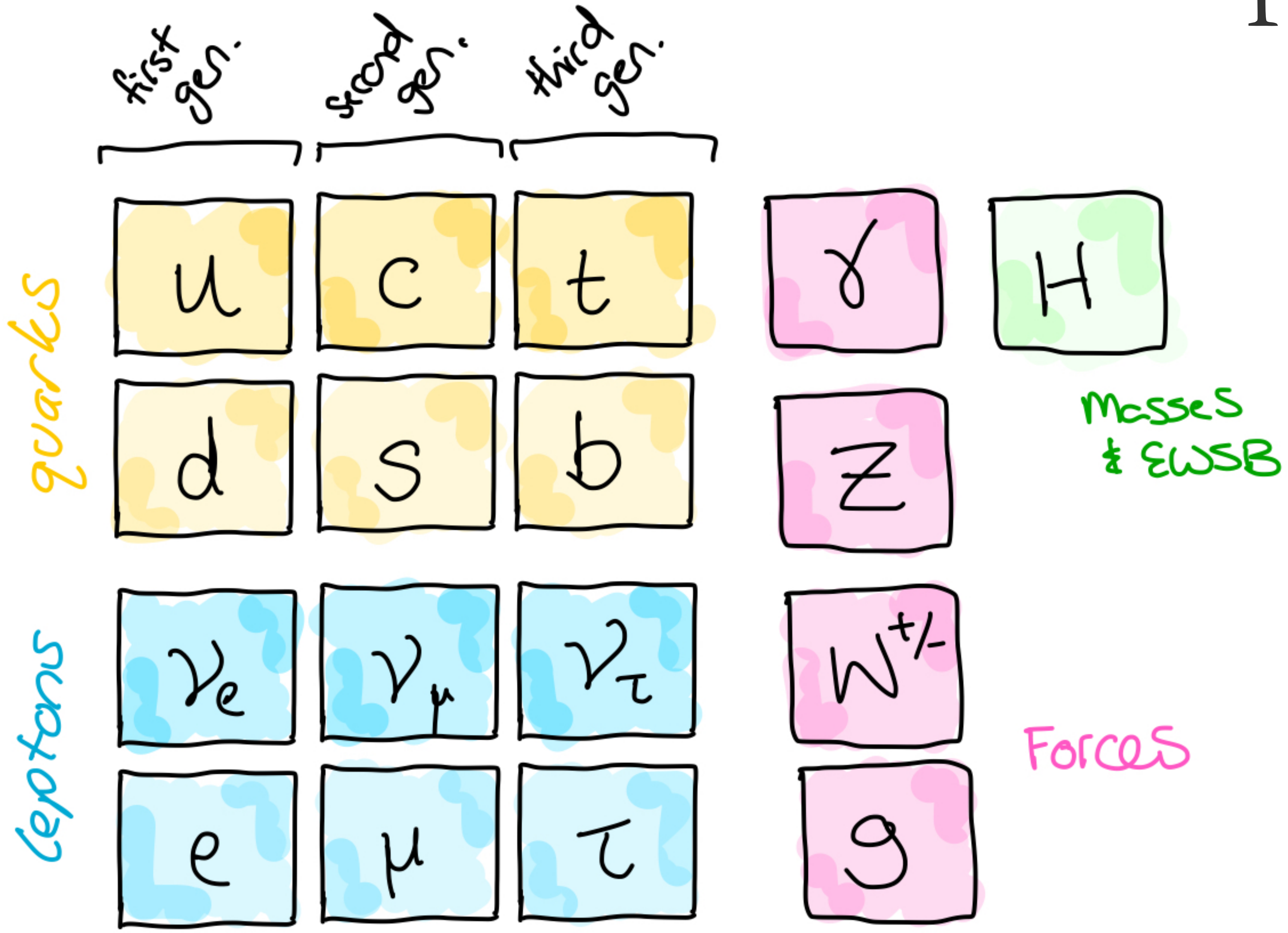
Theory of Muon Charged Lepton Flavour Violation and the Muon g-2

Dr Innes Bigaran

WG4: Muon Physics

Leptons in the Standard Model



$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

- If all the SM leptons were massless, the accidental global symmetry

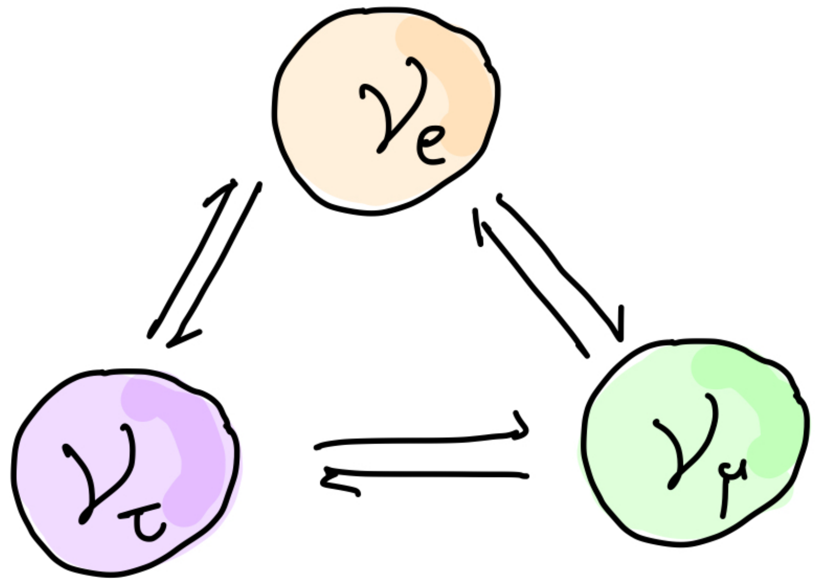
$$U(3)_{E_L} \otimes U(3)_{E_R}$$

- Lepton masses break this symmetry into the subgroup

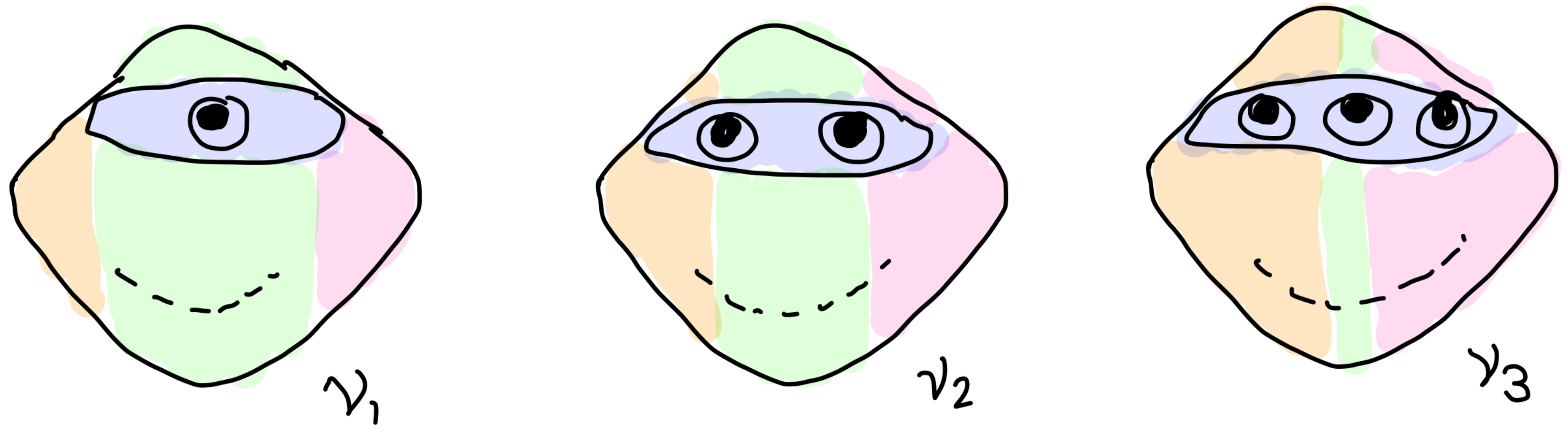
$$\mathcal{G}_{\rm L} = U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$$

- Corresponds to a [perturbative] conservation of lepton flavour. Total lepton number (L) , the sum of flavoured lepton number, is also conserved.

Leptons in the SM + neutrino masses



Mass eigenstates are linear combinations of flavour eigenstates

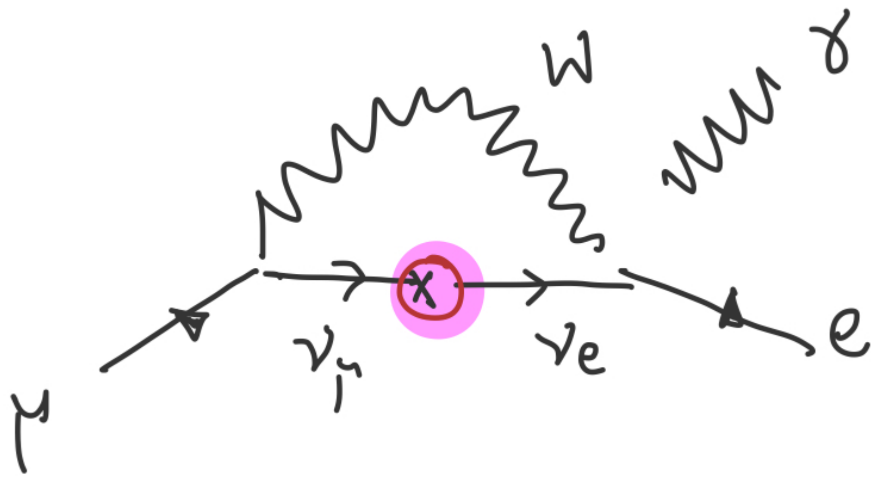


- In the vanilla SM, neutrinos are massless.
- Neutrino and neutrino physics provide a probe of lepton flavour symmetries
- Neutrino flavour oscillations tell us that beyond the SM lepton flavour *is* violated.

Neutrino masses/oscillation mean

1. *We need BSM physics (extended theory)*
2. *This new physics violates lepton flavour symmetry*

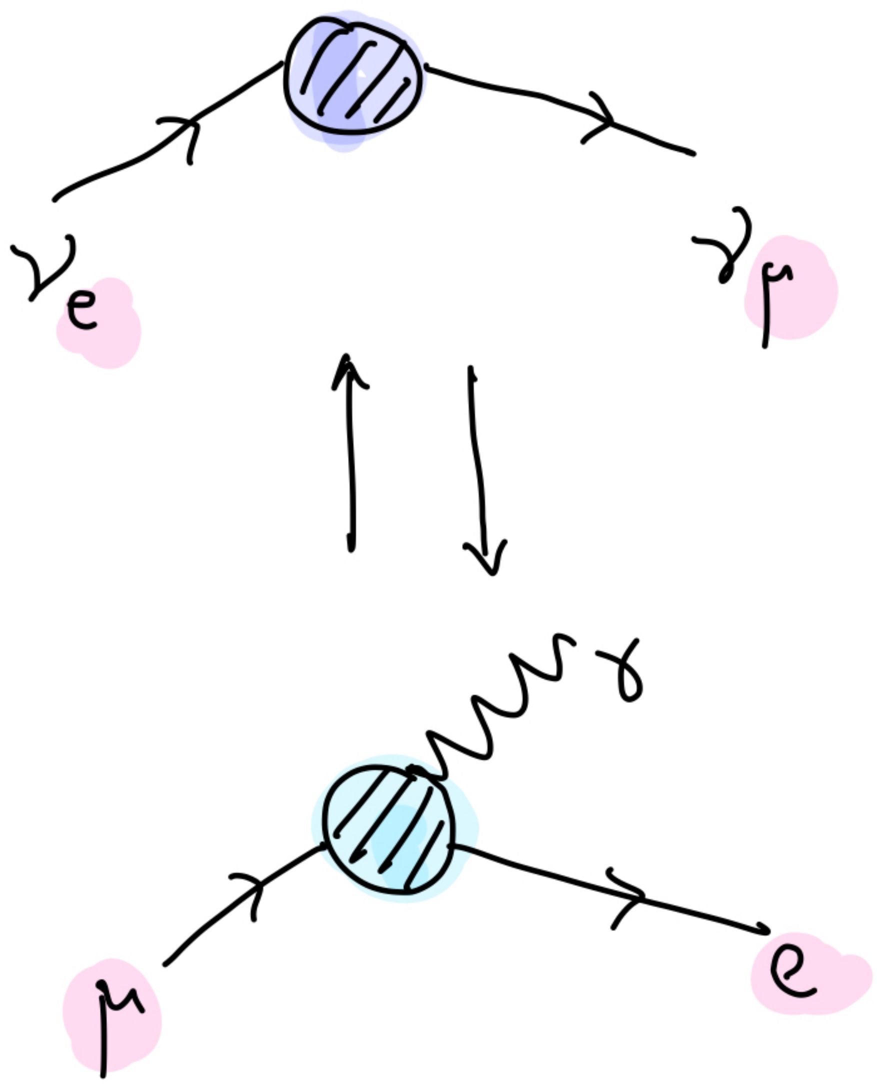
Lepton flavour violation



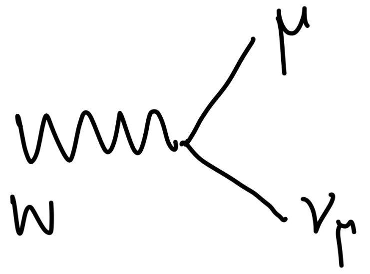
< O(1e-54)

Petcov, 1977

For example, in many BSM models:

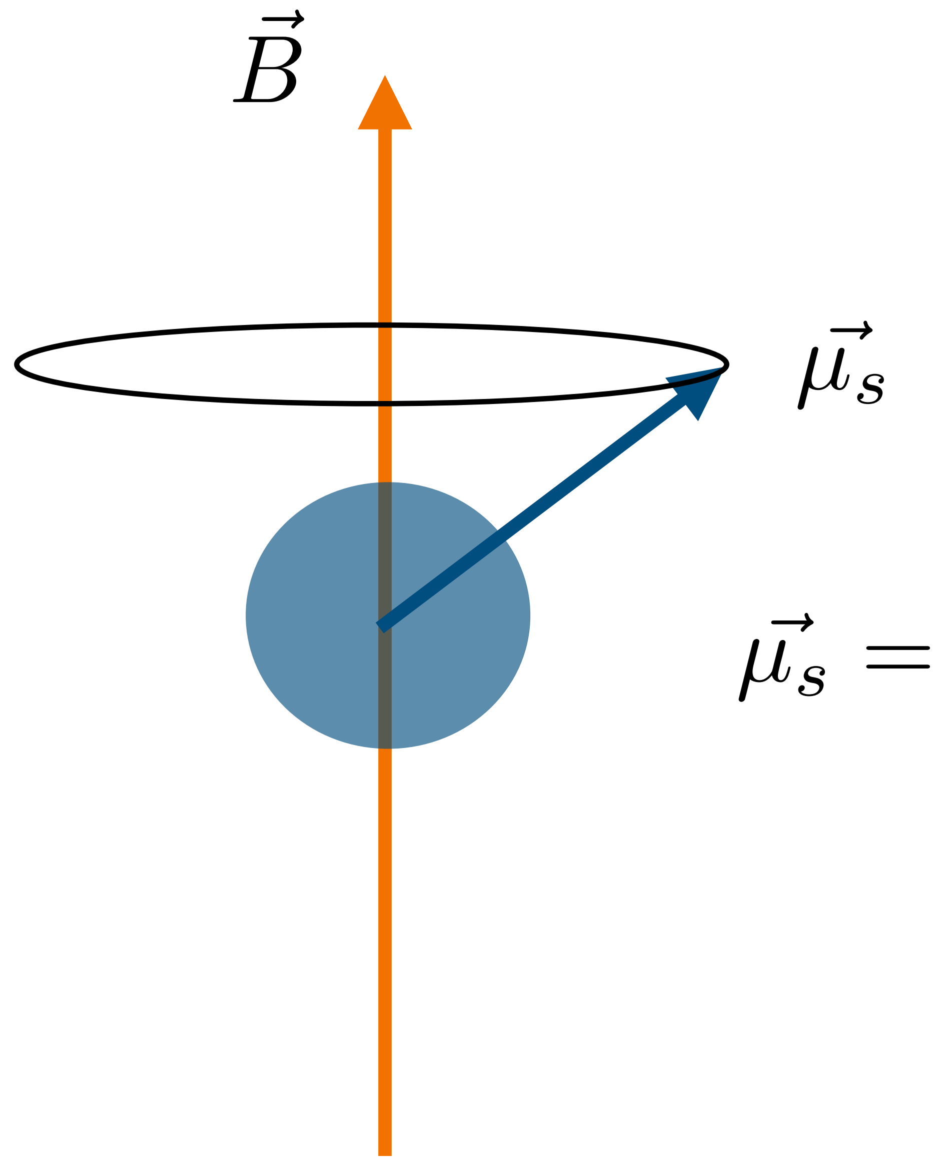


$$L_L \sim \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$



- SU(2) symmetry of the SM links neutrinos and charged leptons
- New physics introduced to the SM to explain neutrino mass generically leads to charged lepton flavour violation (cLFV) signals
- SM + *only* massive neutrinos predicts a very small branching ratio (BR) of cLFV decays
- Any sizeable value of this BR would be a genuine signal of new physics

Lepton Flavour Universality (violation?)



$$\vec{\mu_s} = \frac{g\ e}{2\ m}\vec{s}$$

e.g. of other LFU tests, via electroweak:

[Francesca Dordei, Friday 18.00 WG4]

