Giuseppe Dattoli

Enea Frascati Dip. Fusione

X-Ray Compton Backscattering Sources

Outline

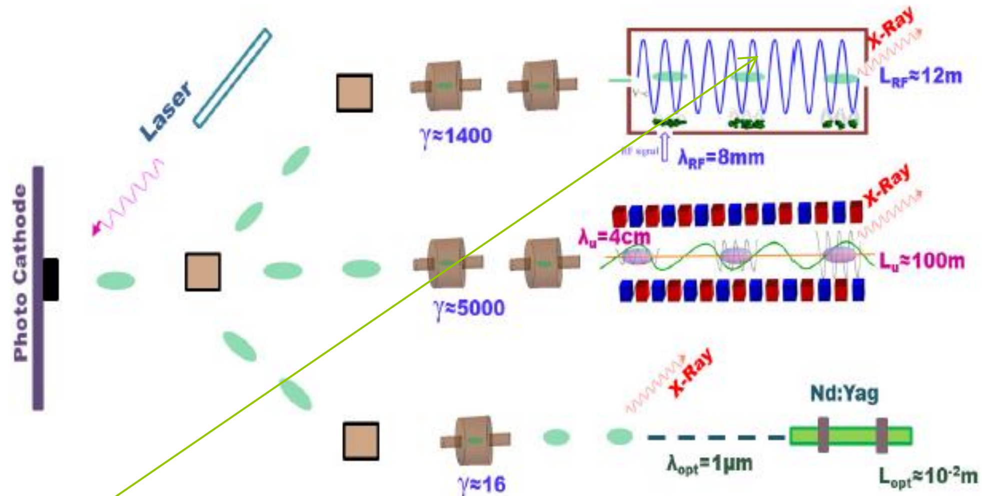
- Pre-Hystory and Hystory of CBS and of associated sources
- 0-th order (kynematics) description of the process
- CBS and undulator (wiggler) radiation
- Kinematics and Thomson cross section
- Design elements, definition of brightness…
- mention of non-linear effects, Harmonic et al.
- Some idea for next developments…

Motivations

- Interest In
- Wave* undulators
- Compact Synchrotron light sources
- Compact FEL devices

- * Wave undulator=Laser or microwave field

…*Reduction of costs, size….*

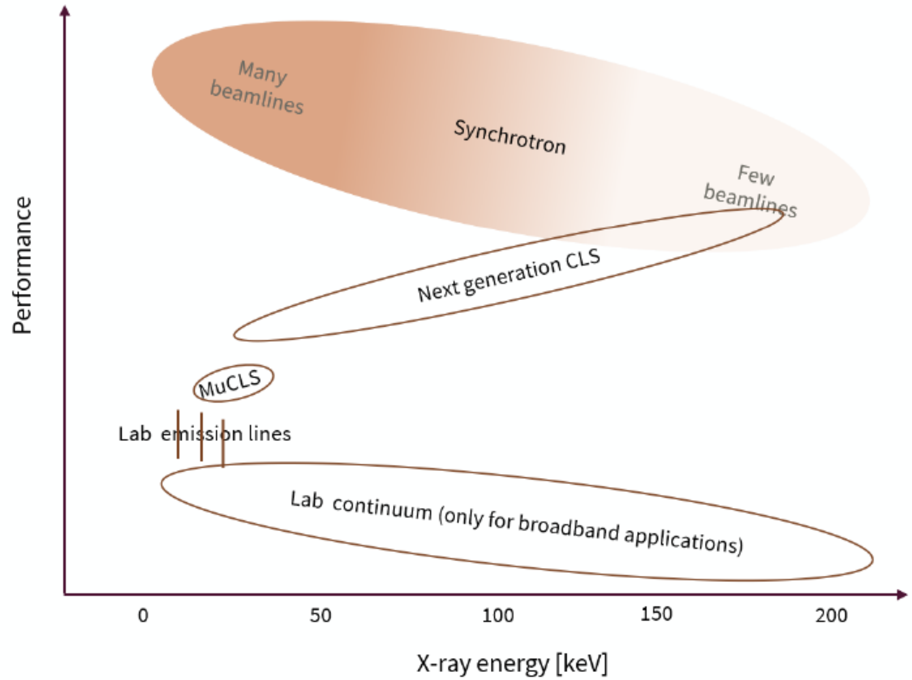


Radio-Frequency Undulators, Cyclotron Auto Resonance Maser and Free Electron Lasers

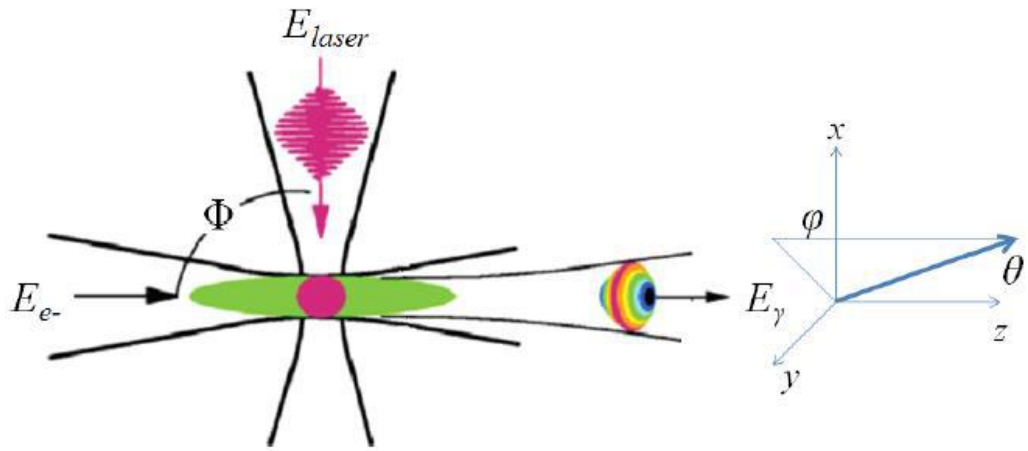
Emanuele Di Palma *,†, Silvio Ceccuzzi †, Gian Luca Ravera †, Elio Sabia † and Ivan Spassovsky †
and Giuseppe Dattoli †

???

- *Con's : Few light lines, powerful laser/microwaves fields…*



Compton Scattering (CS) e Inverse (Back) Compton Scattering (CBS)



First Suggestions

- R. H. Milburn, Phys. Rev. Lett. 10 (1963) 75.
- F. R. Arutyunyan and V. A. Tumanian, Phys. Lett. 4 (1963) 176
- Varfolomeyev Said «do not publish it!…»

First Experiment

HIGH-ENERGY PHOTONS FROM COMPTON SCATTERING OF LIGHT ON 6.0 GEV ELECTRONS

C. Bemporad, R.H. Milburn, and N. Tanaka

Tufts University

and

M. Fotino

Cambridge Electron Accelerator

January 30, 1965

3/10/65

LADON-INFN Frascati (seventies-eighties XX-th)

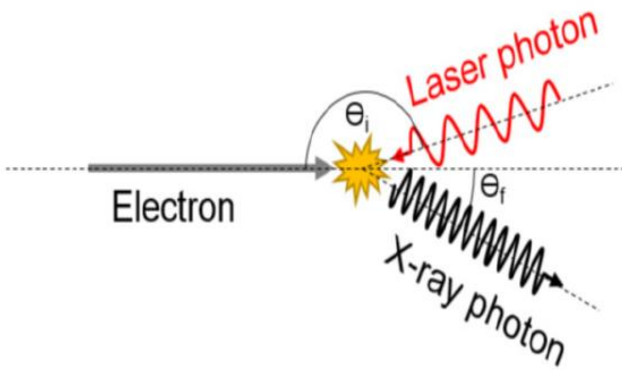
- Experimental results for the Ladon photon beam at Frascati (1981)
- L. Federici, G. Giordano, G. Matone, G. Pasquariello, and P. Picozza
- INFN-Laboratori Nazionali di Frascati
- R. Caloi, L. Casano, M. P. De Pascale, M. Mattioli, E. Poldi, C. Schaerf, M. Vanni, P. Pelfer, D. Prosperi, S. Frullani and B. Girolami
- Collaboration
- INFN-Rome & Napoli University-Istituto Superiore di Sanità

Compton and Thomson

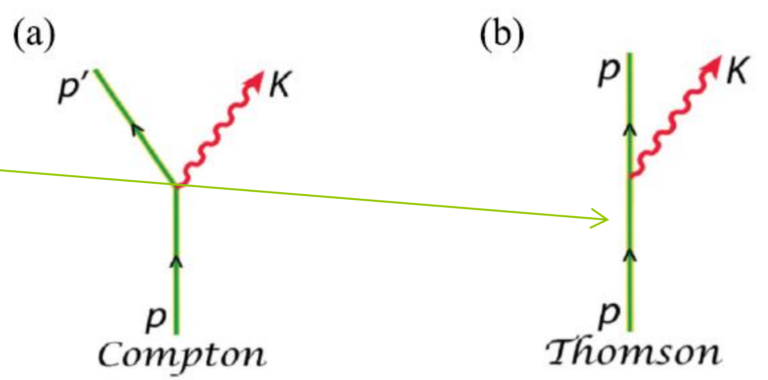
$$E_s = \frac{(1-\beta\cos(\vartheta_i))E_l}{(1-\beta\cos(\vartheta_f)) + \frac{E_l}{E_e}\left[1-\cos(\vartheta_f-\vartheta_i)\right]}$$

$$E_{l,s} = \hbar\omega_{l,s}, E_e = m_e\gamma c^2$$

$$\beta \cong 1-\frac{1}{2\gamma^2}, \cos(\vartheta_f) \cong 1-\frac{\vartheta_f^2}{2}, \vartheta_i \cong \pi$$

