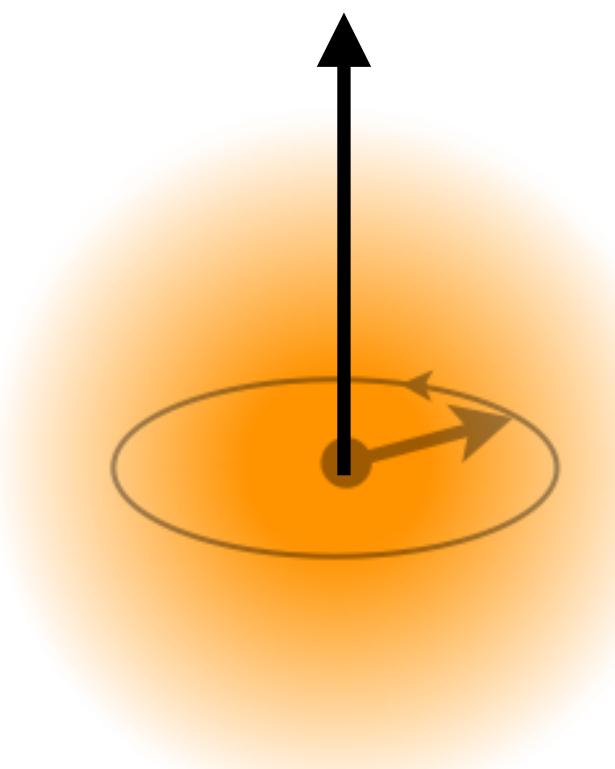
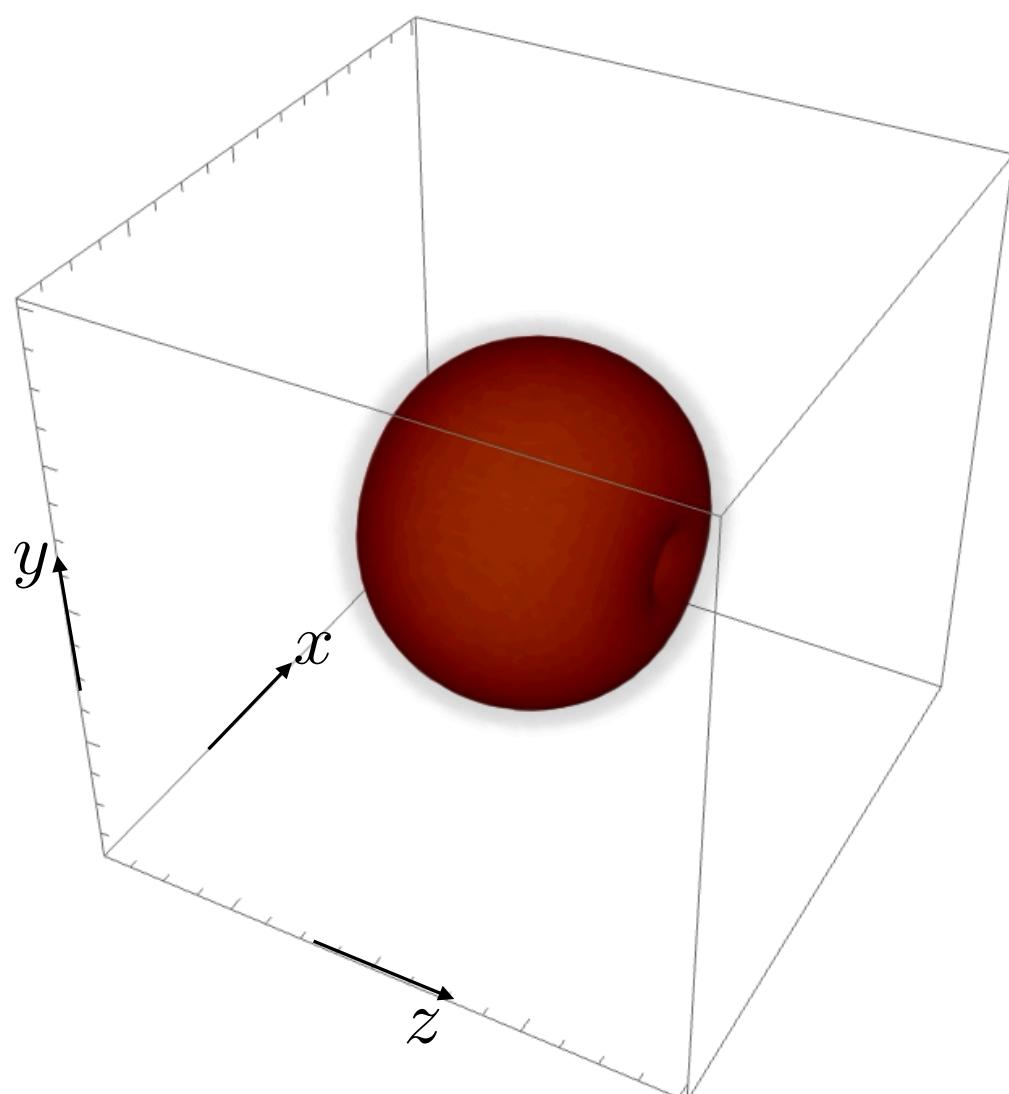


# ‘Spinor’ Bose Einstein Condensates

from Cosmos to Lab

Mudit Jain



[Phys.Rev.D 105 \(2022\) 5, 056019](#)

[Phys.Rev.D 105 \(2022\) 9, 096037](#)

[JCAP 08 \(2022\) 08, 014](#)

[Phys.Rev.D 106 \(2022\) 8, 8](#)

[JCAP 04 \(2023\) 053](#)

[arXiv:2304.01985](#)

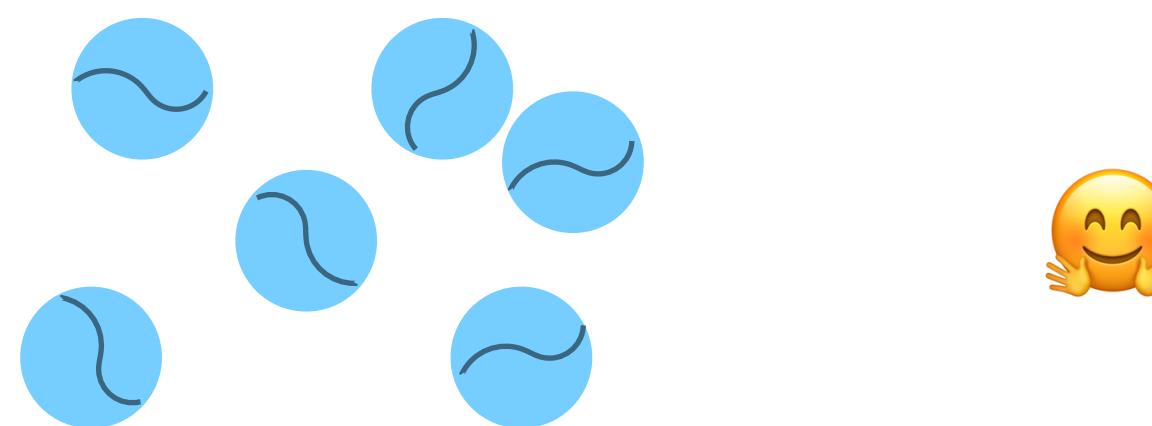
[arXiv:2305.01675](#)

+ ongoing work

Collaborators:

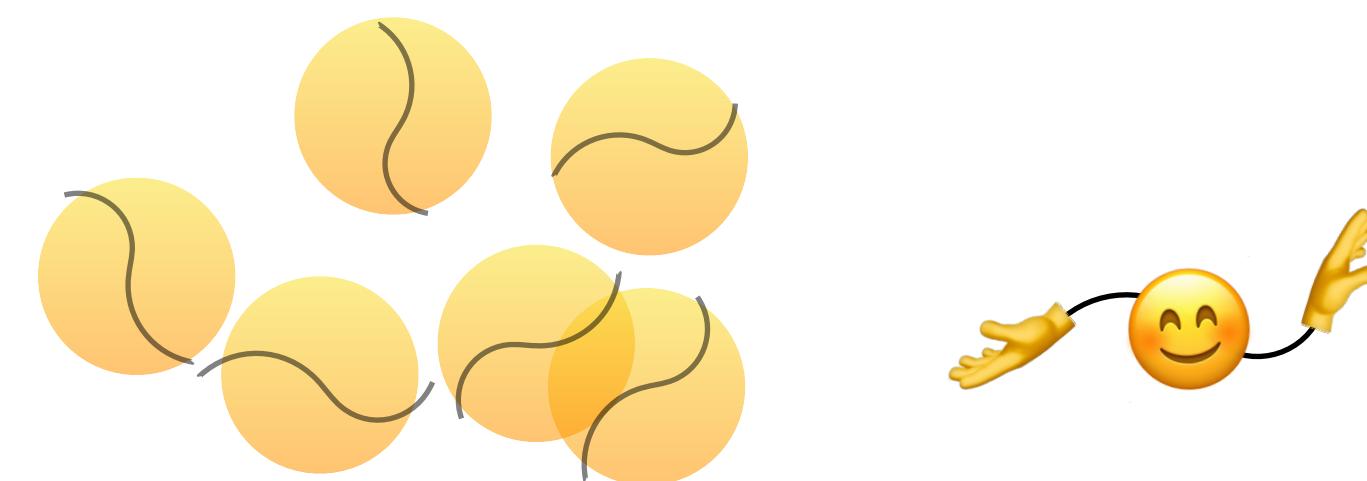
Amin, Karur, Mocz, Pu, Thomas, Wanichwecharungruang, Zhang.

# A quick recap



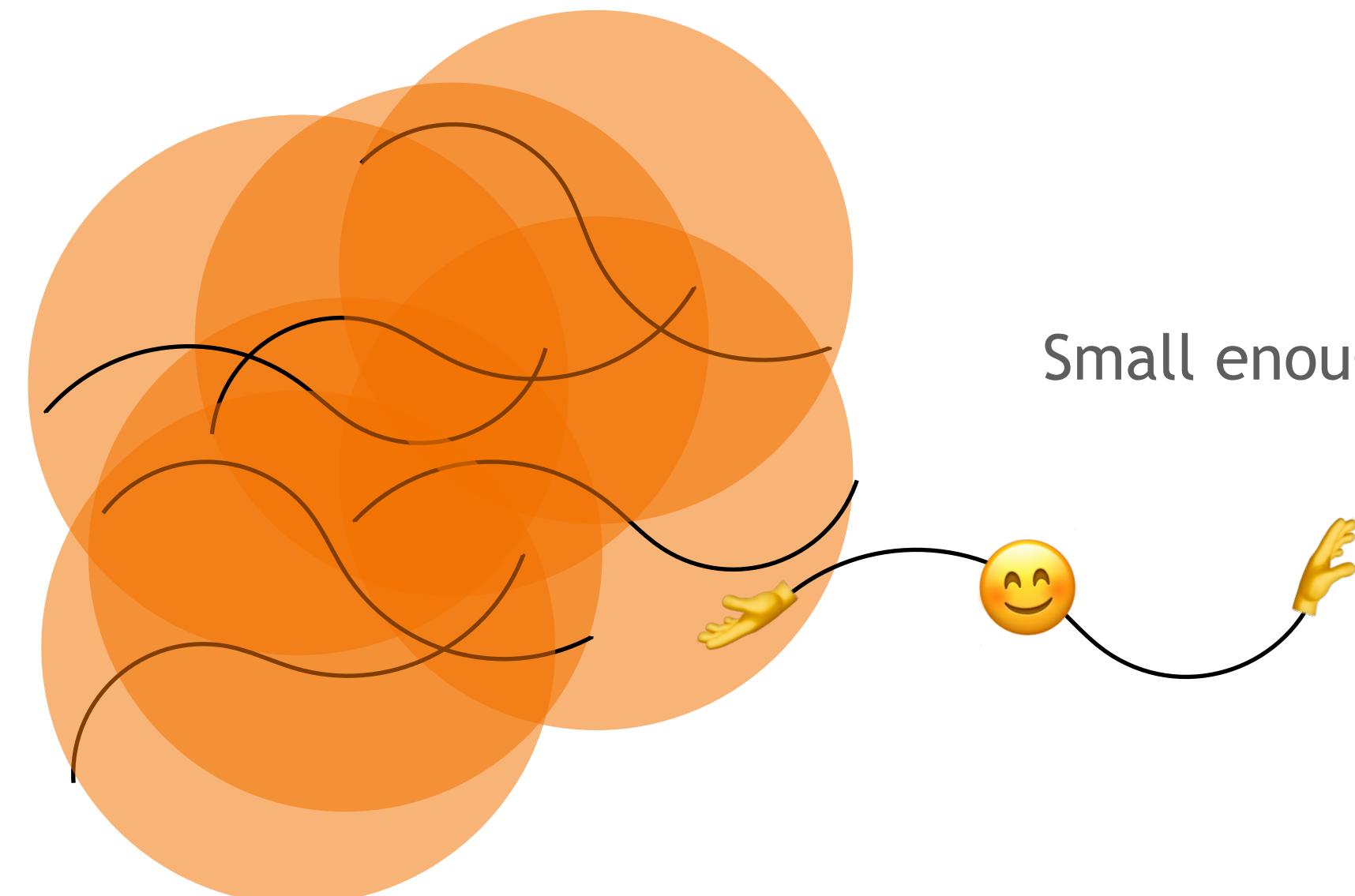
At high temperatures / typical energy scale in the system

