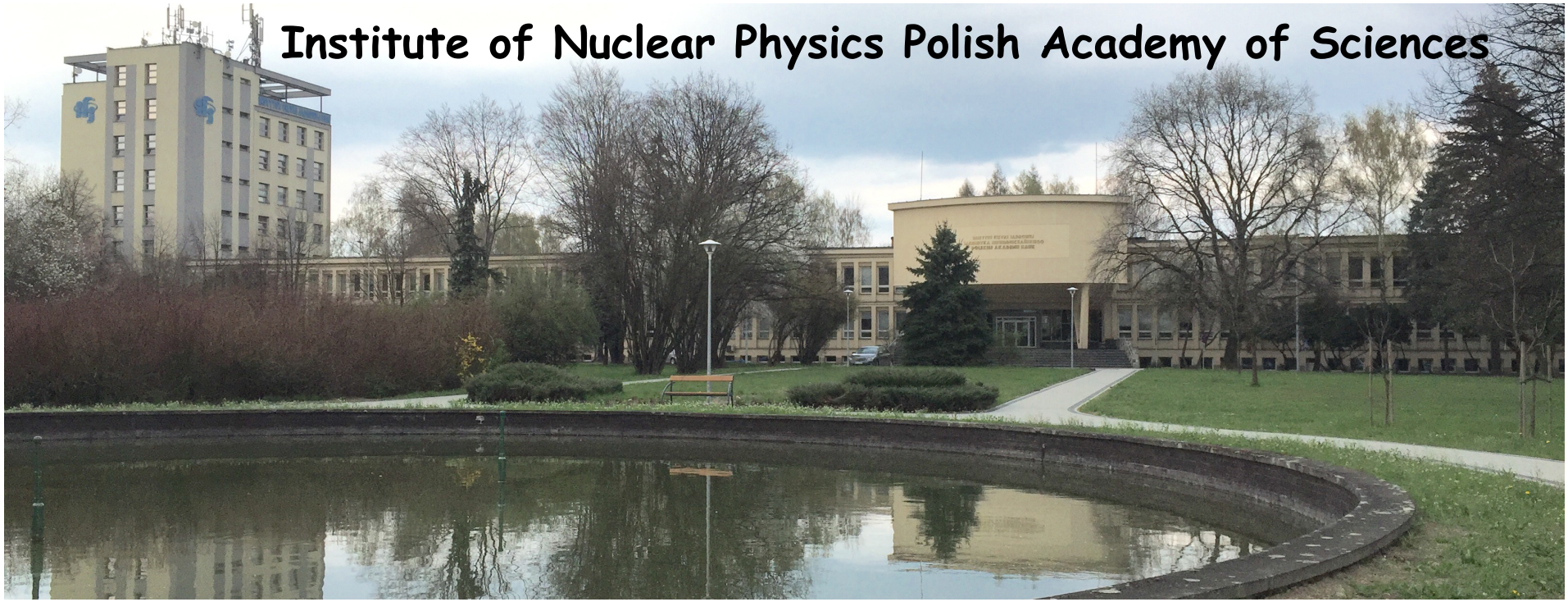
Theory uncertainties for e+e- luminosity measurements

Stanisław Jadach

In collaboration with M. Skrzypek, W. Płaczek, Z. Wąs, B.F.L. Ward and S.A. Yost



Presented at ECFA-WG1 meeting Dec. 16-th, 2022

Partly supported by CERN FCC Design Study Programme.

Three recent papers

Upgrade of LEP luminosity precision (2020)

Forecast of lumi precision at FCCee (2019)

Luminosity theory precision at other h.e. future electron colliders (2021)

Physics Letters B 803 (2020) 135319

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Improved Bhabha cross section at LEP and the number of light neutrino species ☆

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Physics Letters B 790 (2019) 314–321

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Eur. Phys. J. C (2021) 81:1047
https://doi.org/10.1140/epjc/s10052-021-09860-9

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Study of theoretical luminosity precision for electron colliders at higher energies

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ARTICLE INFO

Article history:
Received 17 December 2019
Received in revised form 13 February 2020
Accepted 19 February 2020
Available online 25 February 2020
Editor: L. Rolandi

ABSTRACT

In e^+e^- collisions, the integrated luminosity is generally measured from the rate of low-angle Bhabha interactions $e^+e^- \to e^+e^-$. In the published LEP results, the inferred theoretical uncertainty of $\pm 0.061\%$ on the predicted rate is significantly larger than the reported experimental uncertainties. We present an updated and more accurate prediction of the Bhabha cross section in this letter, which is found to reduce the Bhabha cross section by about 0.048%, and its uncertainty to $\pm 0.037\%$. When accounted for, these changes modify the number of light neutrino species (and its accuracy), as determined from the LEP measurement of the hadronic cross section at the Z peak, to $N_\nu = 2.9963 \pm 0.0074$. The 20-years-old 2σ tension with the Standard Model is gone.