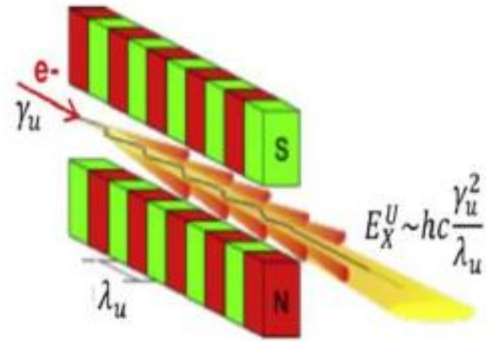


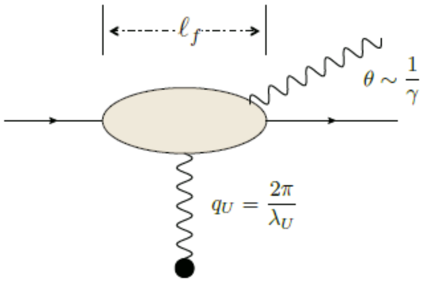
$$\frac{E_l}{E_e} = \frac{\lambda_e}{\gamma\lambda_l} << 1$$

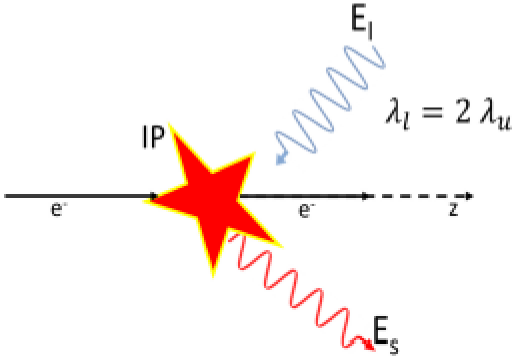


$$\omega_s \cong \frac{4\gamma^2\omega_l}{1+\gamma^2\vartheta^2}, \lambda_s = \frac{\lambda_l}{4\gamma^2}\left(1+\gamma^2\vartheta^2\right)$$

Undulators and backscattering





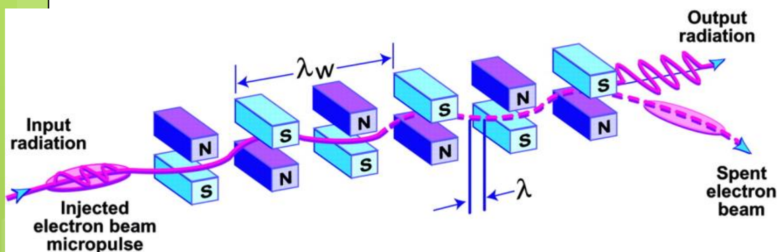


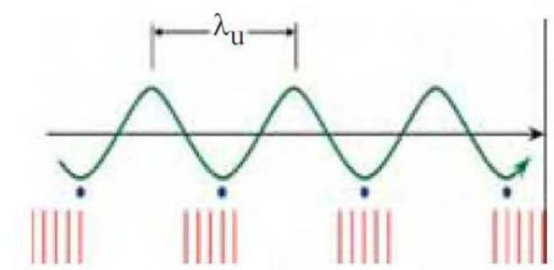
Fermi-Weiszacker Williams Approximation

- Dattoli & Renieri, Theoretical and Experimental Aspects of Free Electron Laser, North Holland (1985)
- Dattoli & Nguyen Progress In Particle and Nuclear Physics (2018)
- E. Di Palma, G. Dattoli, S. Sabchevsky, Comments on the Physics of microwaves Undulators, MDPI (2022)

- (Inverse FWW-A and link with non zero mass photon fields)

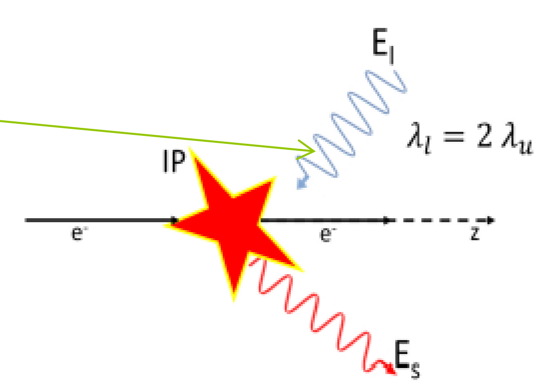
F-W-W-U





$$\delta \sim (c - v_z)\frac{\lambda_u}{c}\qquad \delta \cong (1-\beta_z)\lambda_u \cong \frac{\lambda_u}{2\gamma^2}\qquad \delta = n\lambda_r$$

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\qquad \lambda^* = 2\lambda_u$$



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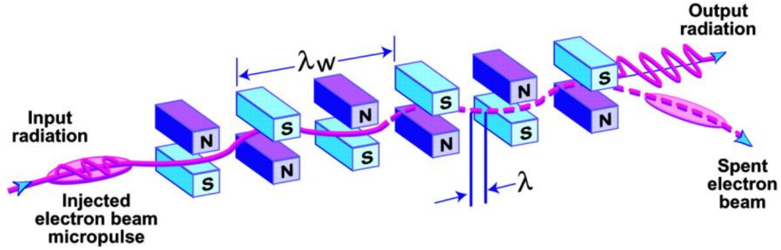


$$v_x \cong \frac{cK}{\sqrt{2}\gamma},\quad v_z \cong c[1-\frac{1}{2\gamma*^2}],\quad \gamma* = \frac{\gamma}{\sqrt{1+\frac{K^2}{2}}}, K = \frac{eB_0\lambda_u}{2\pi m_0 c^2}$$

$$\lambda_r = \frac{\lambda_u}{2\gamma*^2} = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

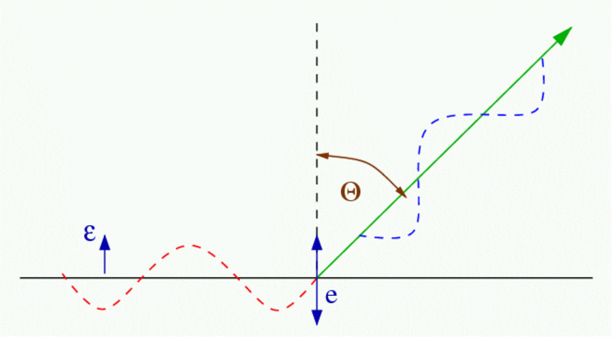
$$K_l \cong 8.5\cdot 10^{-15}\lambda_l[nm]\sqrt{I_l\left[\frac{W}{m^2}\right]}.$$

F-W-W-U

- a) $\lambda^{*} \to 2\lambda_u,$
- b) $K \to$ $K_l \cong 8.5 \cdot 10^{-15} \lambda_l [nm] \sqrt{I_l \left[\frac{W}{m^2}\right]}.$
- c) $\lambda_s = \frac{\lambda_l}{4\gamma^2}\left(1 + \frac{K_l^2}{2} + \gamma^2 \vartheta^2\right)$

Thomson scattering cross section

- Charged particle acceleration
- Cause :
- electric field component of the incident wave
- Direction of motion:
- that of the oscillatingel electric f

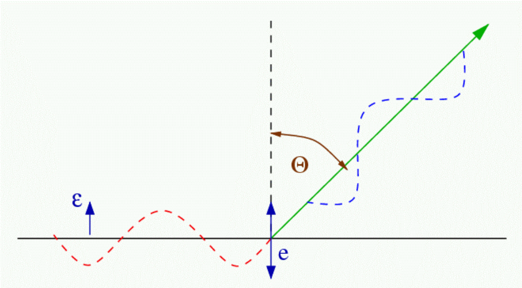


- Consequences:
- electromagnetic dipole radiation

$$\vec{F} = e\hat{\varepsilon}E_o \sin\omega_o t$$

$$\vec{d} = e\vec{r} \quad \rightarrow \quad \ddot{\vec{d}} = e\ddot{\vec{r}} = \frac{e^2 E_o}{m}\hat{\varepsilon}\sin\omega_o t$$

arized wave



$$\vec{d} = -\left(\frac{e^2 E_o}{m\omega_o^2}\right)\hat{\varepsilon}\sin\omega_o t = \vec{a}(t)$$

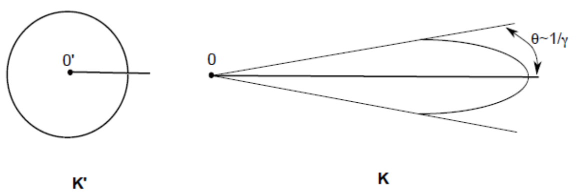
- Larmor Formula

$$\left\langle\frac{dW}{d\Omega}\right\rangle = \left\langle\frac{2}{3}\frac{\ddot{\vec{d}}^2}{c^3}\right\rangle = \left\langle\frac{e^4 E_o^2}{8\pi m^2 c^3}\sin^2\Theta\right\rangle = \langle S\rangle\frac{d\sigma}{d\Omega}$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m^2c^4}\sin^2\Theta = r_o^2\sin^2\Theta$$

Inverse Thomson backscattering

- Transtion from Rest to lab frame: Beaming effect



$\theta \sim 1/\gamma.$

Radiation in the lab frame is «beamed» to

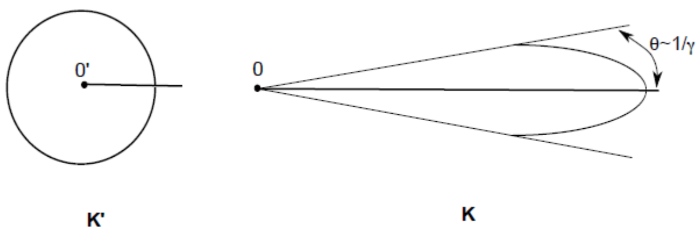
$$\vartheta' = \frac{\pi}{2}$$

$$\tan\theta = \frac{c}{\gamma v} \quad \text{and} \quad \cos\theta = \frac{v}{c}$$

$$\sin\theta = \sqrt{1 - \cos\theta^2} = \frac{1}{\gamma}$$

From K' to K frames

$$\epsilon_f = \frac{\epsilon_i}{1 + \frac{\epsilon_i}{m_e c^2}(1 - \cos\theta)}$$



$$\epsilon_i = D\epsilon'_i \quad \leftrightarrow \quad \epsilon'_i = \epsilon_i \gamma(1 - \beta\cos\theta_i)$$

$$\epsilon_f = \frac{\epsilon'_f}{\gamma(1 - \beta\cos\theta_f)} = \epsilon'_f \gamma(1 + \beta\cos\theta'_f)$$

$$D = 1/(\gamma[1 - \beta\cos\theta])$$

$$\cos\theta'_{i,f} = \frac{\cos\theta_{i,f} - \beta}{1 - \beta\cos\theta_{i,f}}$$

$$\begin{aligned}\epsilon_f &= \gamma^2\epsilon_i(1 - \beta\cos\theta_i)(1 + \beta\cos\theta'_f) = \gamma^2\epsilon_i(1 - \beta\cos\theta_i)\left(1 + \beta\frac{\cos\theta_f - \beta}{1 - \beta\cos\theta_f}\right) \\ &= \gamma^2\epsilon_i\frac{(1 - \beta\cos\theta_i)}{(1 - \beta\cos\theta_f)}(1 - \beta^2) = \epsilon_i\frac{(1 - \beta\cos\theta_i)}{(1 - \beta\cos\theta_f)}\end{aligned}$$

$$\frac{\epsilon_{f,max}}{\epsilon_i} = \frac{(1 + \beta)}{(1 - \beta)} = \gamma^2(1 + \beta)^2 \simeq 4\gamma^2 .$$

