Channeling tools for high energy and particle physics

High energy nuclear optics of polarized particles

V.G. Baryshevsky
Photon plane rotation in optical anisotropy matter

Let photons with the linear polarization
\[ \vec{e}_l = -\frac{\vec{e}_+ - \vec{e}_-}{\sqrt{2}} \]
fall in a matter

Polarization vector of a photon in matter

\[ \vec{e}'_1 = -\frac{\vec{e}_+}{\sqrt{2}} e^{ikN_+L} + \frac{\vec{e}_-}{\sqrt{2}} e^{ikN_-L} = \]

\[ = e^{\frac{1}{2}ik(N_+ + N_-)L} \left\{ \vec{e}_1 \cos \frac{1}{2}k(N_+ - N_-)L - \vec{e}_2 \sin \frac{1}{2}k(N_+ - N_-)L \right\} \]

\( L \) is the photon propagation length in a medium, \( \vec{e}_2 \perp \vec{e}_l \)

Photon polarization plane rotates in a matter and the angle of rotation

\[ \vartheta = \frac{1}{2} k \, Re(N_+ - N_-) \, L \]

\( \vartheta > 0 \) corresponds to the right rotation of the light polarization plane (clockwise rotation)

\( \vartheta < 0 \) corresponds to the left rotation (i.e. counter-clockwise rotation)
Spin rotation and dichroism in the polarized target

Could the similar effect exist for particles?
Scattering amplitude in the polarized target, caused by strong interaction

\[ f(0) = A + \sigma \cdot \overrightarrow{g}, \text{ where } \overrightarrow{g} = A_1 \overrightarrow{P} + A_2 \overrightarrow{n} \left( \overrightarrow{n} \cdot \overrightarrow{P} \right) \]

\[ \overrightarrow{g} = (A_1 + A_2) \overrightarrow{P} \quad \text{for } \overrightarrow{n} \parallel \overrightarrow{P} \]

\[ \overrightarrow{g} = A_1 \overrightarrow{P} \quad \text{for } \overrightarrow{n} \perp \overrightarrow{P} \]

\[ f(0)_{\uparrow\uparrow} = A + g \quad \text{for nucleon with spin parallel to } \overrightarrow{P} \]

\[ f(0)_{\uparrow\downarrow} = A - g \quad \text{for nucleon with spin antiparallel to } \overrightarrow{P} \]

Hence, the corresponding refractive indices are not equal to each other:

\[ f(0)_{\uparrow\downarrow} \neq f(0)_{\uparrow\uparrow} \quad \rightarrow \quad N_{\uparrow\uparrow} \neq N_{\uparrow\downarrow} \]
The index of refraction and effective potential energy of particles in matter

The wave number of the particle in vacuum is denoted \( k \), \( k' = kN \) is the wave number of the particle in medium.

\[
E = \frac{\hbar^2k^2}{2m} \quad \quad E' = \frac{\hbar^2k^2N^2}{2m}
\]

From the energy conservation condition we immediately obtain the necessity to suppose that a particle in medium possesses effective potential energy.

\[
N_{\uparrow\uparrow(\uparrow\downarrow)} = 1 + \frac{2\pi\rho}{k^2} f(0)_{\uparrow\uparrow(\uparrow\downarrow)}
\]

\[
N_{\uparrow\uparrow} \neq N_{\uparrow\downarrow}
\]

\[
f(0) = A + \vec{\sigma} \cdot \vec{g}
\]

Spin splitting levels in the nuclear pseudo-magnetic field goes the same way as in the magnetic field. Spin rotation appears.

\[
E = E' + U \quad U = E - E' = -\frac{2\pi\hbar^2}{m} \rho f(0)
\]

Neutron nuclear precession (nuclear pseudomagnetism)


\[
\psi(r) = \left( \begin{array}{c} c_1 \psi_+ (r) \\ c_2 \psi_- (r) \end{array} \right) = c_1 e^{i\vec{k}_\perp \cdot \vec{r}_\perp} e^{i k_z n_z} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{i\vec{k}_\perp \cdot \vec{r}_\perp} e^{i k_z n_{-z}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

\[p_{nx} = \cos[k_z \text{Re}(n_+ - n_-)z] e^{-k_z \text{Im}(n_+ + n_-)z} \langle \psi | \psi \rangle^{-1},\]
\[p_{ny} = -\sin[k_z \text{Re}(n_+ - n_-)z] e^{-k_z \text{Im}(n_+ + n_-)z} \langle \psi | \psi \rangle^{-1},\]
\[p_{nz} = \frac{1}{2} (e^{-2k_z \text{Im}n_+ z} - e^{2k_z \text{Im}n_- z}) \langle \psi | \psi \rangle^{-1},\]
\[p_x^2 + p_y^2 + p_z^2 = 1.\]

\[
\theta = k_z \text{Re}(n_+ - n_-)z = \frac{2\pi \rho}{k_z} \text{Re}(f_+ - f_-)z
\]
Neutron nuclear precession: experiments


Forte, M, Nuovo Cimento A18,4 (1973) 726


Murman Tsulaya, Nuclear precession in a proton target, DUBNA. our days.

Birefringence effect

Could the similar effect exist for particles with the nonzero mass?