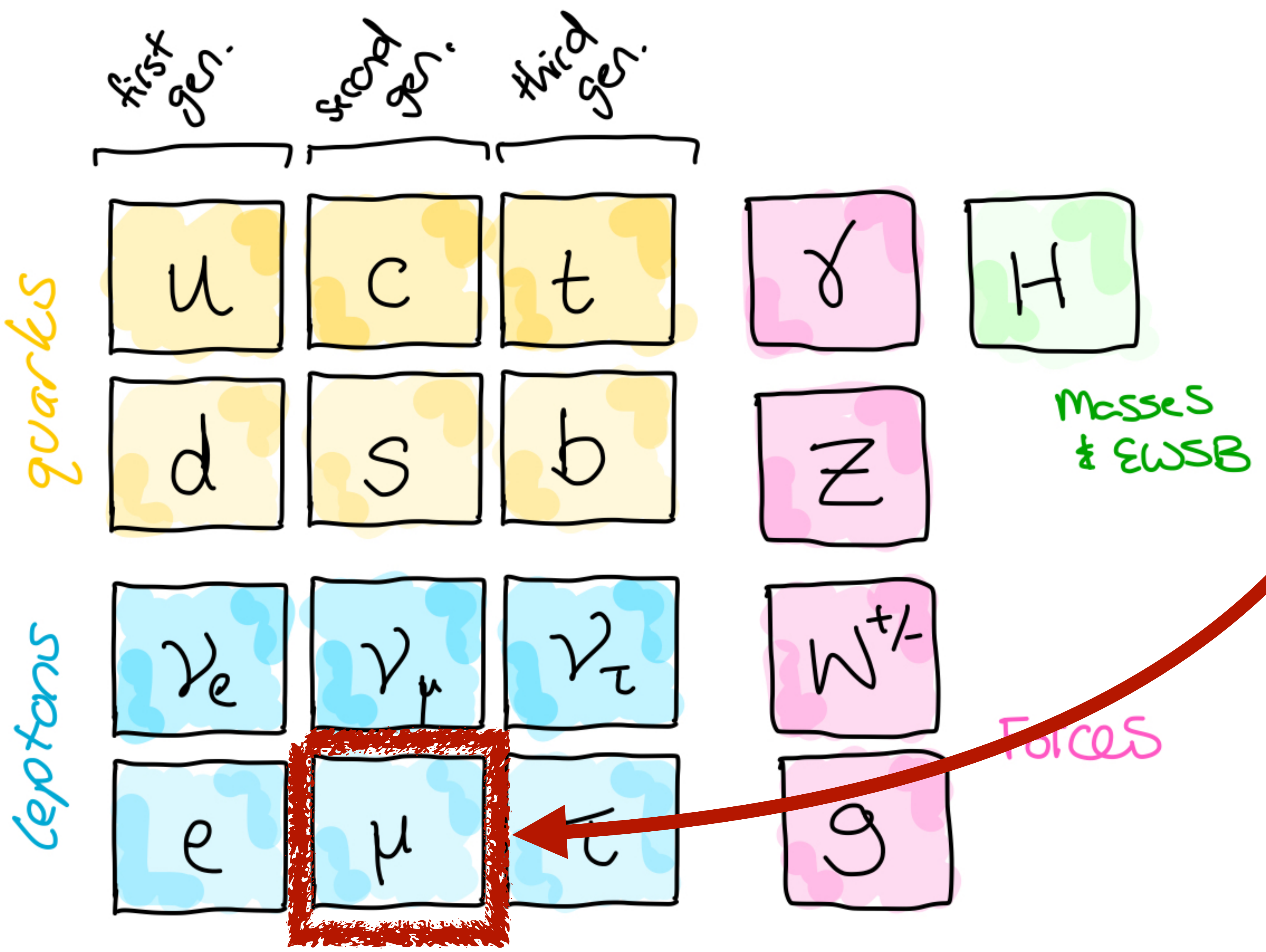
$$R_{D^{(*)}} = \frac{Br(B \to D^{(*)}\tau\nu_\tau)}{Br(B \to D^{(*)}\ell\nu_\ell)}$$ ~ 3 sigma anomaly

$$R_{K^{(*)}} = \frac{Br(B \to K^{(*)}\mu^+\mu^-)}{Br(B \to K^{(*)}e^+e^-)}$$ ~ SM-like

- In the SM, all particles *apart from the Higgs* couple in the same way to all three flavours of lepton [different masses]
- If it seems that SM forces interact with the different flavours of lepton in different ways not explained by masses, then we have a violation of lepton flavour universality, and a signal of new physics.
- e.g. deviation in the way the photon interacts with muons? Muon g-2!

Why do we care about the muon?

("Who ordered that? "- I.I. Rabi)



- Our focus today: the muon —

  the second generation charged lepton

  - Muon mass ~ 105 MeV and lifetime ~ 2.2 μs

    'Goldilocks mass': no hadronic decays, but does decay

    We can make and manipulate in experiments
  - Main decay mode: $\mu \to e\nu\bar{\nu}$.
  - We can access exciting new physics on the energy and precision frontier via muon study now and in the future

Two main parts of this talk

1. Muon flavour violation theory

   Experimental overview: Satoshi Mihara (just after this talk!)

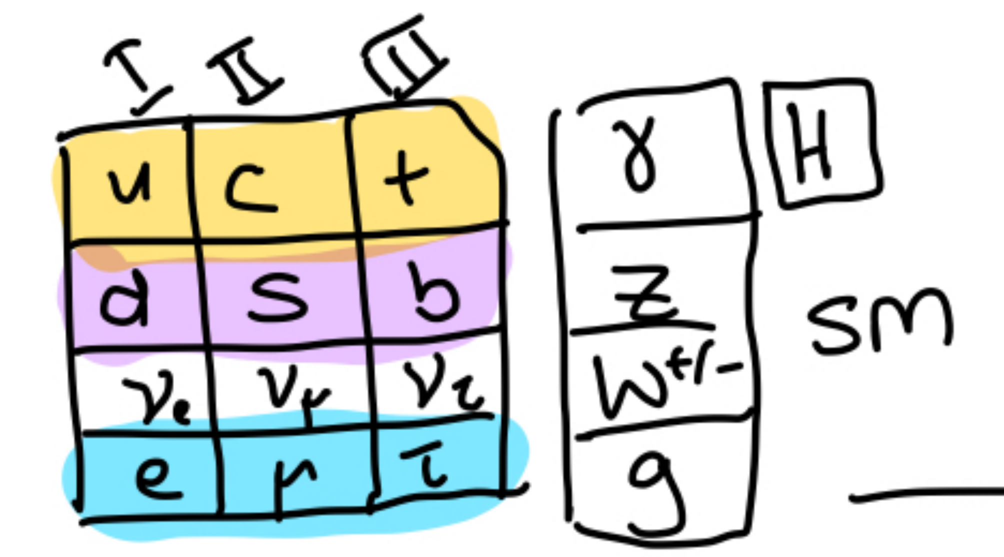
2. Muon g-2 theory (caveat: I am *not* a lattice theorist)

   Experimental overview: Matteo Sorbara (just before this talk!)

Muon cLFV theory landscape

SM particle masses and flavour

- SM particle masses are driven by the values of their Yukawa couplings to the Higgs
- This gives SM fermions ***Dirac-type* masses**
- *Flavour physics* is the study of the differences between the types of quarks and leptons in the SM (and beyond), masses and mixing

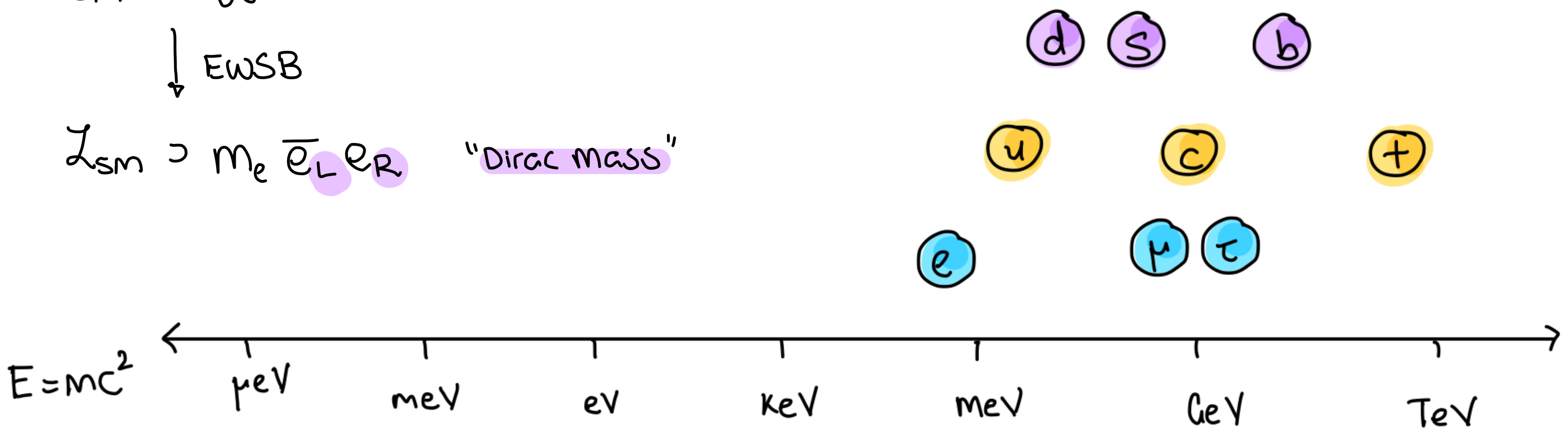


$$\mathcal{L}_{SM} \supset y_e^{ij}\, \bar{L}_L e_R H$$

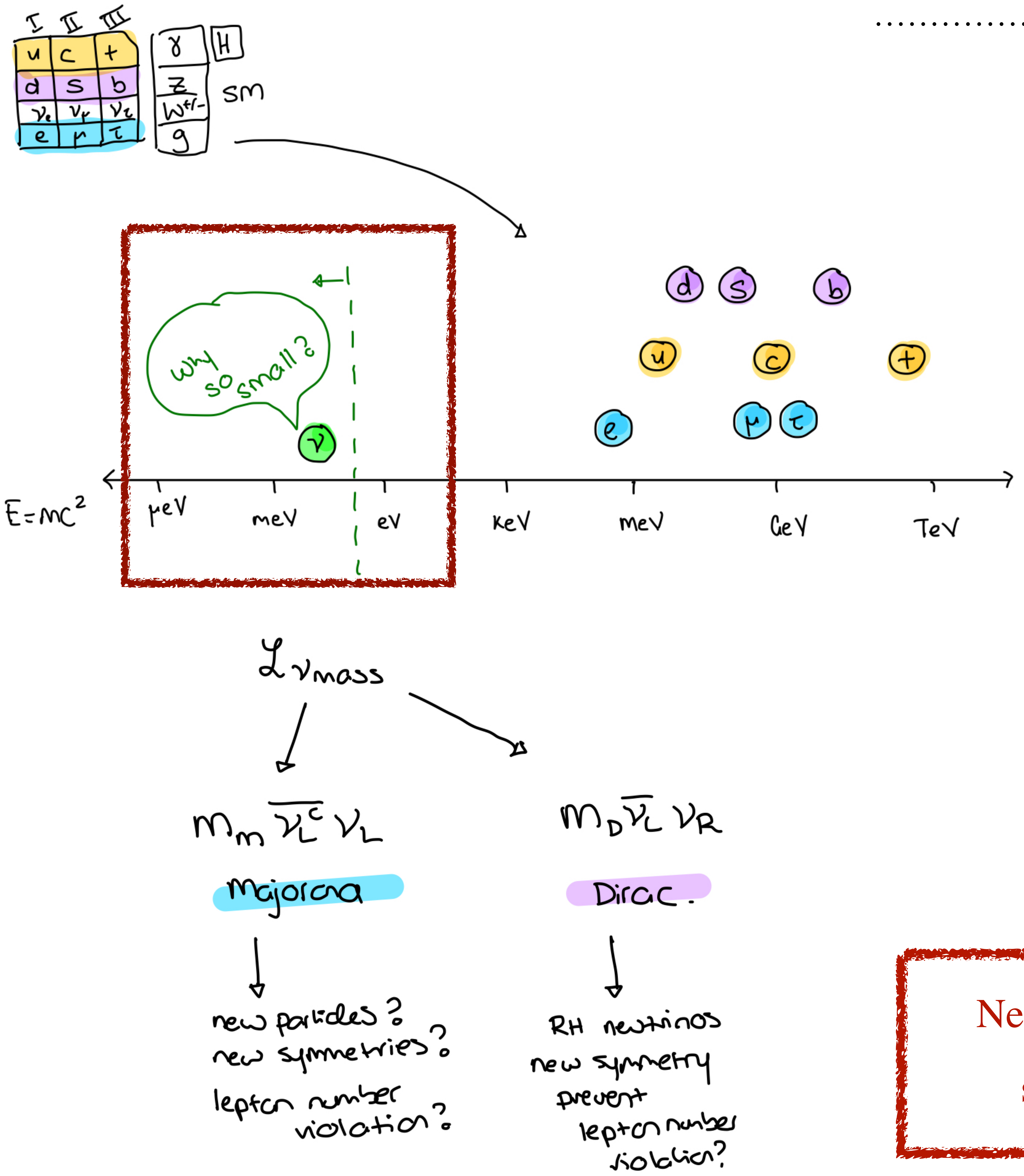
$\downarrow$ EWSB

$$\mathcal{L}_{SM} \supset m_e\, \bar{e}_L e_R$$

"Dirac mass"



Neutrino masses



- Neutrinos are massless in the SM. Measured neutrino masses are much smaller than other fermions
- Neutrino masses could be Dirac or Majorana — motivates different types of new physics
- Both types of neutrino mass violate lepton flavour. Dirac masses don't require a violation of lepton number, Majorana do
$$L = L_e + L_\mu + L_\tau$$
- Dirac mass models require a mechanism/symmetry enforced to prevent ΔL

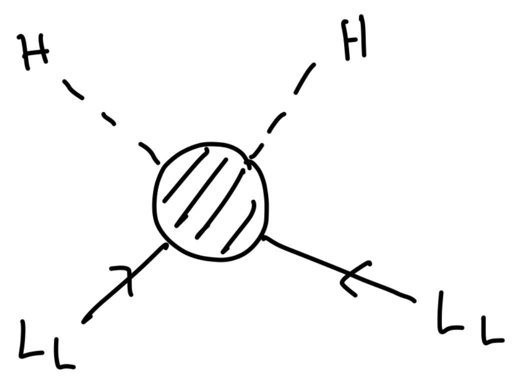
Neutrino masses may be so small because they correspond to broken L symmetry. Maybe the mass mechanism is different for neutrinos?

Neutrino masses via EFT

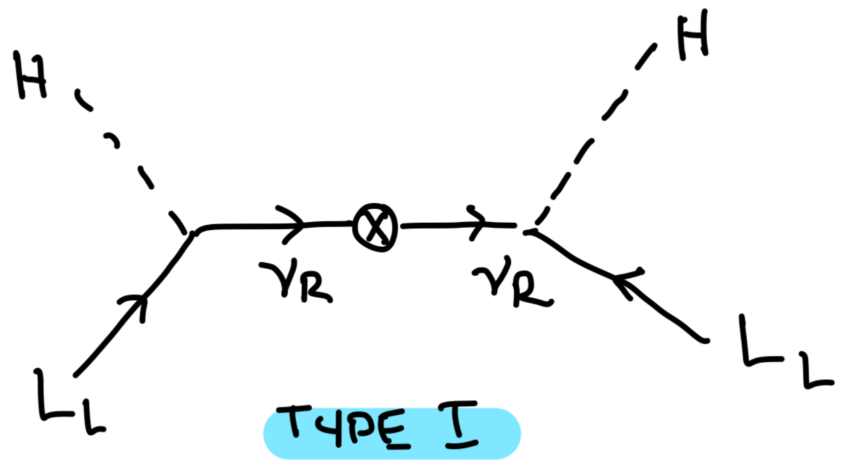
- To systematically study the generation of neutrino masses, we can consider *effective* interactions of neutrinos
- Interactions above mass-dimension (D) four don't appear as bare terms in the SM Lagrangian, but are generated via higher-order, incl. BSM, effects
- "Opening up" EFT — write down UV-complete models that generate these interactions: new particles and fields
- Many new physics models exist which can generate observed neutrino masses (incl. Seesaw and beyond)

e.g. Weinberg operator (D=5), $\Delta L = 2$

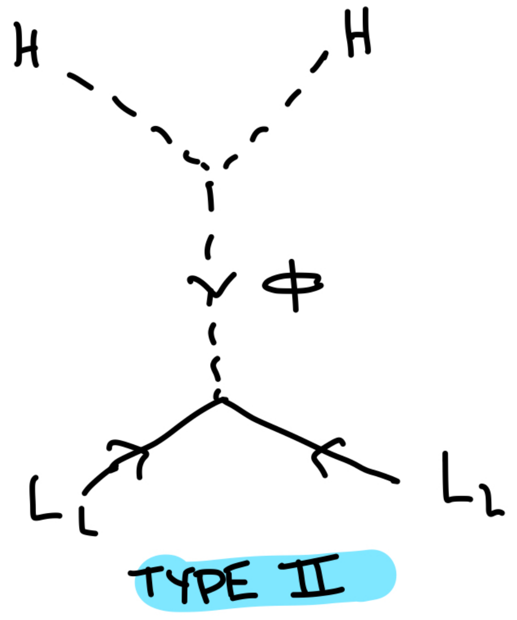
$$\mathcal{L}_{\text{effective}} \supset \frac{\lambda}{\Lambda} L_L L_L H H$$