Non Linear-Frequency shift

- H. R. Reiss, (1962).
- A. I. Nikishov and V. I. Ritus, (1963),
- A. I. Nikishov and V. I. Ritus, (1964)
- A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. 47, 1130 (1964)
- I. I. Goldman, Phys. Lett. (1964).
- N. B. Narozhnyi, A. Nikishov, and V. Ritus, Zh. Eksp. Teor. Fiz. 47, 930 (1964).
- L. S. Brown and T. W. B. Kibble, (1964).
- T. W. B. Kibble, (1965).

....

$$\lambda_s = \frac{\lambda_l}{4\gamma^2}\left(1 + \frac{K_l^2}{2} + \gamma^2 \vartheta^2\right), \omega_s = \frac{4\gamma^2 \omega_l}{1 + \frac{K_l^2}{2} + \gamma^2 \vartheta^2}$$

$$K_l \equiv a_0 \equiv \eta = \frac{eE_l \lambda_l}{2\pi m_e c^2} = \frac{e\sqrt{A_\mu A^\mu}}{m_e c^2}$$

$$m^* \equiv m_e \sqrt{1 + K_l^2}$$

- ....

$$\frac{eE_l ƛ_l}{m_e c^2}, ƛ_l = \frac{\lambda_l}{2\pi}$$

Work done by E_l *on the electron in a reduced wave-length normalized to the electron mass energy .*

It is a classical quantity usually associated with

$$\frac{eE_l ƛ_e}{m_e c^2} = \frac{E_l}{E_{ss}}$$

$$E_{ss} = \frac{m_e c^2}{e}\frac{1}{ƛ_e} \cong 1.323 \cdot 10^{18}\frac{V}{m}$$

Sauter (1931)-Schwinger (1951) critical field

A Few Wise calculations to get the working point of the deviced

- Tools: Thomson Cross Section and basic arithmetic

$$\dot{N}_x \cong \sigma_{Th} \cdot L_0,$$

$$\sigma_{Th} = \frac{8}{3}\pi r_0^2, \quad r_0 = \frac{e^2}{4\pi\varepsilon_0 mc^2} \equiv \text{electron classical radius},$$

$$L_0 \cong \frac{f}{2\pi}\frac{N_l N_e}{\Sigma_{e,l}},$$

Collision Rate

$$\Sigma_{e,l} = \sigma_e^2 + \sigma_l^2,$$

$$\dot{N}_x \cong kf\frac{E_l\overline{Q}_e\lambda_l}{(1+d)},$$

$$k = \frac{8}{3}\pi\frac{r_0}{\hbar I_0},$$

$$\overline{Q}_e = \frac{Q_e}{2\pi\sigma_e^2} \equiv \text{electron beam charge density},$$

$$2\pi\sigma_e^2 \equiv \text{ e-beam transverse area},$$

$$I_0 = \frac{ec}{r_0} \cong 1.7\cdot 10^4\text{A} \equiv \text{ Alfvèn current},$$

$$d = \frac{\sigma_l^2}{\sigma_e^2}.$$

$$1.317\cdot 10^{16}$$

$$\dot{N}_x \cong 1.317\cdot 10^{16}\, f\,\frac{E_l\overline{Q}_e\lambda_l}{(1+d)}$$

Ideal conditions:

Large Charge, Large Electric field, Large collision rate, Small transverse sections

$\dot{N}_x\,[s^{-1}]$	$1.656 \cdot 10^{14}$
$f\,[Hz]$	10^{8}
$E_l\,[J]$	10^{-2}
$\lambda_l\,[m]$	10^{-6}
γ	50
$\beta_T\,[m^{-1}]$	$5 \cdot 10^{-3}$
$\varepsilon_n\,[mm \cdot mmrad]$	0.1

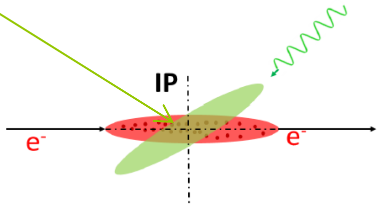


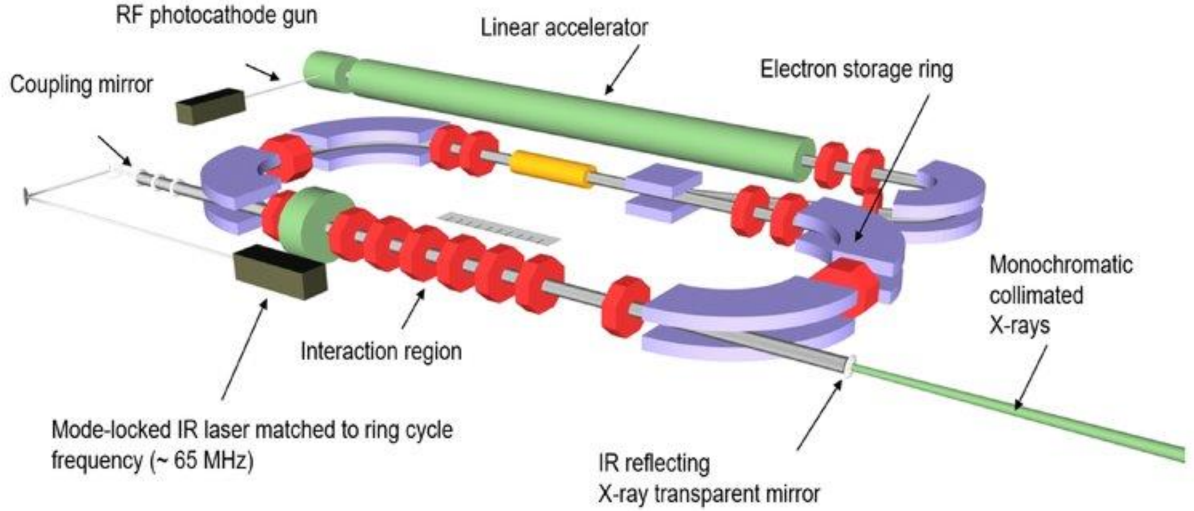
Figure 7. Geometry of the electron (red) and photon (green) bunches interaction at the IP.

$$\overline{Q}_e \cong \frac{2}{k f\, E_l\, \lambda_l} \dot{N}_x .$$

$$\overline{Q}_e\,[C/m^2] \cong 0.025, \quad \text{X} \quad \sigma_e^2 = \beta_T \frac{\varepsilon_n}{\gamma}, \longrightarrow Q_e = 1.579\ pC,$$

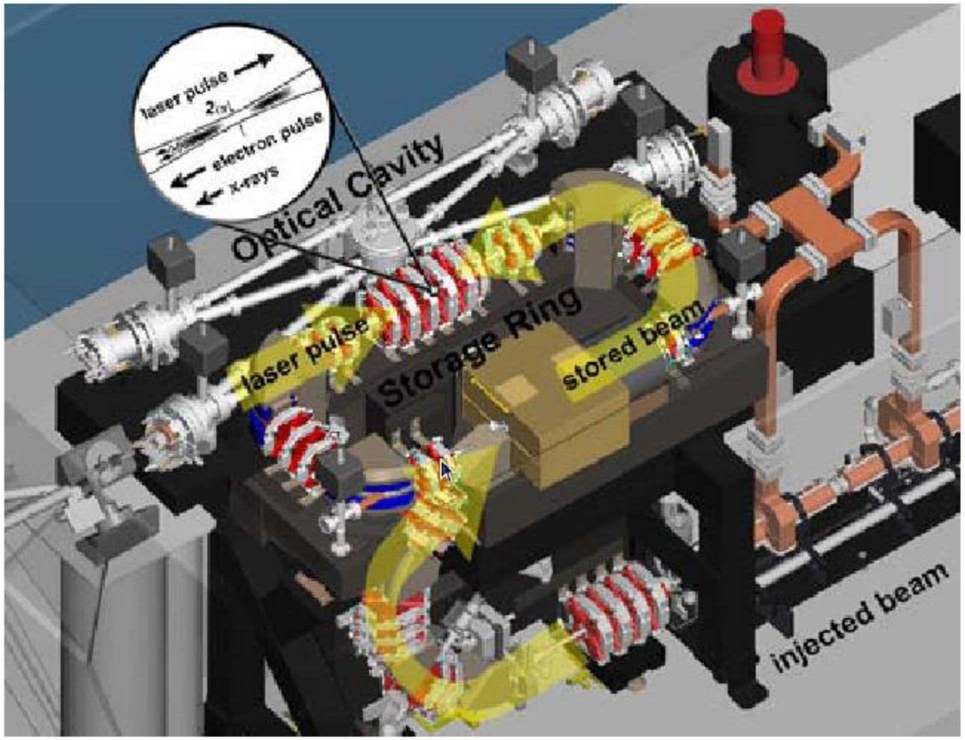
Compact BCS X-ray source

Simplified schematic of the Lyncean Compact Light Source. Electron bunches are generated in the RF photocathode gun, accelerated in the linear accelerator and injected into the electron storage ring. A mode-locked IR laser resonantly drives an optical cavity, shown here schematically as a two-mirror cavity. Electrons and laser pulses collide in the interaction region and generate a collimated, quasi-monochromatic X-ray beam.