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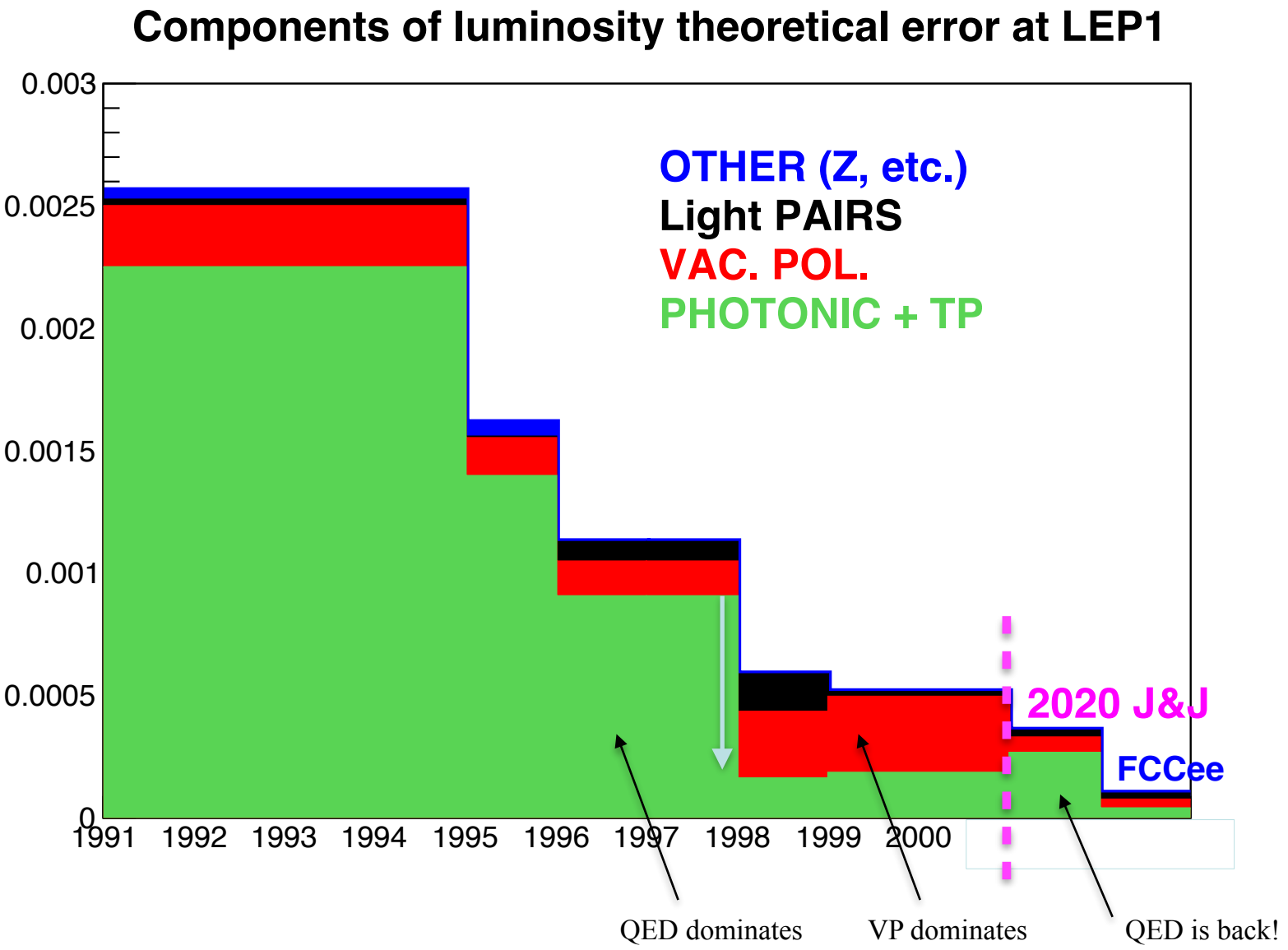
2019 Upgrade of LEP lumi mainly due to big improvements in vacuum polarization

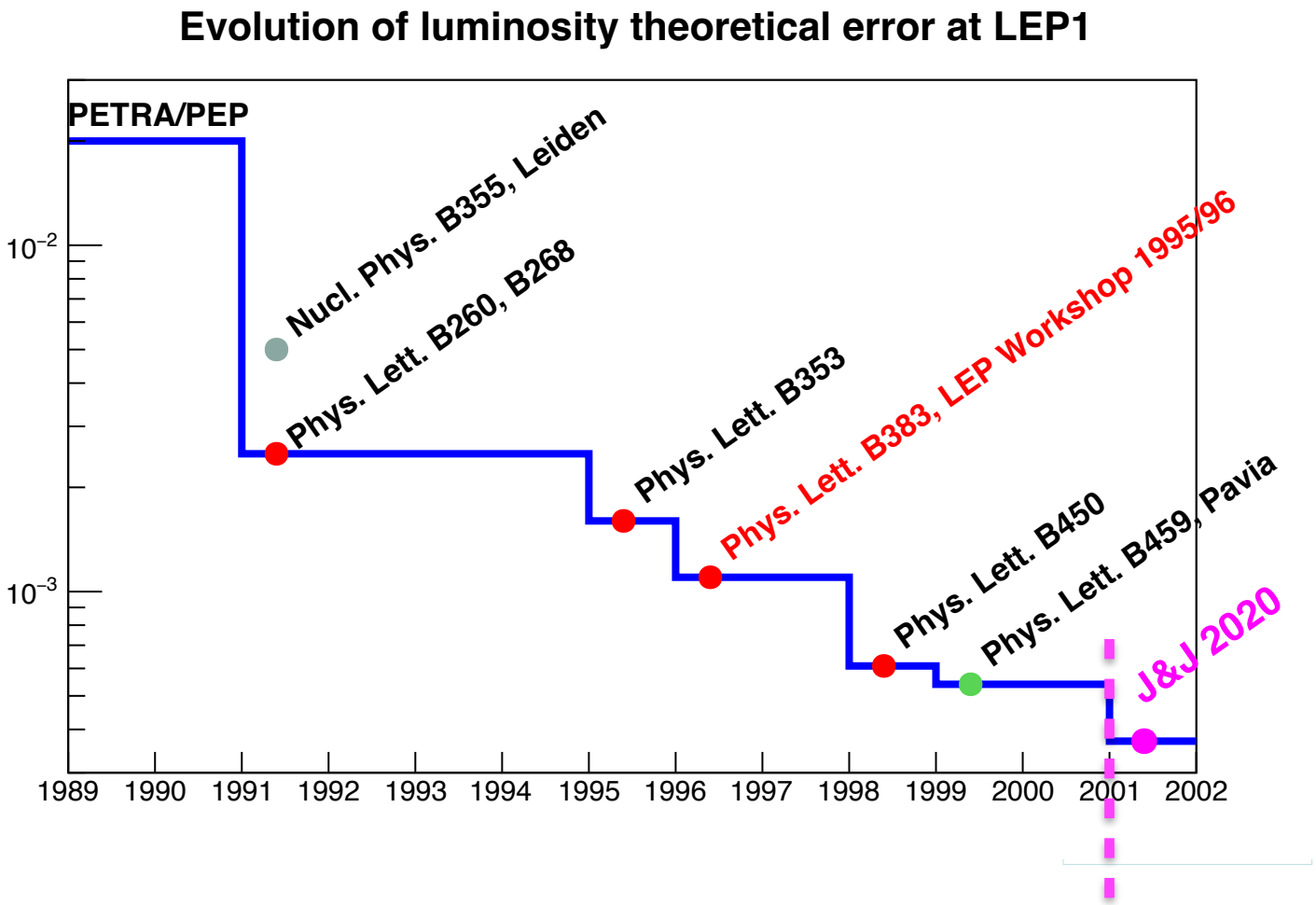
Table 3
Inspired from Refs. [28,29,25]: Summary of the theoretical uncertainties for a typical LEP luminosity detector covering the angular range from 58 to 110 mrad (first generation) or from 30 to 50 mrad (second generation). The total uncertainty is the quadratic sum of the individual components.