$$N \simeq n\lambda_{dB}^3 \sim n(mv)^{-3} \ll 1$$



Decreasing temperature

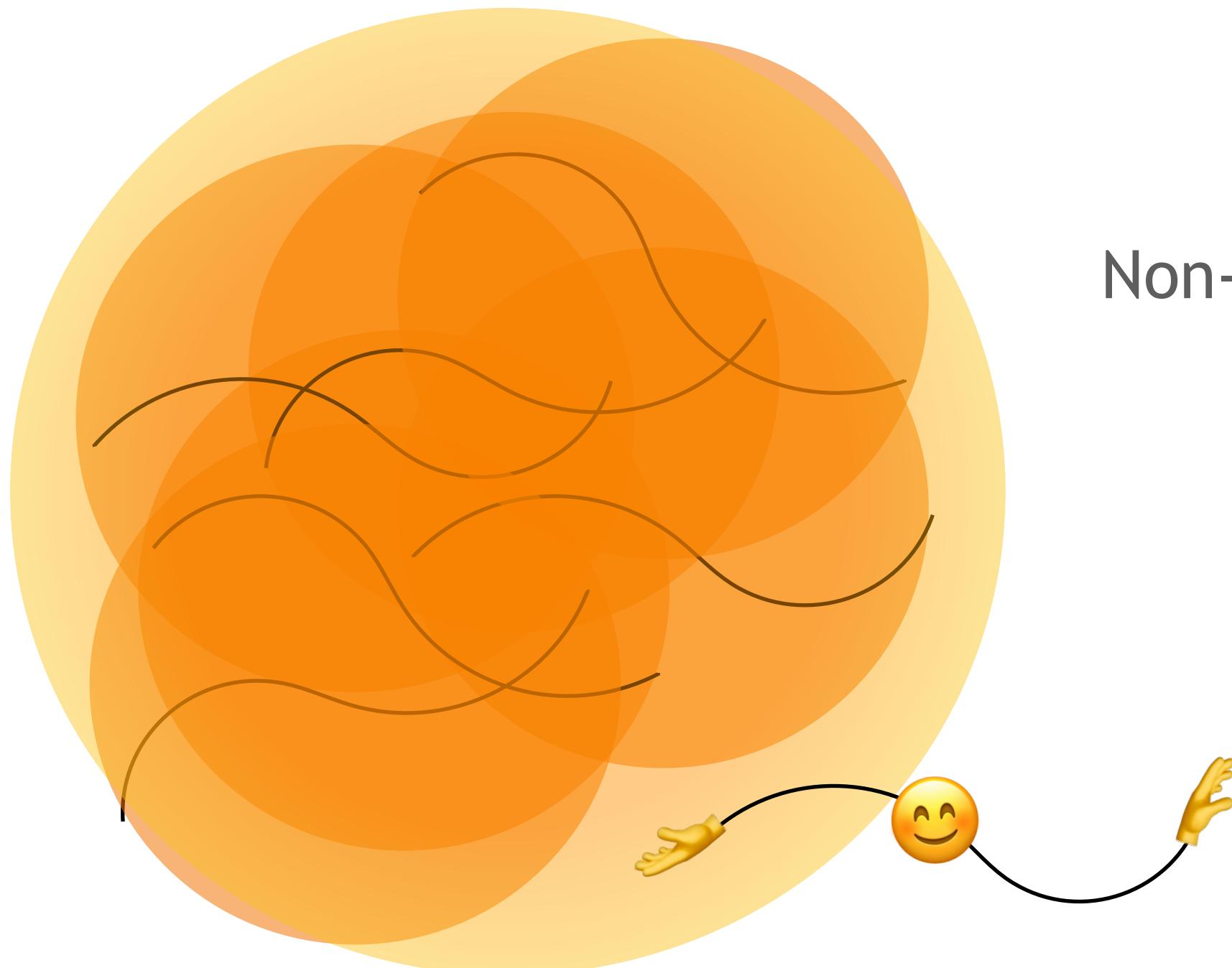
$$N \simeq n\lambda_{dB}^3 \sim n(mv)^{-3} \sim 1$$



Small enough temperatures : lot of particles accumulate towards lower energy states

$$N \simeq n\lambda_{dB}^3 \sim n(mv)^{-3} \gg 1$$

$$T \sim mv^2$$



Non-linear Schrödinger / Gross-Pitaevski equation (wave equation)

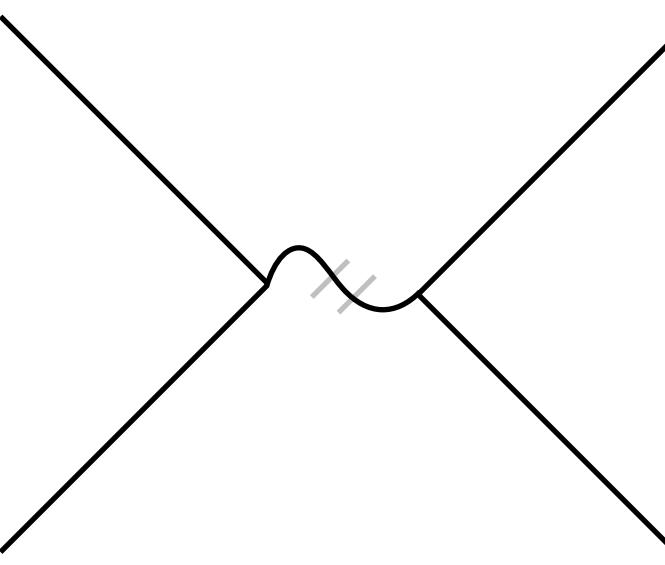
$$i \frac{\partial \Psi}{\partial t} = -\frac{1}{2m} \nabla^2 \Psi + \partial_{\Psi^*} \mathcal{H}_{\text{int}}$$

( $\Psi = \langle \text{large} | \hat{\Psi} | \text{large} \rangle$  = mean field)

$$N \simeq n \lambda_{\text{dB}}^3 \sim n(mv)^{-3} \gg 1$$

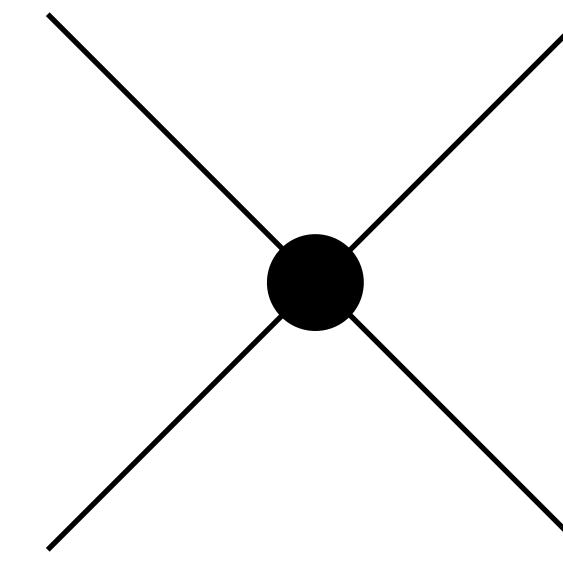
Gravity

$$(4\pi G m^2) \Psi^* \Psi \nabla^{-2} \Psi^* \Psi$$

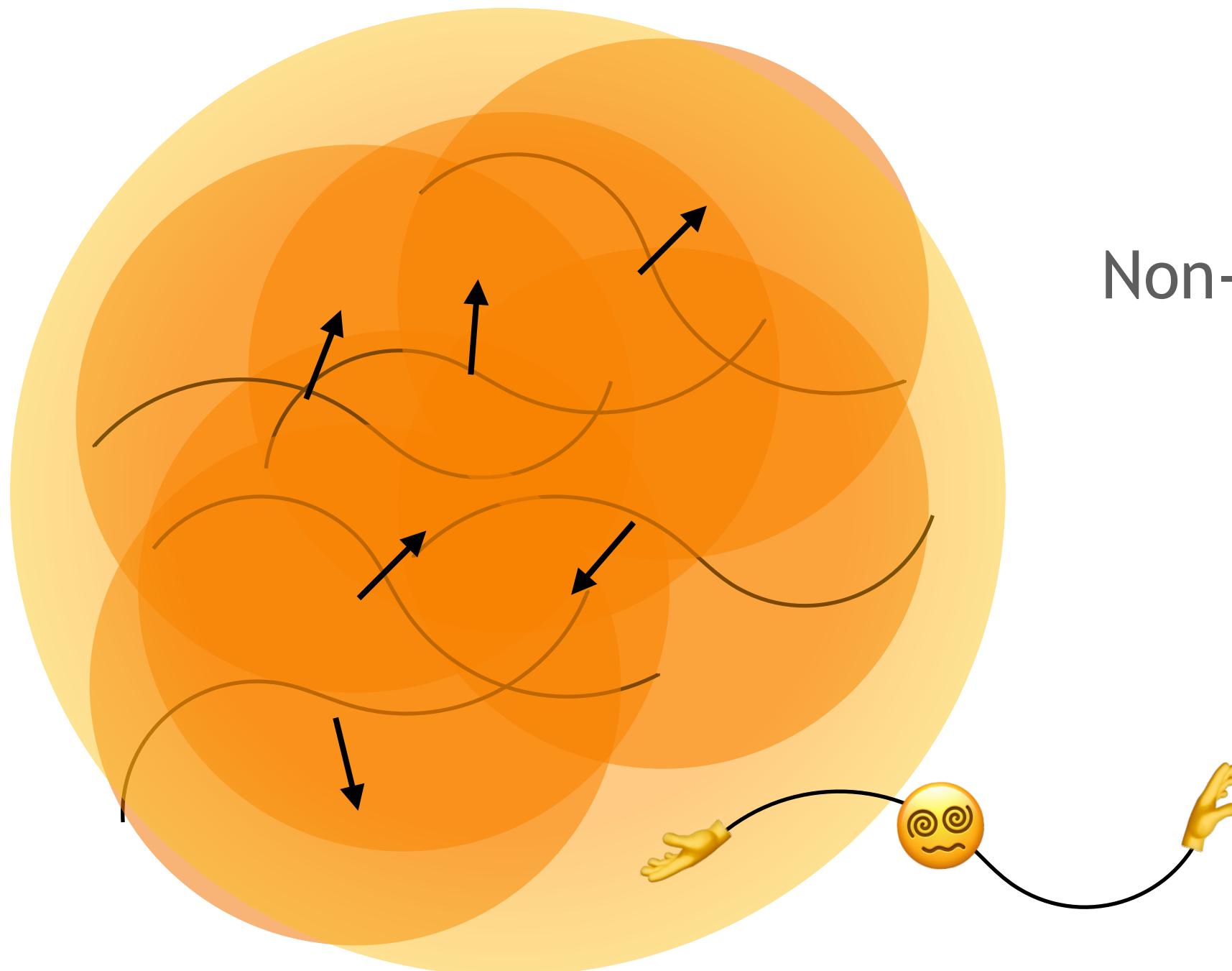


Short-range

$$\lambda |\Psi^* \Psi|^2$$



# Intrinsic spin



$$N \simeq n\lambda_{dB}^3 \sim n(mv)^{-3} \gg 1$$

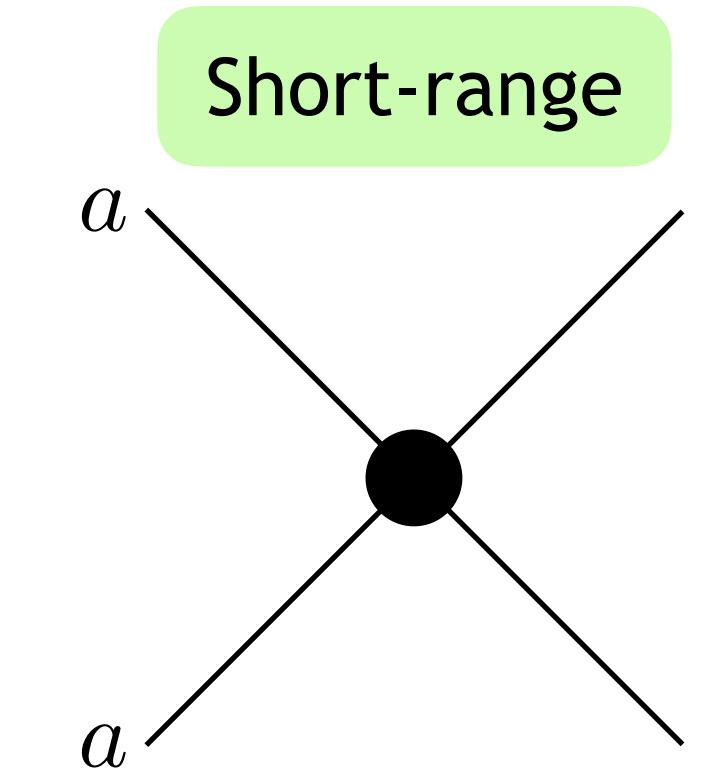
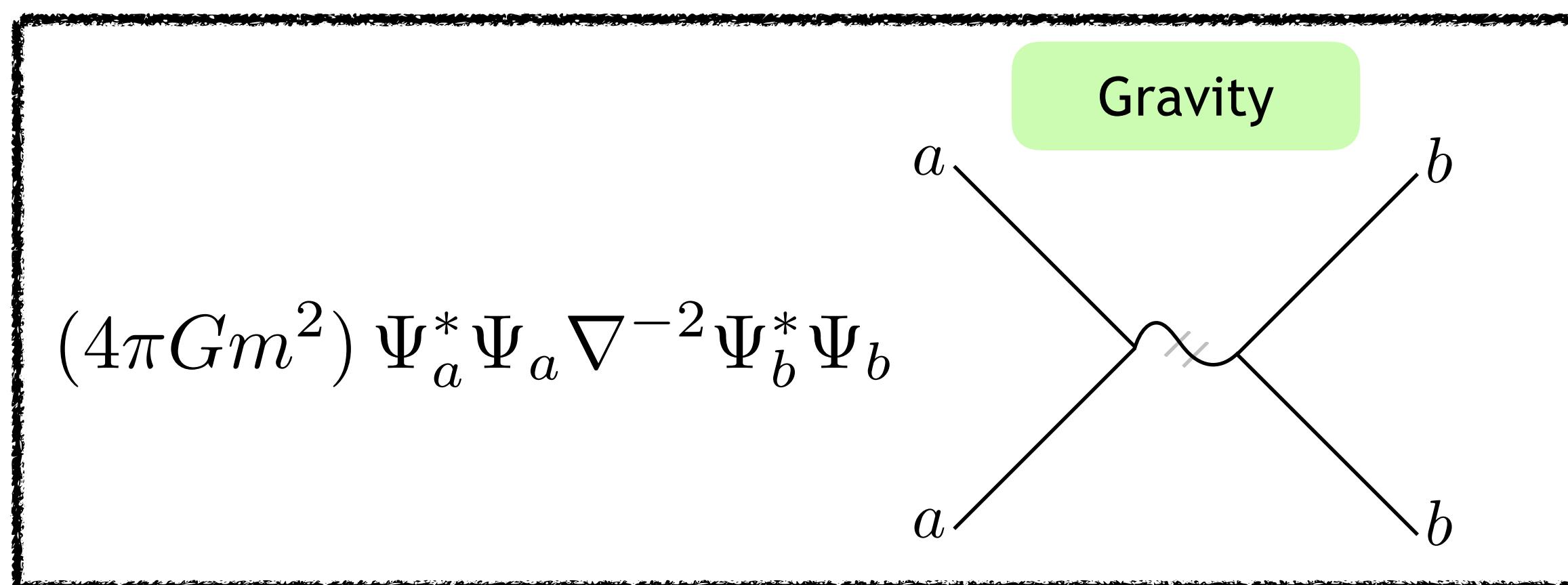
Non-linear Schrödinger / Gross-Pitaevski equation (wave equation)

$$i\frac{\partial\Psi_a}{\partial t} = -\frac{1}{2m}\nabla^2\Psi_a + \partial_{\Psi_a^*}\mathcal{H}_{\text{int}}$$