Schematic drawing of the Lyncean compact light source illustrating the laser–electron pulse interaction. The storage ring has a footprint of approximately 1m by 2m

- .

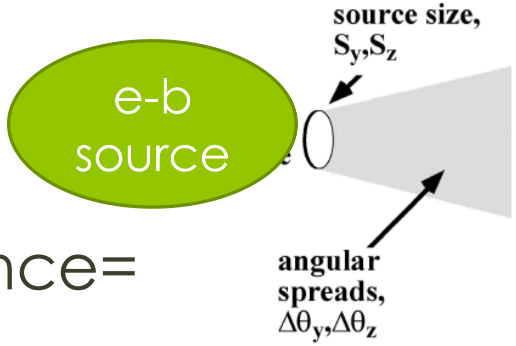


Collision Rate

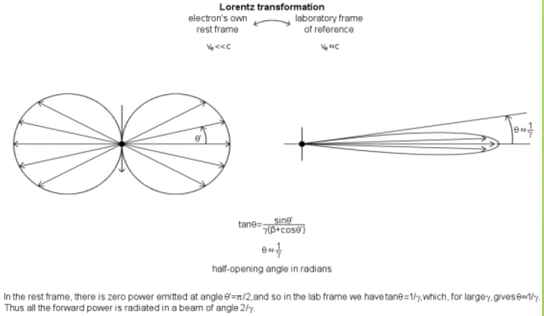
- *Order of many* MHz (LCLS= Lyncean Compact Light Source)
- E-Beam recirculation and laser Cavity Stacking technology

...

- Emittance=



$$\varepsilon_n = S \cdot \Delta \vartheta$$



$$Q_e \cong \frac{2}{kf\,E_l\,\lambda_l} 2\pi \frac{\beta_T \varepsilon_n}{\gamma} \dot{N}_x$$

$$\hat{I}_e = \frac{Q_e}{\sqrt{2\pi}\sigma_\tau}$$

$$\varepsilon = \frac{\varepsilon_n}{\gamma}$$

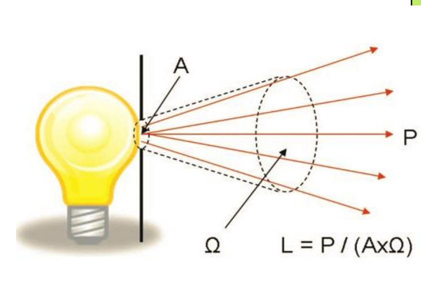
$\sigma_\tau \equiv$ e- bunch duration,

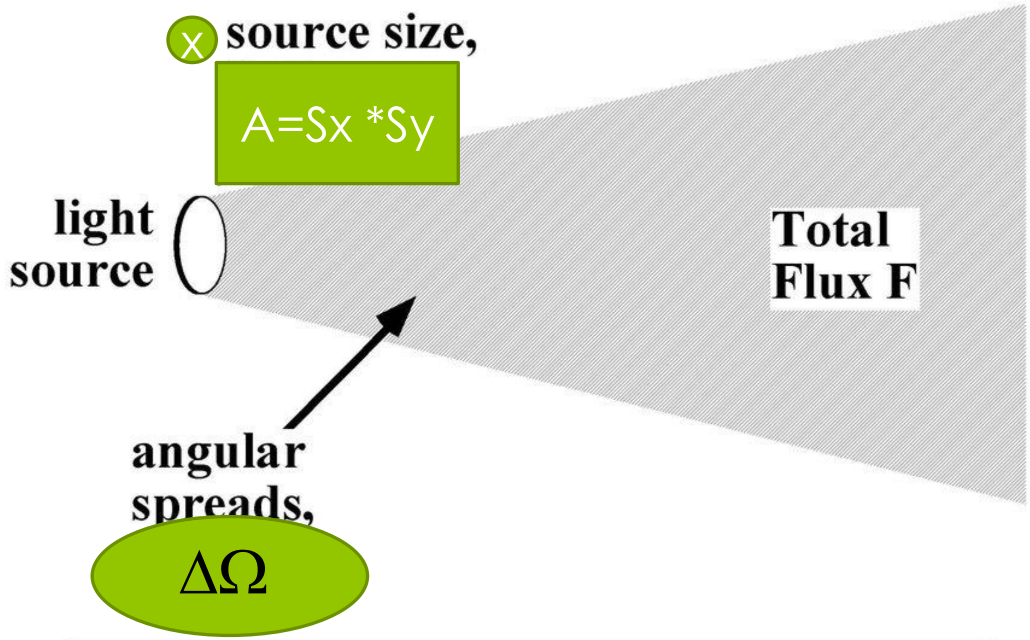
$$\hat{I}_e \cong \frac{2\sqrt{2\pi}}{kf\sigma_\tau E_l \lambda_l} \frac{\beta_T \varepsilon_n}{\gamma} \dot{N}_x$$

$$\hat{P}_e[W] \cong \frac{1.944 \cdot 10^{-10}}{f\sigma_\tau E_l \lambda_l} \beta_T \varepsilon_n \dot{N}_x.$$

Brightness

- Of a «light Source»





$$\text{Brightness} - \text{constant} \times \frac{F}{A^{*}\,\Delta\Omega}$$

Spectral Brigtness
Brightness per frequency bandwidth

- .

$$SB = \frac{\Delta B}{\Delta \omega}$$

For the photons emitted per 0.1% bandwidth, we define

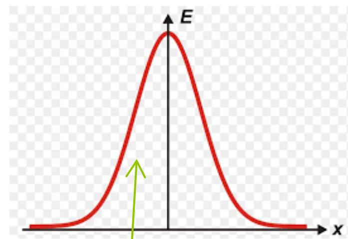
$$\dot{N}_{x,0.1\%} = 1.5 \cdot 10^{-3} \dot{N}_x.$$

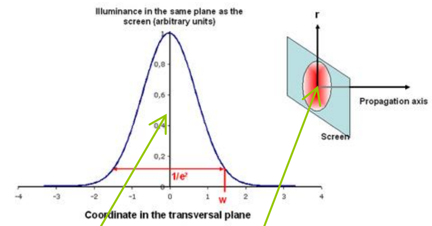
For a non diffracted beam the brightness is defined as

$$B[s^{-1}/mm \cdot mrad \cdot mm \cdot mrad/10^{-3} bw] = \frac{\dot{N}_{x,0.1\%}}{4\pi^2 \varepsilon_{n,x} \varepsilon_{n,y}} . \gamma^2$$

So far we have forgotten the e-beam qualities (namely the relevant brightness)

- Energy Spread
- Angular divergence
- Transverse dimensions





$$\hbar\omega_s = \frac{4\gamma^2\hbar\omega_l}{1+\frac{K_l^2}{2}+(\gamma\vartheta)^2}.$$

$$\frac{\delta\omega}{\omega} = \left(\frac{\delta\omega}{\omega}\right)_\varepsilon + \left(\frac{\delta\omega}{\omega}\right)_{K_l} + \left(\frac{\delta\omega}{\omega}\right)_\vartheta$$

$$\varepsilon = \frac{\delta\gamma}{\gamma} \equiv \text{relative energy deviation}$$

$$S(\nu) = \left[\frac{\sin\left(\frac{\nu}{2}\right)}{\left(\frac{\nu}{2}\right)}\right]^2 = 2Re\left(\int_0^1 (1-t)e^{-i\nu t}dt\right),$$

$$\nu = 2\pi\,\mathrm{N}_l \frac{\omega_x - \omega}{\omega_x},$$

…

- *Average on the beam distribution*

$$\langle S(\nu)\rangle = \int_D S(\nu+\delta\nu) f(x,x';y,y';\varepsilon,t) d^5x,$$

$$\delta\nu = 2\pi\, \mathrm{N}_l \frac{\delta\omega}{\omega},$$

$$\langle S(\nu)\rangle = 2Re \int_0^{\tau} (1-t) \frac{e^{-i\nu t - \frac{1}{2}(\pi\mu_\varepsilon t)^2}}{\sqrt{R_x(t)R_y(t)}} dt,$$

$$R_\eta(t) = (1+\alpha_\eta^2)(1-\imath\pi\mu_\eta t)(1-\imath\pi\mu_\eta t) - \alpha_\eta^2,$$