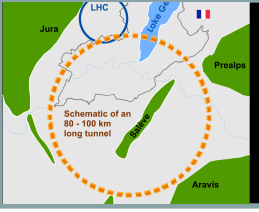
LEP Publication in:	1994		2000		2019	
LumiCal generation	1st	2nd	1st	2nd	1st	2nd
Photonic $\mathcal{O}(\alpha^2 L_e)$	0.15%	0.15%	0.027%	0.027%	0.027%	0.027%
Photonic $\mathcal{O}(\alpha^3 L_e^3)$	0.09%	0.09%	0.015%	0.015%	0.015%	0.015%
Z exchange	0.11%	0.03%	0.09%	0.015%	0.090%	0.015%
Vacuum polarization	0.10%	0.05%	0.08%	0.040%	0.015%	0.009%
Fermion pairs	0.05%	0.04%	0.05%	0.040%	0.010%	0.010%
Total	0.25%	0.16%	0.13%	0.061%	0.100%	0.037%

Evolution of components in TH lumi (BHLUMI) error



Evolution of the luminosity TH error





QED prediction for Low angle Bhabha (luminosity) xsection at FCCee/TeraZ

Physics Letters B 790 (2019) 314–321

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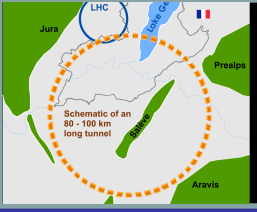
[e] The Citadel, Charleston, SC, USA

- **LEP lumi update 2018**

Type of correction / Error	1999	Update 20(
(a) Photonic $O(L_e\alpha^2)$	0.027% [5]	0.027%
(b) Photonic $O(L_e^3\alpha^3)$	0.015% [6]	0.015%
(c) Vacuum polariz.	0.040% [7,8]	0.013% [25]
(d) Light pairs	0.030% [10]	0.010% [18,19]
(e) s-channel Z-exchange	0.015% [11,12]	0.015%
(f) Up-down interference	0.0014% [27]	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061% [13]	0.038%

- **By the time of FCC-ee VP contribution will be merely 0.006%**
- **QED corrections and Z contrib. come back to front!**
- **Z contrib. easy (?) to master**
- **Improvements on photonics cors. will be difficult, new MC will be needed!**
- **Our FCC-ee forecast is 0.01% provided QED m.e. and VP are improved.**

Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4\alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $O(L_e^2\alpha^3)$	0.015%	0.1×10^{-4}
(c) Vacuum polariz.	0.014% [25]	0.6×10^{-4}
(d) Light pairs	0.010% [18,19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}



BHLUMI 4.x (1995) is always the reference

The exponentiated total cross cross section for $e^+(p_1)+e^-(q_1)\to e^+(p_2)+e^-(q_2)+\gamma(k_1)+\dots\ \gamma(k_n)+\gamma(k'_1)+\dots\ \gamma(k'_{n'})$ reads as follows:

$$\sigma^{(r)}=\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\frac{1}{n!}\frac{1}{n'!}\int\frac{d^3p_2}{p_2^0}\int\frac{d^3q_2}{q_2^0}\prod_{j=1}^{n}\int\limits_{k_j\notin\Omega_U}\frac{d^3k_j}{k_j^0}\tilde{S}_p(k_j)\prod_{l=1}^{n'}\int\limits_{k'_l\notin\Omega_L}\frac{d^3k'_l}{k'^0_l}\tilde{S}_q(k'_l)$$
$$\delta^{(4)}\left(p_1-p_2+q_1-q_2-\sum_{j=1}^{n}k_j-\sum_{l=1}^{n'}k'_l\right)e^{Y_p(\Omega_U)+Y_q(\Omega_L)}$$
$$\left\{\bar{\beta}_0^{(r)}+\sum_{j=1}^{n}\frac{\bar{\beta}_{1U}^{(r)}(k_j)}{\tilde{S}_p(k_j)}+\sum_{l=1}^{n'}\frac{\bar{\beta}_{1L}^{(r)}(k'_l)}{\tilde{S}_q(k'_l)}+\sum_{n\geq j>k\geq 1}\frac{\bar{\beta}_{2UU}^{(r)}(k_j,k_k)}{\tilde{S}_p(k_j)\tilde{S}_p(k_k)}\right.$$
$$\left.+\sum_{n'\geq l>m\geq 1}\frac{\bar{\beta}_{2LL}^{(r)}(k_l,k_m)}{\tilde{S}_q(k'_l)\tilde{S}_q(k'_m)}+\sum_{j=1}^{n}\sum_{l=1}^{n'}\frac{\bar{\beta}_{2UL}^{(r)}(k_j,k'_l)}{\tilde{S}_p(k_j)\tilde{S}_q(k'_l)}\right\}. \tag{8}$$

where the YFS formfactor is defined as in Ref. [2] and the various components of the differential distribution $\bar{\beta}_0^{(r)}, r=0,1,2$, are defined according to the usual YFS scheme in terms of the $D^{(r)}$ distributions:

$$\bar{\beta}_0^{(r)}=\left\{D_{[0,0]}^{(r)}\exp(-Y_p(\Omega_U)-Y_q(\Omega_L)\right\}\Big|_{\mathcal{O}(\alpha^r)}, \tag{9}$$

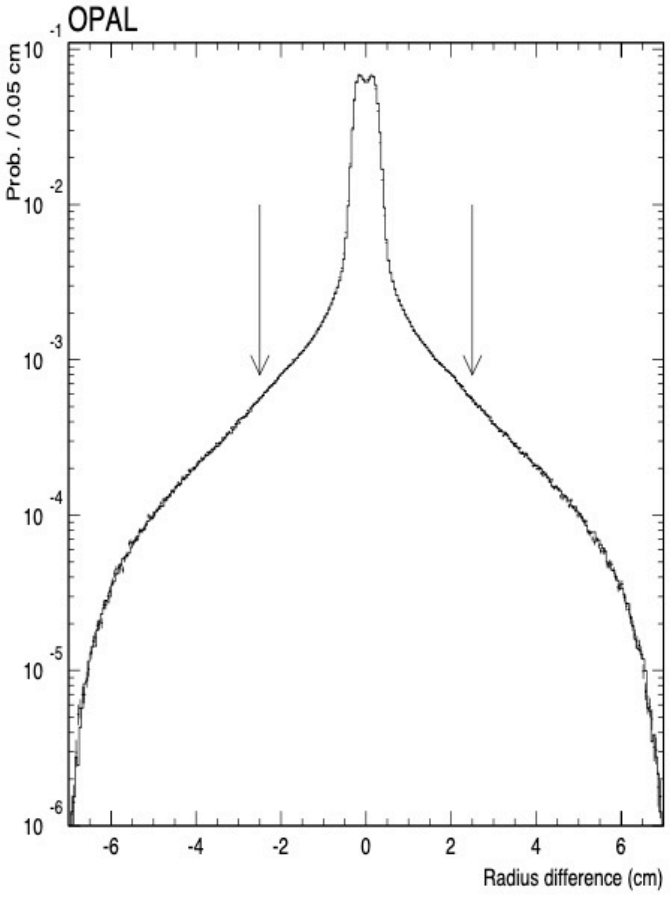
$$\bar{\beta}_{1U}^{(r)}(k_i)=\left\{D_{[1,0]}^{(r)}(k_i)\exp(-Y_p(\Omega_U)-Y_q(\Omega_L)\right\}\Big|_{\mathcal{O}(\alpha^r)}-\tilde{S}_p(k_i)\,\bar{\beta}_0^{(r-1)}, \tag{10}$$

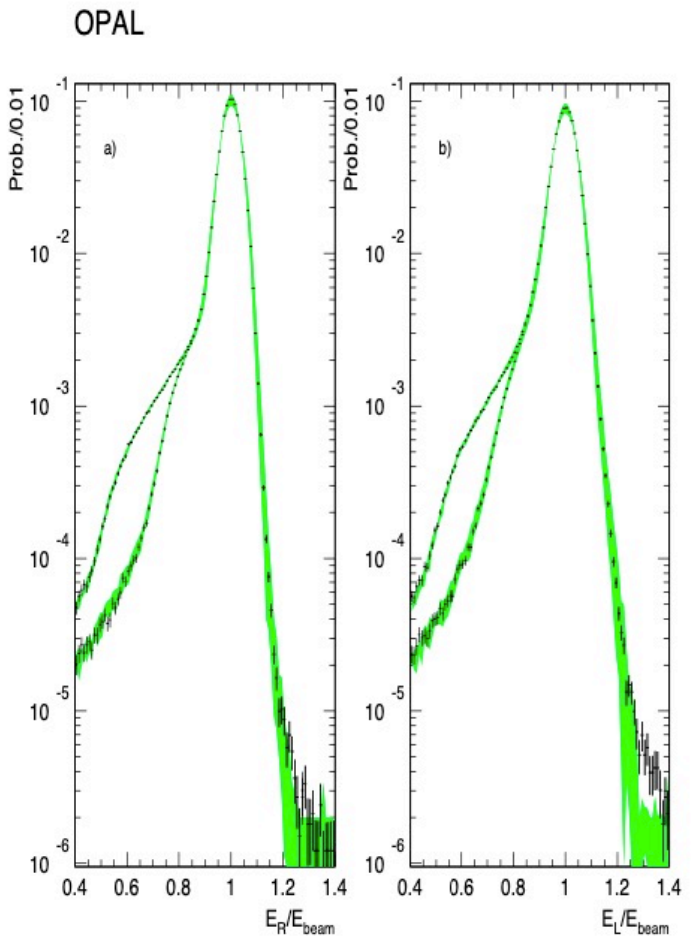
$$\bar{\beta}_{2UU}^{(2)}(k_i,k_j)=D_{[2,0]}^{(2)}(k_i,k_j)-\bar{\beta}_{1U}^{(1)}(k_i)\tilde{S}_p(k_j)-\bar{\beta}_{1U}^{(1)}(k_j)\tilde{S}_p(k_i)-\bar{\beta}_0^{(0)}\tilde{S}_p(k_i)\tilde{S}_p(k_j), \tag{11}$$