( $\Psi = \langle \text{large} | \hat{\Psi} | \text{large} \rangle = \text{mean field}$ )

$$\Psi = (\Psi_1, \Psi_2, \dots \Psi_{2s+1})$$

vector in the  $(2s+1)$ -dim Unitary irrep. of  $\text{SO}(3)$

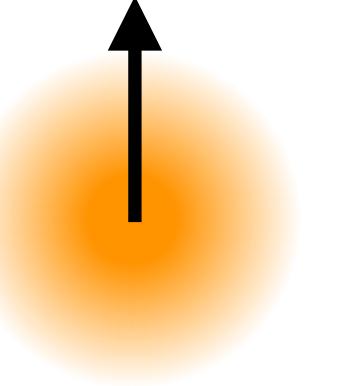


Spin matrices (3 of them)  
 $\downarrow$   
 $\lambda|\Psi_a^* \delta_{ab} \Psi_b|^2, \alpha |\Psi_a^* \hat{S}_{ab} \Psi_b|^2$   
 (from UV complete models such as Abelian Higgs for spin-1 ; relevant for DM)

+ Zeeman interactions + spin singlet interactions + spin orbit couplings, in the case of spinor BECs in lab.

# Spinor Condensates

(Macroscopic intrinsic spin!)



## Cosmos : (Ultra-)light bosonic dark matter

- Spin-0, e.g. axion or ALP
- **Spin-1, dark photon DM**
- Spin-2, bi-gravity DM

Kinetic relaxation of gravitating BECs

Reduced interference in DM halos → relaxed mass bound

Gravitational atoms,  
Black hole superradiance, etc.

## Laboratory : Ultra-cold atoms, AMO systems

- Spin 0 BECs, e.g. superfluid helium
- **Spinor BECs with hyperfine ↔ spin**

Zeeman interaction ; Effective monopoles → Dirac Strings

Spin textures, quantum spin hall effect,  
topological insulators, atomic lasers, etc.

# COSMOS

Spin-1 / Dark photon (ultra-)light dark matter

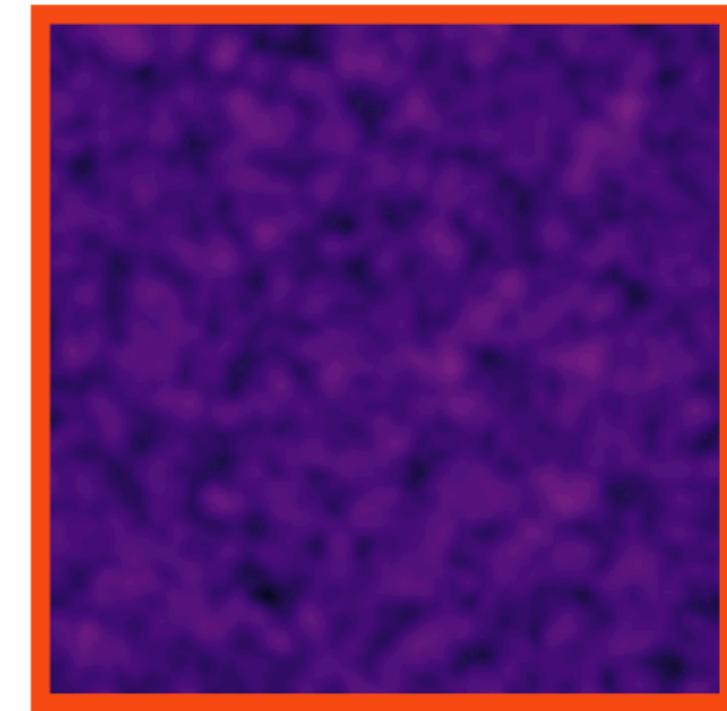
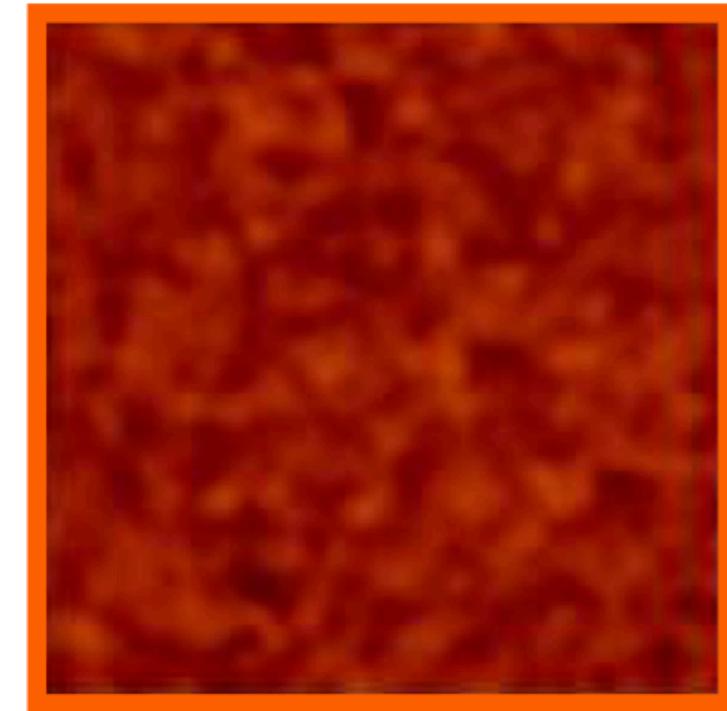
# Kinetic condensation of (spin-1) gravitating BECs / solitons

[arXiv:2304.01985](https://arxiv.org/abs/2304.01985)

MJ, Amin, Thomas, Wanichwecharungruang

$$i \frac{\partial \Psi_a}{\partial t} = -\frac{1}{2m} \nabla^2 \Psi_a + m\Phi\Psi_a$$

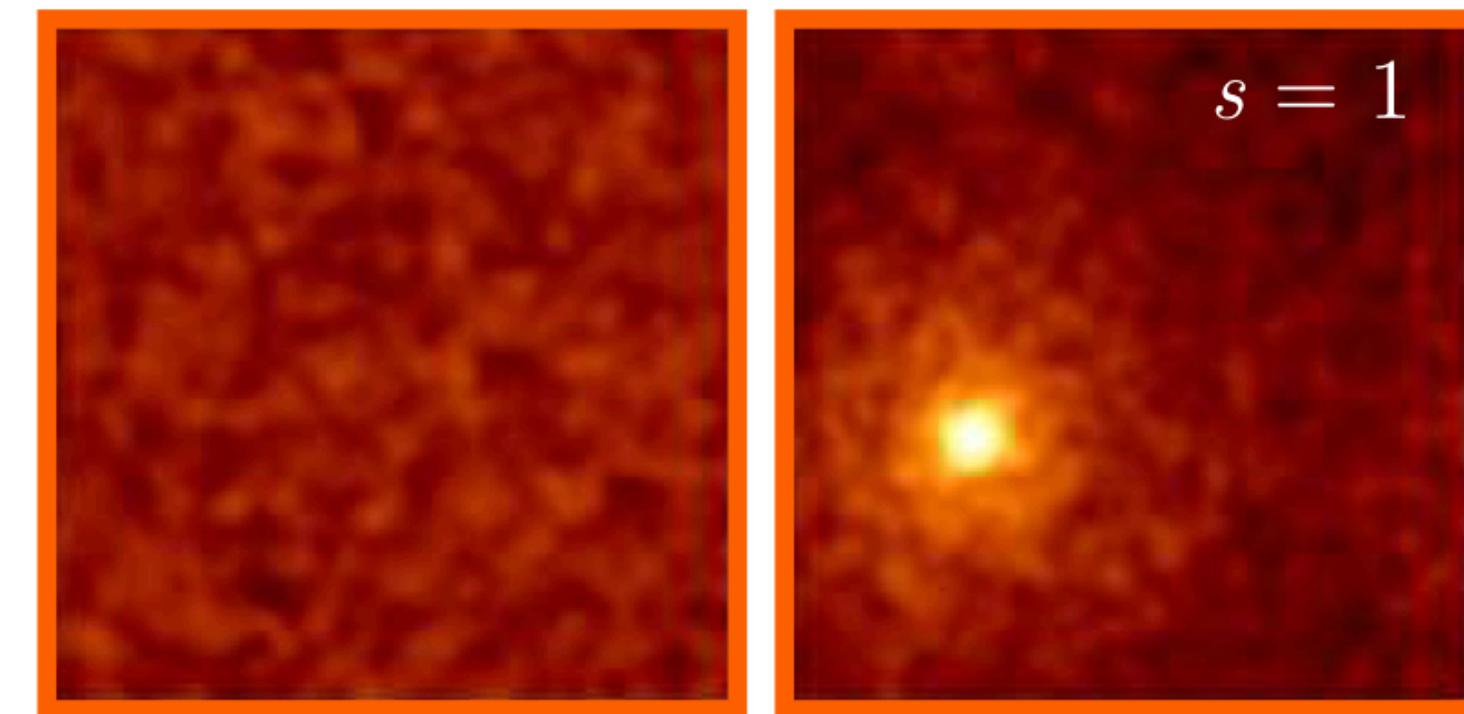
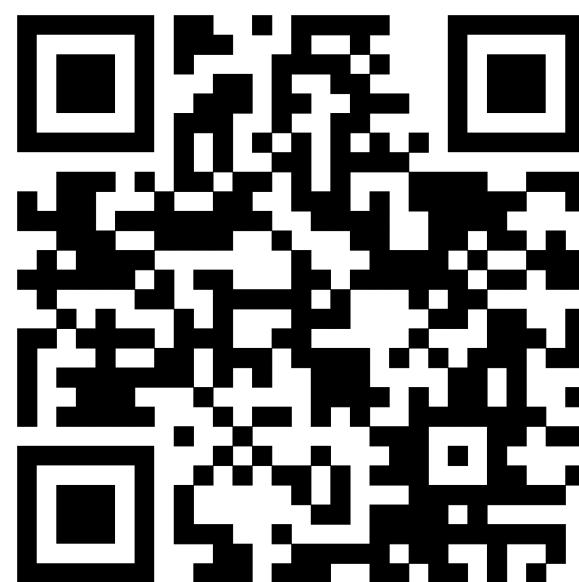
$\Phi$  = Newtonian Potential