…

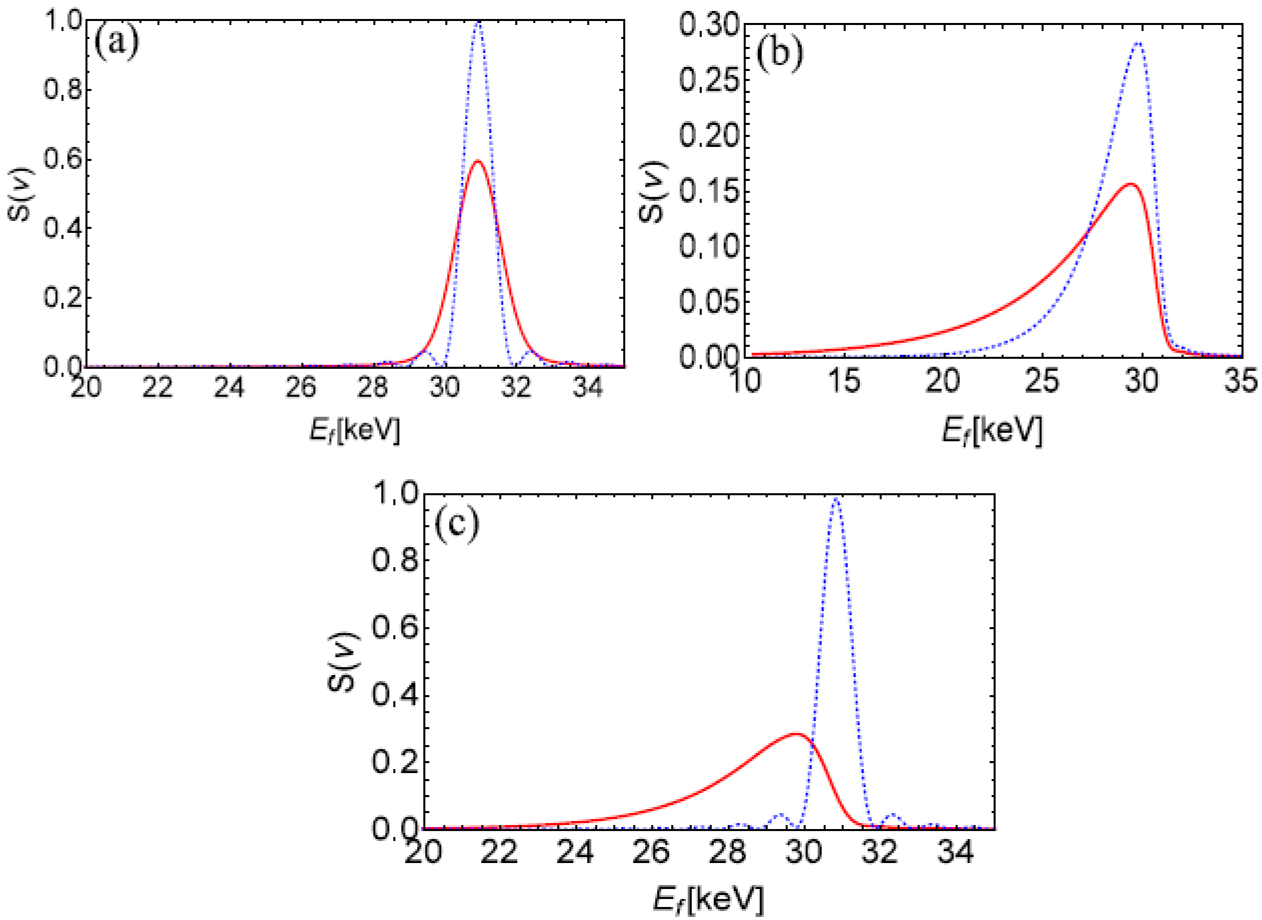
- *Inhomogeneous Broadening Parameters*

$$\mu_\varepsilon = 4N_l\sigma_\varepsilon,$$

$$\mu_{\eta'} = \frac{4N_l\gamma^2\varepsilon_\eta}{\left(1+\frac{K^2}{2}\right)\beta_\eta} \quad \mu_\eta = \frac{4\,N_l\gamma^2\varepsilon_\eta}{\left(1+\frac{K^2}{2}\right)\gamma_\eta}k_\beta^2,$$

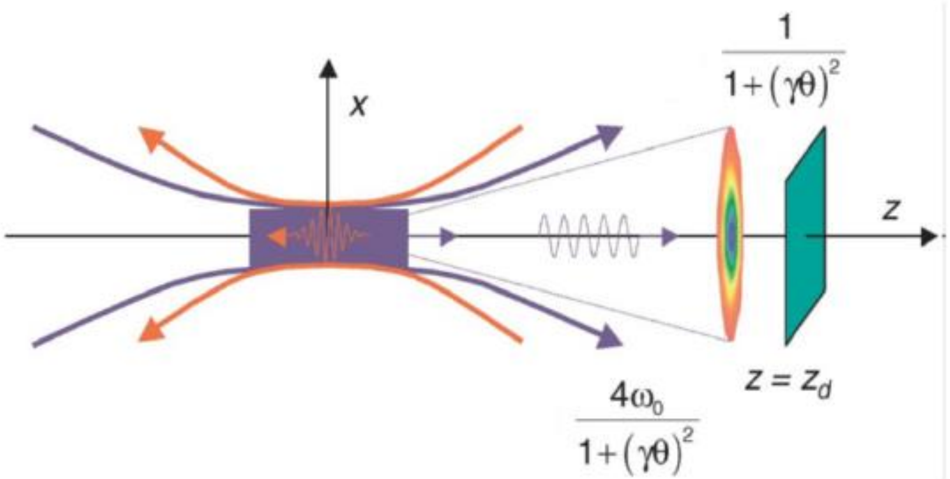
$$\varepsilon_\eta = \frac{\varepsilon_n}{\gamma}\ \eta = x, y\ \ ,$$

$$k_\beta = \frac{\pi K_l}{\gamma\lambda_l} \equiv \text{betatron motion wave number.}$$


(a)
S(ν)
E_f[keV]
(b)
S(ν)
E_f[keV]
(c)
S(ν)
E_f[keV]

Well Educated Computations are needed…but

- …



$$\frac{d^4N_x(x_\nu)}{d^4x_\nu} = \frac{\sigma}{ec} j_\mu(x_\nu)\Phi^\mu(x_\nu) = \frac{\sigma c}{\gamma\omega} n_e(x_\nu)n_\lambda(x_\nu)u_\mu k^\mu.$$

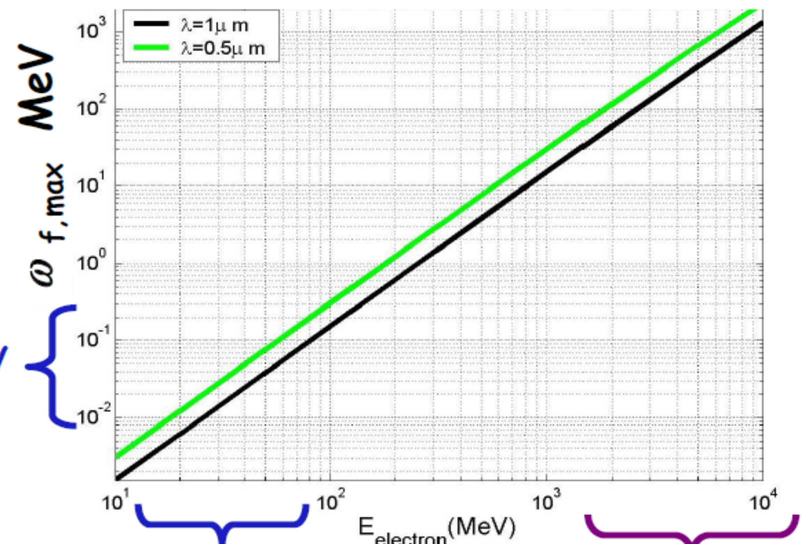
$$\frac{d^4N_x(x, y, z, t)}{dxdydzcdt} = \sigma n_e(x, y, z, t)n_\lambda(x, y, z, t)\left(1 - \beta \cdot \frac{c\mathbf{k}}{\omega}\right).$$

Alessandro Variola 2015

γ ~100 MeV

γ ~ 1MeV

·X ray ~10-100keV



· Elec.~20-100MeV

· Elec. 1GeV

·Elec.~100-500MeV

·*Low energy applications*
·Medical:
radiography
&radiotherapy
·Museology
·Material science
·crystallography

·*Nuclear fluorescence applications*
·Nuclear survey
·Nuclear waste management

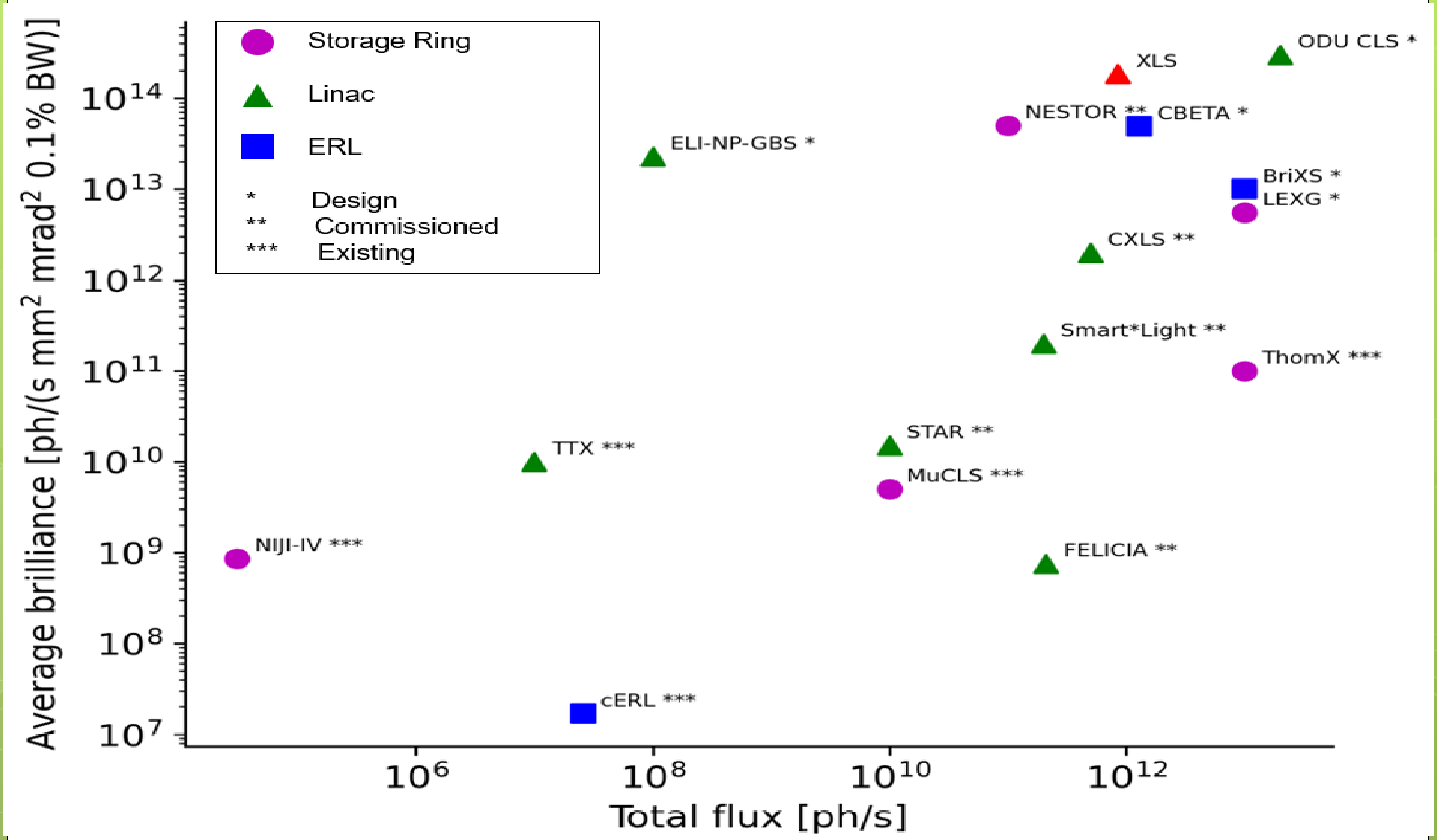
High energy applications
- Compton polarimeter ➔LEP energy measurement
- Laser wire
- gg collider