$$\bar{\beta}_{2UL}^{(2)}(k_i,k'_j)=D_{[1,1]}^{(2)}(k_i,k'_j)-\bar{\beta}_{1U}^{(1)}(k_i)\tilde{S}_q(k'_j)-\bar{\beta}_{1L}^{(1)}(k'_j)\tilde{S}_p(k_i)-\bar{\beta}_0^{(0)}\tilde{S}_p(k_i)\tilde{S}_q(k'_j). \tag{12}$$

Perfect agreement between LEP data and BHLUMI 4.x (see next slide)
is thanks to the above QED matrix element with YFS soft photon resummation.
Photonic correction $\mathcal{O}(L_e\alpha)$, $\mathcal{O}(\alpha)$, $\mathcal{O}(L_e^2\alpha^2)$ are complete. Most of $\mathcal{O}(L_e\alpha^2)$, $\mathcal{O}(L_e^3\alpha^3)$.

Extraordinary agreement of OPAL data vs. BHLUMI MC, collinearity and energy distrs.





BHLUMI 4.04 Monte Carlo was used by all four LEP experiments. Not only controls luminosity normalization $d\sigma/\sigma = 0.06\%$, but also perfectly agrees with all experimental spectra, with NO "TUNING" to experimental data! (Only one bug in 1995.)

Recent (2021) study of the prospects of low angle Bhabha (lumi) theoretical precision at all future H.E. electron colliders

Eur. Phys. J. C (2021) 81:1047
https://doi.org/10.1140/epjc/s10052-021-09860-9

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Study of theoretical luminosity precision for electron colliders at higher energies

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Received: 20 September 2021 / Accepted: 21 November 2021 / Published online: 30 November 2021
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Abstract We present an estimation of the theoretical precision of low angle Bhabha scattering at the proposed future ILC collider at 500 GeV. The analysis is an extension of the previous analysis done for the FCCee collider at $\sqrt{s} = M_Z$. As the state-of-the-art and the reference point we use the BHLUMI Monte Carlo event generator. Based on the current precision status of BHLUMI for LEP analysis, we estimate how various error components evolve from the LEP to ILC setups. The conclusion of our work is that for the ILC the precision of the current version of BHLUMI 4.04 deteriorates to 0.5%, by more than an order of magnitude w.r.t. the present precision for LEP. With the expected future improvements, the precision of BHLUMI can change to 0.016%, nearly as good as for the FCCee at the M_Z setup (0.01%). Based on the developed methodology we present also results for ILC_{1000}, FCCee_{350} and CLIC_{3000} setups.

actions, thus minimizing the higher order EW and QCD corrections.

In the previous paper [1] we scrutinized the LEP legacy and the current state-of-the-art situation concerning the theoretical precision of LABH calculations as implemented in the BHLUMI [2,3] Monte Carlo code. Based on this background we analysed the future prospects of theoretical precision of luminosity measurement at the FCCee collider at the Z peak. By analysing various components of the higher order corrections we came to the conclusion that the desired relative precision of 1×10^{-4} is feasible.

In this paper we extend further the analysis of [1] to another future e^+e^- collider project, the ILC. Technical parameters of the ILC and its detector differ from FCCee, but the essence of the measurement remains the same. In the following we will discuss all the corrections to LABH scattering for the ILC setup. We will demonstrate, that the

Most of correction to be estimated or known at $\bar{t}_{LEP}$ are of order $\alpha^n L_e^r$, $L_e = \ln(\bar{t}/m_e^2)$, $\bar{t} = \sqrt{t_{\min} t_{\max}}$.
Average t-channel transfer $\bar{t}$ is the key parameter!

Table 1 Basic parameters of the detectors at e^+e^- past and future machines. The $\theta_{\min} - \theta_{\max}$ are the generic LABH detector acceptances. The $\bar{t}$ is defined in Eq. (3)

Machine	$\theta_{\min} - \theta_{\max}$ (mrad)	$\sqrt{s}$ (GeV)	$\bar{t}/s$	$\sqrt{\bar{t}}$ (GeV)
LEP	28–50	M_Z	3.5×10^{-4}	1.70
FCCee	64–86	M_Z	13.7×10^{-4}	3.37
FCCee	64–86	350	13.7×10^{-4}	13.0
ILC	31–77	500	6.0×10^{-4}	12.2
ILC	31–77	1000	6.0×10^{-4}	24.4
CLIC	39–134	3000	13.0×10^{-4}	108

At ILC (500 GeV) we can assume that the $\mathcal{O}(\alpha^3 \ln^3(|t|/m_e^2))$ term grows by a factor of $\ln^3(\bar{t}_{ILC}/m_e^2)/\ln^3(\bar{t}_{LEP}/m_e^2) = 1.9$, i.e. it increases to 0.028%. Including an additional factor of $(\alpha/\pi)\ln(\bar{t}_{ILC}/m_e^2) = 0.049$ and a safety factor of 2 we end up with the estimate of the missing term $\mathcal{O}(\alpha^4 \ln^4(|t_{ILC}|/m_e^2)) \sim 0.27 \times 10^{-4}$.