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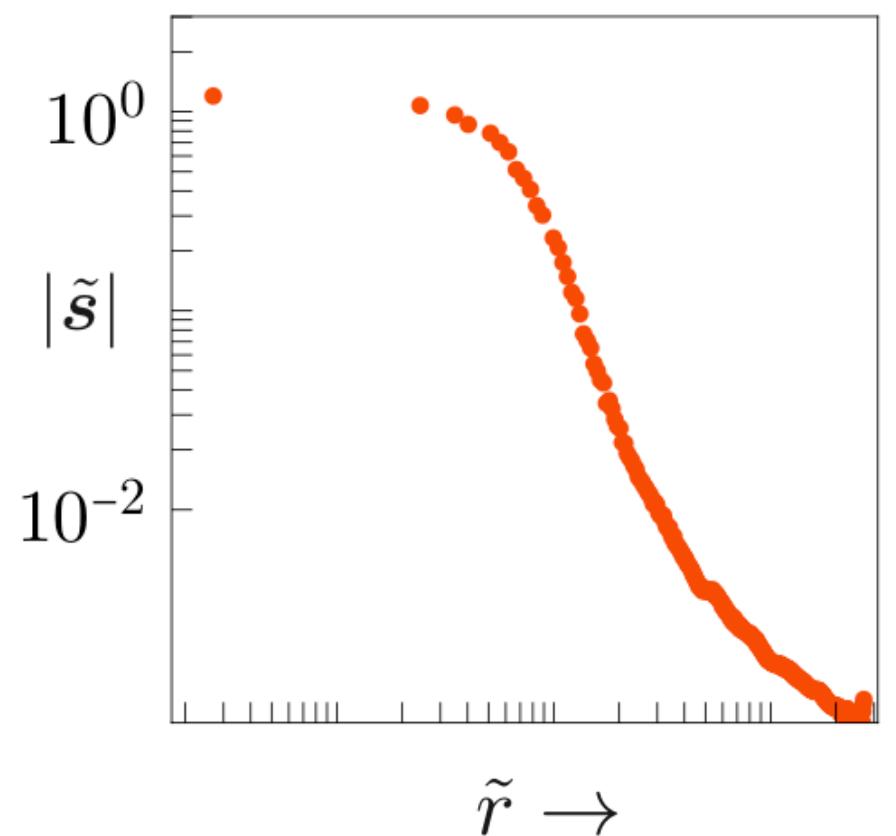
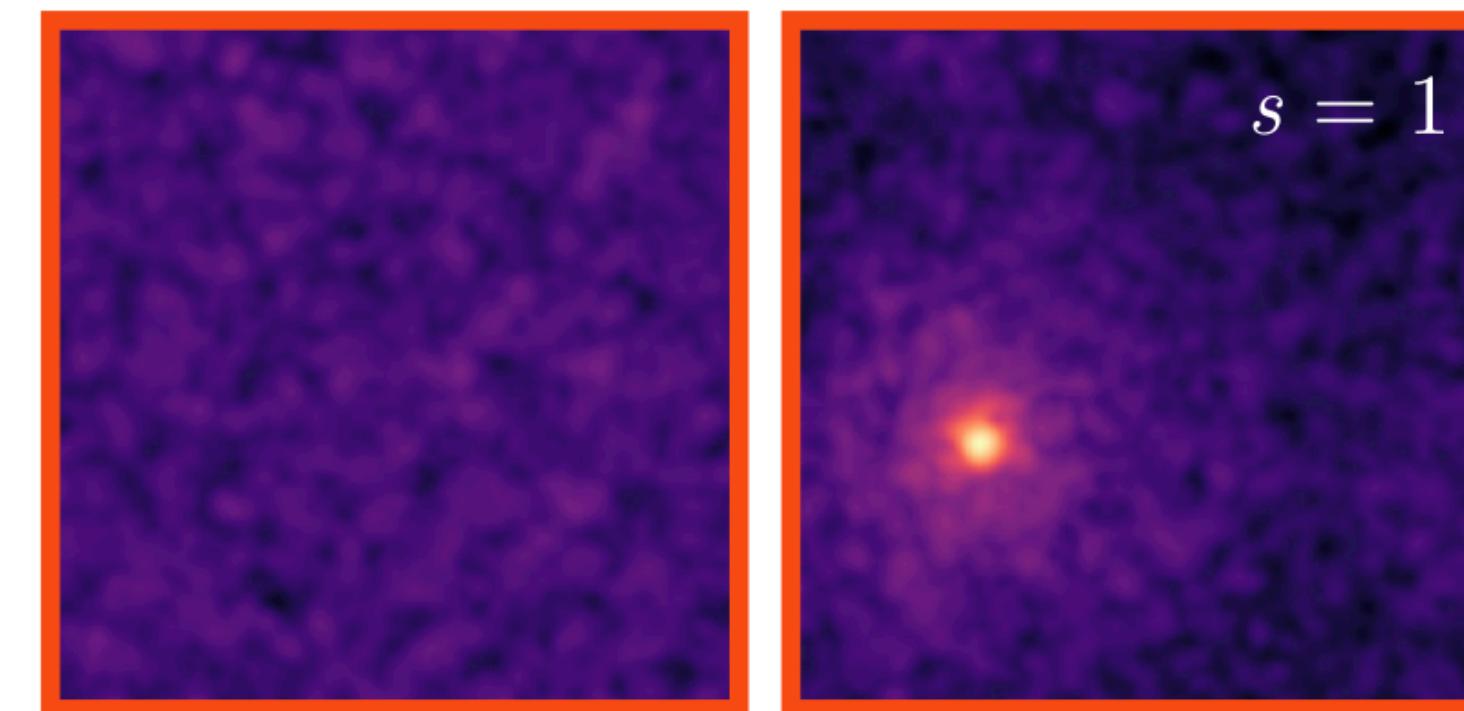
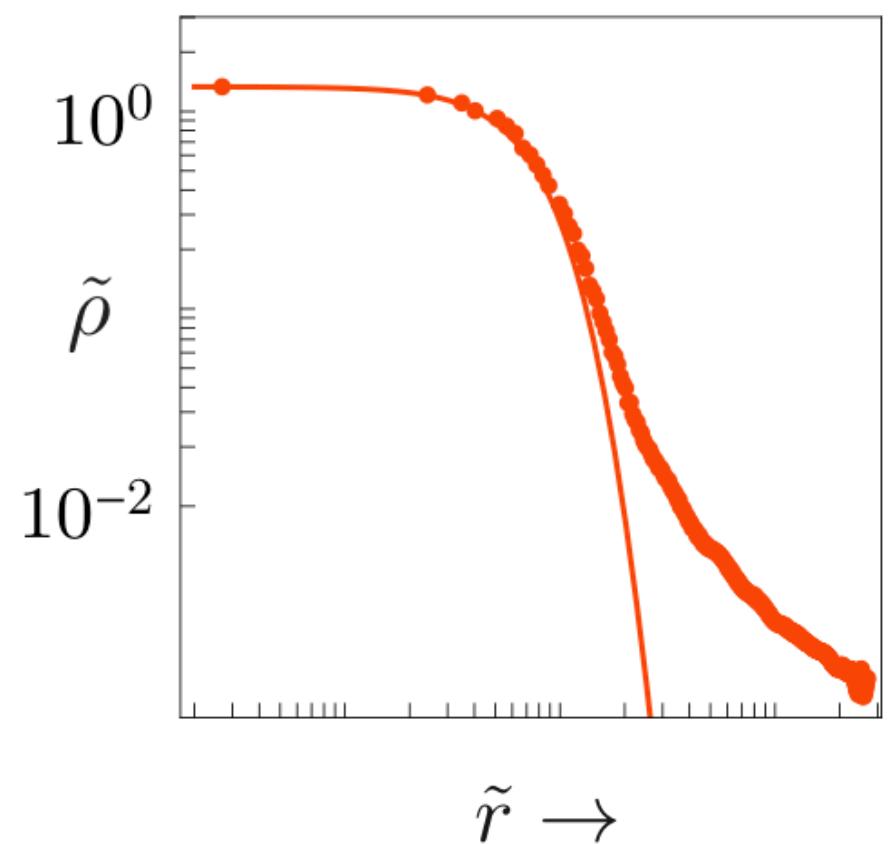
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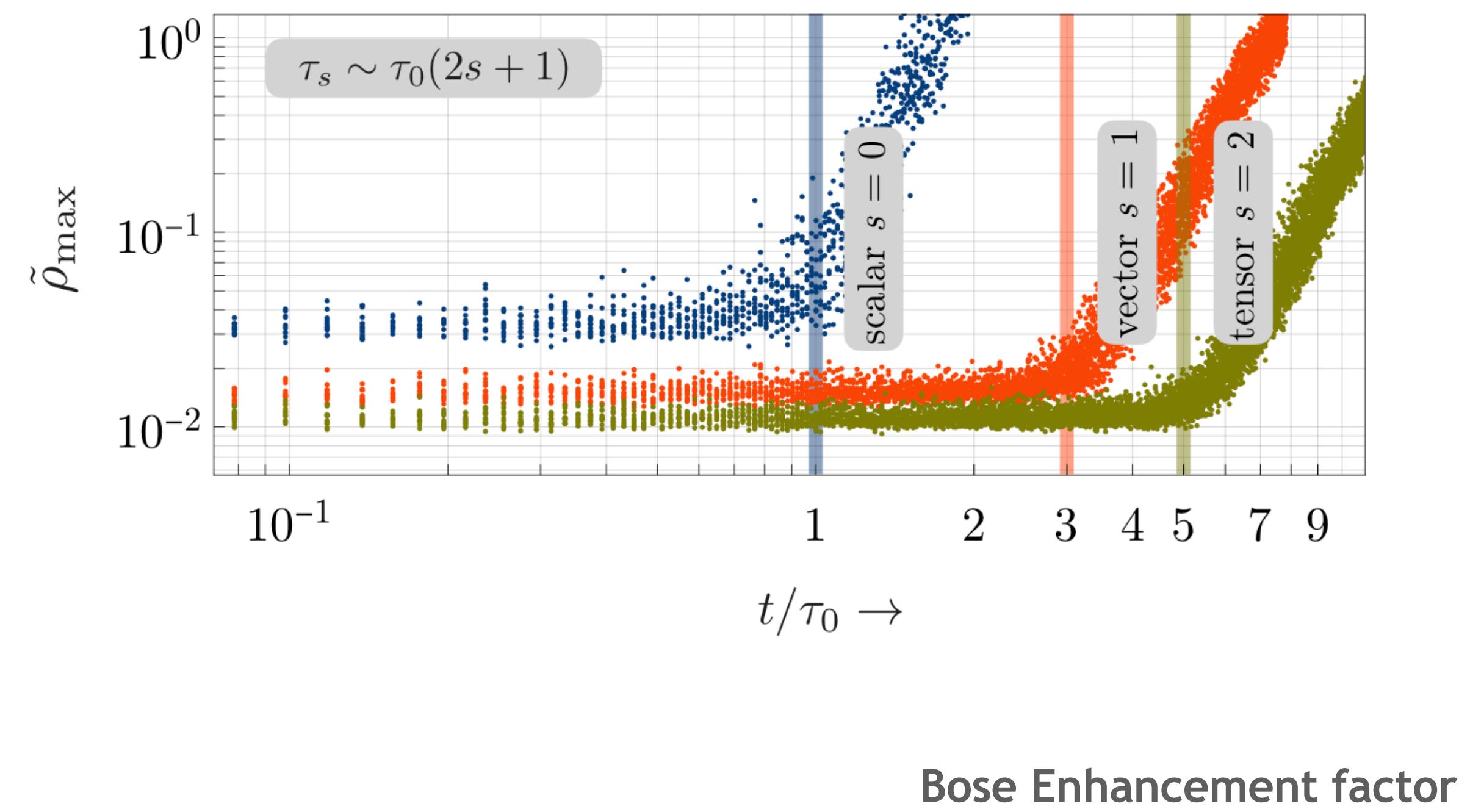
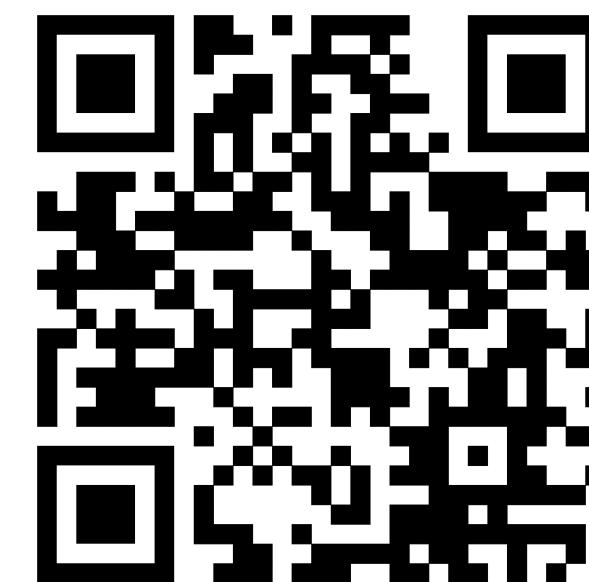
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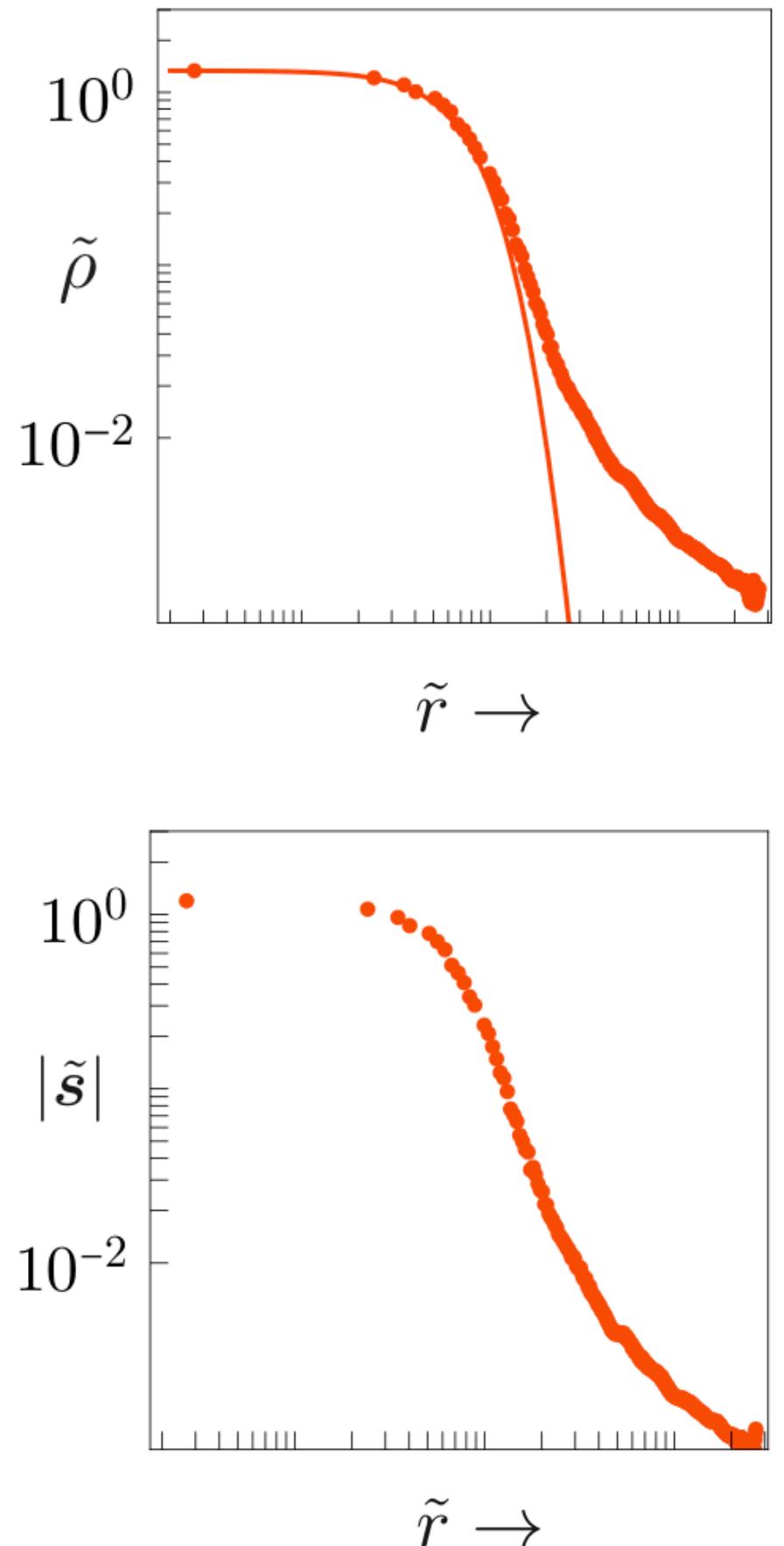
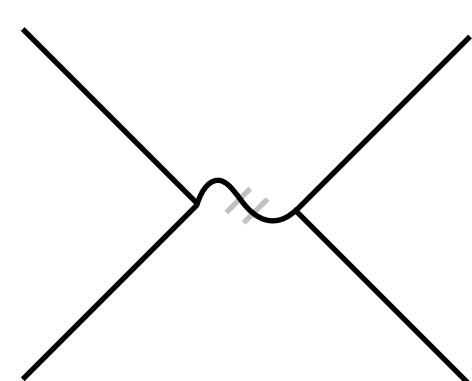
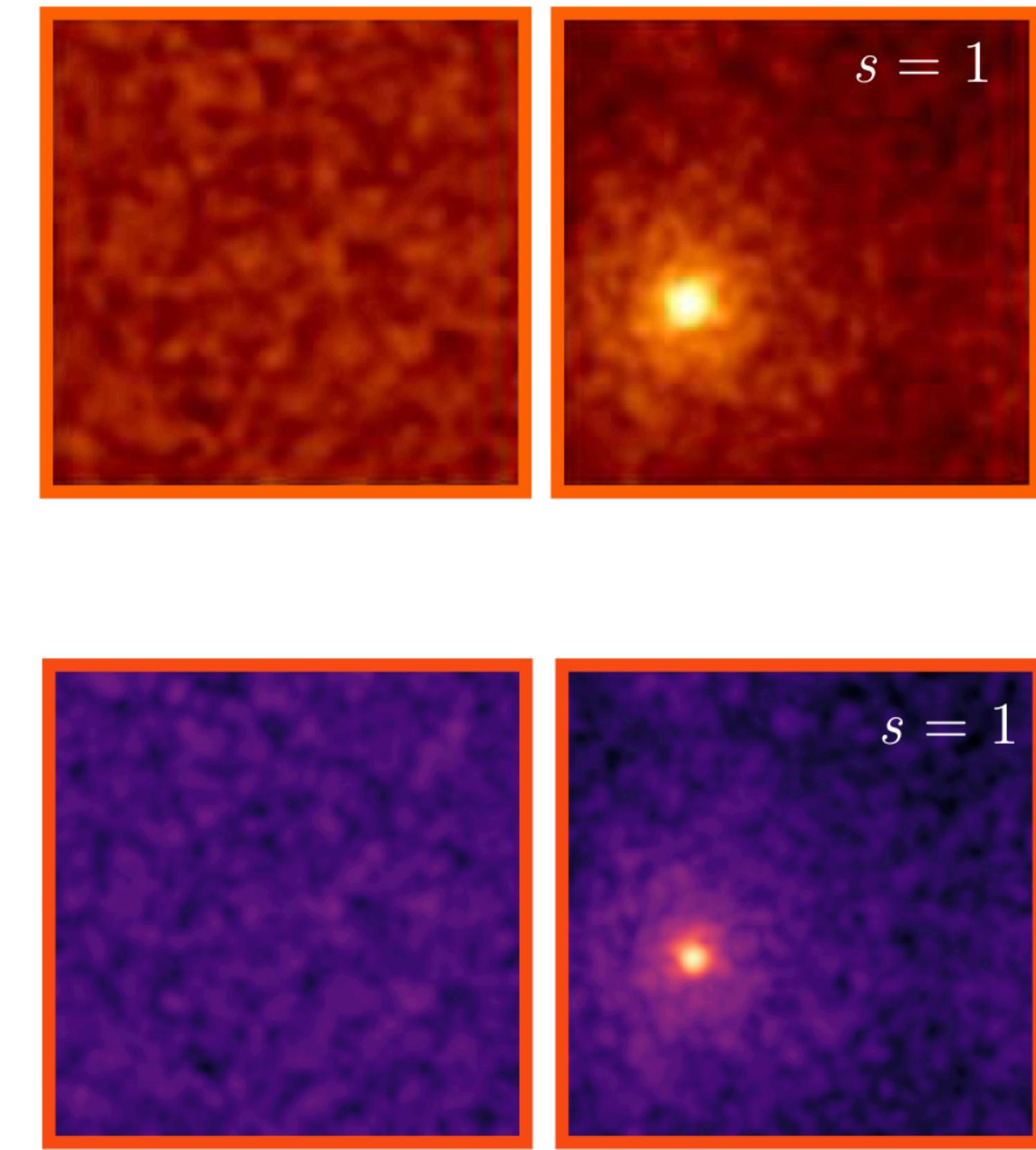
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$\Phi$  = Newtonian Potential