Deuteron (spin = 1) passing through a nonpolarized target

The phenomenon of birefringence in matter

Appearance of two refraction indices of deuteron can be easily explained

As the ground state of a deuteron is non-spherical, then the scattering cross-section depends on the angle between the spin and momentum of the deuteron

\[ \text{Im} f_{||}(0) = \frac{k}{4\pi} \sigma_{||} \neq \text{Im} f_{\perp}(0) = \frac{k}{4\pi} \sigma_{\perp} \]

According to the dispersion relation

\[ \text{Re} f(0) \sim \Phi(\text{Im} f(0)), \text{ therefore} \]

\[ \text{Re} f_{\perp}(0) \neq \text{Re} f_{||}(0) \]

Unlike optical birefringence, the birefringence effect for particles appear in isotropic matter (and even the spin of matter nuclei is either zero or unpolarized !). Anisotropy is provided by the particle itself (a particle with the spin \( S \geq 1 \) and mass \( M \neq 0 \) has the intrinsic anisotropy).
Deuteron spin dichroism: experiments

**First observation of spin dichroism with deuterons up to 20 MeV in a carbon target** (LANL arxive: hep-ex/0501045 (2005)

V. Baryshevsky, A. Rovba (Research Institute of Nuclear Problems, Minsk, Belarus)

R. Engels, F. Rathmann, H. Seyfarth, H. Stroher, T. Ullrich (Institut fur Kernphysik, Forschungszentrum Julich, Germany)

C. Duweke, R. Emmerich, A. Imig, J. Ley, H. Paetz gen. Schieck, R. Schulze, G. Tenckhoff, C. Weske (Institut fur Kernphysik, Universität zu Koln, Germany)

M. Mikirtychiants, A. Vassiliev (PNPI, Russia)


Channeling and High energy nuclear optics of polarized particles in crystals

What is new?
Birefringence effect

\[ W^H \sim (\text{\varepsilon} \cdot \text{\vec{H}})(\text{\varepsilon} \cdot \text{\vec{H}}) \]

\[ W^{HE} \sim (\text{\varepsilon} \cdot \text{\vec{H}})(\text{\varepsilon} \cdot \text{\vec{E}}) \]

replace $T$ by $(-T)$

\[ W^{HE} \sim -(\text{\varepsilon} \cdot \text{\vec{H}})(\text{\varepsilon} \cdot \text{\vec{E}}) \]
High-energy gamma-quanta birefringence in crystals


V.G. Baryshevsky and V.V. Tikhomirov, Birefringence of the high-energy $\gamma$-quanta in monocrytsals, Phys.Lett. A 90, 3 (1982) 153
High-energy gamma-quanta birefringence in crystals

Energy dependence of birefringence effects for a linearly polarized beam and length of the quarter–wave plates for (110) planes of Ge and W crystals at $T = 293$ K, and also Si at $T = 0$ and at $T = 293$ K.
Nuclear optics in crystals for high-energy electrons and positrons

Effects with Participation of Polarized $e^{\pm}$ . . . . . . . . .
Radiative Self-Polarization of $e^{\pm}$ in the Intense Fields of Crystals . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
Effect of Variation of the Anomalous Magnetic Moment of $e^{\pm}$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .


Anomalous magnetic moment dependence on $E$

\[
\frac{\mu'(\chi)}{\mu_B} = \frac{\alpha}{\pi} \int_0^\infty du \left( \frac{u}{\chi} \right)^{2/3} \frac{1}{(1+u)^3} \int_0^\infty dt \sin \left[ \left( \frac{u}{\chi} \right)^{2/3} t + \frac{t^3}{3} \right]
\]

\[
\chi = \frac{\gamma E}{E_{cr}}, \quad E_{cr} = 1.32 \cdot 10^{16} \text{ V/cm} = 4.41 \cdot 10^{13} \text{ CGSE}
\]

Plane case

\[
\chi = 1 \text{ if } \gamma = \frac{E_{cr}}{E} = \frac{1,32 \cdot 10^7}{z}
\]

for tungsten $z=74$ \quad $\gamma = 1.78 \cdot 10^5$

Axis case

\[
\gamma_{ax} \approx 1.78 \cdot 10^4
\]

The effect is huge for $e^+/-$. But calculations of $\mu'_B(E)$ are needed for baryons.

Radiative self-polarization

$e^{+/-}$ spin radiative self-polarization in bent crystals.

$$\zeta_z(t) = \zeta_0(0) e^{-t/T_0} + 8(5\sqrt{3})^{-1}(1 - e^{-t/T_0})$$

$$T_0^{-1} = (5\sqrt{3}/8) \alpha \left(\frac{\hbar \gamma}{mc}\right)^2 \left(\frac{\gamma}{R}\right)^3 c$$

During $t > T_0$ independently from initial polarization $\zeta_z = 8(5\sqrt{3})^{-1} = 0.924$.

The beam polarizes along crystal bending axis – one photon is being radiated.

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Parity (P) and time T (CP) non-invariance at LHC and FCC
P non-invariance
The Wu experiment, conducted in 1956 by the Chinese American physicist Chien-Shiung Wu in collaboration with the Low Temperature Group of the US National Bureau of Standards.