Examples of estimates in EPJC81

At LEP the $\mathcal{O}(\alpha^4 \ln^4(|t_{LEP}|/m_e^2))$ corrections were estimated based on the size of known $\mathcal{O}(\alpha^3 \ln^3(|t_{LEP}|/m_e^2))$ corrections. The latter ones are of the size of 1.5×10^{-4} at LEP [9, 20]. Multiplied by the generic $(\alpha/\pi)\ln(\bar{t}_{LEP}/m_e^2) = 0.039$ it gave an estimate of 6×10^{-6} for the $\mathcal{O}(\alpha^4 \ln^4(|t_{LEP}|/m_e^2))$ terms.

The case of ILC 500

Table 3 Forecasts of the total (physical+technical) theoretical uncertainty for the ILC 500 luminosity calorimetric detector with the angular range of 31–77 mrad. Description of photonic corrections in square brackets is related to the 2nd column. The numbers in curly brackets in lines (e) and "Total" refer to a fictitious situation with all Born-level interferences included in BHLUMI. The total error is summed in quadrature. A technical error is not included in "Total"

ILC 500 setup

Type of correction/error	Update 2019	Forecast
(a) Photonic $[\mathcal{O}(L_e\alpha^2)]$ $\mathcal{O}(L_e^2\alpha^3)$	0.033%	0.13×10^{-4}
(b) Photonic $[\mathcal{O}(L_e^3\alpha^3)]$ $\mathcal{O}(L_e^4\alpha^4)$	0.028%	0.27×10^{-4}
(c) Vacuum polariz.	0.022% [34]	1.1×10^{-4}
(d) Light pairs	0.010% [7]	0.4×10^{-4}
(e) Z and s-channel γ exchange	0.5% (0.06%)	1.0×10^{-4}
(f) Up-down interference	0.004% [13]	$<0.1 \times 10^{-4}$
(g) Technical Precision	(0.027%)	0.1×10^{-4}
Total	0.5% (0.078%)	1.6×10^{-4}

The lumi TH precision forecast for three future electron colliders

Table 4 Forecast of the total (physical) theoretical uncertainty for the FCCee$_{350}$, ILC$_{1000}$ and CLIC luminosity calorimetric detectors with angular acceptances as defined in the text. Number (*) is likely overestimated. The total error is summed in quadrature. A technical error is not included

Forecast			
Type of correction/error	FCCee$_{350}$	ILC$_{1000}$	CLIC$_{3000}$
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exchange	$1.0 \times 10^{-4(*)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	0.1×10^{-4}	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

Summary

- The estimate (forecast) of the theory uncertainty in luminosity measurements using low angle Bhabha is varying between 1×10^{-4} to 16×10^{-4} depending on scattering energy and angular range of the luminometer

- Achieving such precision levels is feasible but it will require a lot of dedicated work and new MC event generators better than BHLUMI 4.x

- BHLUMI may serve as a starting point for the future improved MC event generator, CEEX class m.e. similar to that in KKMC should be first step.

- RESERVE

Recent revival of the interest in Bhabha luminometry and N_ν measurements

Physics Letters B 790 (2019) 314–321

Contents lists available at ScienceDirect

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FCC-ee oriented studies have triggered new interest in re-analysing LEP data

JHEP

Published for SISSA by Springer

JHEP10(2019)225

Received: *August 7, 2019*
Revised: *October 2, 2019*
Accepted: *October 13, 2019*
Published: *October 22, 2019*

Beam-beam effects on the luminosity measurement at FCC-ee

Georgios Voutsinas,[a] Emmanuel Perez,[a] Mogens Dam[b] and Patrick Janot[a]

Physics Letters B 800 (2020) 135068

Contents lists available at ScienceDirect

Physics Letters B

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Beam-beam effects on the luminosity measurement at LEP and the number of light neutrino species

Georgios Voutsinas [a], Emmanuel Perez [a], Mogens Dam [b], Patrick Janot [a,*]

Physics Letters B 803 (2020) 135319

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Improved Bhabha cross section at LEP and the number of light neutrino species ☆

Patrick Janot [a,*], Stanisław Jadach [b]

New study by P.Janot and S.J.

arXiv:1912.02067 and Phys. Letters B803 (2020) 135319

Correcting LEP 1990-95 data for low angle Bhabha of ALEPH, DELPHI, L3 and OPAL for realistic event selection, emulating all eight variants of small angle Bhabha LEP detectors.

MC programs used by LEP collaborations

	ALEPH	DELPHI	L3	OPAL
1990	BABAMC (0.320%)	2.01 (0.300%)	2.01 (0.250%)	2.01 (0.300%) later scaled to 4.04
1991-92	2.01 (0.210%)			
Fall 92	2.01 (0.160%)			
1993	4.04 (0.061%)	4.02 (0.170%)	4.04 (0.061%)	4.04 (0.054%)
1994-95		4.03 (0.061%)		