- Time of condensation  $\tau_{gr} \simeq \sum n \langle \sigma v \rangle F \simeq (2s+1) \frac{m^3 v^6}{\Lambda (4\pi G)^2 \bar{\rho}^2}$
- Macroscopic intrinsic spin / polarized solitons



- also see [Levkov et al](#) , [Chen et al](#)

# Small scale structure in (ultra-)light spin-1 dark matter

JCAP 08 (2022) 08, 014

Amin, MJ, Karur, Mocz

$$i \frac{\partial \Psi_a}{\partial t} = -\frac{1}{2m} \nabla^2 \Psi_a + m\Phi\Psi_a$$

$\Phi$  = Newtonian Potential

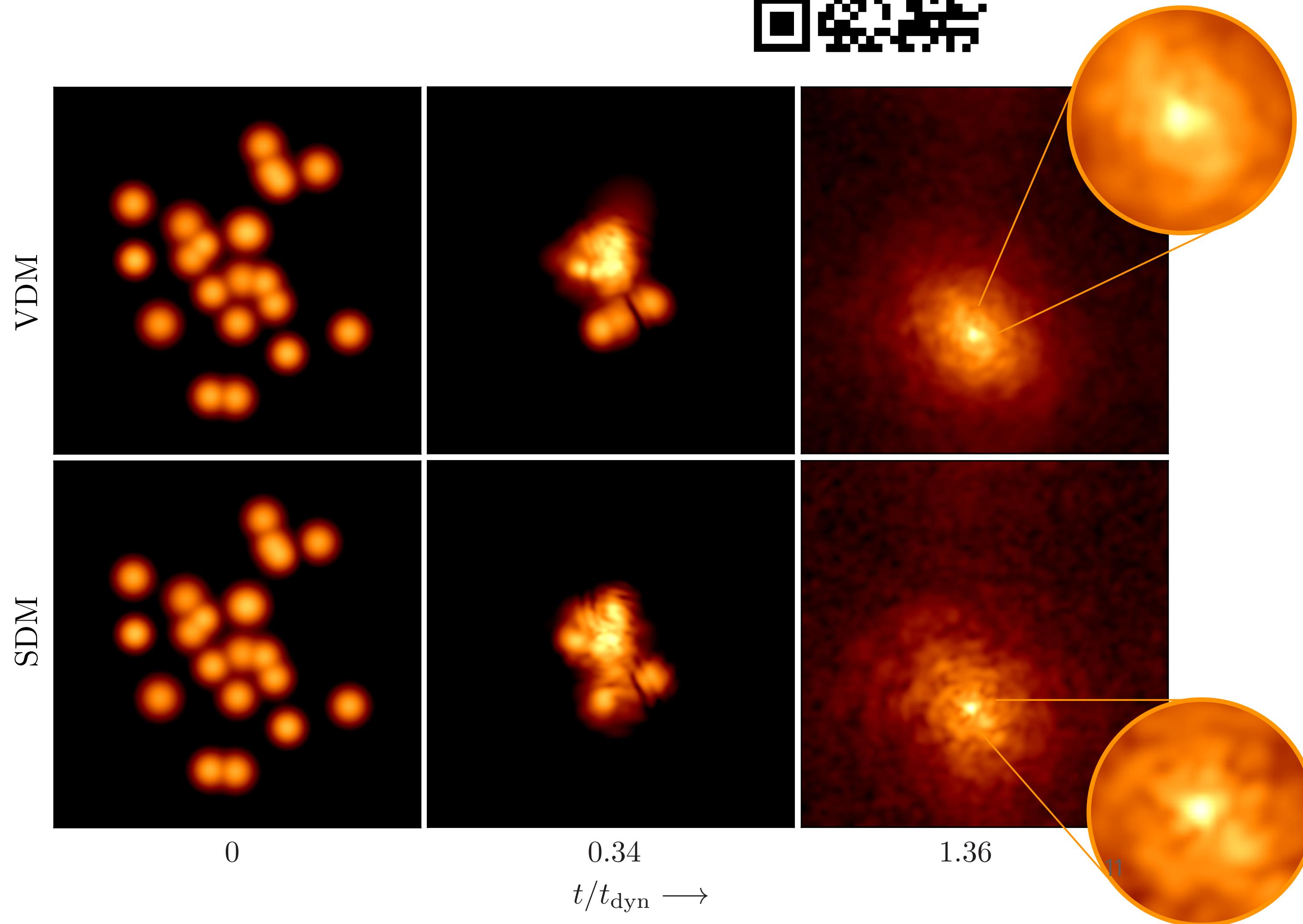
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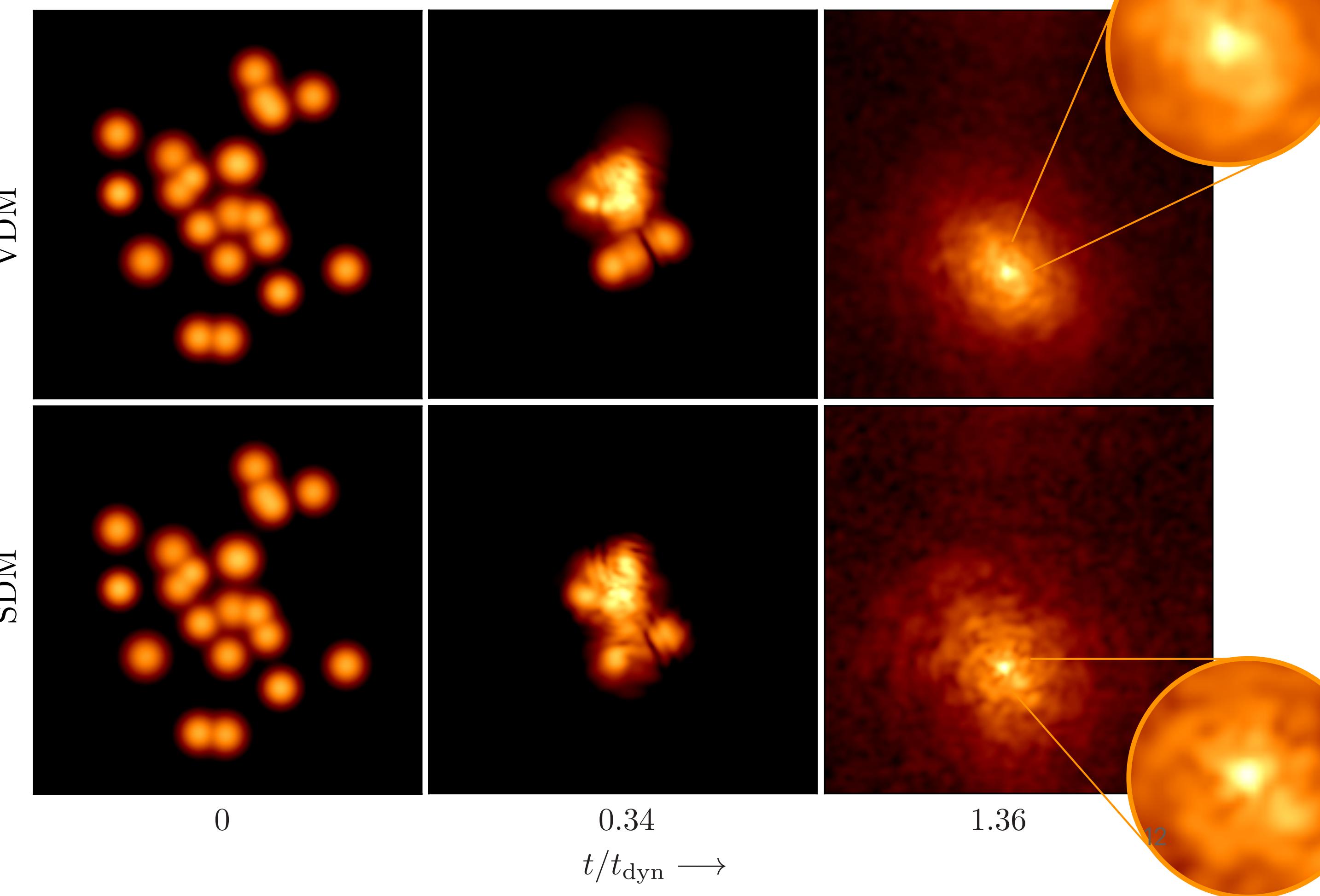
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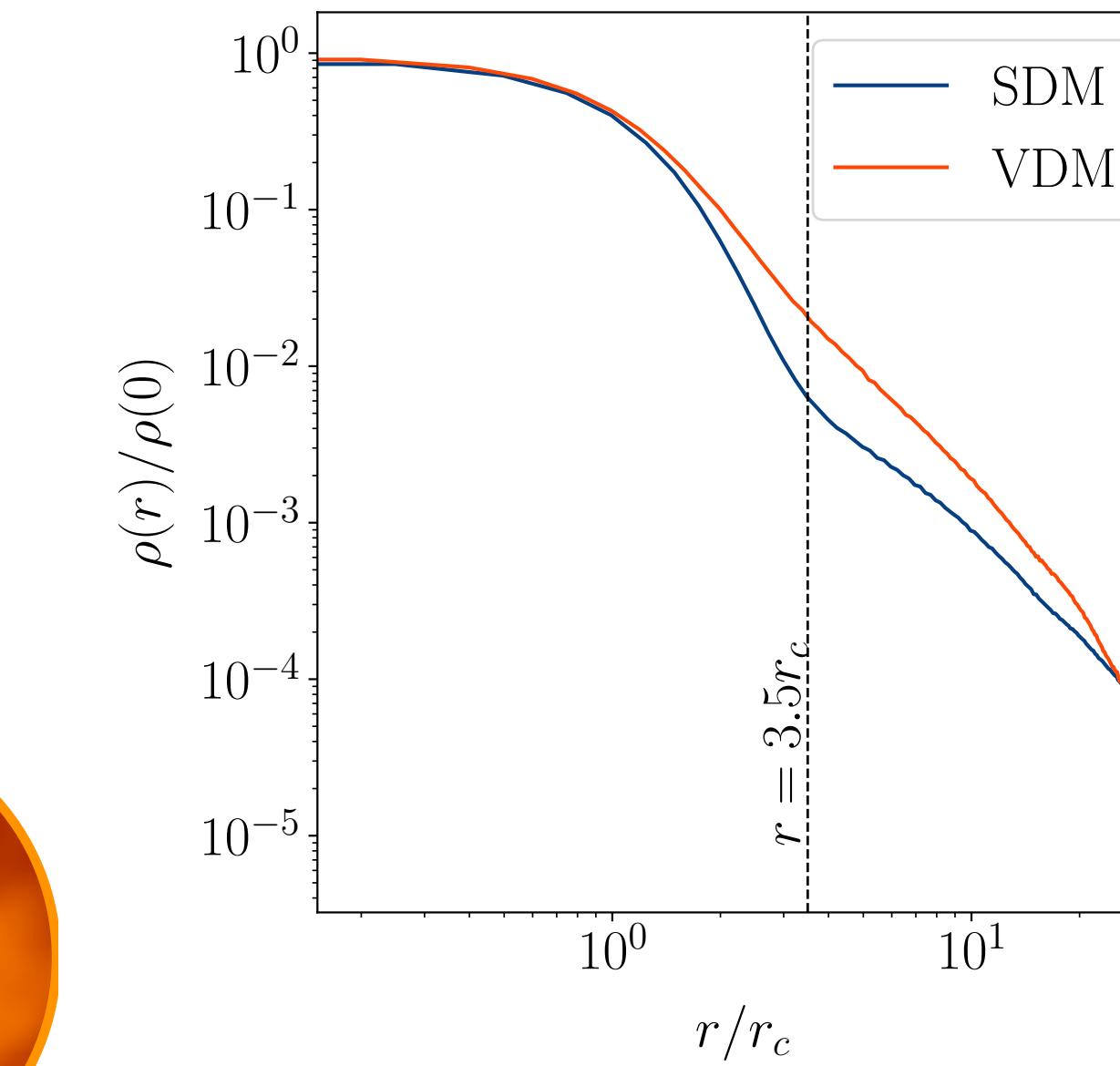
- less interference in VDM