Tsung-Dao Lee and Chen-Ning Yang, the theoretical physicists who originated the idea of parity nonconservation and proposed the experiment, received the 1957 Nobel Prize in physics for this result.
Optical gyrotropy caused by $P$-violating interactions

1957 Zeldovich I.B.
1978 Barkov L.M., Zolotarev M.S.

Neutral weak currents were later discovered in SLAC at deeply nonelastic electrons scattering by deutons.
Neutron optical spin rotation

\[ \psi(\vec{r}) = \begin{pmatrix} c_1 & \psi_+ (\vec{r}) \\ c_2 & \psi_- (\vec{r}) \end{pmatrix} = c_1 e^{i \vec{k}_\perp \cdot \vec{r}_\perp} e^{i k_z n+} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{i \vec{k}_\perp \cdot \vec{r}_\perp} e^{i k_z n-} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

\[ p_{nx} = \cos[k_z \text{Re}(n_+ - n_-) z] e^{-k_z \text{Im}(n_+ + n_-) z} \langle \psi | \psi \rangle^{-1} \]
\[ p_{ny} = -\sin[k_z \text{Re}(n_+ - n_-) z] e^{-k_z \text{Im}(n_+ + n_-) z} \langle \psi | \psi \rangle^{-1} \]

\[ \theta = k_z \text{Re}(n_+ - n_-) z \]

T non-invariance

To be or not to be
T non-invariance

Non-Relativistic Hamiltonian

\[ H = -\mu \vec{B} - \vec{d}\vec{E} \]

Assume \( \vec{\mu} = \mu \frac{\vec{J}}{J} \) and \( \vec{d} = d \frac{\vec{J}}{J} \)
Role of CP violation in the matter/antimatter asymmetry of the Universe

Sakharov Criteria:

Particle Physics can produce matter/antimatter asymmetry in the early universe IF there is:
• Baryon Number Violation
• CP & C violation
• Departure from Thermal Equilibrium
First result for neutron EDM

E.M. Purcell and N.F. Ramsey, Phys. Rev. 78, 807 (1950)

Pioneered Neutron Beam Magnetic Resonance
n-EDM vs Time

Theoretical Prediction:
- Electromagnetic
- Milliweak
- Weinberg
  - Multi-Higgs
- Supersymmetry
- Cosmology
- Superweak
- Standard Model
Electrical dipole moment of heavy charm and beauty baryons

It was recently stated that heavy baryons EDM can be as great as the value $d \approx 10^{-17}$.

• F. Sala, JHEP 03, 061, (2014)
**Characteristics. MDM and EDM = ?**

**Charmed baryons**

\[
\begin{align*}
\Lambda_{c}^{+} & : \tau = 0.2 \cdot 10^{-12} \text{s}; \quad m = 2286.46 \text{MeV}; \quad l_{d} = l_{\text{decay}} = \tau c \gamma = 6 \text{cm}. \\
\Xi_{c}^{+} & : \tau = 0.44 \cdot 10^{-12} \text{s}; \quad m = 2467.8 \text{MeV}; \quad l_{d} = 13.2 \text{cm}. \\
\Xi_{c}^{0} & : \tau = 0.1 \cdot 10^{-12} \text{s}; \quad m = 2470.88 \text{MeV}; \quad l_{d} = 3.3 \text{cm}; \quad \gamma = 10^{3} \\
\Omega_{c}^{0} & : \tau = 7 \cdot 10^{-14} \text{s}; \quad m = 2695 \text{MeV}; \quad l_{d} = 2.1 \text{cm}.
\end{align*}
\]

**Bottom baryons**

\[
\begin{align*}
\Lambda_{b}^{0} & : \tau = 1.425 \cdot 10^{-12} \text{s}; \quad m = 5619.4 \text{MeV}; \quad l_{d} = 42.7 \text{cm}; \quad \gamma = 10^{3}. \\
\Xi_{b}^{0} & : \tau = 1.49 \cdot 10^{-12} \text{s}; \quad m = 5788 \text{MeV}; \quad l_{d} = 44.7 \text{cm}. \\
\Xi_{b}^{-} & : \tau = 1.56 \cdot 10^{-12} \text{s}; \quad m = 5791 \text{MeV}; \quad l_{d} = 44.7 \text{cm}. \\
\Omega_{b}^{-} & : \tau = 1.1 \cdot 10^{-12} \text{s}; \quad m = 6071 \text{MeV}; \quad l_{d} = 33 \text{cm}.
\end{align*}
\]
Spin rotation effect of ultrarelativistic particles passing through a crystal


In particle rest frame

\[ B^* \rightarrow \gamma E \]

\[ \omega' = \frac{2\mu' B^*}{\hbar} = \frac{2\mu' \gamma E}{\hbar} \]

In laboratory frame

\[ \omega = \frac{\omega'}{\gamma} = \frac{2\mu' E}{\hbar} \]
First experiment to measure (g-2) rotation

E761 Collaboration, FERMILAB


Energy of $\Sigma^+$: 200 – 300 GeV

D. Chen et all


A.V. Khanzadeev, V.M. Samsonov, R.A. Carrigan, D. Chen

"Experiment to observe the spin precession of channeled relativistic $\Sigma^+$ hyperons" NIM 119 (1996) 266.
Electromagnetic dipole moment and particles spin rotation in bent crystals at Large Hadron Collider

Non-Relativistic Hamiltonian

\[
H = -\mu \hat{B} - d\hat{E}
\]

C-even
P-even
T-even

Relativistic equation

\[
\frac{d\vec{S}}{dt} = -\frac{e(g-2)}{2mc} [\vec{S} \times [\vec{\beta} \times \vec{E}]] + \\
+ \frac{d}{\hbar S} [\vec{S} \times \vec{E}].
\]


Behavior of the spin rotation caused by magnetic moment and EDM. The figure is reprinted from Botella et al., On the search for the electric dipole moment of strange and charm baryons at LHC, *Eur. Phys J.C.* 77, 181 (2017). Black arrows represent spin rotation caused by magnetic dipole moment, red arrows represent spin rotation caused by electric dipole moment.
T non-invariance interactions at LHC and FCC

• **V.G. Baryshesky**, Electromagnetic dipole moment and time reversal invariance violating interactions of high energy short-lived particles in bent and straight crystals, Phys.Rev Accelerators and Beams 22,081004 (2019)

The index of refraction and effective potential energy of relativistic particles in matter

The wave number of the particle in vacuum is denoted $k$, $k' = kn$ is the wave number of the particle in medium.

$$n = 1 + \frac{2\pi N}{k^2} f(0)$$

Expression for $n$ does not contain $\hbar$.