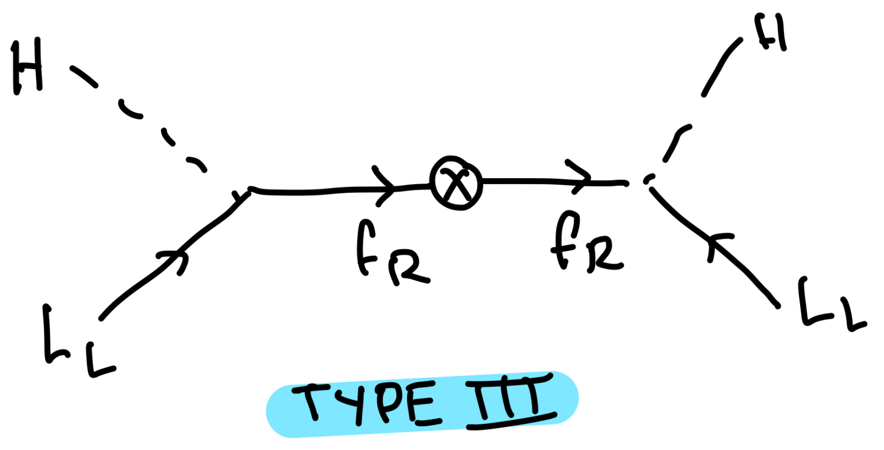
"Opening-up" the Weinberg operator: The Seesaw Models



Minkowski 1977
Yanagida 1979
Gell-Mann, Ramond, Slansky 1979
Mohapatra, Senjanovic 1980



Magg, Wetterich 1980
Schechter, Valle 1980
Cheng, Li 1980
Lazarides, Shafi, Wetterich 1981
Wetterich 1981
Mohapatra, Senjanovic 1981

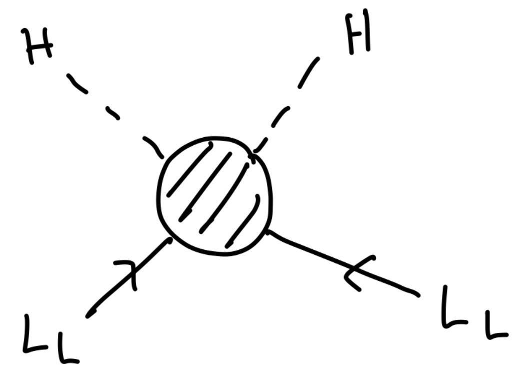


Foot, Lew, He, Joshi 1989

Neutrino masses via EFT

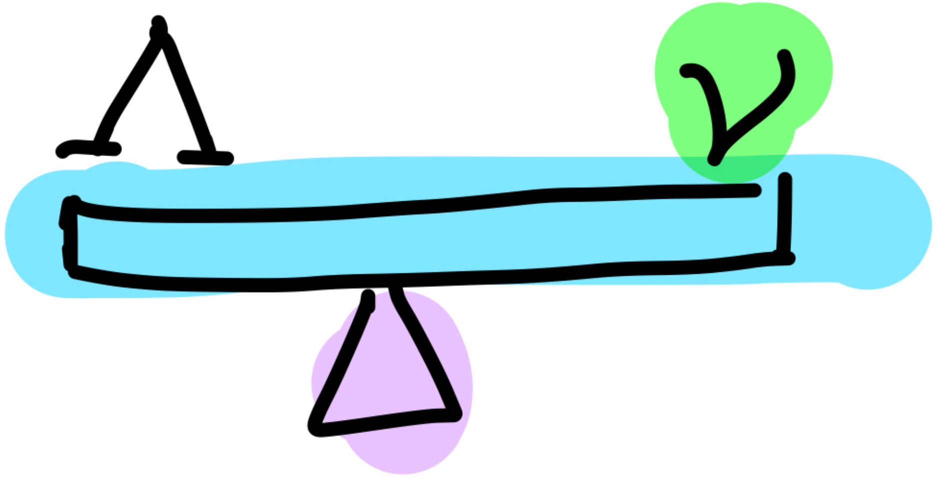
e.g. Weinberg operator (D=5), $\Delta L = 2$

$$\mathcal{L}_{\text{effective}} \supset \frac{\lambda}{\Lambda} L_L L_L H H$$



"The Seesaw Models"

$$m_\nu \sim \lambda \frac{v^2}{\Lambda}$$



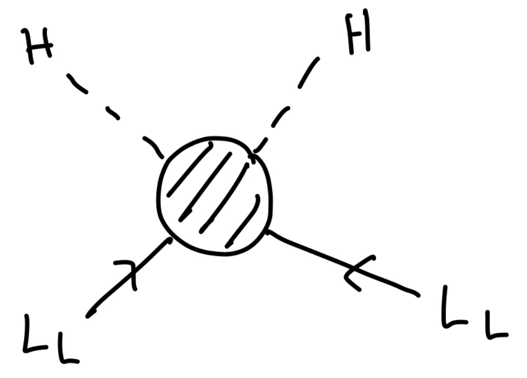
λ is a dimensionless coupling , Λ is a new "mass scale", v is the Higgs vev

- To systematically study the generation of neutrino masses, we can consider *effective* interactions of neutrinos
- Interactions above mass-dimension (D) four don't appear as bare terms in the SM Lagrangian, but are generated via higher-order, incl. BSM, effects
- "Opening up" EFT — write down models that generate these interactions: new particles and fields
- Many new physics models exist which can generate observed neutrino masses (incl. Seesaw and beyond)

Neutrino masses via EFT

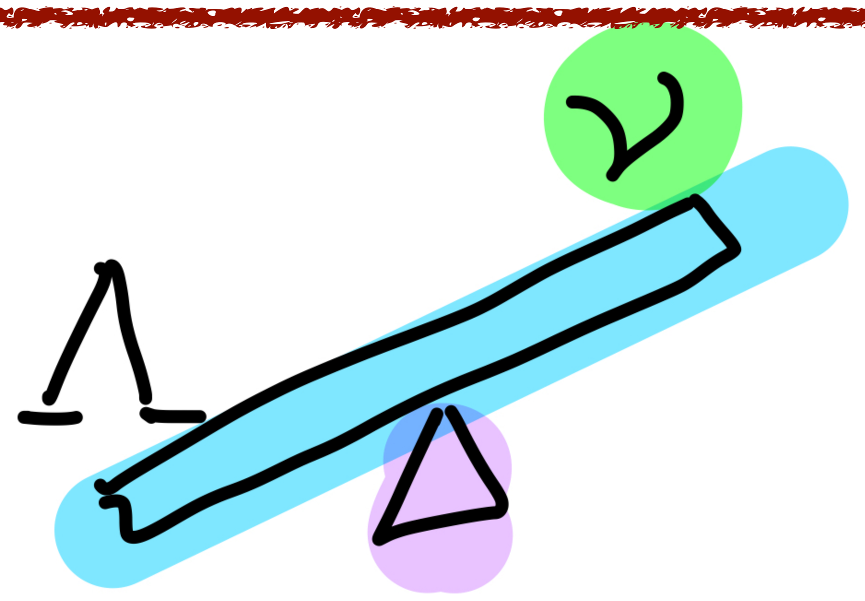
e.g. Weinberg operator (D=5), $\Delta L = 2$

$$\mathcal{L}_{\text{effective}} \supset \frac{\lambda}{\Lambda} L_L L_L H H$$



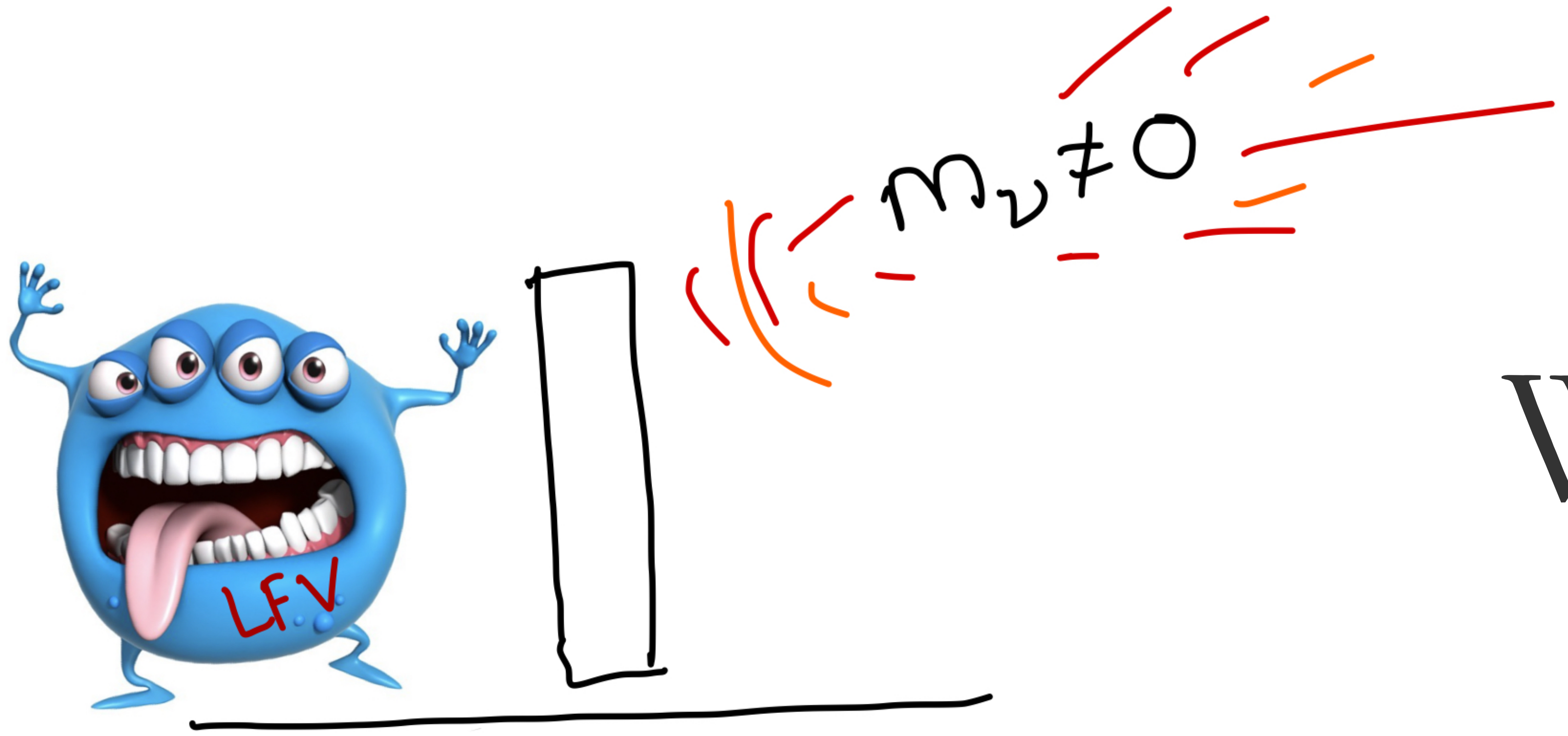
"The Seesaw Models"

$$m_\nu \sim \lambda \frac{v^2}{\Lambda}$$



λ is a dimensionless coupling , Λ is a new "mass scale", v is the Higgs vev

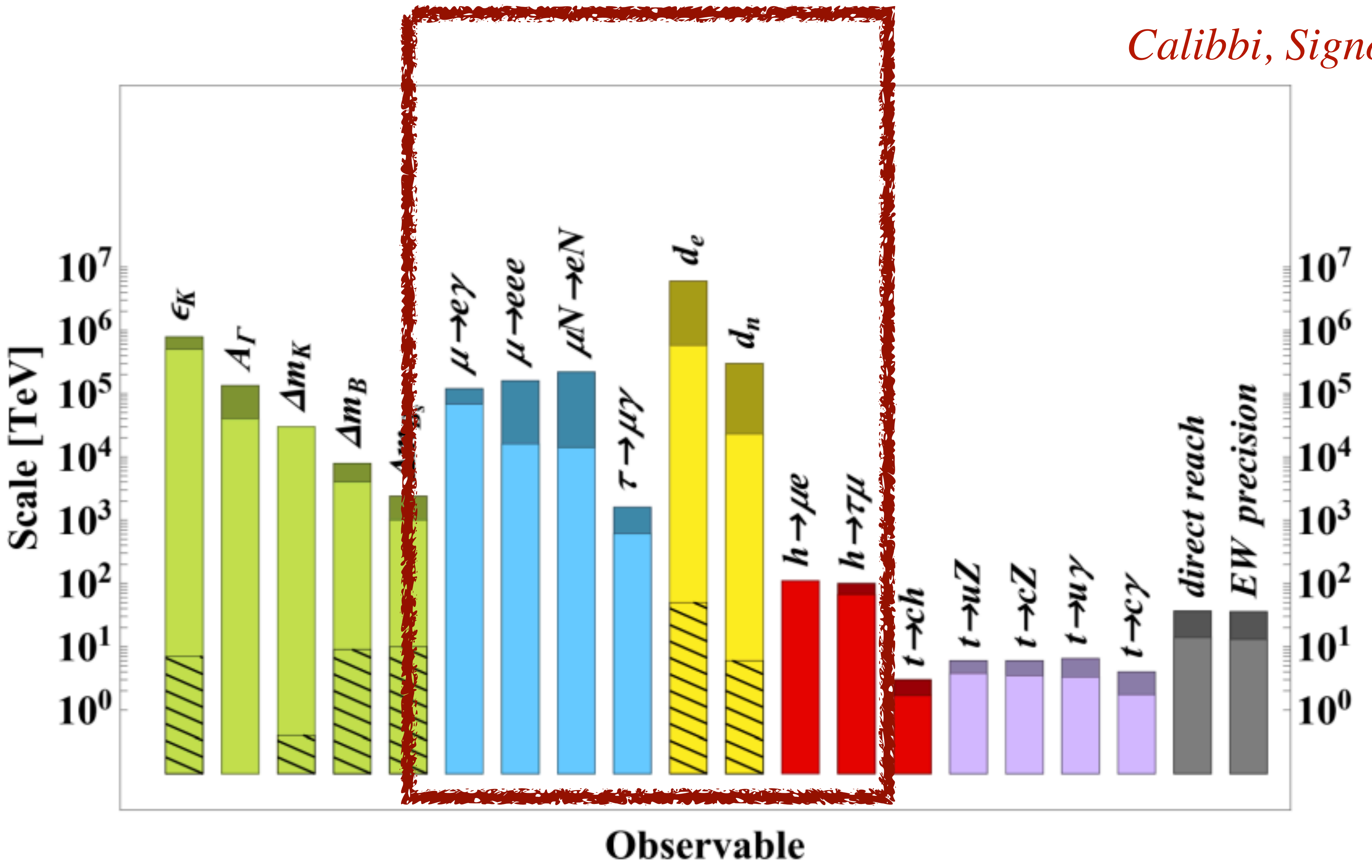
- To systematically study the generation of neutrino masses, we can consider *effective* interactions of neutrinos
- Interactions above mass-dimension (D) four don't appear as bare terms in the SM Lagrangian, but are generated via higher-order, incl. BSM, effects
- "Opening up" EFT — write down models that generate these interactions: new particles and fields
- Many new physics models exist which can generate observed neutrino masses (incl. Seesaw and beyond)



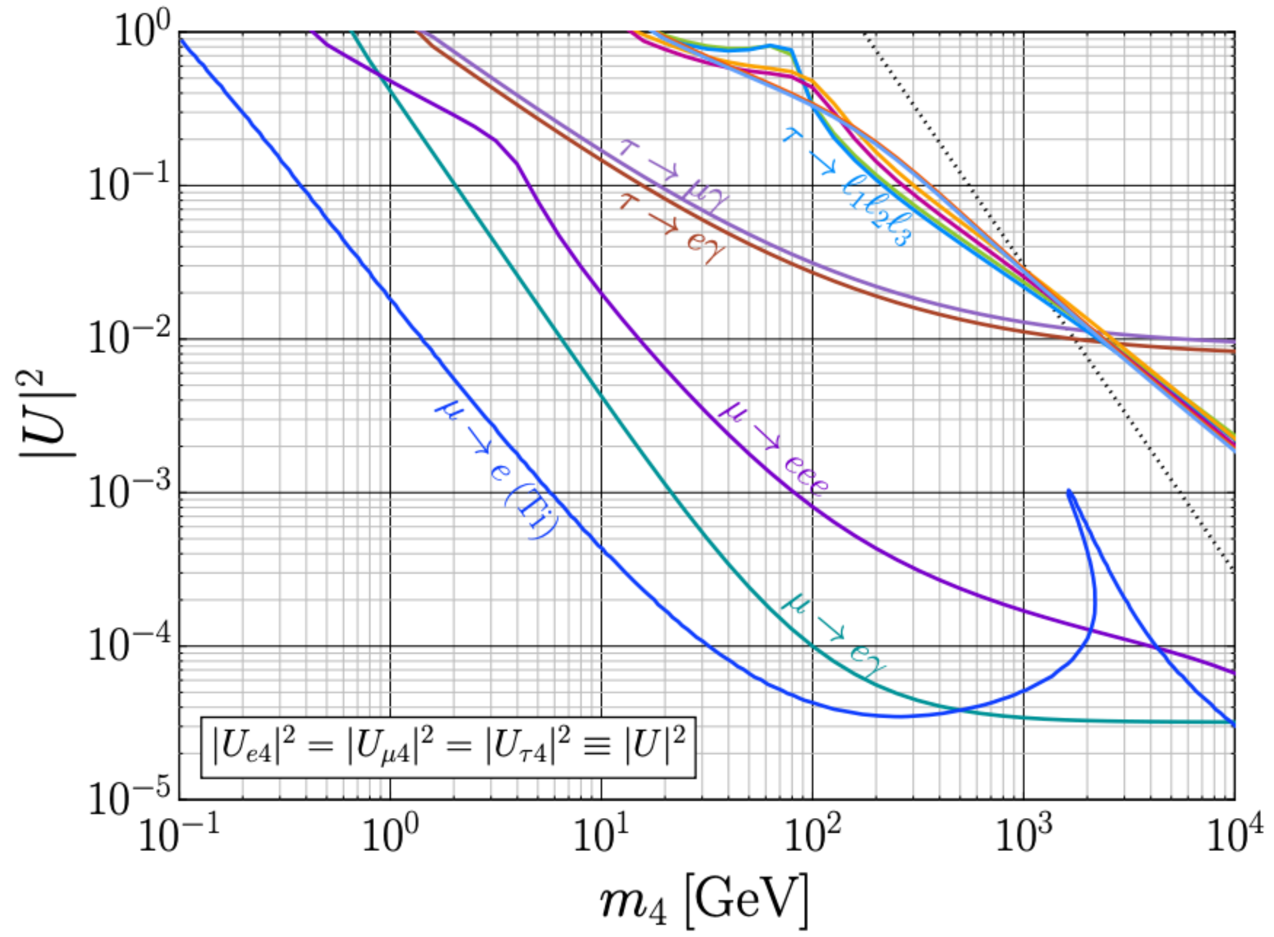
What does this have to do with cLFV?