$$\rho = \rho_1 + \rho_0 + \rho_{-1}$$

$$m \gtrsim 3 \times 10^{-19} \text{ eV}/\sqrt{3}$$

- VDM Halo cores can have huge spin

- Smoother transition to  $r^{-3}$



- also see [Gorghetto et al](#)

# SUMMARY

- Spinor BECs are naturally (and in many respects necessarily) present in the spectrum of non-relativistic integer spin field theories with long and/or short range interactions.
  - Huge macroscopic Spin → various phenomenological avenues (coupling to SM fields → EM signatures etc.)
    - e.g. see [Amin, Long, Schiappacasse](#)
  - Spinor BECs in Lab: Interesting applications, such as Dirac strings, spin textures, topological insulators etc.
- In the context of fuzzy dark matter
  - Kinetic relaxation of gravitating BECs. For dwarf galaxies with  $m \sim 10^{-20}\text{eV}$  ,  $\tau_{gr} \sim H_0^{-1}$
  - Reduced interference in DM halos → Less heating of stars → relaxed mass bound
$$m \gtrsim 3 \times 10^{-19}\text{eV}/\sqrt{3}$$

# LABORATORY

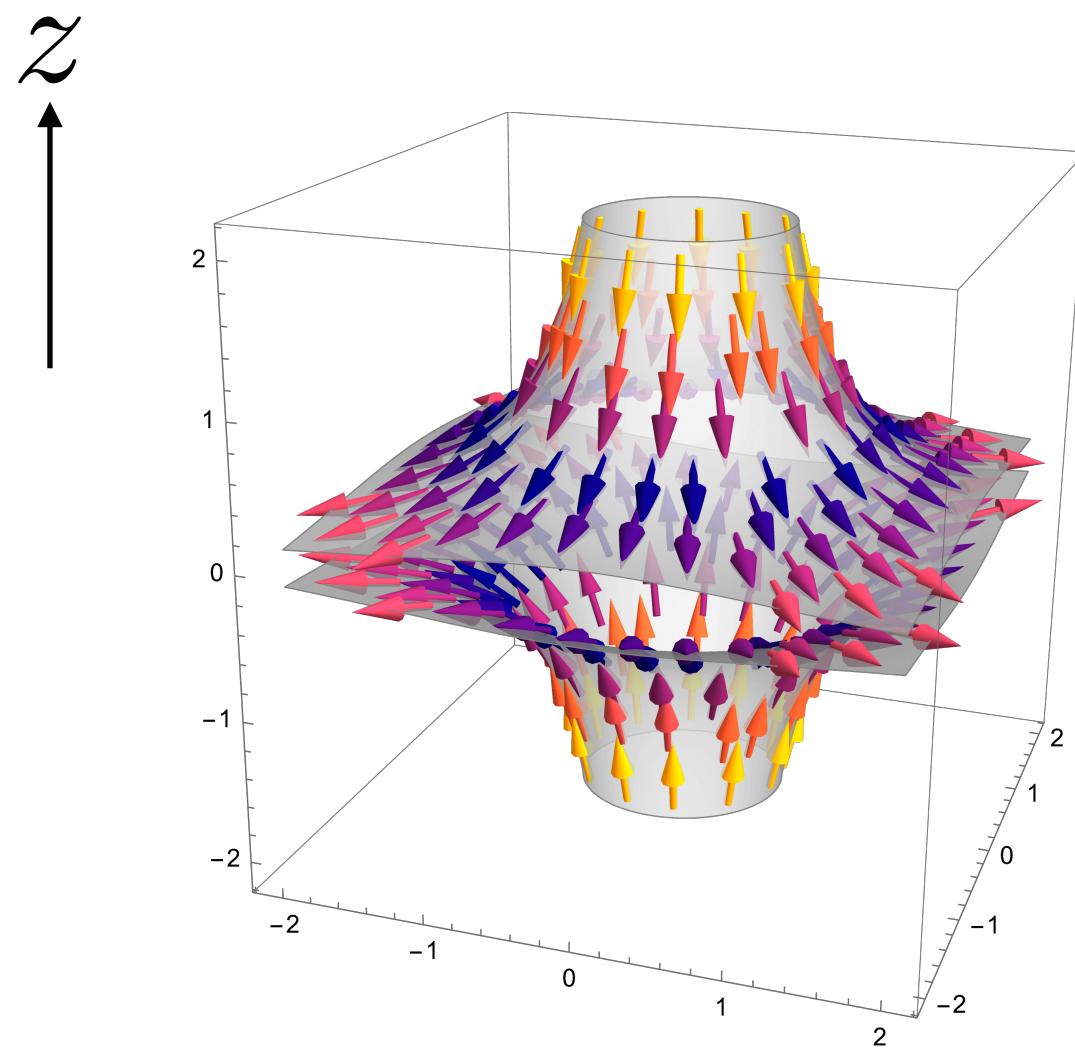
Relevant for ultra-cold atomic gases

Hyperfine levels <-> spin multiplicity levels

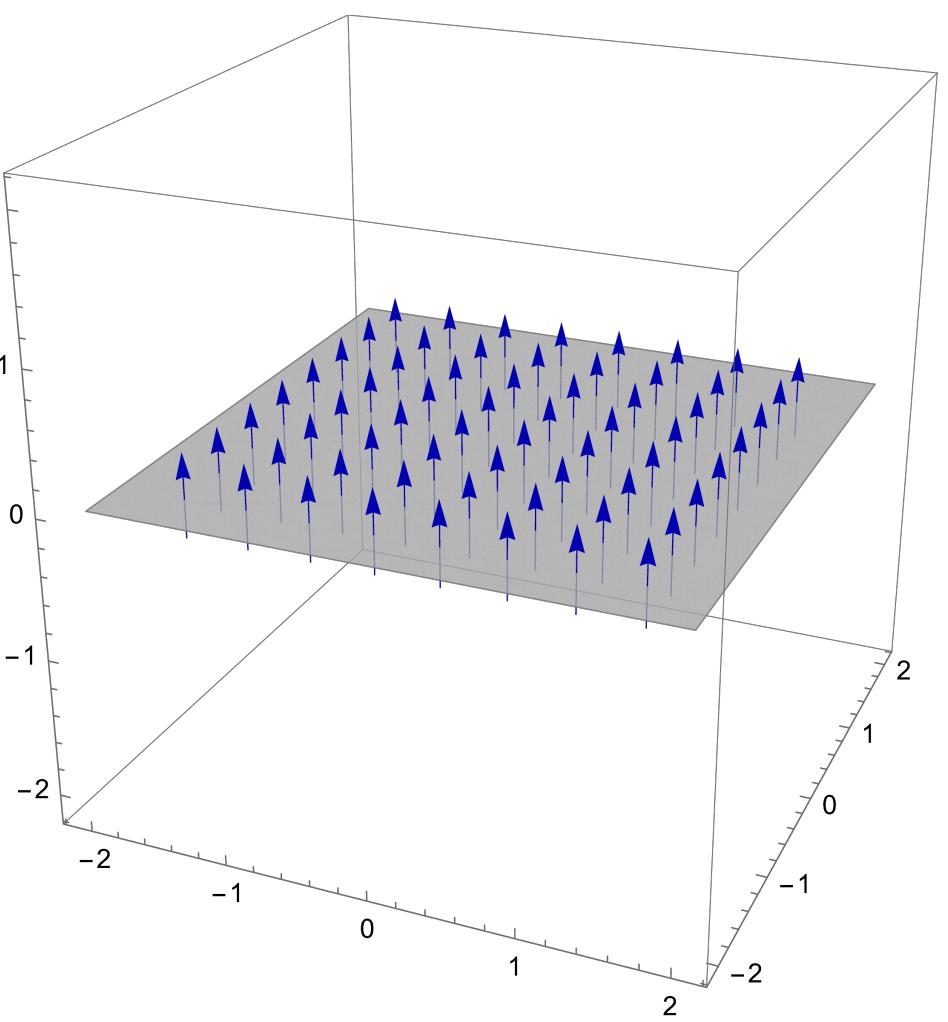
# Synthetic monopole & Dirac Strings

[Phys. Rev. Lett. 120, 130402 \(2018\)](#) Zhou, Wu, Guo, Wang, Pu, and Zhou

[arXiv:2305.01675](#) MJ, Amin, and Pu



Oscillating quadrupolar magnetic field



Constant bias field



Rotating frame

$$\omega = \omega_L$$

$$\vec{B} = B_0 \vec{r}$$

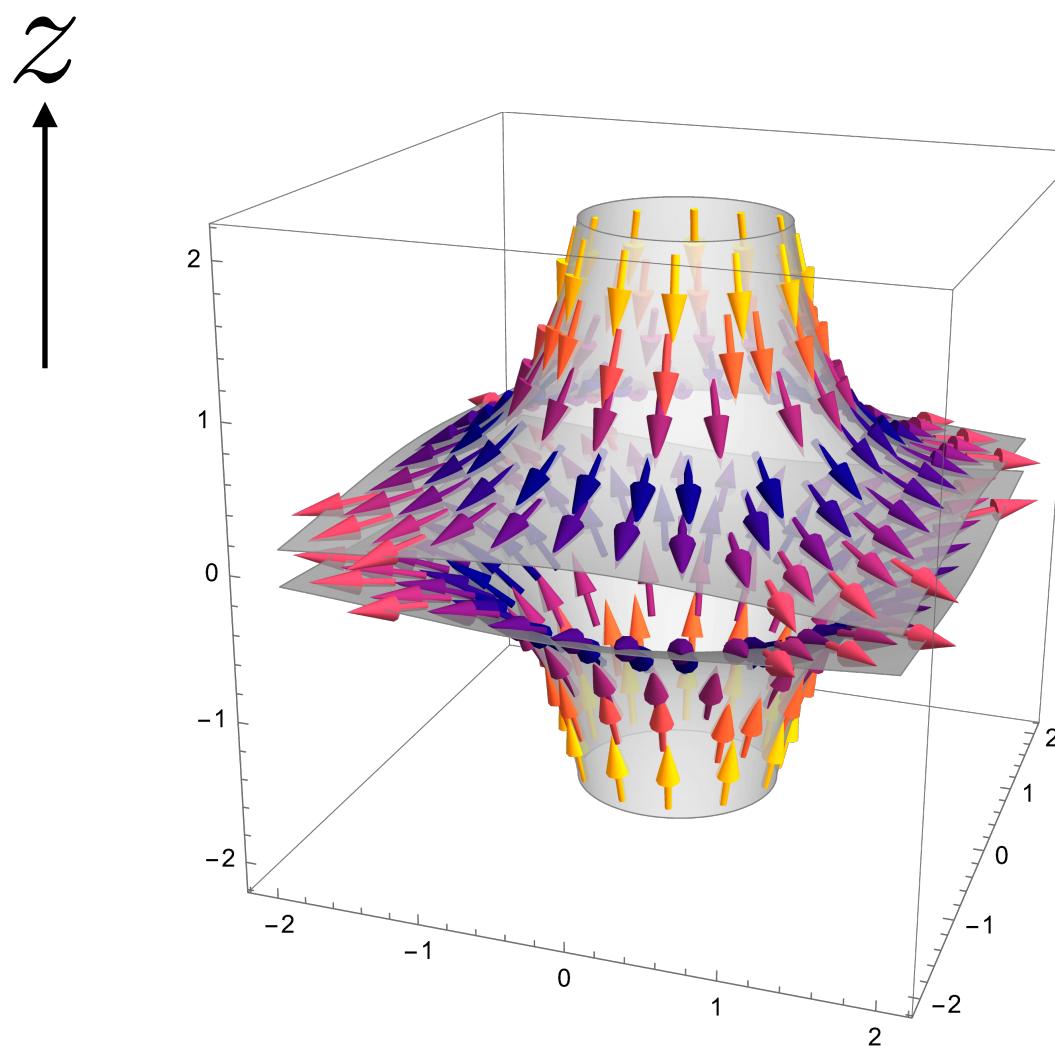
Zeeman term

$$\vec{B} \cdot \vec{S} = B_0 \Psi_a^* (\vec{r} \cdot \hat{S}_{ab}) \Psi_b$$

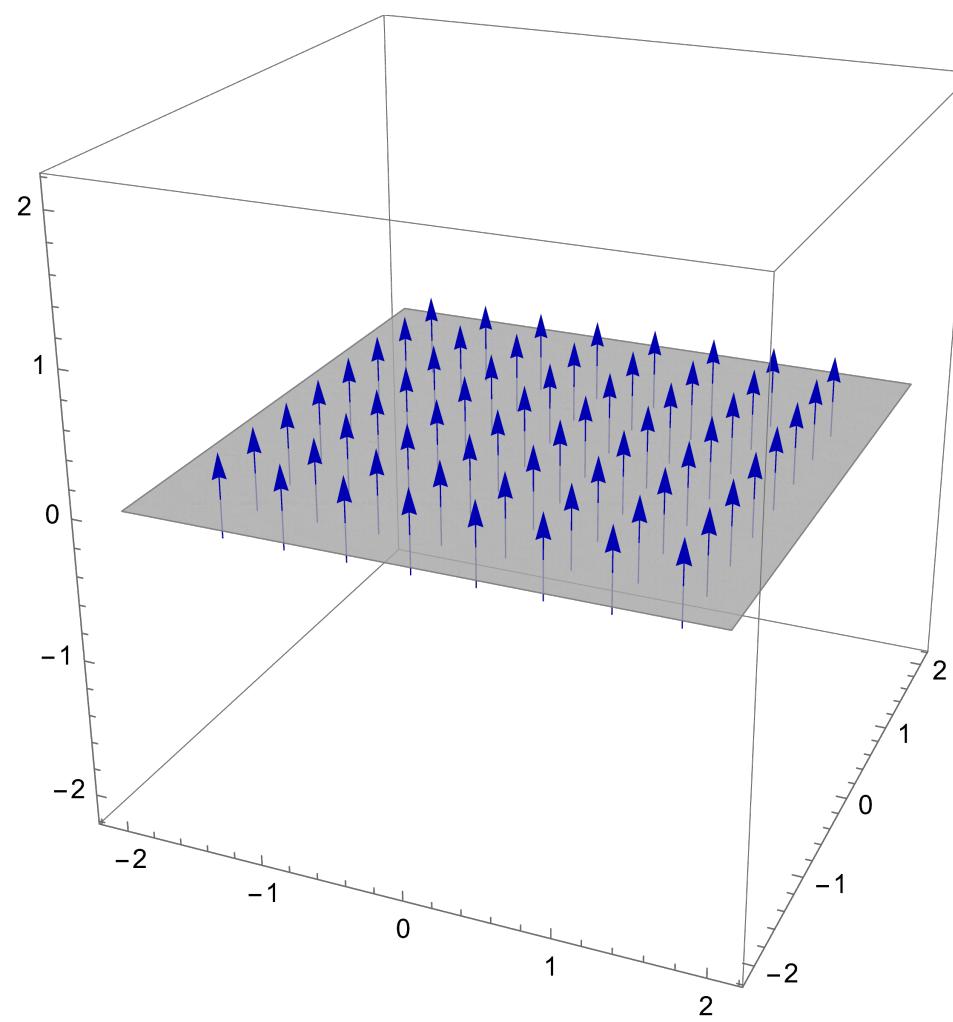
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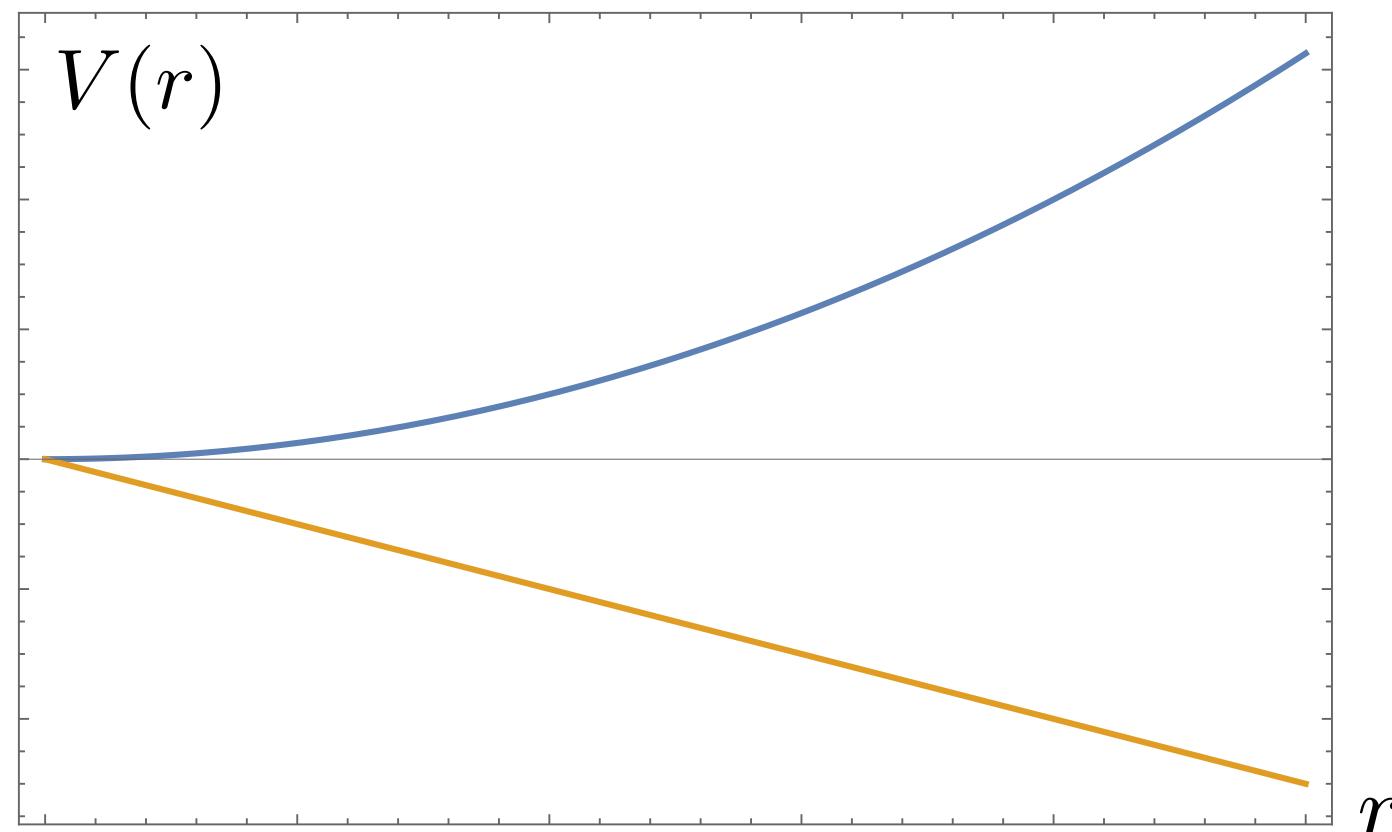
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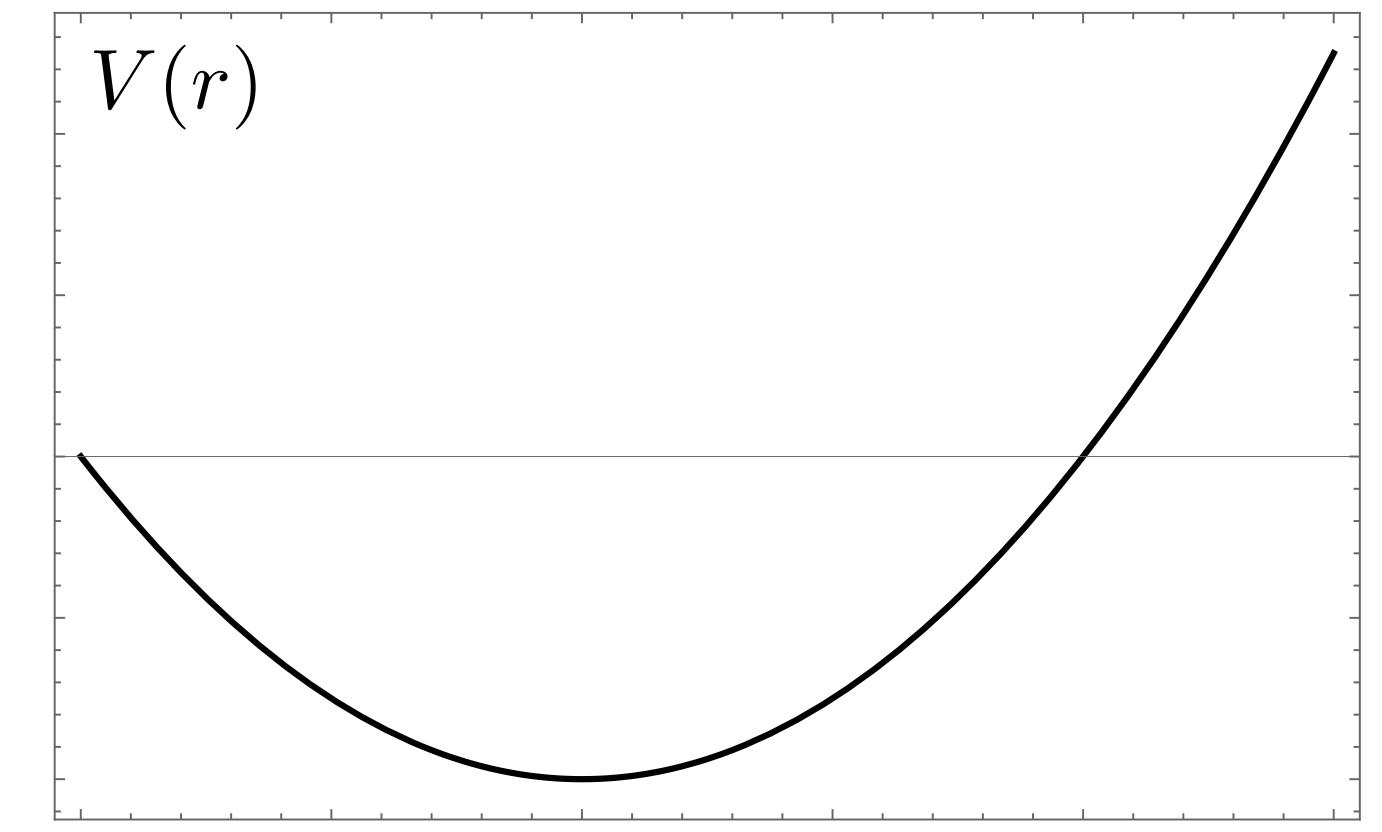
$$\Psi_a = \phi \chi_a \quad \leftarrow m_s = -1 \text{ radially}$$