Boundary vacuum-medium

vacuum

$$E = \sqrt{\hbar^2 k^2 c^2 + m^2 c^4}$$

medium

$$E_{med} = \sqrt{\hbar^2 k^2 n^2 c^2 + m^2 c^4}$$

Kinetic energy of a particle in vacuum is not equal to that in medium.
Effective potential energy of particle interaction in matter

From the energy conservation condition we immediately obtain the necessity to suppose that a particle in medium possesses effective potential energy. This energy can be found easily from the evident equality:

\[ E = E_{med} + U_{\text{eff}} \]

\[ U_{\text{eff}} = E - E_{med} = -\frac{2\pi\hbar^2}{m\gamma} Nf (E, 0) = (2\pi)^3 N T_{aa} (\vec{k}' - \vec{k} = 0) \]

\[ f (E, 0) = -(2\pi)^2 \frac{E}{c^2\hbar^2} T_{aa} (\vec{k}' - \vec{k} = 0) = -(2\pi)^2 \frac{m\gamma}{\hbar^2} T_{aa} (\vec{k}' - \vec{k} = 0) \]
Effective potential energy of particle interaction with crystal

\[ U(\vec{r}) = \sum_{\vec{\tau}} U(\vec{\tau}) e^{i\vec{\tau} \cdot \vec{r}} \quad \quad U(\vec{\tau}) = \frac{1}{V} \sum_{j} U_j(\vec{\tau}) e^{i\vec{\tau} \cdot \vec{r}_j} \]

\[ U_j(\vec{\tau}) = -\frac{2\pi \hbar^2}{m\gamma} F_j(\vec{\tau}) \]

\[ F_j(\vec{k}' - \vec{k}) = f_j(\vec{k}' - \vec{k}) - i \frac{k}{4\pi} \int f_j^*(\vec{k}'' - \vec{k}') f_j(\vec{k}'' - \vec{k}) d\Omega_{k''} \]
Effective potential energy of particle interaction with plane and axis

For plane:

\[
\hat{U}(x) = -\sum_{\tau_x} \frac{2\pi \hbar^2}{m\gamma V} \hat{F}(q_x = \tau_x, q_y = q_z = 0)e^{i\tau_x x} = \\
= -\frac{2\pi \hbar^2}{m\gamma V d_y d_z} \sum_{X_n} \hat{F}(x - X_n, q_y = q_z = 0)
\]

\[
\hat{F}(\vec{q}) = \int \hat{F}(\vec{r}')e^{-i\vec{q} \cdot \vec{r}'} d^3 r'
\]

For axis:

\[
\hat{U}(\rho) = -\frac{2\pi \hbar^2}{m\gamma V} \sum_{\rho_x, \rho_y} \hat{F}(q_x = \tau_x, q_y = \tau_y, q_z = 0)e^{i\tau_x \rho} = \\
= -\frac{2\pi \hbar^2}{m\gamma d_z} \sum_{R_{n\perp}} \hat{F}(\rho - R_{n\perp}, q_z = 0)
\]
Scattering amplitude of a particle with spin 1/2

$$\hat{F}(\vec{q}) = A_{coul}(\vec{q}) + A_s(\vec{q}) + (B_{magn}(\vec{q}) + B_S(\vec{q}))\vec{\sigma}[\vec{n} \times \vec{q}] +$$
$$+ (B_{we}(\vec{q}) + B_{wnuc}(\vec{q}))\vec{\sigma}\vec{N}_w + (B_{EDM}(\vec{q}) + B_{Te}(\vec{q}) + B_{Tnuc}(\vec{q}))\vec{\sigma}\vec{q}$$

$$\vec{q} = \vec{k}' - \vec{k}, \quad \vec{n} = \frac{\vec{k}}{k}, \quad \vec{N}_w = \frac{\vec{k}' + \vec{k}}{|\vec{k}' + \vec{k}|}$$
Effective potential energy determined by the anomalous magnetic moment

\[ \hat{F}_{\text{magn}}^{(1)}(q) = B_{\text{magn}}(q)\vec{\sigma} \left[ \vec{n} \times \vec{q} \right] \]

\[ \hat{U}_{\text{magn}}^{(1)} = -\frac{e\hbar}{2mc} \frac{g - 2}{2} E_{\text{plane}}(x) \vec{\sigma} \vec{N} \]

\[ \vec{N} = [\vec{n}_x \times \vec{n}], \quad \vec{n}_x \parallel \vec{E}(x), \quad \vec{n}_x \perp \vec{n}, \quad \vec{n} = \frac{\vec{k}}{k} \]
Effective potential energy determined by the anomalous magnetic moment

\[
\hat{F}^{(2)}(\vec{q} = \vec{\tau}) = i \frac{k}{4\pi \hbar^2 c^2} \int \int e^{-i \vec{r}_1} \left\{ \left[ \int \hat{V}(\vec{r}_1, z) dz \right]^2 - \left[ \int \hat{V}(\vec{r}_1, z) dz \right]^2 \right\} d^2 r_1
\]

\[
\hat{V}(\vec{r}_1, z) = V_{\text{coul}}(\vec{r}_1, z) + \hat{V}_{\text{magn}}(\vec{r}_1, z)
\]

\[
\hat{U}^{(2)}_{\text{magn}}(x) = -i \frac{1}{4d_y d_z mc^2} \left( \frac{g - 2}{2} \right) \frac{\partial}{\partial x} \delta V^2_{\text{coul}}(x) \vec{\sigma} \vec{N}
\]

\[
\hat{U}_{\text{magn}}(x) = -(\alpha_m(x) + i \delta_m(x)) \vec{\sigma} \vec{N}
\]
Effective potential energy determined by spin-orbit interaction

\[ \hat{F}_{ssp\text{-}orb}(q = \vec{\tau}) = B_s(\vec{\tau})\vec{\sigma}[\vec{n} \times \vec{\tau}] \]

\[ \hat{U}_{ssp\text{-}orb} = -(\alpha_s + i\delta_s)\vec{\sigma}\vec{N} \]