Model	$\mu \to eee$	$\mu N \to eN$	$\frac{BR(\mu\to eee)}{BR(\mu\to e\gamma)}$	$\frac{CR(\mu N\to eN)}{BR(\mu\to e\gamma)}$
MSSM	Loop	Loop	$\approx 6 \times 10^{-3}$	$10^{-3} - 10^{-2}$
Type-I seesaw	Loop*	Loop*	$3 \times 10^{-3} - 0.3$	$0.1 - 10$
Type-II seesaw	Tree	Loop	$(0.1 - 3) \times 10^{3}$	$\mathcal{O}(10^{-2})$
Type-III seesaw	Tree	Tree	$\approx 10^{3}$	$\mathcal{O}(10^{3})$
LFV Higgs	Loop†	Loop*†	$\approx 10^{-2}$	$\mathcal{O}(0.1)$
Composite Higgs	Loop*	Loop*	$0.05 - 0.5$	$2 - 20$

Calibbi, Signorelli 1709.00294

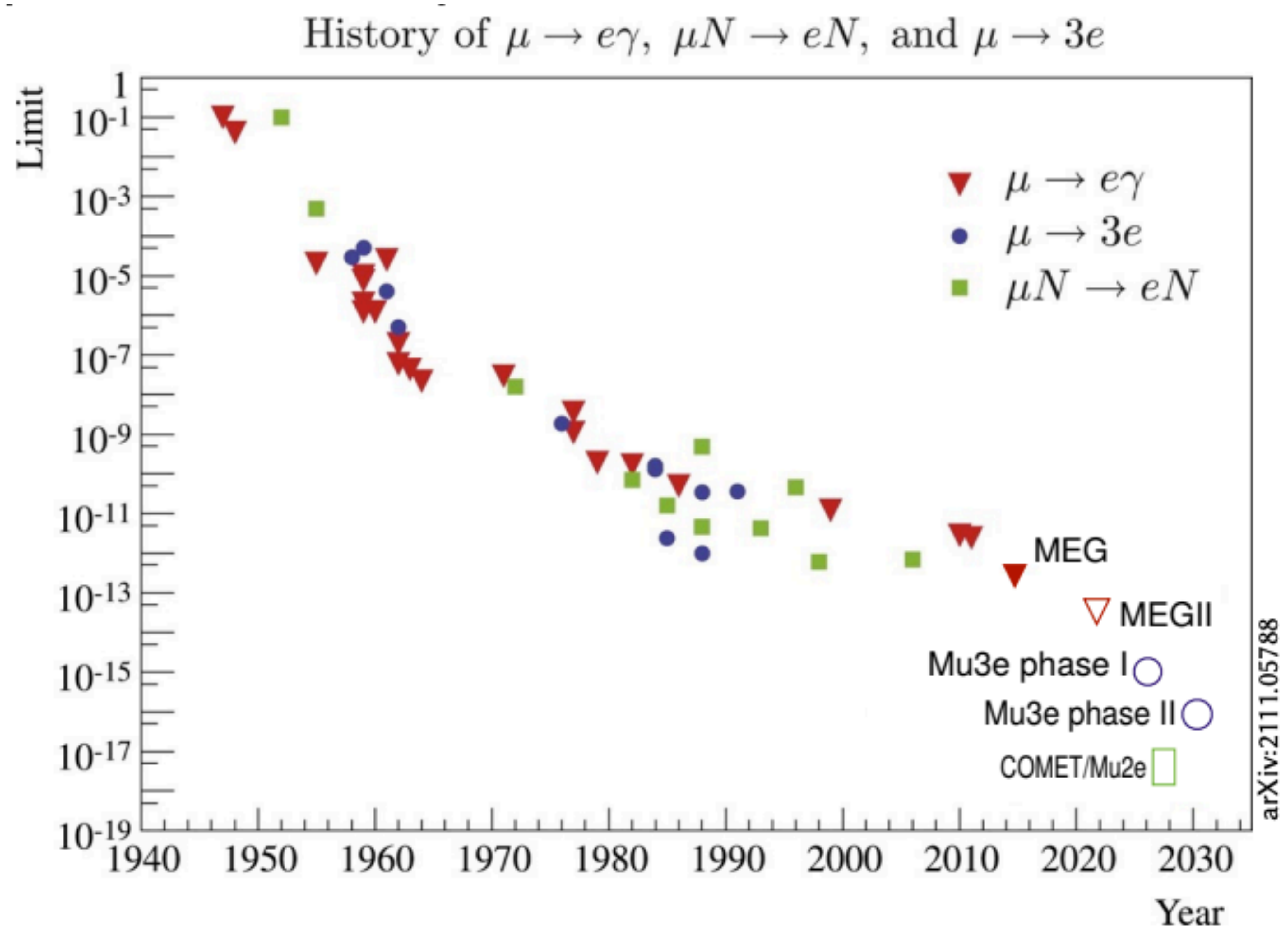


- Once we have allowed for LFV to generate neutrino masses and flavour mixing, it's hard to avoid LFV in charged-lepton processes.
- In a UV complete model, with new particles, cLFV can be the strongest constraints (and most likely places to look for model-discriminating signals!)



de Gouvêa, Kobach 1511.00683

cLFV and BSM physics



- With increased experimental precision we are narrowing in on parameter space for many BSM models **[Many talks in WG4 on experimental progress of muon cLFV]**
- Muon cLFV signals are also key features of BSM models for low-scale leptogenesis
  [Juraj Klaric, Tuesday 08:55 Plenary]
- Complementary probes to these cLFV tests from high-energy experiments (incl. muon colliders!)
  [Michael Schmidt, Theory, Friday 14:00, WG5]
  [Yongwan Kim, CMS , Friday 17:00]
  [Francesca Dordei, LHCb, Friday 18:00]

The muon g-2 theory landscape

The muon g-2 theory landscape

Statement on muon g-2 theory status

https://muon-gm2-theory.illinois.edu/

arXiv.org > hep-ph > arXiv:2006.04822

Search...

Help | Advanced

High Energy Physics – Phenomenology

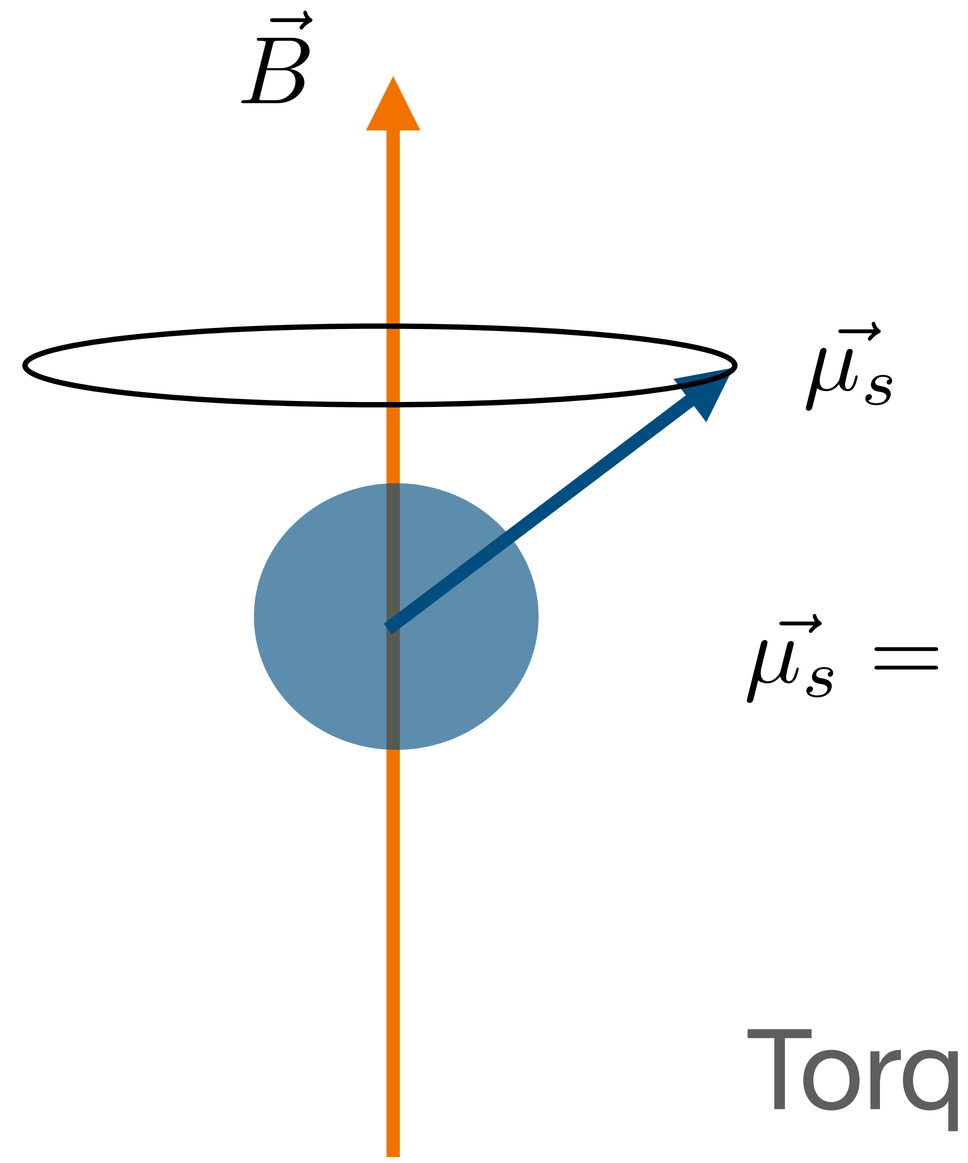
[Submitted on 8 Jun 2020]

The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama, N. Asmussen, M. Benayoun, J. Bijnens, T. Blum, M. Bruno, I. Caprini, C. M. Carloni Calame, M. Cè, G. Colangelo, F. Curciarello, H. Czyż, I. Danilkin, M. Davier, C. T. H. Davies, M. Della Morte, S. I. Eidelman, A. X. El-Khadra, A. Gérardin, D. Giusti, M. Golterman, Steven Gottlieb, V. Gülpers, F. Hagelstein, M. Hayakawa, G. Herdoíza, D. W. Hertzog, A. Hoecker, M. Hoferichter, B.-L. Hoid, R. J. Hudspith, F. Ignatov, T. Izubuchi, F. Jegerlehner, L. Jin, A. Keshavarzi, T. Kinoshita, B. Kubis, A. Kupich, A. Kupść, L. Laub, C. Lehner, L. Lellouch, I. Logashenko, B. Malaescu, K. Maltman, M. K. Marinković, P. Masjuan, A. S. Meyer, H. B. Meyer, T. Mibe, K. Miura, S. E. Müller, M. Nio, D. Nomura, A. Nyffeler, V. Pascalutsa, M. Passera, E. Perez del Rio, S. Peris, A. Portelli, M. Procura, C. F. Redmer, B. L. Roberts, P. Sánchez-Puertas, S. Serednyakov, B. Shwartz, S. Simula, D. Stöckinger, H. Stöckinger-Kim, P. Stoffer, T. Teubner, R. Van de Water, M. Vanderhaeghen, G. Venanzoni, G. von Hippel, H. Wittig, Z. Zhang, M. N. Achasov, A. Bashir, N. Cardoso, B. Chakraborty, E.-H. Chao, J. Charles, A. Crivellin, O. Deineka, A. Denig, C. DeTar, C. A. Dominguez, A. E. Dorokhov, V. P. Druzhinin, G. Eichmann, M. Fael, C. S. Fischer, E. Gámiz, Z. Gelzer, J. R. Green, S. Guellati-Khelifa, D. Hatton, N. Hermansson-Truedsson et al. (32 additional authors not shown)

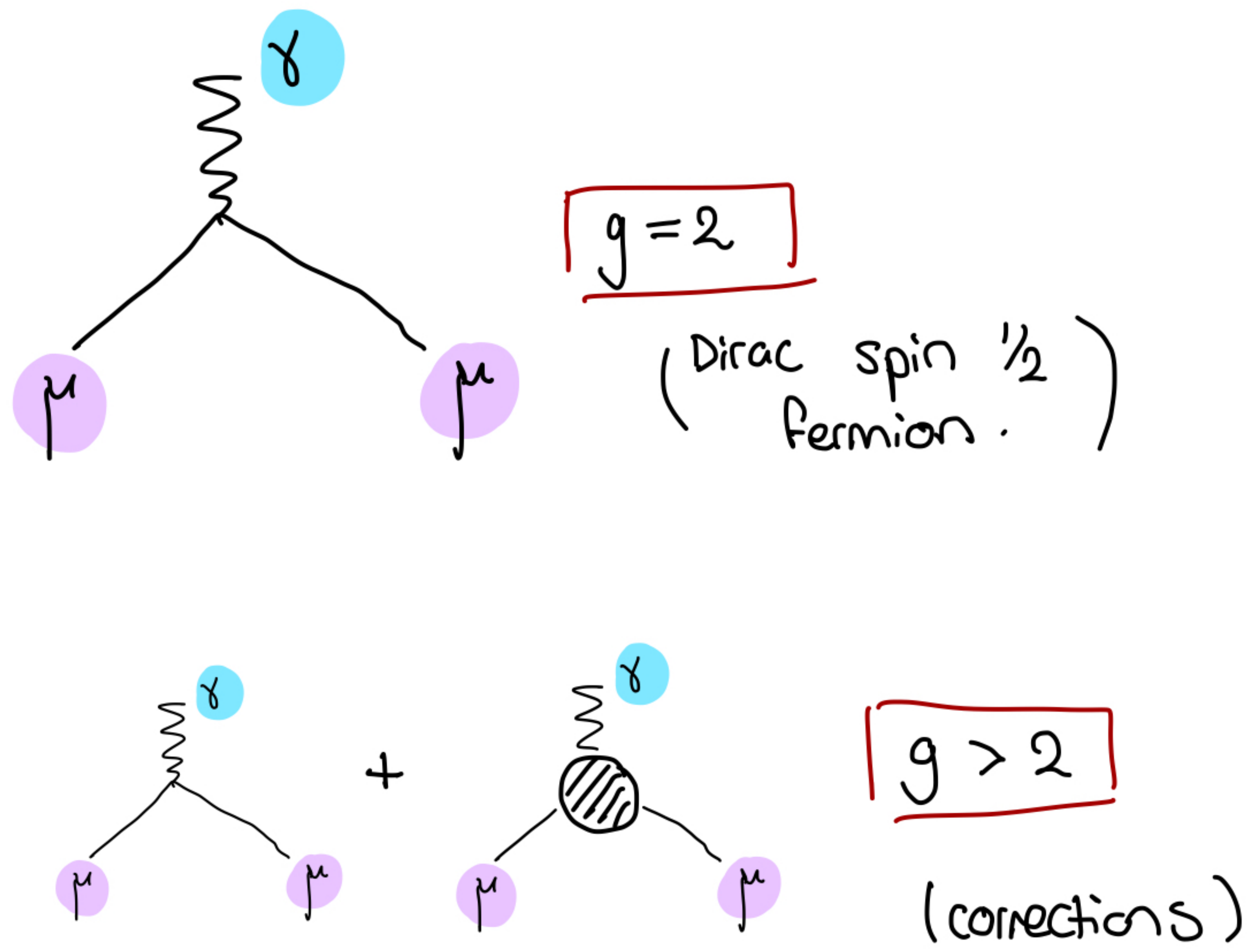
The Muon g-2 Theory Initiative

What is a muon g-2?