Eight luminosity detectors used at LEP

Expt / LumiCal	Period	Narrow (mrad)	Wide (mrad)
ALEPH LCAL [5]	01/90 → 08/92	57 – 107	43 – 125
DELPHI SAT [6, 7]	01/90 → 12/93	56.0 – 128.6	52.7 – 141.8
L3 BGO [8]	01/90 → 12/92	31.2 – 65.2	25.2 – 71.2
OPAL FD [9]	01/90 → 12/92	65.0 – 105.0	55.0 – 115.0
ALEPH SiCAL [10]	09/92 → 12/95	30.4 – 49.5	26.1 – 55.9
DELPHI STIC [11]	01/94 → 12/95	43.6 – 113.2	37.2 – 126.8
L3 SLUM [12]	01/93 → 12/95	32.0 – 54.0	27.0 – 65.0
OPAL SiW [13]	01/93 → 12/95	31.3 – 51.6	27.2 – 55.7

Bhabha event selections in LEP

Experiment	ALEPH [16]	DELPHI [17, 18, 35]	L3 [8, 36]	OPAL [9, 37, 19]
$E_{1,2}^{\min}/E_{\text{beam}}$ $E_{1,2}^{\max}/E_{\text{beam}}$	> 0.44	> 0.65	> 0.40 > 0.80	> 0.45 ($\rightarrow 92$) > 0.50 ($93 \rightarrow$)
$\frac{(E_1+E_2)}{2E_{\text{beam}}}$	> 0.60 ($\rightarrow 93$) > 0.78 (in 94) > 0.84 (in 95)	–	–	> 0.67 ($\rightarrow 92$) > 0.75 ($93 \rightarrow$)
$\Delta\phi$ (mrad)	< 175 ($\rightarrow 8/92$) < 525 ($9/92 \rightarrow$)	< 350	< 175	< 350 ($\rightarrow 92$) < 200 ($93 \rightarrow$)
$\Delta\theta$ (mrad)	–	–	–	– ($\rightarrow 92$) < 10 ($93 \rightarrow$)

All the above was implemented in the mixture of new C++ and old F77 code on top of BHLUMI 4.x
Corrections due to improvements in the BHLUMI matrix element were calculated using MC samples of 160M events generated at seven energies $\sqrt{s}$ = 88.471, 89.444, 90.216, 91.227, 91.959, 93.00, 93.710 GeV.

LEP lumi TH milestones

Main TH precision improvements marked in red:

[1] W. Beenakker, F. A. Berends and S. C. van der Marck, Nucl. Phys. B **355** (1991) 281.
Photonic $\mathcal{O}(\alpha^2 L_e^2)$ and vacuum polarization (VP)

[2] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **253** (1991) 469.
Technical precision 0.02% establishe for the "baseline" $\mathcal{O}(\alpha^1)$ MC

[3] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **260** (1991) 438.
First reliable estimate of the precision of $\mathcal{O}(\alpha^1)_{expon}$ multiphoton BHLUMI MC

[4] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **353** (1995) 362 [Erratum-ibid. B **384** (1996) 488].
Inclusion of $\mathcal{O}(\alpha^2 L_e^2)$ and $\mathcal{O}(\alpha^3 L_e^3)$, new estimate of $\mathcal{O}(\alpha^2 L)$

[5] A. Arbuzov *et al. LEP Working Group 1996*, Phys. Lett. B **383** (1996) 238
New estimate of missing $\mathcal{O}(\alpha^2 L)$ in BHLUMI

[6] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Proc. of ICHEP 98, Vancouver
arXiv:hep-ph/9811245 and Phys. Lett. B **450** (1999) 262
New calculation of missing $\mathcal{O}(\alpha^2 L)$ in BHLUMI

[7] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini and F. Piccinini, Phys. Lett. B **459** (1999) 649
New calculation of missing light real and virtual pairs

Vacuum polarization

- Appears in t-channel photon propagator
  - Dressed with a loop of lepton or quarks
    - Hadronic part obtained from $\sigma(e^+e^- \to \text{hadrons})$
    - Recent progress from huge data samples with ISR
      - ➔ At B and Φ factories
  - Recent code made available by Fred Jegerlehner in 2019
    - Uncertainty reduced by a factor 4
    - Bhabha cross section reduced too
      - ➔ Reduction in units of 10^{-4} :