Spin structure of \( \hat{U}_s(x) \) is similar to the one of \( \hat{U}_{magn}(x) \).

\[ \vec{N} = [\vec{n}_x \times \vec{n}] \]

\[ \alpha_s = \frac{2\pi\hbar^2}{m\gamma d_y d_z} \frac{\partial N_{nuc}}{\partial x} B'' \]

\[ \delta_s = \frac{2\pi\hbar^2}{m\gamma d_y d_z} B' \frac{\partial N_{nuc}}{\partial x} \]
Effective potential energy determined by P-odd and T-even interactions

\[
\hat{F}_w(\vec{q}) = (B_{we}(\vec{q}) + B_{wnuc}(\vec{q}))\vec{\sigma}\vec{N}_w
\]

\[
\hat{U}_w(x) = \hat{U}_{we}(x) + \hat{U}_{wnuc}(x) = -(\alpha_w(x) + i\delta_w(x))\vec{\sigma}\vec{N}_w
\]

\[
\alpha_w(x) = \alpha_{we}(x) + \alpha_{wnuc}(x)
\]

\[
\delta_w(x) = \delta_{we}(x) + \delta_{wnuc}(x)
\]

\[
\alpha_w(x) = \frac{2\pi\hbar^2}{m\gamma d_y d_z} (\tilde{B}'_{we}(0)N_e(x) + \tilde{B}'_{wnuc}(0)N_{nuc}(x))
\]

\[
\delta_w(x) = \frac{2\pi\hbar^2}{m\gamma d_y d_z} (\tilde{B}''_{we}(0)N_e(x) + \tilde{B}''_{wnuc}(0)N_{nuc}(x))
\]
Effective potential energy determined by the electric dipole moment and other T-nonivariant interactions

\[ \hat{F}_T(q) = (B_{EDM}(q) + B_{Te}(q) + B_{Tnuc}(q))\tilde{\sigma}\tilde{q} \]

\[ \tilde{q} = \vec{k}' - \vec{k} \]

\[ \hat{U}_T(x) = \hat{U}_{EDM} + \hat{U}_{Te} + \hat{U}_{Tnuc} = -(\alpha_T(x) + i\delta_T(x))\tilde{\sigma}\tilde{N}_T \]

\[ \hat{U}_{EDM}(x) = -(\alpha_{EDM}(x) + i\delta_{EDM}(x))\tilde{\sigma}\tilde{N}_T, \quad \tilde{N}_T = \tilde{n}_x \]

\[
\alpha_T(x) = \alpha_{EDM} + \alpha_{Te} + \alpha_{Tnuc} \\
\delta_T(x) = \delta_{EDM} + \delta_{Te} + \delta_{Tnuc}
\]

\[
\alpha_{Te(nuc)} = \frac{2\pi\hbar^2}{m\gamma d_y d_z} \tilde{B}''_{Te(nuc)} \frac{dN_{e(nuc)}(x)}{dx} \\
\delta_{Te(nuc)} = \frac{2\pi\hbar^2}{m\gamma d_y d_z} \tilde{B}'_{Te(nuc)} \frac{dN_{e(nuc)}(x)}{dx}
\]
P and CP violating spin rotation in bent crystals

\[ i \hbar \frac{\partial | \Psi(t) \rangle}{\partial t} = \hat{U}_{\text{eff}} | \Psi(t) \rangle \]

\[ \vec{\xi} = \frac{\langle \Psi(t) | \hat{\sigma} | \Psi(t) \rangle}{\langle \Psi(t) | \Psi(t) \rangle} \]
P and CP violating spin rotation in bent crystals

\[
\frac{d\vec{\xi}}{dt} = \left[ \vec{\xi} \times \vec{\Omega}_{msso} \right] - \frac{2}{\hbar} \left( \delta_m (x) + \delta_{so} (x) \right) \{ \vec{N}_m - \vec{\xi} (\vec{N}_m \vec{\xi}) \} + \\
+ \left[ \vec{\xi} \times \vec{\Omega}_T \right] + \frac{2}{\hbar} \left( \delta_{EDM} (x) + \delta_{Te} (x) + \delta_{Tnuc} (x) \right) \{ \vec{N}_T - \vec{\xi} (\vec{N}_T \vec{\xi}) \} + \\
+ \left[ \vec{\xi} \times \vec{\Omega}_w \right] - \frac{2}{\hbar} \delta_w \{ \vec{n} - \vec{\xi} (\vec{n} \vec{\xi}) \}.
\]

\[
\vec{\Omega}_{msso} = \vec{\Omega}_{MDM} + \vec{\Omega}_{so} = - \left( \frac{e(g-2)}{2mc} E_x (x) + \frac{2}{\hbar} \alpha_{so} (x) \right) \vec{N}_m, \\
\vec{\Omega}_T = \vec{\Omega}_{EDM} + \vec{\Omega}_{Ten} = \frac{2}{\hbar} \left( dE_x (x) + \alpha_{Te} (x) + \alpha_{Tnuc} (x) \right) \vec{N}_T, \\
\vec{\Omega}_w = \frac{2}{\hbar} \alpha_w \vec{n}.
\]