$$\vec{\mu_s} = \frac{g\,e}{2\,m}\vec{s}$$

$$\text{Torque} = \vec{\mu_s} \times \vec{B}$$

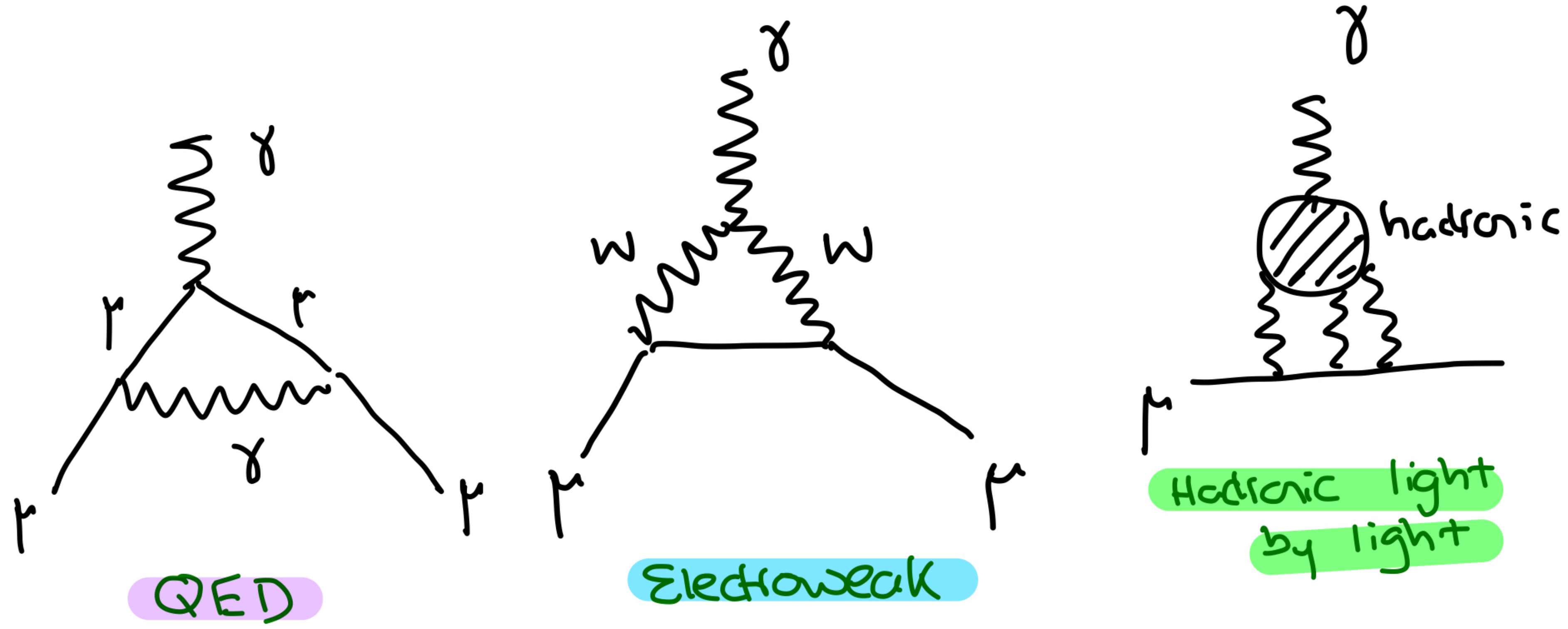


- The value ‘g’ is for gyromagnetic ratio: how does the spin of the muon precess in a magnetic field?
- Virtual corrections lead to deviations from the semiclassical g=2 value, parameterised by:

$$a_\mu = \frac{g-2}{2}$$

"muon g minus 2"
or
"anomalous magnetic moment"

- Theorists *very* precisely calculate the SM value for the muon g-2



11 digits.

$$a_\mu = \frac{g-2}{2} = 0.00116584719$$ QED — Perturbative (5 loop)

$$+\ 0.00000000154$$ Electroweak — Perturbative (2 loop)

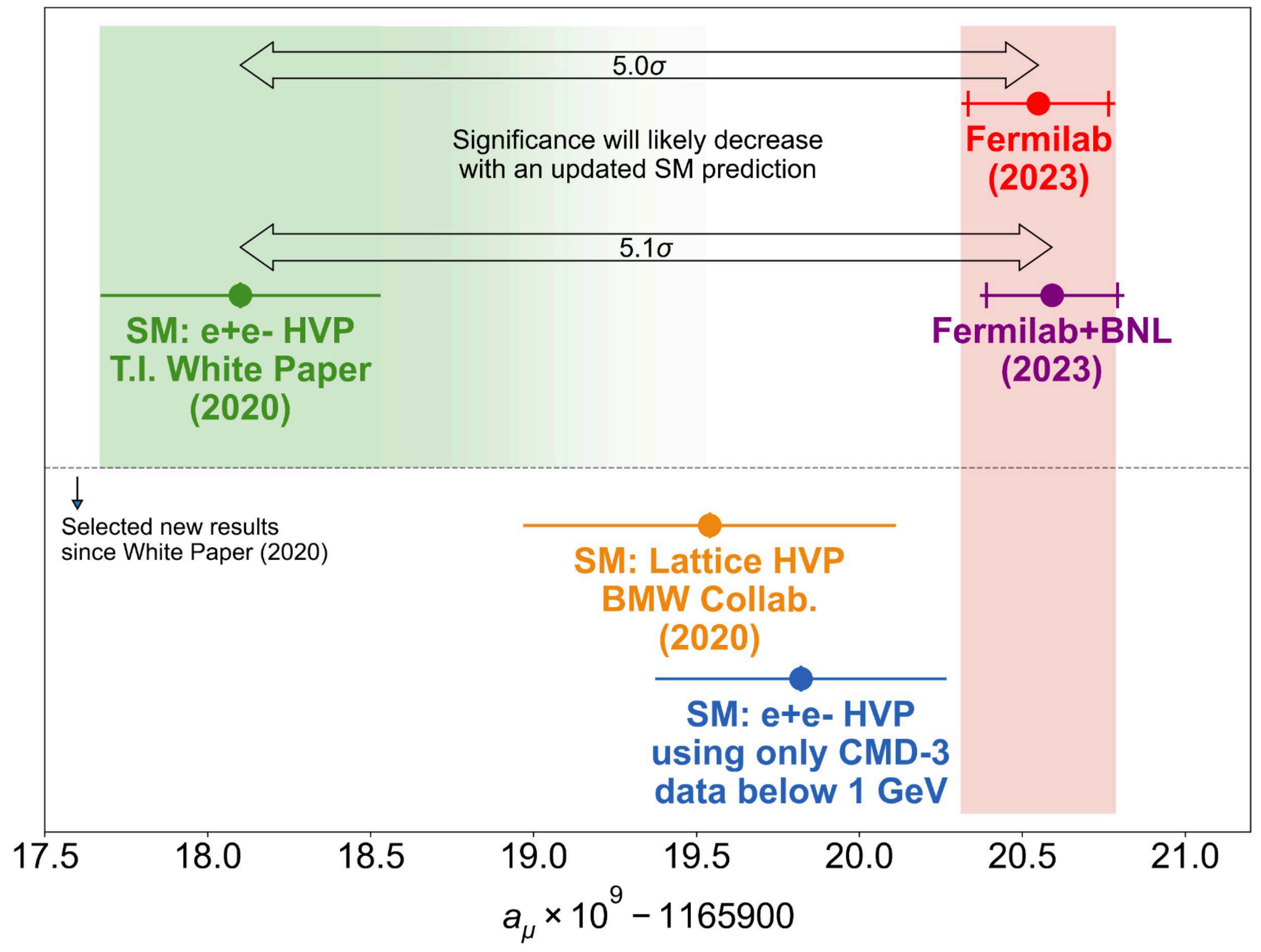
$$+\ 0.00000000092$$ hadronic light by light — Non-perturbative (Data driven/ Lattice)

$$+\ 0.0000000684$$ HVP — Non-perturbative (Data driven/ Lattice)

SM prediction for g-2

Muon g-2 theory initiative White paper, 2006.04822

- Value of SM prediction dominated by QED
- Error in SM dominated by QCD: hadronic light-by-light (~ 16%) and HVP (~84%)
- Presently, data-driven and lattice calculations do not agree for HVP

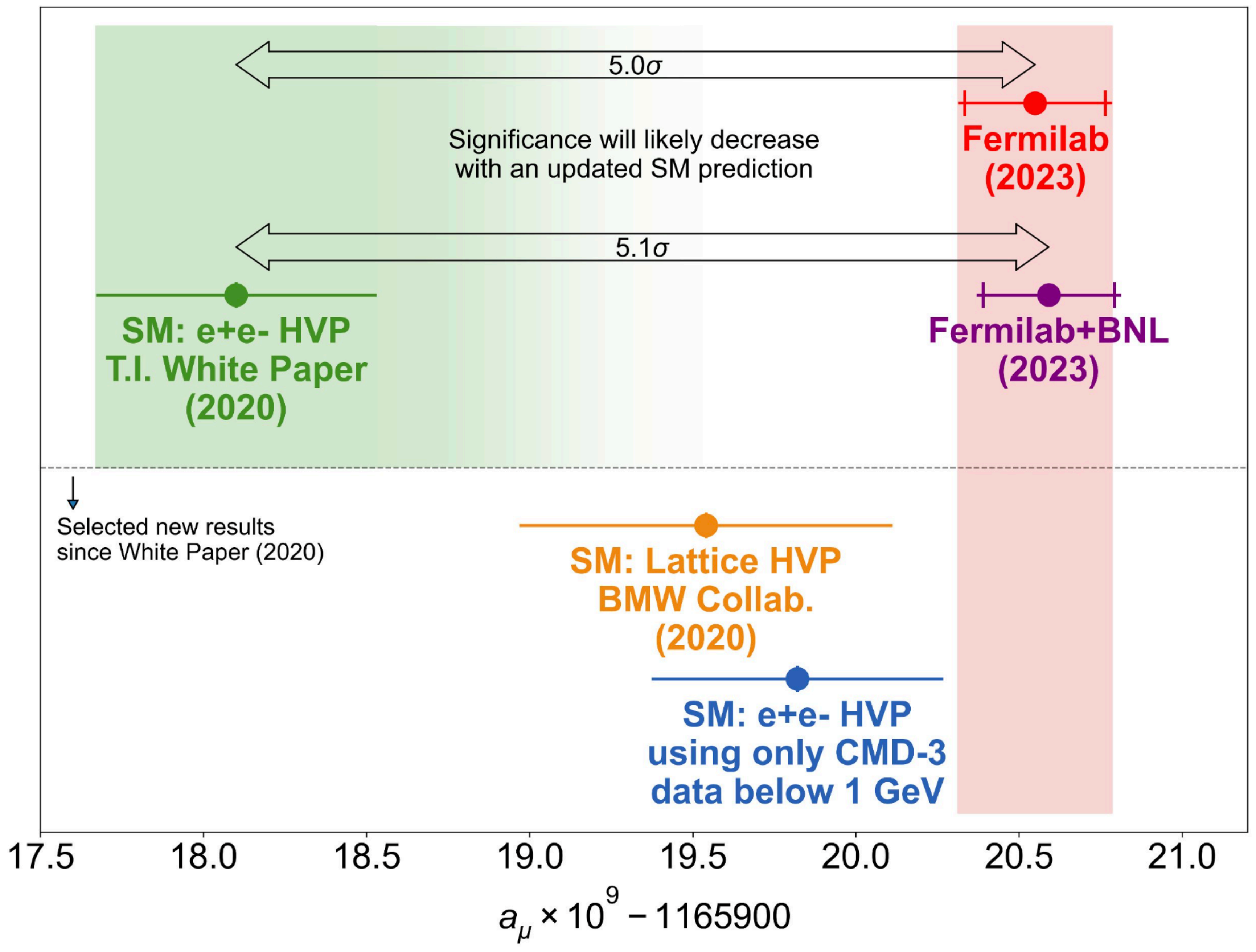


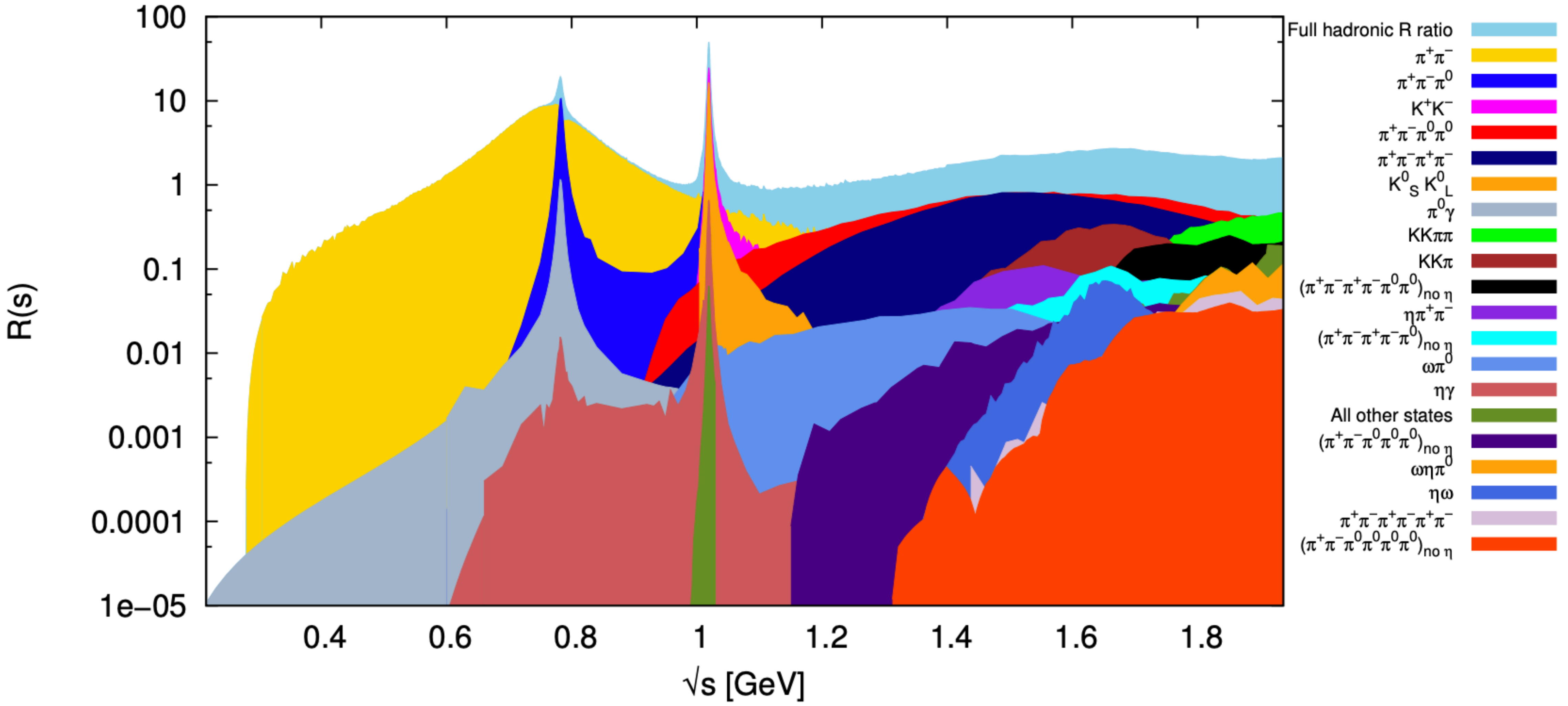
Muon g-2 announcement, FNAL (experiment: 2308.06230)

* N.B. the experimental uncertainty is now in the 10th and 11th decimal places.

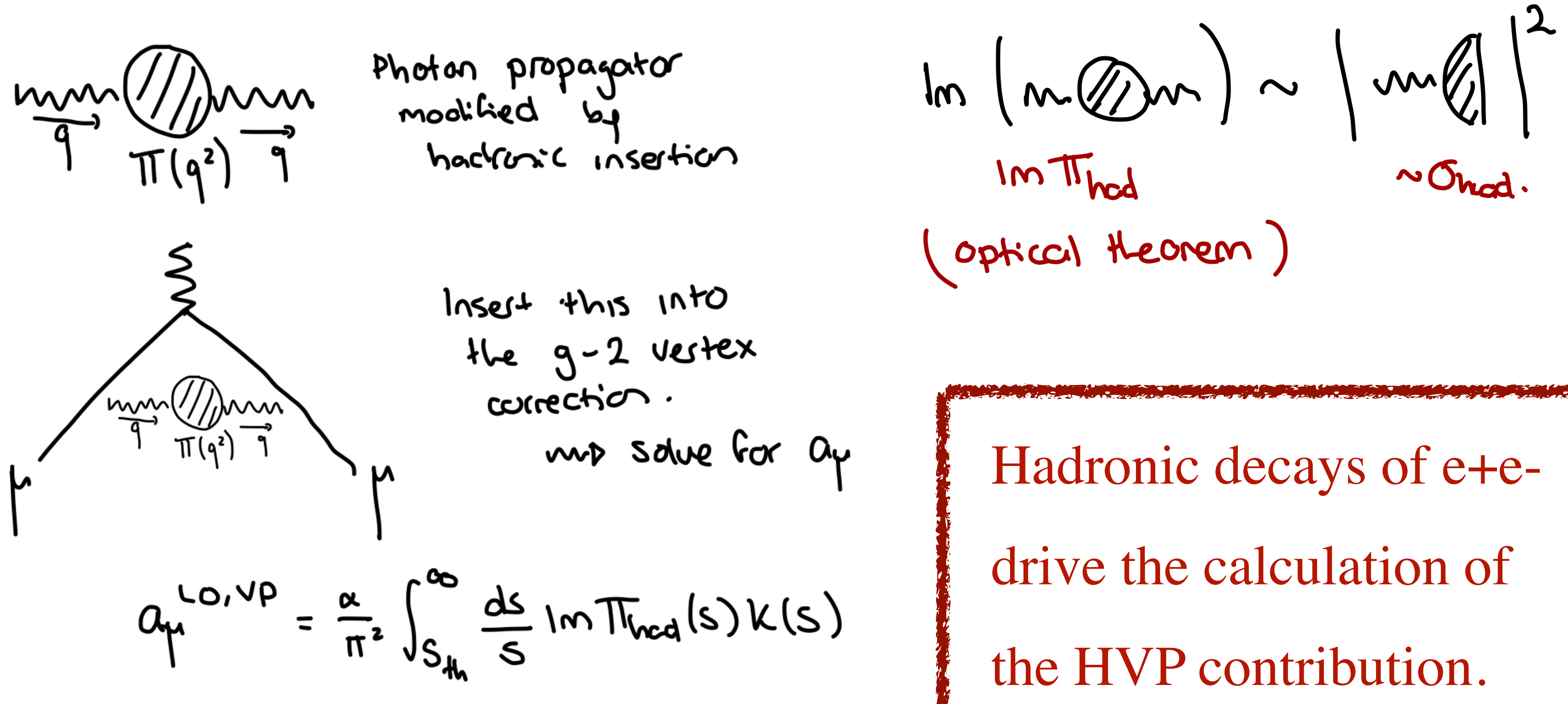
The elephant in the room: HVP

- In 2020, the Muon g-2 theory initiative came together to agree on the employed SM value (2006.04822). Utilised data-driven HVP values: “R-ratio” method