·Polarised positron source

Existing and planned facilities

Photonics **2022**, 9, 308.

https://doi.org/10.3390/photonics9050308

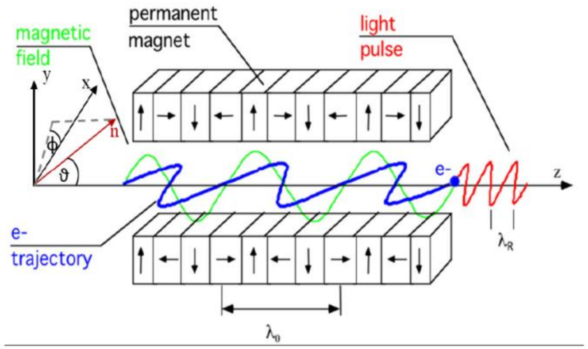


Suggested Design Strategy

- A) Use simple Scaling Formulae
- B) Fix The working Point
- C) Use Massive Computation to refine the design details

Lienard-Wiechert Potentials

- Lienard Wiechert integral



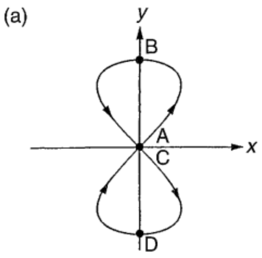
$$\frac{\partial^2 I}{\partial\omega\,\partial\Omega} \propto \left| \int_{-\infty}^{+\infty} \left[\vec{n} \times (\vec{n} \times \vec{\beta})\right] \exp\left[i\,\omega\,(t - \frac{\vec{n}\cdot\vec{r}}{c})\right] dt \right|^2,$$

$$\frac{\partial^2 I}{\partial\omega\,\partial\Omega} \propto \sum_{m=-\infty}^{\infty} \left[\vartheta\cos(\phi) J_m(A\omega,\, B\omega) + \frac{K}{2\gamma}\left[J_{m-1}(A\omega, B\omega) + J_{m+1}(A\omega, B\omega)\right]\right] S_n\left(\frac{\omega}{\omega_1}\right),$$

$$S_n(x) = \frac{2N\pi}{\omega_u}\,\mathrm{sinc}[N\,\pi\,(x-n)]\,e^{i\,N\pi\,(x-n)},$$

On-axis emission (larger Kl values Strong Field Regime)

- *Odd harmonics (non Dipolar emission)*



$$\frac{d^2 I_{x,m}}{d\omega d\Omega} = \frac{16\,\alpha}{\pi} N_e (\gamma\, \mathrm{N}_l)^2 \xi_l^2 m^2 f_{b,m}^2(\xi_l) \langle S_m(\nu_m) \rangle,$$

$$f_{b,m}(\xi_l) = (-1)^{\frac{m-1}{2}} \left[J_{\frac{m-1}{2}}(m\xi_l) - J_{\frac{m+1}{2}}(m\xi_l) \right],$$

$$\xi_l = \frac{1}{4} \frac{K_l^2}{1 + \frac{K_l^2}{2}}.$$

Plans For Future work

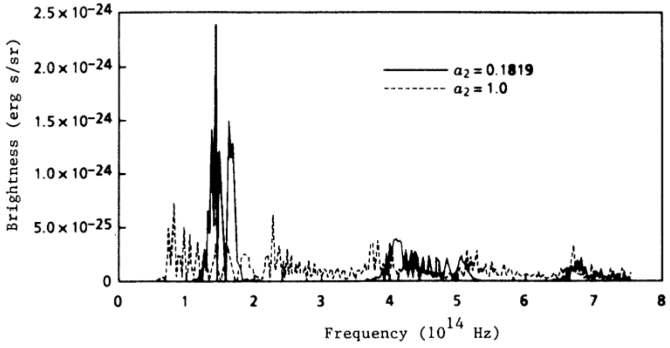
- Exhotic configuration:
- BCS of two photon beams with different wave-lengths
- Bi-Harmonic Laser field

Two Frequency Undulators

- F. Ciocci et al. Phys Rev. 47 A (1993)

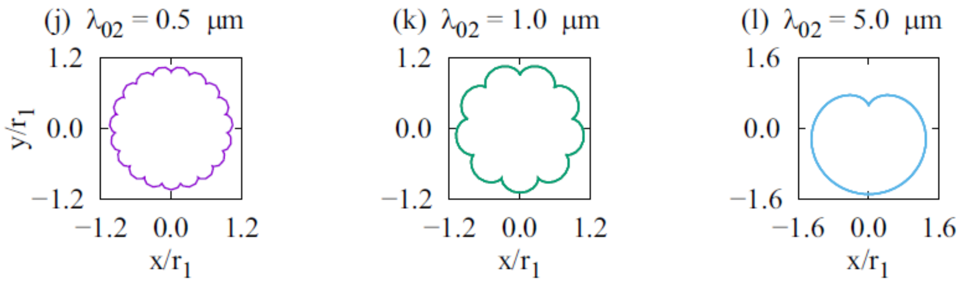
$$\mathbf{B} \equiv B_0(0, b(z), 0) \ ,$$

$$b(z) = a_1 \sin(k_u^{(1)} z) + a_2 \sin(k_u^{(2)} z) \ , \quad k_u^{(\alpha)} = \frac{2\pi}{\lambda_u^{(\alpha)}} \ .$$



Two laser CBS Scattering

- Gamma-ray ***vortices*** emitted from ***nonlinear inverse Thomson scattering*** of a two-wavelength laser beam
- Yoshitaka Taira and Masahiro Katoh
- Phys. Rev. A **98**, 052130 (2019)

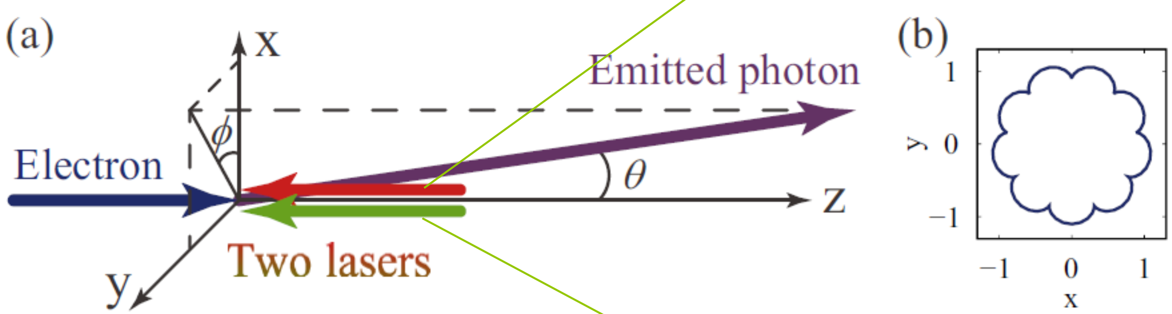


$$\lambda_{l,1} = 10\mu m$$

• • •

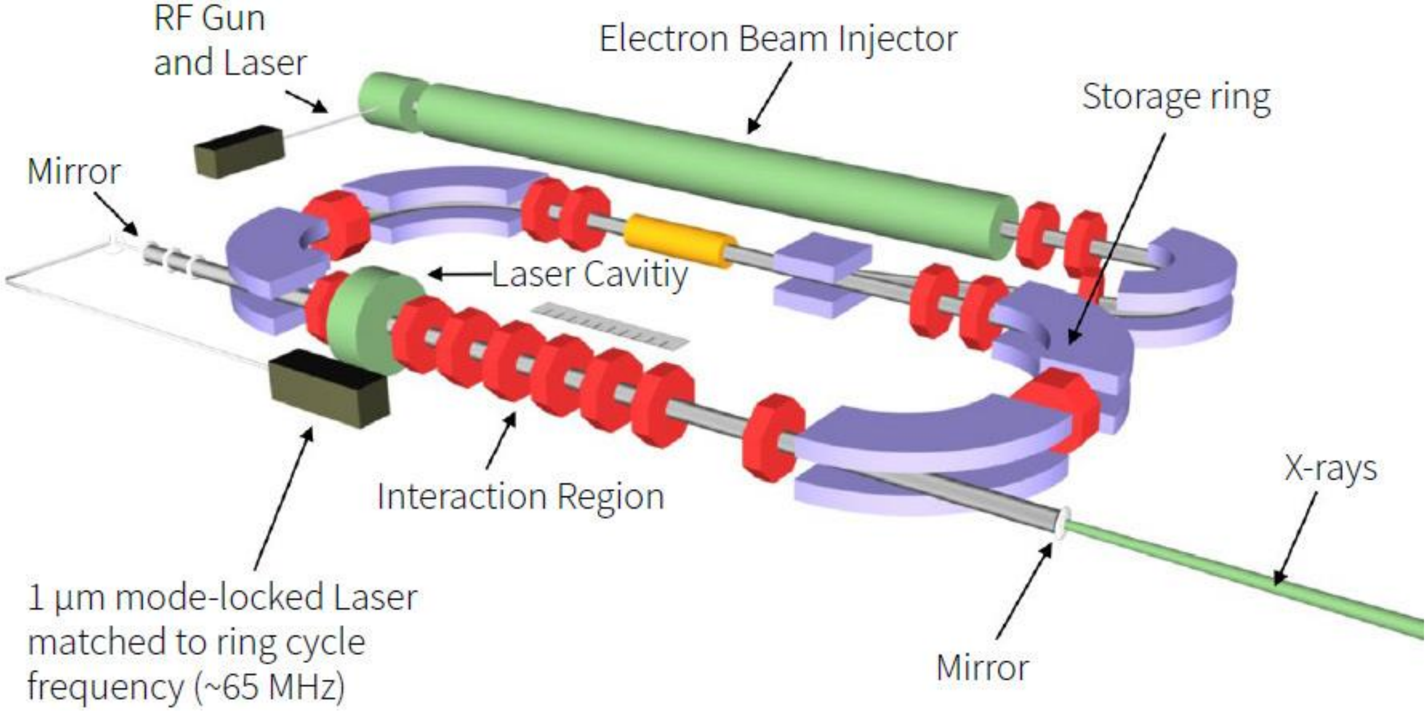
- ***Two Laser Field CBS***

$$\lambda_l \to \parallel$$



$$\frac{\lambda_l}{2} \to \perp$$

Lyncean Technology



What have we done?
A lot of proposals

- IRIDE (2013)