\[
\vec{N}_m = \left[ \vec{n} \times \vec{n}_x \right], \\
\vec{N}_T = \vec{n}_x, \\
\vec{n} = \frac{\vec{k}}{k}
\]
Behavior of the spin rotation caused by magnetic moment and T-reversal violation interactions. Black arrows represent spin rotation about effective magnetic field (about bent axis, direction $\vec{N}_m$), red arrows represent spin rotation about electric field (direction $\vec{N}_T$), purple arrows represent effect of magnetic spin rotation in direction $\vec{N}_m$. (Rotation owing to T-reversal violation and P-violating interactions, is not shown here for simplicity.)
Behavior of the spin rotation caused by magnetic moment, T-reversal violation interactions (including EDM) and P-violation spin rotation about direction $\vec{n}$ and rotation in direction $\vec{n}$ (orange and green arrows). Rotation in direction $\vec{N}_m$ and direction $\vec{N}_T$ is not shown for simplicity. It is obvious that P-odd T-even interactions can imitate EDM rotation.
P and CP violating spin rotation in bent crystals

- Thus baryon spin rotates around three axes: effective magnetic field direction $\vec{N}_m \parallel [\vec{n} \times \vec{E}]$, electric field direction $\vec{N}_T \parallel \vec{E}$ and momentum direction $\vec{n}$.
- Contribution to rotations is determined by several types of interactions.
- Nonelastic processes in crystals result in the addition effects: terms proportional to $\delta$ lead to rotation of the polarization vector in directions of vectors $\vec{N}_m$, $\vec{N}_T$ and $\vec{n}$.
The following estimation for the value $\delta_m$ can be obtained: $\delta_m \equiv 10^8 - 10^9 \text{ sec}^{-1}$. The charm baryon EDM is predicted to be as large as $d \equiv 10^{-17}$. Spin rotation frequency $\Omega_{EDM}$ determined by such charmed baryon EDM is $\Omega_{EDM} \equiv 10^6 - 10^7 \text{ sec}^{-1}$. As a result, the nonelastic processes, which are caused by magnetic moment scattering, can imitate the EDM contribution.
P violating spin rotation in bent crystals

Precession frequency $\Omega_w$ is determined by the real part of the amplitude of baryon weak scattering by an electron (nucleus). This amplitude can be evaluated by Fermi theory for the energies, which are necessary for $W$ and $Z$ bosons production or smaller:

\[
ReB : \quad G_F k = 10^{-5} \frac{1}{m_p^2} k = 10^{-5} \frac{\hbar m\gamma}{m_p c m_p} = 10^{-5} \lambda_{cp} \frac{m\gamma}{m_p}
\]

For different particle trajectories in a bent crystal the value of precession frequency $\Omega_w$ could vary in the range $\Omega_w \in 10^3 - 10^4$ sec$^{-1}$. Therefore, when a particle passes $10 cm$ in a crystal, its spin undergoes additional rotation around momentum direction at angle $\vartheta_p \in 10^{-6} - 10^{-7}$ rad. The effect grows for a heavy baryon as a result of the mechanism similar to that of its EDM growth.
P violating spin rotation in bent crystals

Absorption caused by parity violating weak interaction also contributes to change in spin direction. This rotation is caused by the imaginary part of weak scattering amplitude and is proportional to the difference of total scattering cross-sections $\sigma^{\uparrow\uparrow}$ and $\sigma^{\downarrow\uparrow}$.

$$
\sigma^{\uparrow\uparrow(\downarrow\downarrow)} = \int |f_{c(nuc)} + B_{0w} \pm B_w|^2 \, d\Omega
$$

$$
\sigma^{\uparrow\uparrow} - \sigma^{\downarrow\uparrow} = 2\int[(f_{c(nuc)} + B_{0w})B^* + (f_{c(nuc)} + B_{0w})^* B]d\Omega
$$

When baryon trajectory passes in the area, where collisions with nuclei are important (this occurs in the vicinity of potential barrier for positively charged particles), the value $\delta_w \equiv 10^6 - 10^7 \, \text{sec}^{-1}$. Similar to the real part $ReB$ for the case of heavy baryons the difference in cross-sections grows.
By turning the crystal 180° around the direction of incident baryon momentum One could observe that $P_{\text{odd}}$ spin rotation does not change, while the sign of MDM and $T_{\text{odd}}$ spin rotations does due to change of the electric field direction. Subtracting results of measurements for two opposite crystal positions one could obtain the angle of rotation, which does not depend on $P_{\text{odd}}$ effect.
Separation of the contributions caused by MDM and T-odd spin rotation is possible when comparing experimental results for two initial orientations of polarization vector $\vec{\xi}$. Namely: $\vec{\xi} \parallel \vec{N}_m$ and $\vec{\xi} \parallel \vec{N}_t$, i.e. the initial $\vec{\xi}$ is parallel to the bending axis of the crystal or $\vec{E}$.

In real situation rotating the crystal by $90^\circ$ so that direction of $S_0$ is parallel to $B^*$ can be more convenient.
P and CP violating spin rotation in bent crystals

- Experiment on measuring EDM provides information on contributions of several T noninvariant interactions. Spin precession in bent crystals at LHC gives unique possibility for measurement CP violating and P violating interactions of charm, beauty and strange baryons.