Keshavarzi, Lattice 2023

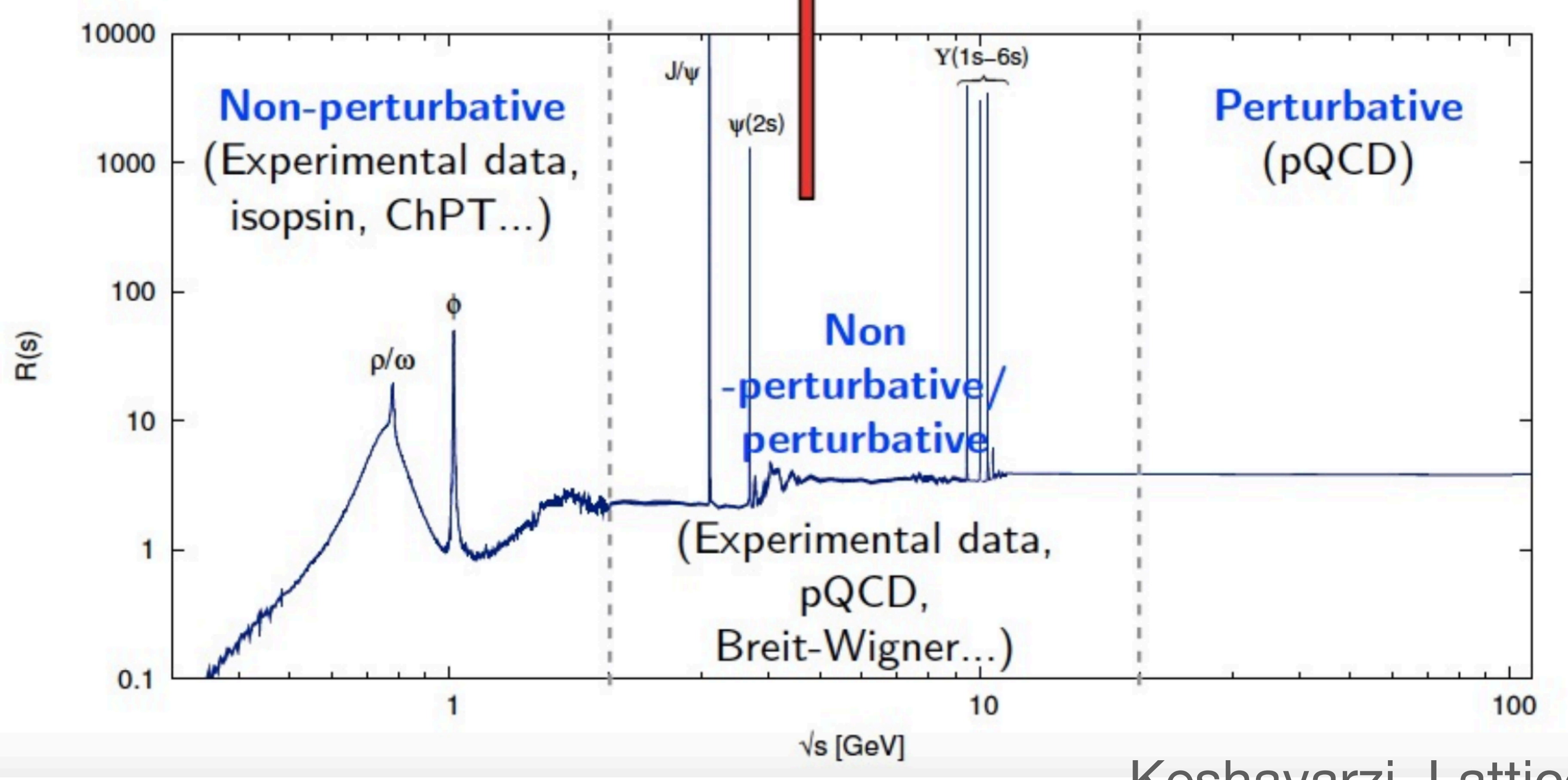


Hadronic decays of e+e- drive the calculation of the HVP contribution.

$$a_\mu^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\text{had},\gamma}(s)}{4\pi\alpha^2/3s}$$

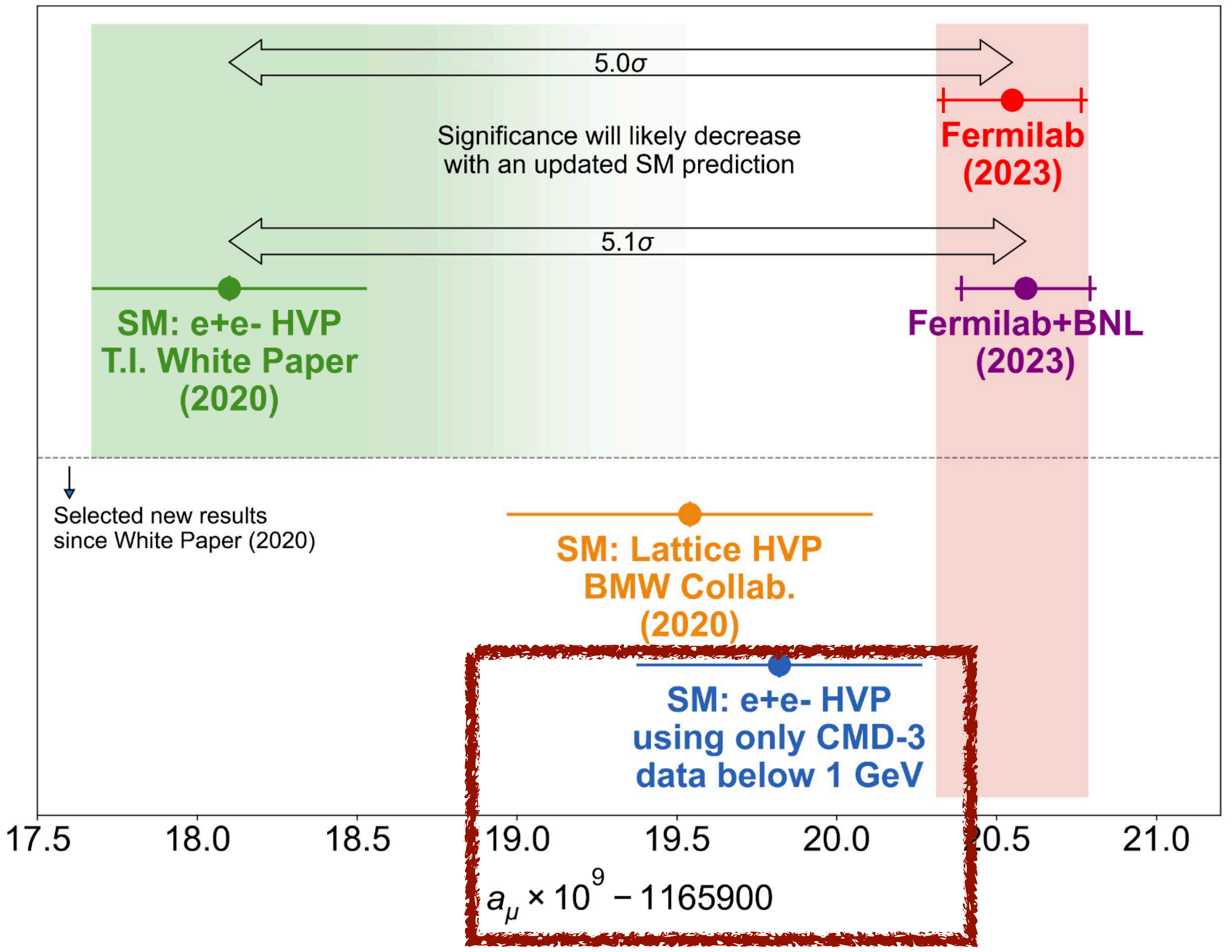
The elephant in the room: HVP

$$a_\mu^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\text{had},\gamma}(s)}{4\pi\alpha^2/3s}$$

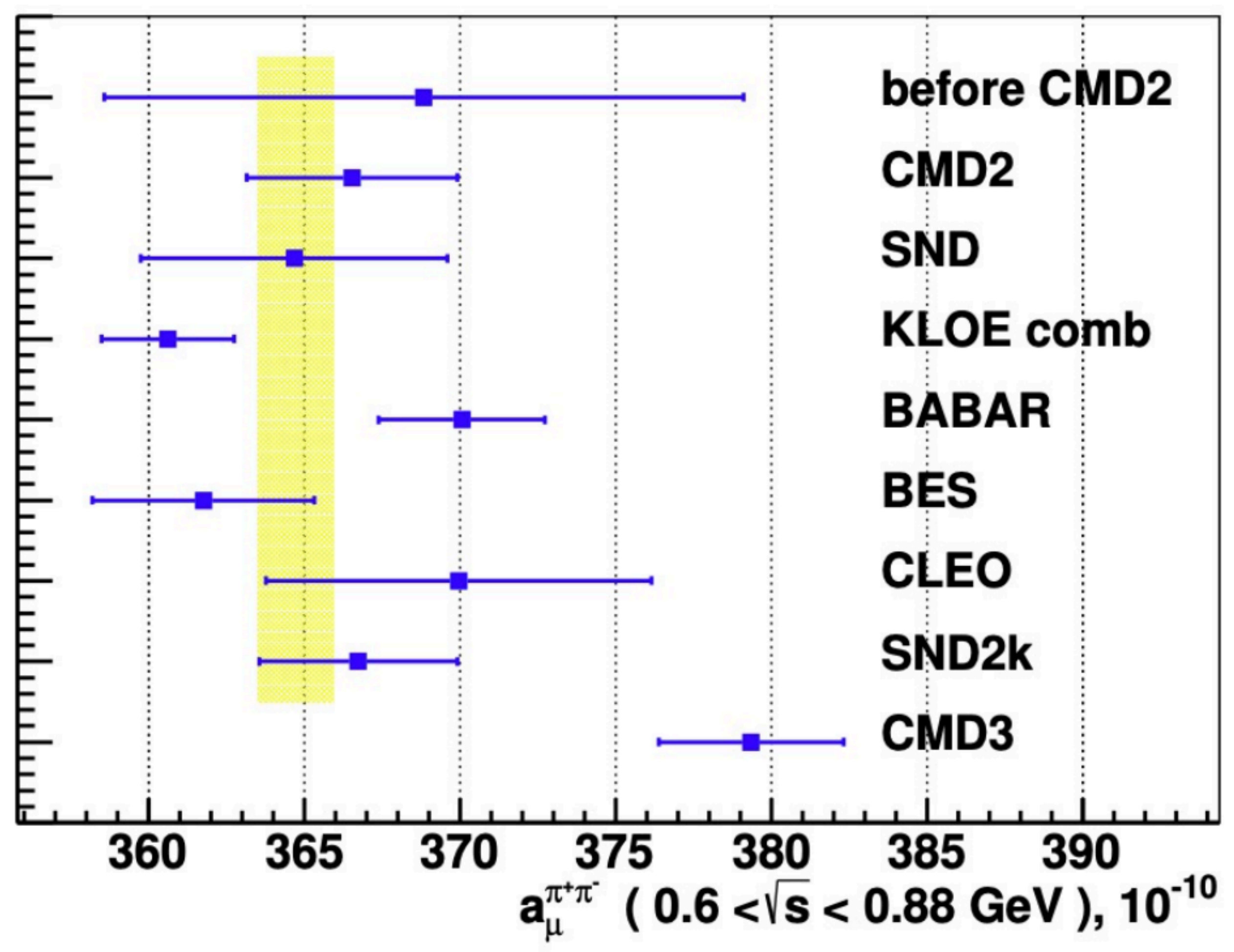


Keshavarzi, Lattice 2023

- The R-ratio utilises data from over 20 years of experiments. 1/s weights low-energy more strongly, e+ e- to pions ~ 73% of the contribution.
  - New CDM-3 measurement indicates stronger agreement with experiment?

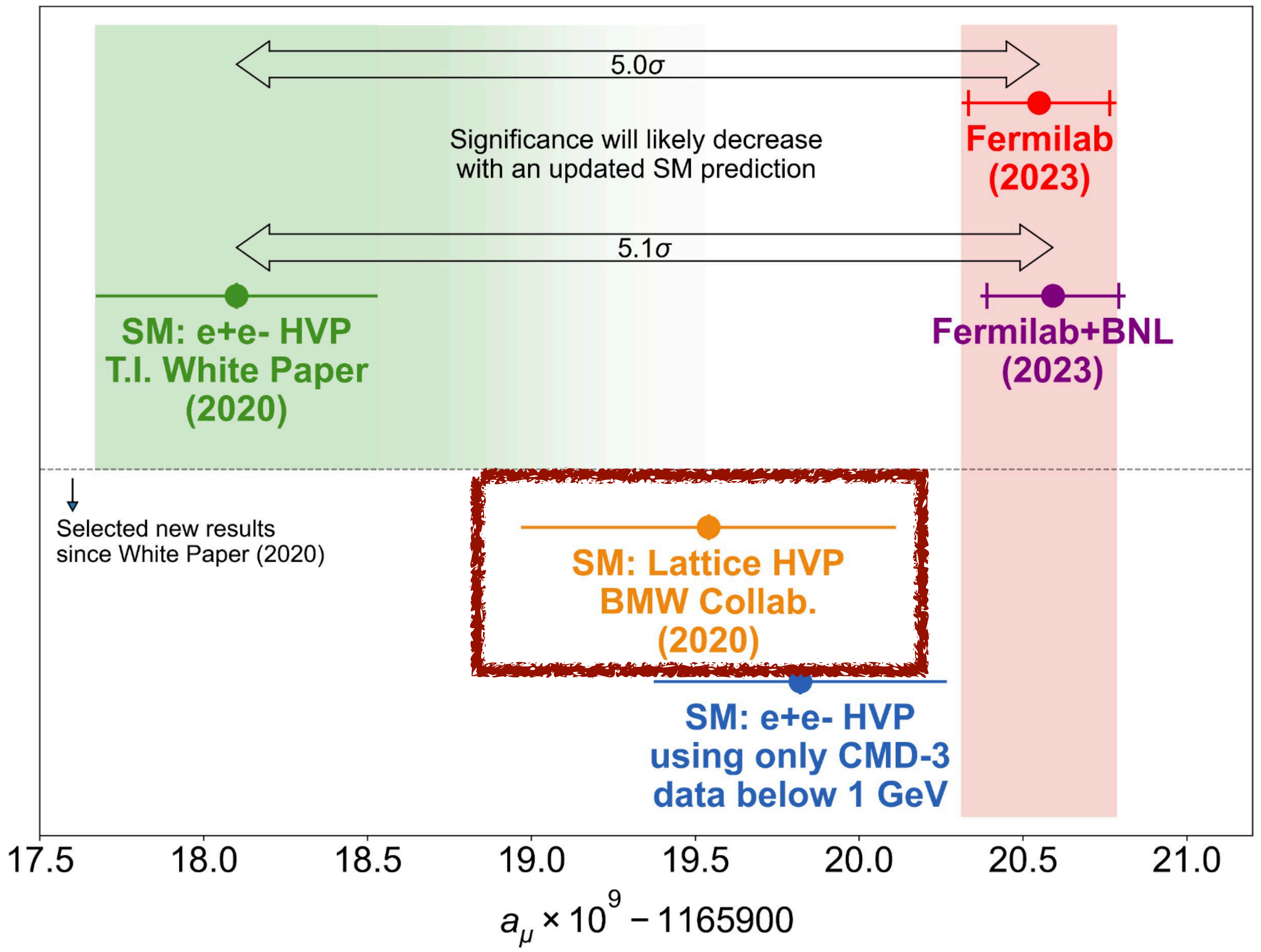


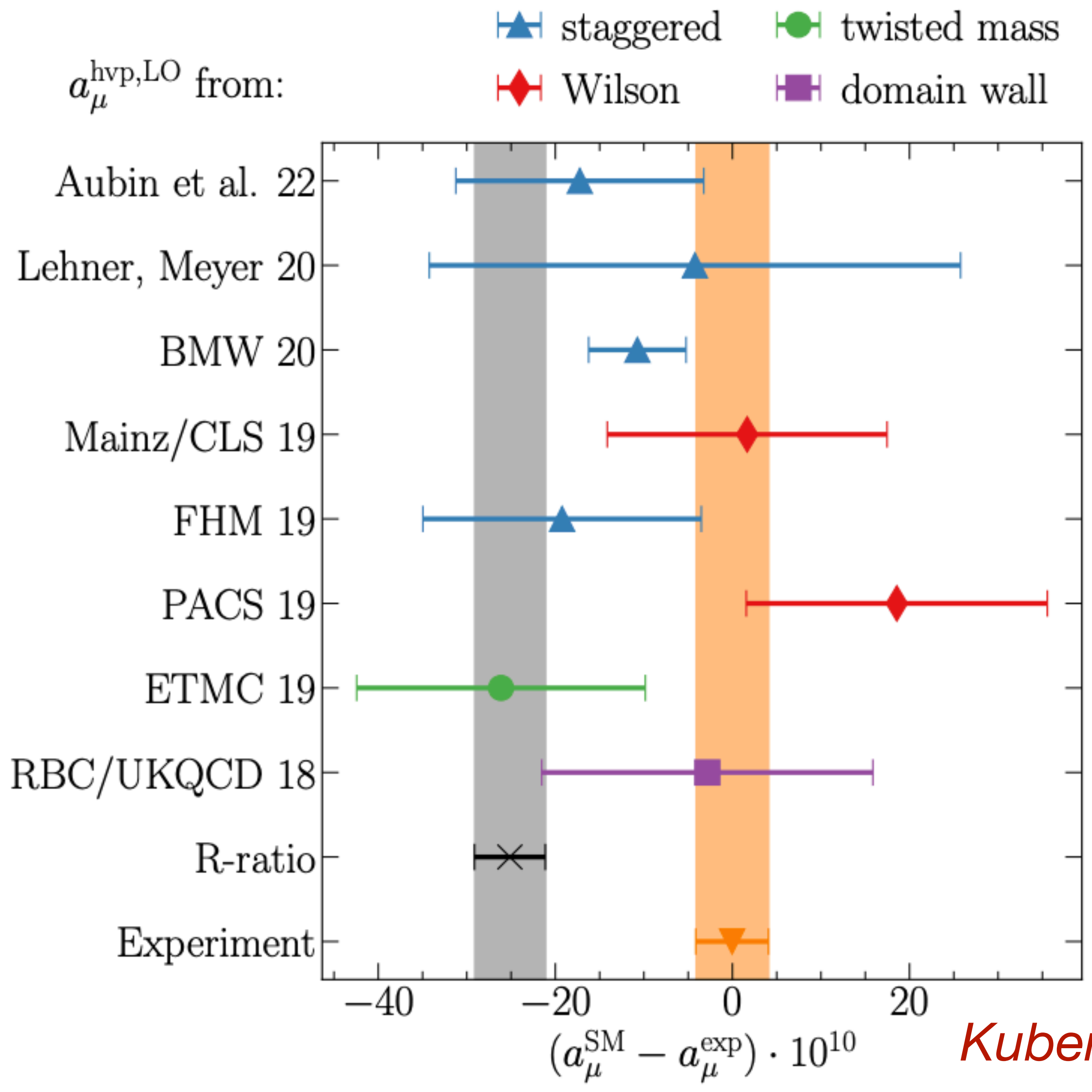
New: from CMD-3 [F. Ignatov et al, arXiv:2302.08834]



DISCLAIMER: these are **NOT** new updates or combinations including the CMD-3 data – simply demonstrations of the impact of the CMD-3 data alone.