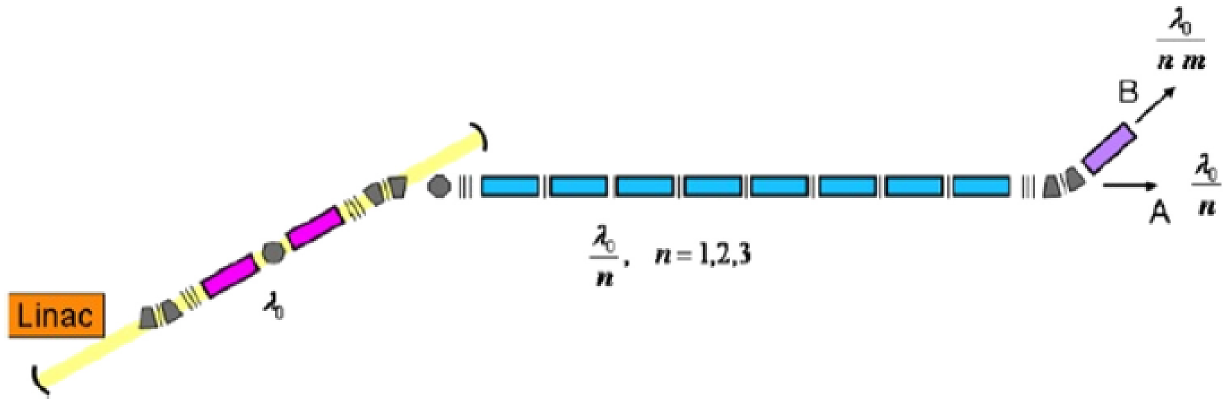


Fig. 3. Undulator chain at the L2 exit, the first component is an oscillator acting also as a micro-buncher driving the downstream SASE FEL.

…g-g scattering

A. Torre, G. Dattoli, I. Spassovsky, V. Surrenti. JOSA-B (2013)

Double FEL scheme

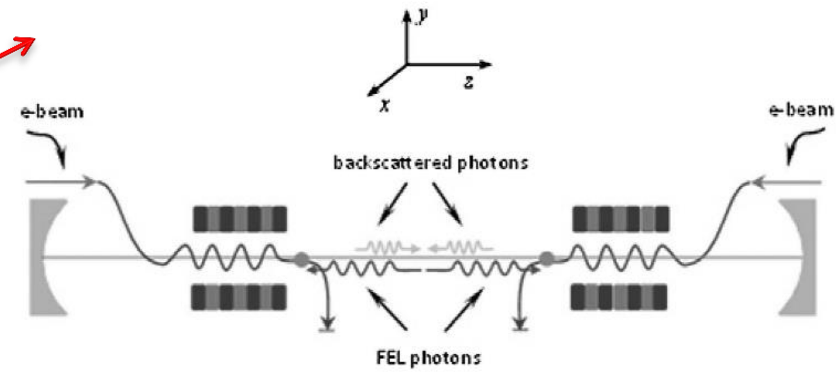


Fig. 2. "Double" FEL oscillator as a possible device for head-on γ-photon collisions.

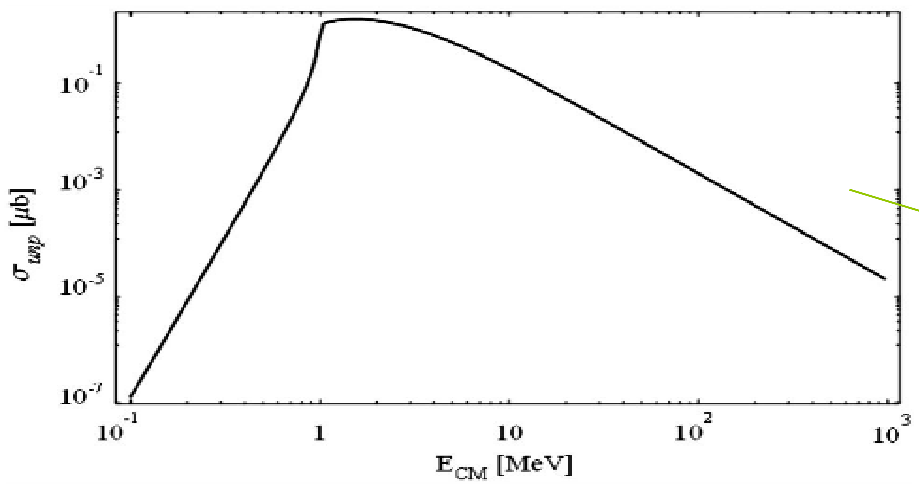
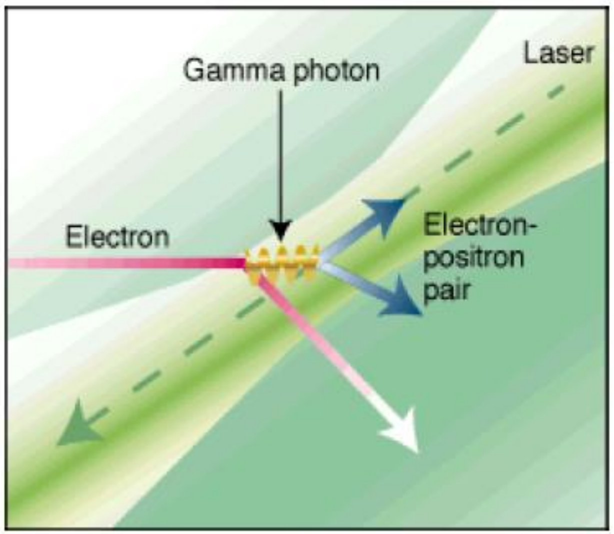
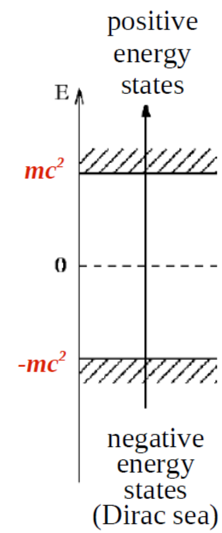


Fig. 1. Total cross section σ_{unp} for unpolarized initial photons versus CM energy $E_{CM} = 2E_{\text{ph}}$. The scale for both axes is logarithmic. We recall that 1 μb = 10^{-30} cm^2.

Cross section
B. De Tollis

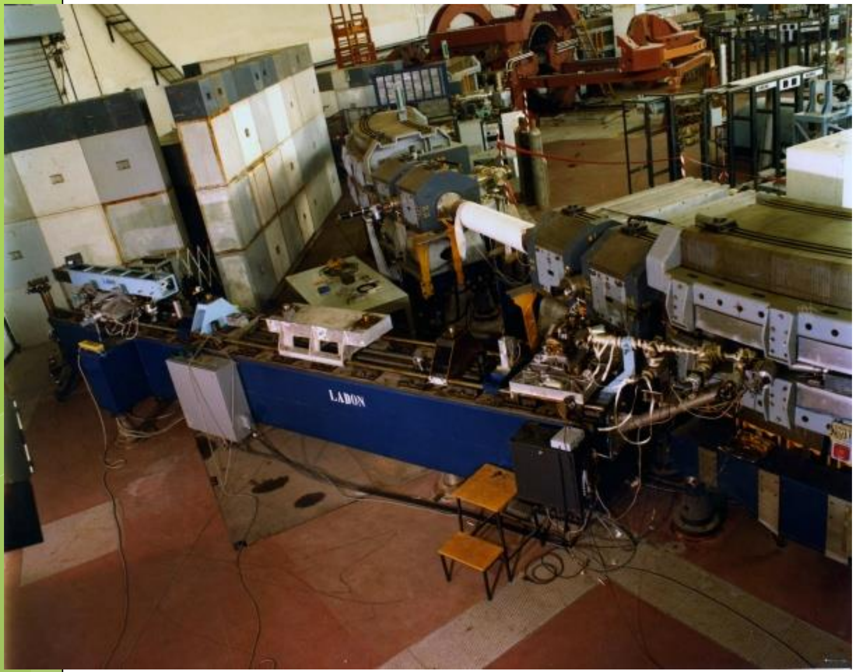
Strong Field





Production of electron-positron pairs according to $E=mc^2$ from laser photons possible, if $\hbar\omega \approx mc^2$ or $eE_L\lambda_C \approx mc^2$ ($I_{cr} \sim 10^{29}$ W/cm^2)

Ladon INFN Frascati



Head on Scattering
laser light (Argon line ~ = 5145 A) on the ADONE high energy electrons (E max= 1.5 GeV), to produce a monoenergetic and polarized photon beam suitable for nuclear research.

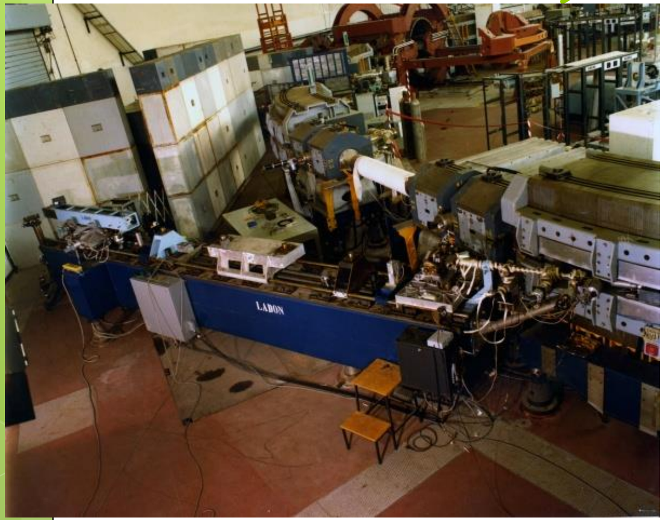
1) a photon energy continously adjustable between -- 5 MeV and ~, 78 MeV, for an electron energy ranging from 0.37 GeV to 1.5 GeV ;
2) a photon intensity 10^4 -10^5 photons/sec,

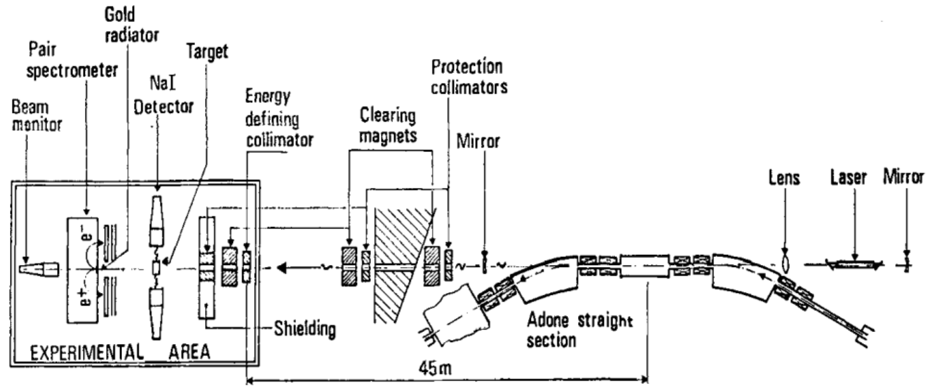
3) an energy resolution between ~,1% and 10%

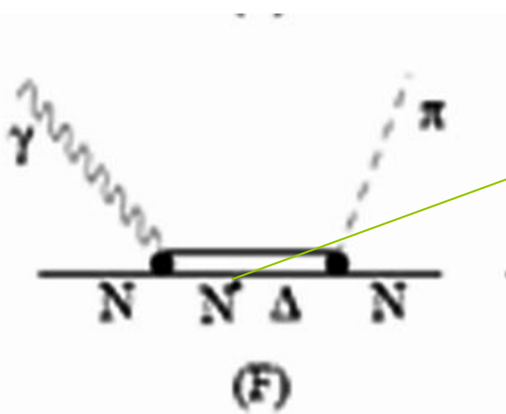
4) linear polarization

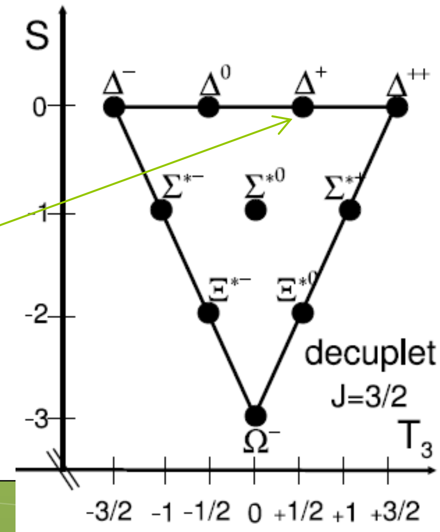
6) a time microstructure similar to that of the electrons in the SR

Ladon INFN Frascati

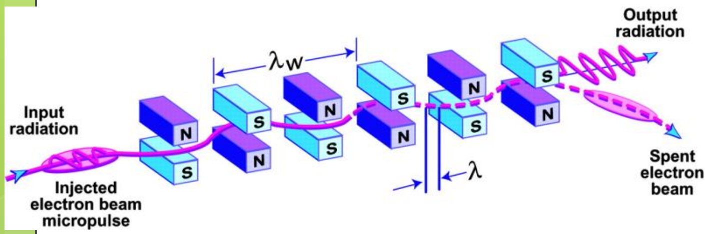


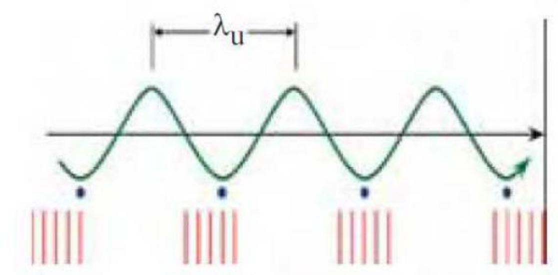






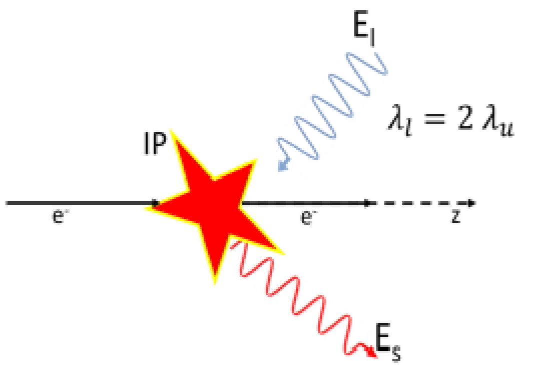
F-W-W_U

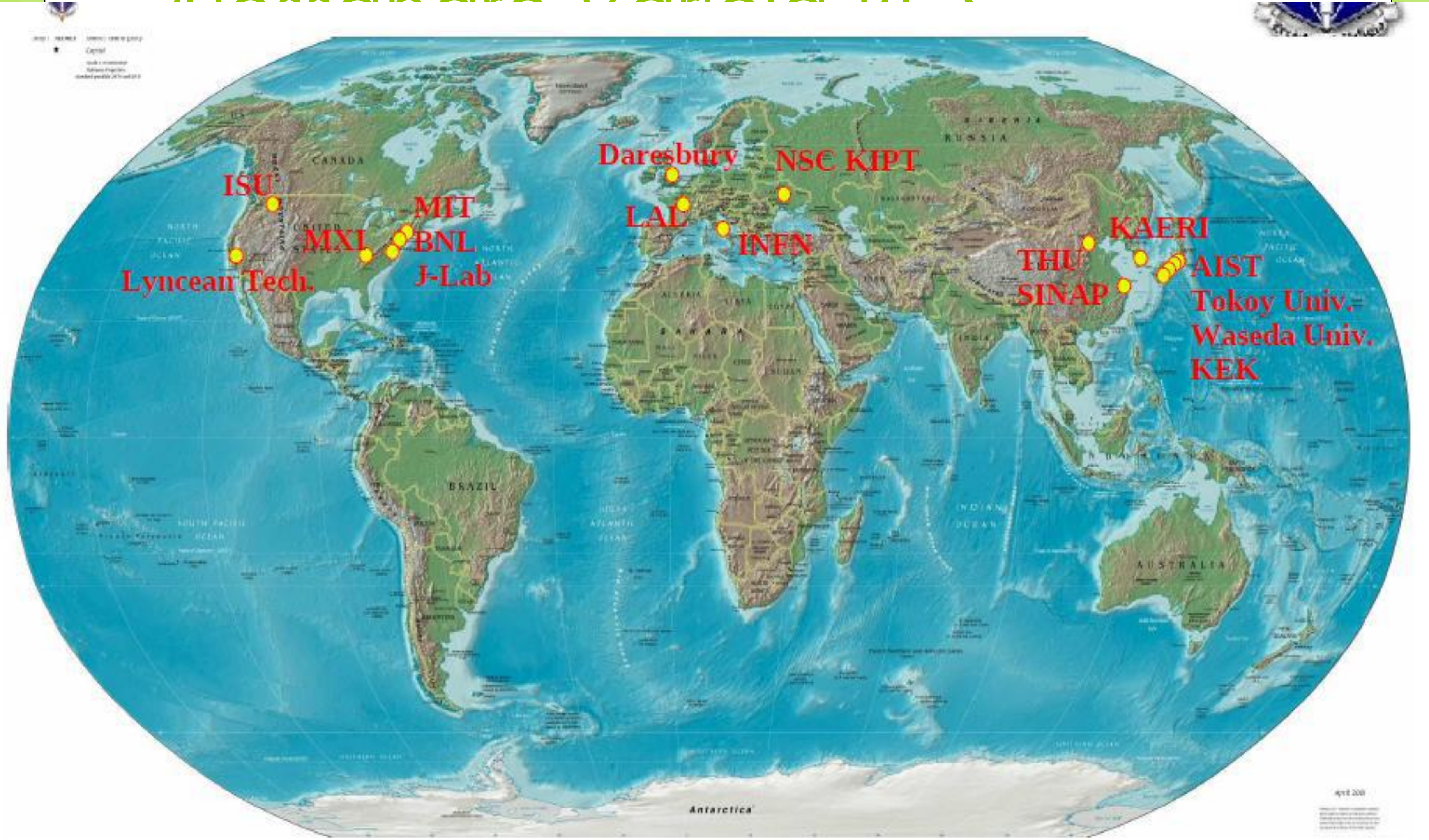




$$\delta \sim (c - v_z)\frac{\lambda_u}{c} \qquad \delta \cong (1-\beta_z)\lambda_u \cong \frac{\lambda_u}{2\gamma^2} \qquad \delta = n\lambda_r$$

$$\lambda_r = \frac{\lambda_u}{2\gamma^2} \qquad \lambda_s = \frac{\lambda_l}{4\gamma^2} \qquad \lambda^* = 2\lambda_u$$

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Interaction point