- Effect, which is caused by nonelastic processes, arises – spin rotation to the direction of the bend axis, the direction of the electric field and the direction of the particle momentum. This effect can imitate T noninvariant rotation.
Spin oscillations and possibility of quadrupole moment measurement, birefringence, P and CP noninvariant interactions for $\Omega^{-+}$ hyperons moving in a crystal

V.G. Baryshevsky, A.G. Shechtman, “Spin oscillations and possibility of quadrupole moment measurement for $\Omega^{-}$ hyperons moving in a crystal” NIM B83 (1993)


Interactions contributing to the spin motion of a particle

\( \Omega^- \) hyperon is an example of a particle with \( S > 1/2 \).

Considering evolution of the spin \( (S \geq 1) \) of a particle we should take into account several addition interactions:

- interaction of the magnetic moment with an electric field of the crystal planes and axes;
- interaction of the quadrupole moment with an inhomogeneous electric field;
- interaction due to birefringence effect
- EDM interaction with a crystal electric field and P and CP-odd interactions
What effects arise from the above interactions?

When a particle with $S=3/2$ ($S>1/2$) moves in a straight and bent crystal the effects of spin rotation and vector-to-tensor (tensor-to-vector) polarization conversion appear due to the interactions $W_Q$, $V_{bir}$, $W_{p(cp)}$ along with the spin rotation caused by $(g-2)$ and EDM.
Interaction of the particle quadrupole moment with an inhomogeneous electric field

For a particle with the quadrupole moment $Q$ the energy of spin interaction with an inhomogeneous electric field $E$

$$\hat{W}_Q = \frac{1}{6} Q_{ik} \frac{\partial E_i}{\partial x_k}, \quad \text{where} \quad Q_{ik} = \frac{3Q}{2S(2S-1)} \left( \hat{S}_{ik} - \frac{2}{3} S(S+1) \delta_{ik} \right)$$

is the quadrupole interaction operator of the particle, $Q$ is the quadrupole moment, $S$ is the value of the particle spin, $\hat{S}_{ik} = \hat{S}_i \hat{S}_k + \hat{S}_k \hat{S}_i$ is the spin operator.
The possibility of quadrupole moment measurement of hyperons moving in crystal

The relativistic equation of spin motion in this case has the following form (EDM, P and CP odd interaction terms is not written):

$$\frac{d\hat{I}_i}{dt} = \varepsilon_{ijk} \left( \Omega_j \hat{I}_k + \frac{1}{3} e \varphi_{il} \hat{Q}_{kl} \right)$$

$$\Omega_j$$ is a component of the vector:

$$\vec{\Omega} = \left[ \frac{1}{2} (g-2) \gamma + \frac{\gamma}{(\gamma+1)} \right] v^2 \vec{\Omega}_0, \quad \vec{\Omega}_0 = -\left( \frac{e}{m} \right) \left( \frac{\gamma}{(\gamma^2-1)} \right) (E \times v), \quad \varphi_{ik} = \frac{d^2U}{dx_i dx_k}$$

$$\frac{d\hat{I}_z}{dt} = \Omega_x \hat{I}_y - \Omega_y \hat{I}_x + \frac{\varphi_{xx} eQ}{2I(2I-1)} \hat{I}_{xy}$$

$$\frac{d\hat{I}_y}{dt} = \Omega_z \hat{I}_x - \Omega_x \hat{I}_z + \frac{\varphi_{xx} eQ}{2I(2I-1)} \hat{I}_{zx}$$

$$\frac{d\hat{I}_x}{dt} = \Omega_x \hat{I}_z - \Omega_z \hat{I}_y$$
The possibility of quadrupole moment measurement of hyperons moving in crystal

When a particle with quadrupole moment moves in a straight or bent crystal, not only spin oscillations and rotation appear, but also the transitions between tensor $P_{ik}$ and vector $\vec{P}$ polarizations of the particle.

In experiments for an $\Omega^-$ hyperon beam with the Lorentz factor $\gamma = 100$, intensity $N \approx 10^6$ particles/s and the beam divergence angle $\theta_{div} < 0.4 \text{ mrad}$, it is possible to measure the quadrupole moment $Q$ of an $\Omega^-$ hyperon on the level $10^{-27} \text{ cm}^2$ in a tungsten crystal of length $l = 20 \text{ cm}$. Time for measurement approximately 250 hour.

For the realization of this measuring procedure it is quite sufficient to have a crystal with a mosaic spread $\chi < 0.4 \text{ mrad}$ or a set of crystals arranged with an exactness of not more than $\chi < 0.4 \text{ mrad}$ relative to the chosen family of crystallographic planes.

So such the measurement possibilities of the $\Omega^-$ hyperon quadrupole moment study requires neither a high quality crystal nor a monochromatic hyperon beam. For $\Omega^+$ hyperon bent crystal can be useful.
Angular asymmetry and polarization of the scattered particles