The elephant in the room: HVP





Kuberski, Lattice 2023

- Alternative method: calculating the HVP using Lattice
- Prior to 2020 (BMWc, 2002.12347), Lattice didn't have competitive precision to the R-ratio method.
- Since 2020, other collaborations have found some consistent results, but none yet rival the precision in full kinematic region. Work in progress from the Lattice community.

Statement on muon g-2 theory status https://muon-gm2-theory.illinois.edu/

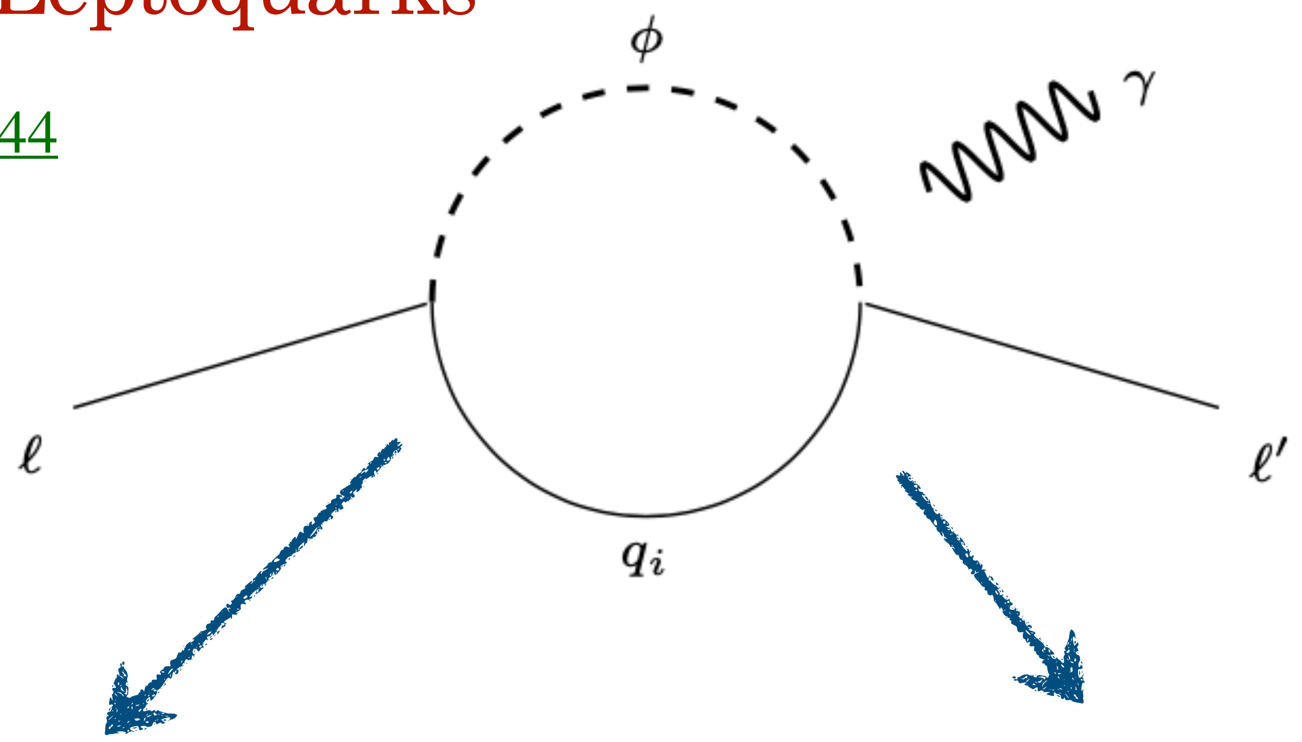
Moral of the story: we nail down HVP, we can make a more concrete statement about status of theory vs. experiment.

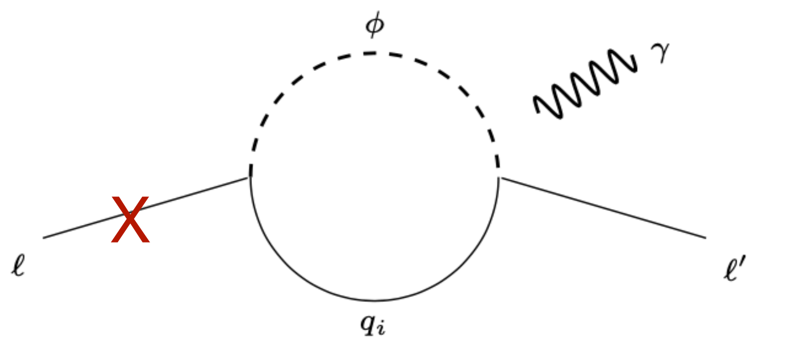
Muon g-2 with BSM

Example: Scalar Leptoquarks

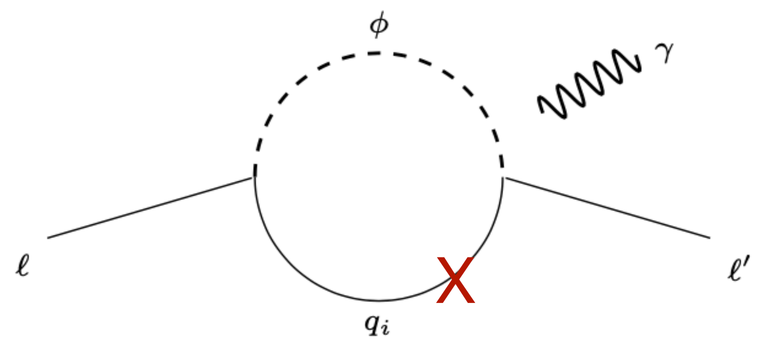
e.g. **IB** and Volkas 2002.12544

Doršner+2006.11624





Same-chiral term



Mixed-chiral term

Symbol	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
$\tilde{S}_1$	$(\mathbf{3}, \mathbf{1}, -4/3)$
S_1	$(\mathbf{3}, \mathbf{1}, -1/3)$
S_3	$(\mathbf{3}, \mathbf{3}, -1/3)$
$\overline{S}_1$	$(\mathbf{3}, \mathbf{1}, 2/3)$
R_2	$(\mathbf{3}, \mathbf{2}, 7/6)$
$\tilde{R}_2$	$(\mathbf{3}, \mathbf{2}, 1/6)$

$$\Delta a_\ell = -\frac{3m_\ell}{8\pi^2 m_\phi^2}\sum_q \Big[m_\ell(|y_\ell^R|^2 + |y_\ell^L|^2)\kappa + m_q \mathrm{Re}(y_\ell^{L*} y_\ell^R)\kappa'\Big]$$

These same scalars appear in solutions for B physics anomalies and neutrino mass generation (e.g. **IB,** Gargalionis, Volkas 1906.01870), electron g-2 (e.g. **IB** and Volkas 2002.12544)

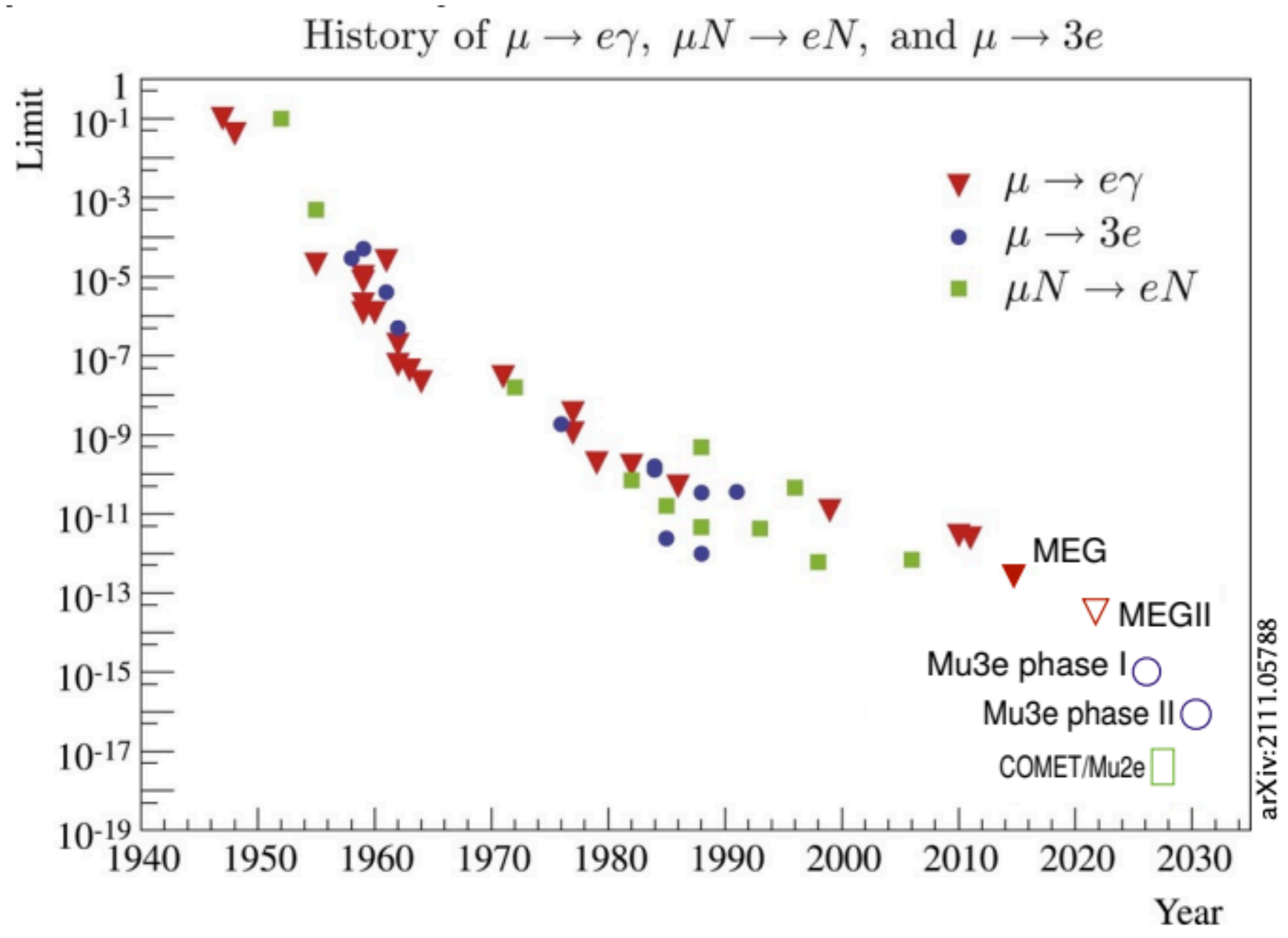
- Could there be new physics in the R-ratio?

  e.g. Coyle and Wagner 2305.02354

- Assume the theory value converges onto a value different to experiment, what BSM could be contributing?
  - New forces? e.g. Z' models
  - New scalars? e.g. Leptoquarks
- Could this same new physics be responsible for solving other BSM problems?
  - Neutrino masses?
  - Electron g-2 ?
  - B physics anomalies?
  - And more…

Conclusion

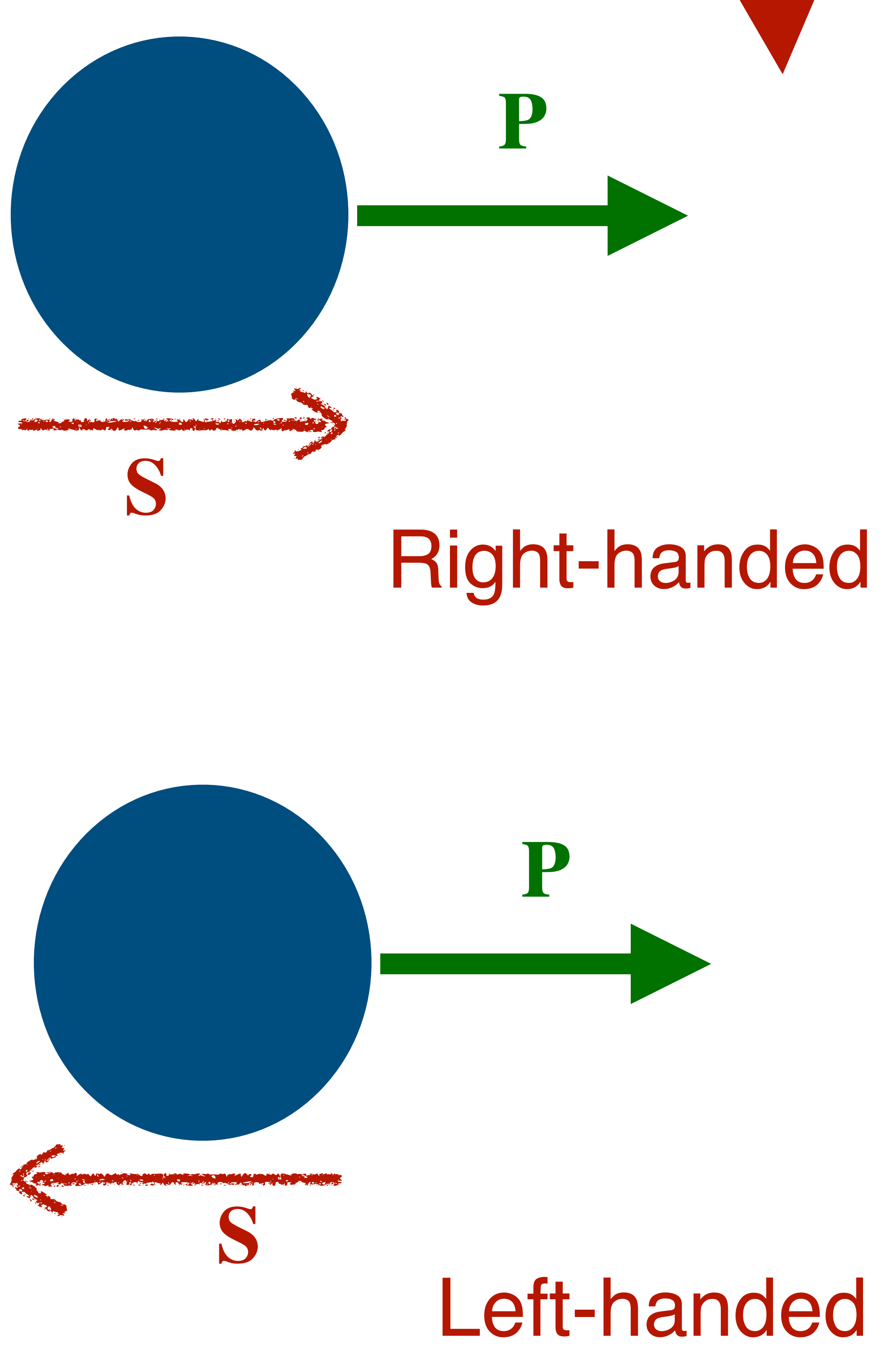
Conclusions



- Neutrino masses means that we need lepton flavour violation
- Muon cLFV and neutrinos are strongly tied together theoretically
- The muon g-2 is an experimental *and* theory effort with state-of-the art precision. Future developments will reveal the nature of the agreement between the two.
- Many models of neutrino masses and other BSM (including for muon g-2) are strongly constrained by muon physics. Even better, muon physics is predicted to be where we are likely to see the first hints of them!
- Lepton flavour physics is a very exciting and active area of theory and experimental developments

Backup slides

Aside: helicity and chirality



- Helicity: 'RH' or 'LH' particles refer to whether the S and P run parallel or anti-parallel
- Chirality: is whether a field is an eigenvector of the LH and RH projection operators

$$P_{\mathrm{R}} = \frac{1+\gamma^5}{2} \qquad P_{\mathrm{L}} = \frac{1-\gamma^5}{2}$$