— Harmonic confining potential

+

— Effective magnetic field

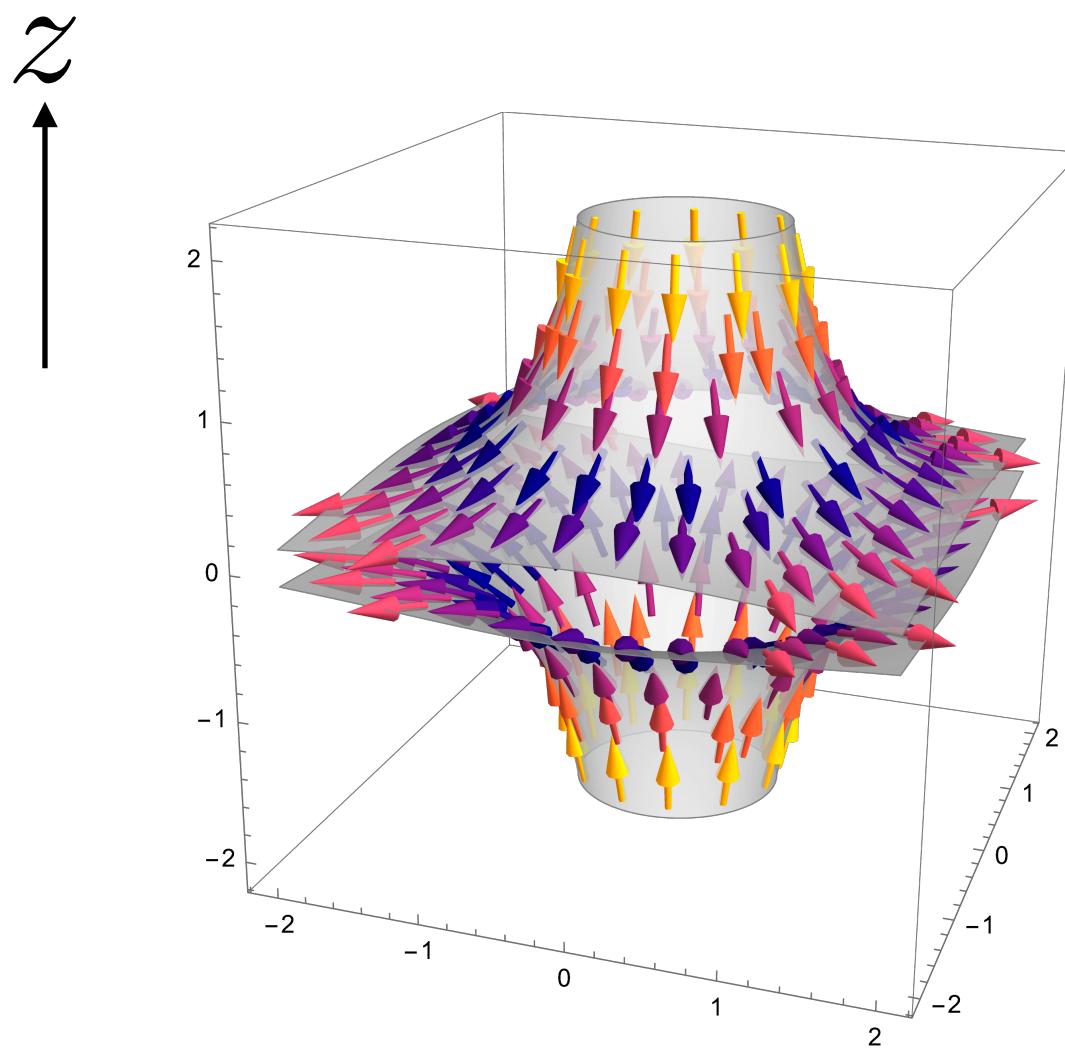


Displaced effective minima

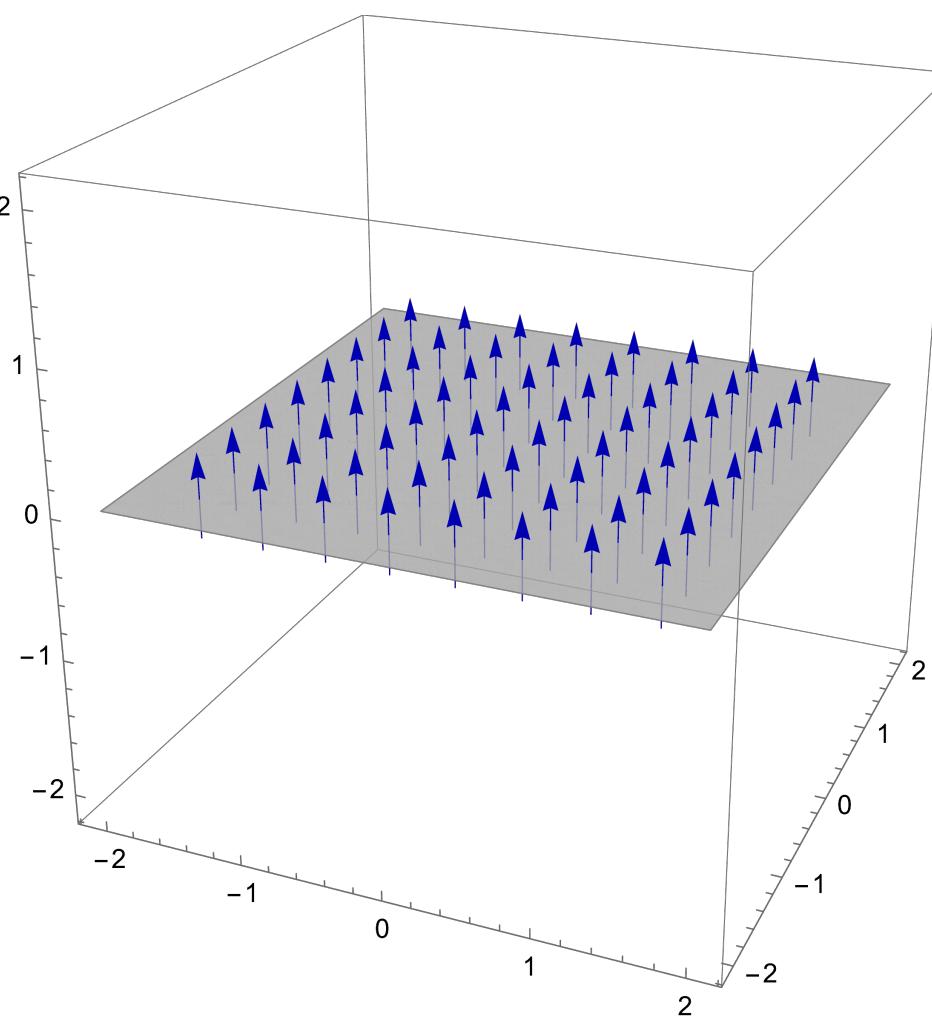
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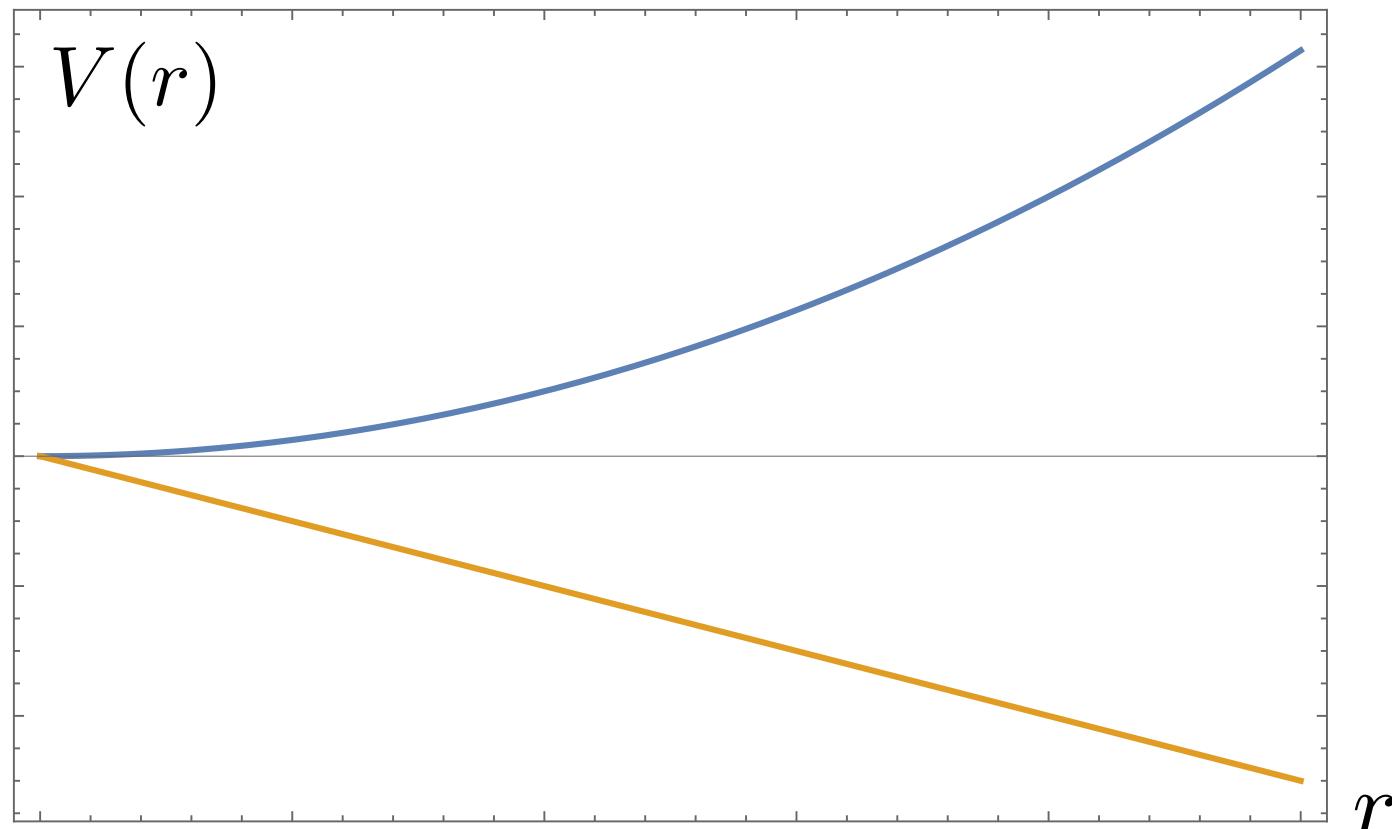
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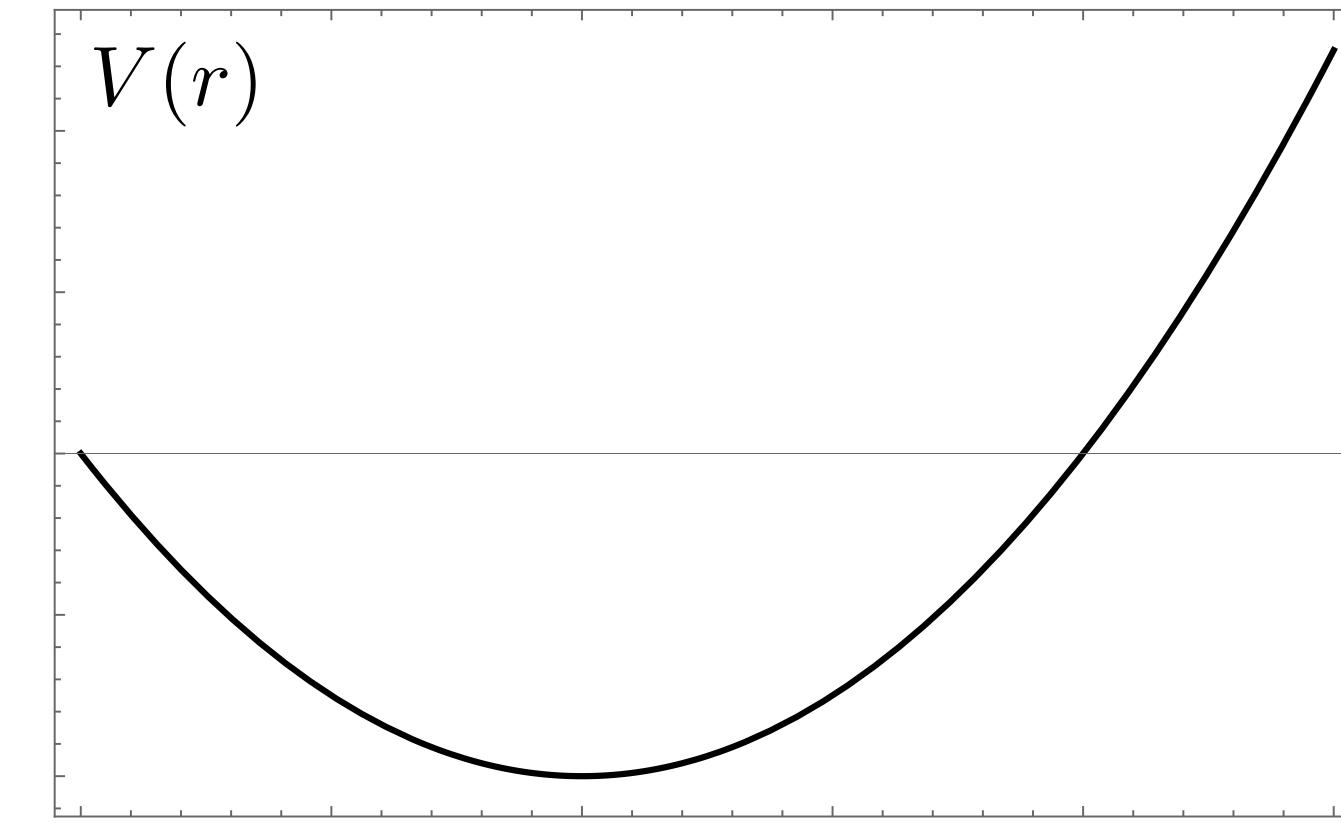
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— Harmonic confining potential

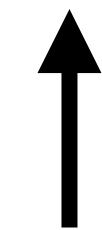
+

— Effective magnetic field



Displaced effective minima

$$\mathcal{H} \approx \frac{1}{2m} \phi^* \left( \nabla - im_s \frac{\cot \theta}{r} \hat{\varphi} \right)^2 \phi + V(r)$$



Synthetic monopole

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$$\mathcal{H} \approx \frac{1}{2m} \phi^* \left( \nabla - im_s \frac{\cot \theta}{r} \hat{\varphi} \right)^2 \phi + V(r)$$

Synthetic monopole

$$+ \frac{1}{m^2} \rho \left( \lambda + \alpha m_s |m_s| \right) \rho$$

↑  
density-density      ↑  
spin-spin