Elastic scattering angular distribution of a particle with spin 1/2
Elastic scattering of a particle with spin 1/2

\[
\hat{F}(\vec{q}) = A(\vec{q}) + B(\vec{q})\vec{\sigma}\vec{N} + B_{0w}(\vec{q}) + B_{w}(\vec{q})\vec{\sigma}\vec{N}_w + B_T\vec{\sigma}\vec{N}_T
\]

\[
\vec{q} = \vec{k}' - \vec{k}, \quad \vec{n} = \frac{\vec{k}}{k}, \quad \vec{N}_w = \frac{\vec{k}' + \vec{k}}{|\vec{k}' + \vec{k}|}, \quad \vec{N} = \frac{[\vec{k} \times \vec{k}']}{\left|\vec{k} \times \vec{k}''\right|}, \quad \vec{N}_T = \frac{\vec{q}}{q}.
\]

\[
\frac{d\sigma}{d\Omega} = tr \rho \hat{F}^+(\vec{q})\hat{F}(\vec{q})
\]

\[
\tilde{\xi} = \frac{tr \rho F^+\vec{\sigma}F}{tr \rho F^+F} = \frac{tr \rho F^+\sigma F}{d\sigma/d\Omega}
\]

\[
\tilde{\xi} = \tilde{\xi}_{so} + \tilde{\xi}_w + \tilde{\xi}_T
\]
Scattering of a particle with spin 1/2 in crystals

\[
\tilde{\xi}_{so} = \left( |A|^2 - |B|^2 \right) \tilde{\xi}_0 + 2\text{Im}(AB^*)[N\tilde{\xi}_0] + 2 |B|^2 \tilde{N}(N\tilde{\xi}_0) + 2\tilde{N}\text{Re}(AB^*) \right) \cdot \left( \frac{d\sigma}{d\Omega} \right)^{-1}
\]

\[
\tilde{\xi}_w = \left( |A|^2 - |B_w|^2 \right) \tilde{\xi}_0 + 2\text{Im}(AB_w^*)[N_w\tilde{\xi}_0] + 2 |B_w|^2 \tilde{N}_w(N_w\tilde{\xi}_0) + 2\tilde{N}_w\text{Re}(AB_w^*) \right) \cdot \left( \frac{d\sigma}{d\Omega} \right)^{-1}
\]

\[
\tilde{\xi}_T = \left( |A|^2 - |B_T|^2 \right) \tilde{\xi}_0 + 2\text{Im}(AB_T^*)[N_T\tilde{\xi}_0] + 2 |B_T|^2 \tilde{N}_T(N_T\tilde{\xi}_0) + 2\tilde{N}_T\text{Re}(AB_T^*) \right) \cdot \left( \frac{d\sigma}{d\Omega} \right)^{-1}
\]

\[
\frac{d\sigma}{d\Omega} = \text{tr} \rho F^+ F = |A|^2 + |B|^2 + |B_w|^2 + |B_T|^2 + 2\text{Re}(AB^*)\tilde{N}\tilde{\xi}_0 + \\
+ 2\text{Re}(AB_w^*)\tilde{N}_w\tilde{\xi}_0 + 2\text{Re}(AB_T^*)\tilde{N}_T\tilde{\xi}_0
\]

Both rotation around \( \tilde{N}, \tilde{N}_w, \tilde{N}_T \) and components in directions of \( \tilde{N}, \tilde{N}_w, \tilde{N}_T \) appear.
Scattering of particles in crystals

Scattering of particles on crystal axes (planes) gives unique possibility for measurement of EDM and constants determining $T_{odd}$, $P_{odd}$ (CP) violating interactions and $P_{odd}$, $T_{even}$ interactions of negative, positive and neutral baryons with electrons and nucleus (nucleons).

V.G. Baryshesky, Electromagnetic dipole moment and time reversal invariance violating interactions of high energy short-lived particles in bent and straight crystals, Phys.Rev Accelerators and Beams 22,081004 (2019)

Spin depolarization of particles in crystals


Baryshevsky V.G. Spin rotation and depolarization of high-energy particles in crystals at Hadron Collider (LHC) and Future Circular Collider (FCC) energies and the possibility to measure the anomalous magnetic moments of short-lived particles, arXiv:1504.06702 [hep-ph].

In amorphous target electric fields possess various values and directions.

The degree of depolarization is small: less than 1% on nuclear absorption length.

Spin depolarization in crystals

Mean-square angle of multiple scattering $\left\langle g_p^2 \right\rangle_{cr}$ becomes much bigger in crystals

$$\zeta_{\parallel} = \zeta_z (0) e^{-\frac{1}{2} \left\langle g_{s1}^2 \right\rangle_{cr} l}$$

$$\left\langle g_{s1}^2 \right\rangle_{cr} = \left( \frac{g - 2}{2} \right)^2 \gamma^2 \left\langle g_{p1}^2 \right\rangle_{cr}$$

Mean-square angle of multiple scattering per unit length

Does not depend on $\gamma$

Spin depolarization in crystals

Depolarization in crystal is increasing up to dozens percent on 1 cm.

\[ |g - 2| = \sqrt{\frac{8}{\gamma^2} \left\langle \mathcal{J}_p^2 (l) \right\rangle_{ch} \ln \frac{\left\langle \zeta_z (0) \right\rangle}{\left\langle \zeta_z (l) \right\rangle}} \]

Enables measurement of magnetic moment of negative beauty baryons and neutral baryons.


Baryshevsky V.G. Spin rotation and depolarization of high-energy particles in crystals at Hadron Collider (LHC) and Future Circular Collider (FCC) energies and the possibility to measure the anomalous magnetic moments of short-lived particles, arXiv:1504.06702 [hep-ph]

Conclusion

High-energy particles channeling and scattering on crystal axes (planes) gives unique possibility for measurement of MDM, EDM and constants determining $T_{odd}$, $P_{odd}$ (CP) violating interactions and $P_{odd}$, $T_{even}$ interactions of negative, positive and neutral baryons with electrons and nucleus (nucleons).


V.G. Baryshesky, Electromagnetic dipole moment and time reversal invariance violating interactions of high energy short-lived particles in bent and straight crystals,

V.G. Baryshesky, Electromagnetic dipole moments and time reversal violating interactions for high energy charged baryons in bent crystals,
Thank you for your attention