- These notions are *equivalent* in the massless-particle limit

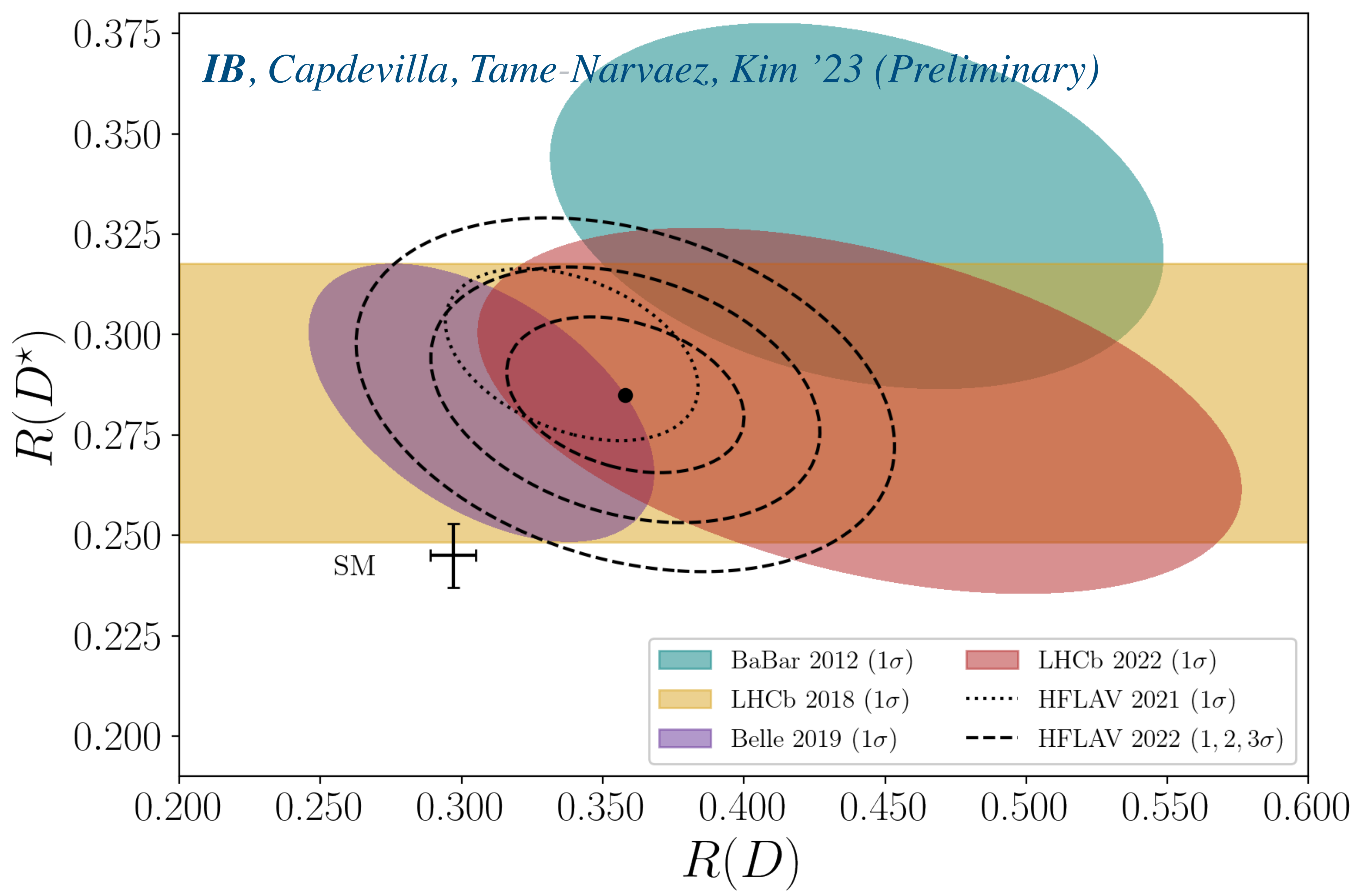
In this talk, when I talk about RH and LH, I am referring to chirality

$$R_{K^{(*)}} = \frac{Br(B \to K^{(*)}\mu^+\mu^-)}{Br(B \to K^{(*)}e^+e^-)}$$

LFU (violation?)

Prior to 2022, a fit to BSM in $b \to s\mu^+\mu^-$ gave a ~ 5 sigma pull from SM

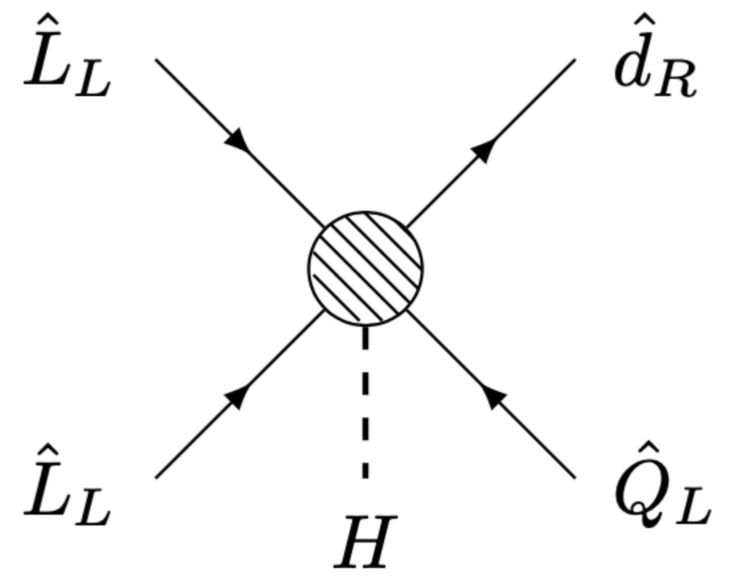
e.g. Kriewald et al, 2104.00015



- Ratios of processes mediated by SM gauge bosons and with different lepton final states allow cancellation of uncertainties and probe deviation from a universal coupling
- Prior to 2022, there was a significant conspiracy of measurements which indicated that new physics really really loved muons… now less compelling
- A strong preference for muons would imply the BSM sector is not coupling flavour-universally
- Still existing tests which show a discrepancy, less muon focussed (e.g. in RD and RDstar)

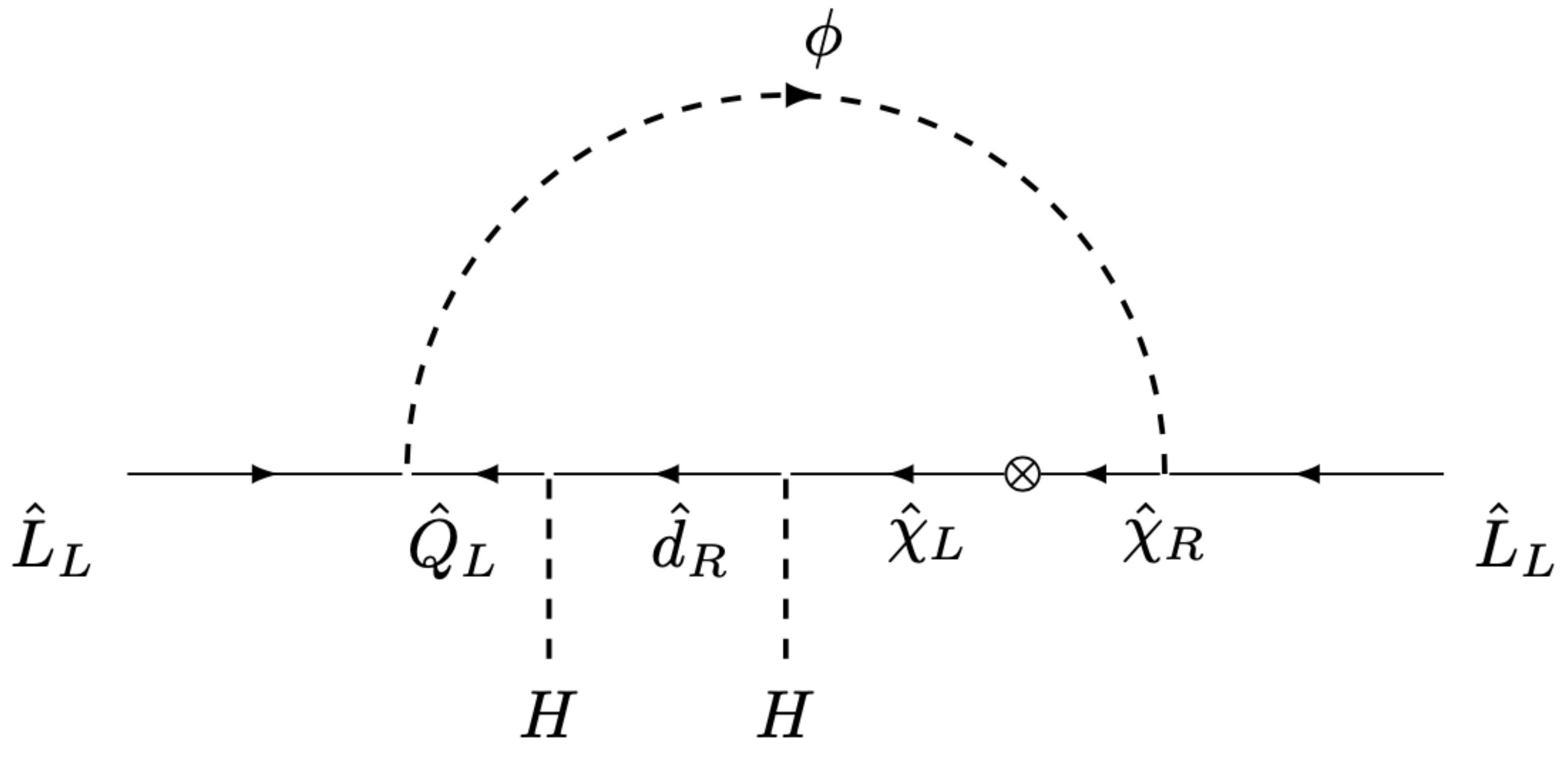
[See more Francesca Dordei, Friday 18.00 WG4]

Neutrino masses constrained by cLFV

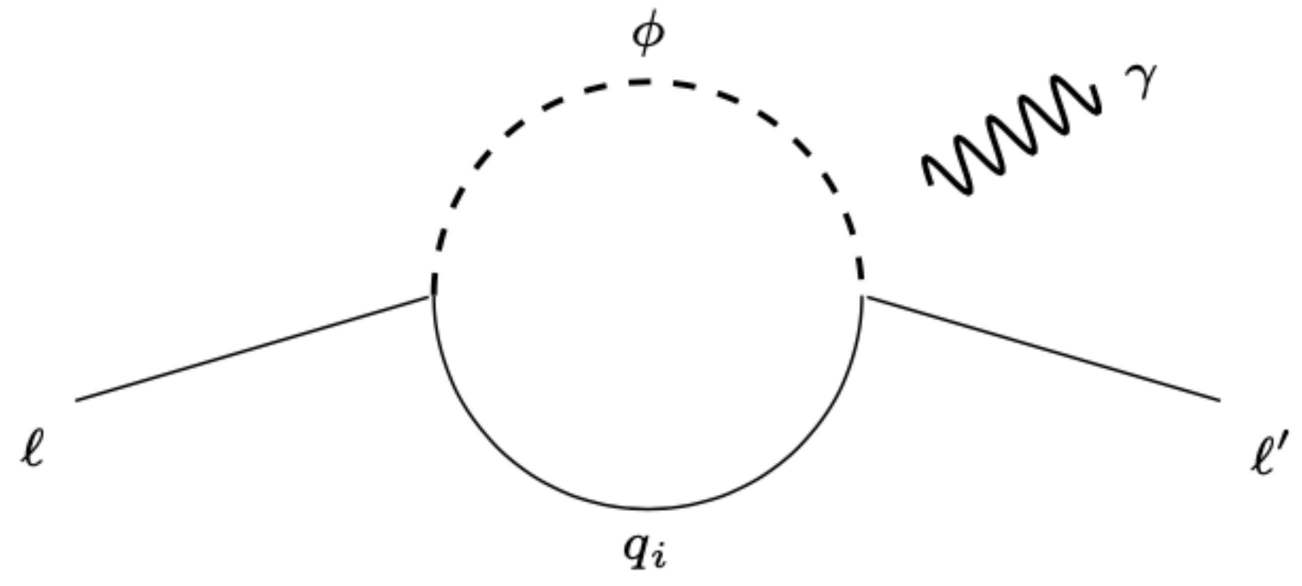


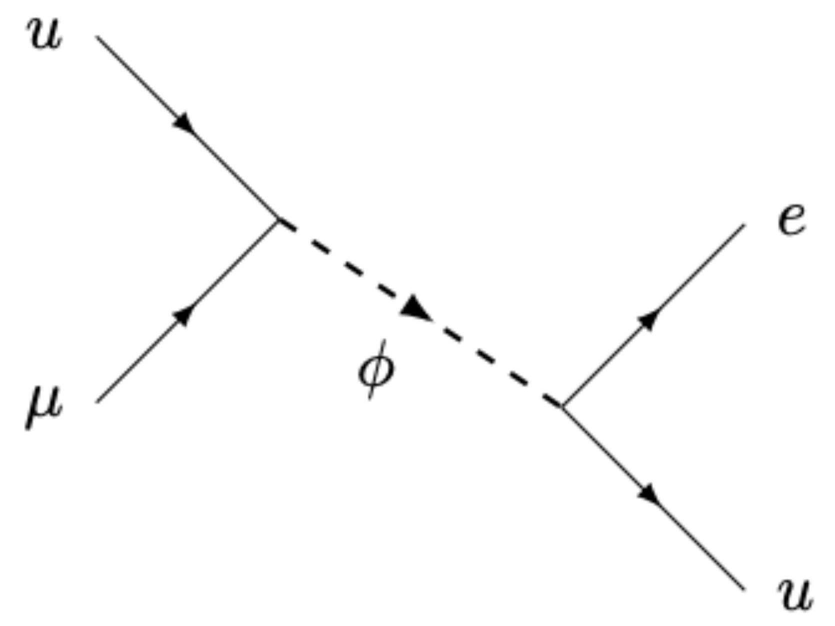
$\phi \sim (\mathbf{3}, \mathbf{1}, -1/3)$

$\chi \sim (\mathbf{3}, \mathbf{2}, -5/6)$



- Example: the completion of a D=7 operator involving a scalar leptoquark and a vector-like quark
- Strong constraints on the same couplings that generate neutrino mass from radiative muon decay, and muon to electron conversion





Cai + '17. ***IB,*** *Volkas, Gargalionis '19.* ***IB,*** *Felkl, Schmidt, Hagedorn '23 (TBD)*

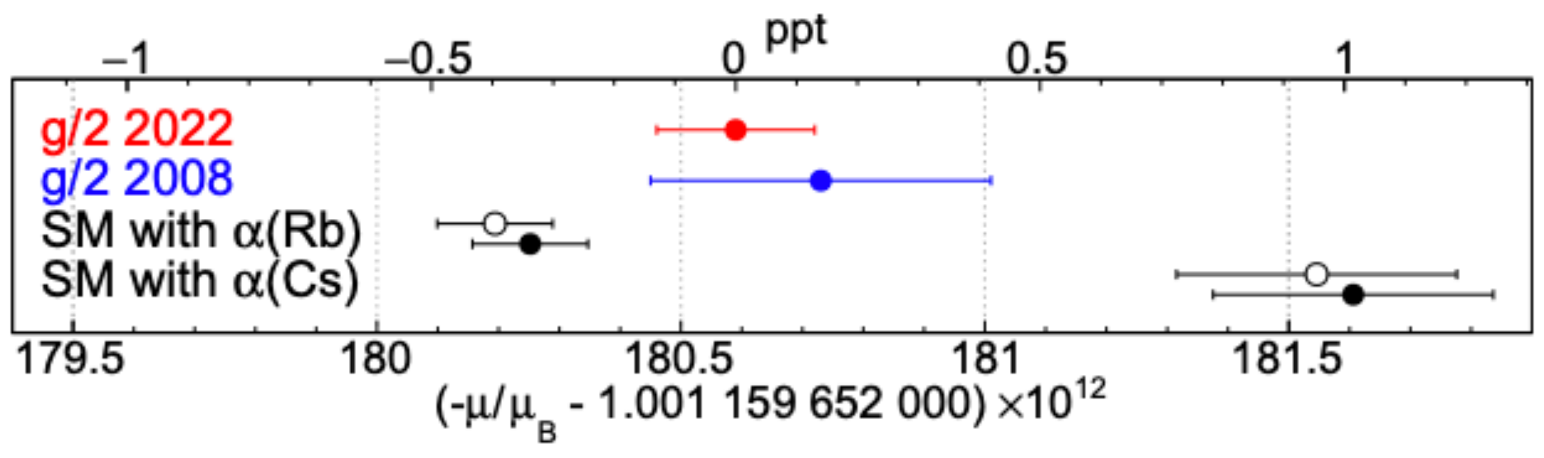


FIG. 1. This Northwestern determination (red) and our 2008 Harvard determination (blue) [37]. SM predictions (solid and open black points for slightly differing C_{10} [38, 39]) are functions of discrepant α measurements [40, 41]. A ppt is 10^{-12}.

Fan et al 2209.13084 (Gabrielse group)

Deviation from SM prediction	Significance
$\Delta a_e^{\mathrm{Cs}} = -(0.88 \pm 0.36) \times 10^{-12}$ Parker et al 1812.04130	2.5σ
$\Delta a_e^{\mathrm{Rb}} = (4.8 \pm 3.0) \times 10^{-13}$ Morel et al 2020, INSPIRE: 1837309	1.6σ

So far, no resolution to this disagreement

The electron g-2 problem

- Electron magnetic moment: long-term precision test of the SM and QED
- Measuring the fine-structure constant historically via

$$a_e^{\mathrm{SM}}(\alpha) = a_e^{\mathrm{Exp}} \implies \alpha$$

- To relax the assumption about BSM physics, and provide complementary tests of the SM we need independent measurements of α
- Two existing measurements disagree on derived value of the electron magnetic moment