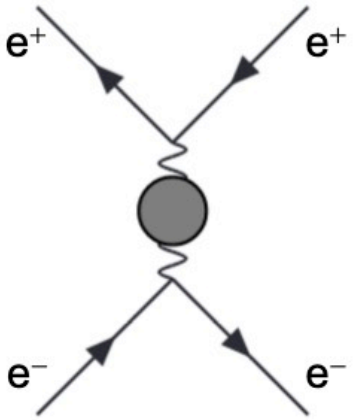


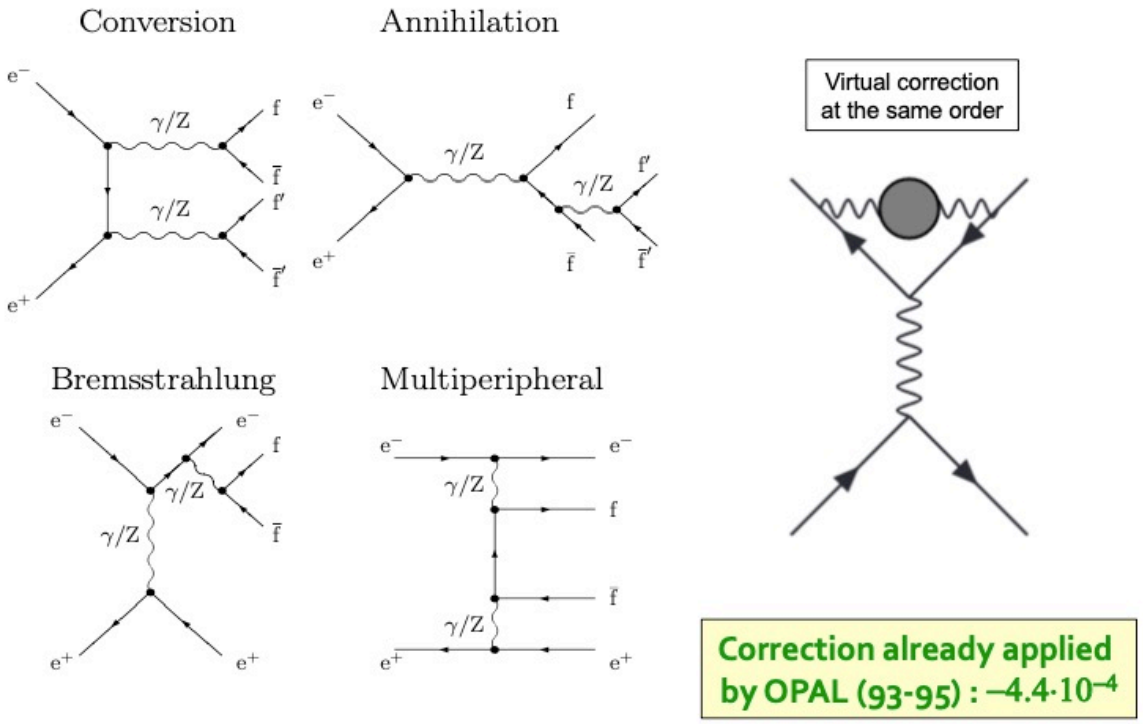
Table 6
Vacuum polarization correction relative to the Bhabha cross section at the Z peak (switching off the Z boson contribution) in the LumiCals of the four LEP experiments, as a function of time. Each entry value, expressed in units of 10^{-4}, corresponds to the change between the 2019 evaluation of the vacuum polarization in the t-channel photon propagator, and the default function proposed in the BHLUMI versions used in the LEP experiments' publications at the time. When relevant, the entries also include non-soft $\mathcal{O}(\alpha^2 L_e^2)$ corrections implemented only in BHLUMI 4.0. Statistical uncertainties are negligible and are not quoted in the table. The numbers display the quasi-linear excursion of the correction when varying $\sqrt{s}$ from 88.471 GeV (subscript) to 93.710 GeV (superscript).

Experiment	ALEPH	DELPHI	L3	OPAL
01/90 → 08/92	$-2.00^{-0.18}_{+0.21}$	$-1.02^{-0.18}_{+0.18}$	$+1.57^{-0.09}_{+0.10}$	$-4.60^{-0.03}_{+0.04}$
09/92 → 12/92	$+1.22^{-0.12}_{+0.12}$			
01/93 → 12/93	$-2.12^{-0.08}_{+0.09}$	$-4.62^{-0.06}_{+0.07}$	$-2.36^{-0.09}_{+0.11}$	$-2.24^{-0.09}_{+0.10}$
01/94 → 12/94		$-3.86^{-0.11}_{+0.12}$		
01/95 → 12/95				

Vacuum polarisation of Fred Jegerlehner (2019) was cross-checked with the private vacuum-polarisation code from the DHMZ team (Michel Davier, Andreas Hoecker, Bogdan Malaescu, Zhiqing Zhang) and the KNT team (Alexander Keshavarzi, Daisuke Nomura, Thomas Teubner)

Light fermion-pair production

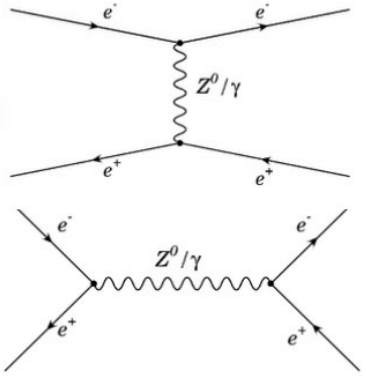
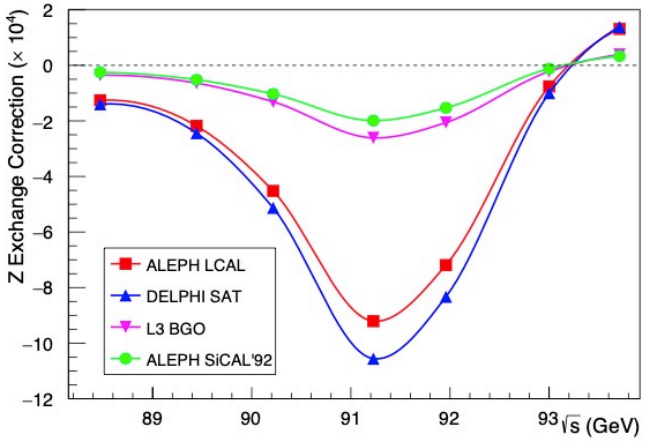
- **Four-fermion final state may be accepted by Bhabha selection**



- Only OPAL corrected its data. Others included fermion pair corrs. in the error budget.
- Now we do the same as OPAL for all luminosity LEP data.
- Real pair emissions using FERMISV MC (Leiden) and KORALW MC (Krakow)
- Virtual pair corrs. added using S. Actis et.al. Phys.Rev.D78 (2008) 085019.

Update of Z-exchange correction

QED corrections to Z exchange

- **QED corrections to Z-exchange were not state of the are until 1994**
  - BHLUMI 2.01 contained Born-level diagrams
  - O(α) corrections of the order of 50% (implemented in BABAMC)
    - Need higher-orders, implemented in only BHLUMI 4.04
      - ➔ Early LEP data (until 1992) need to be corrected

        Except for OPAL (correction already included)
  - Higher-orders reduce the Bhabha cross section until 1992: