The Primordial Black Holes that Disappeared: Connections to Dark Matter and MHz-GHz Gravitational Waves

Thomas C. Gehrman

The UNIVERSITY of OKLAHOMA

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With Barmak Shams Es Haghi, Kuver Sinha, and Tao Xu

Motivation

- First order phase transition (FOPT) and curvature perturbation comparison and dependencies
- Generate dark matter (DM) relic abundance and standard model (SM) particles from PBH Hawking evaporating before BBN
- Light PBHs are difficult to detect ($<10^9$g) since they Hawking evaporate before BBN, but High frequency gravitational waves (GW) serves as a new channel for detection
- Determine PBH origin from either mechanism using High frequency stochastic GW signals

PBH Mass Range & GW Signal Range

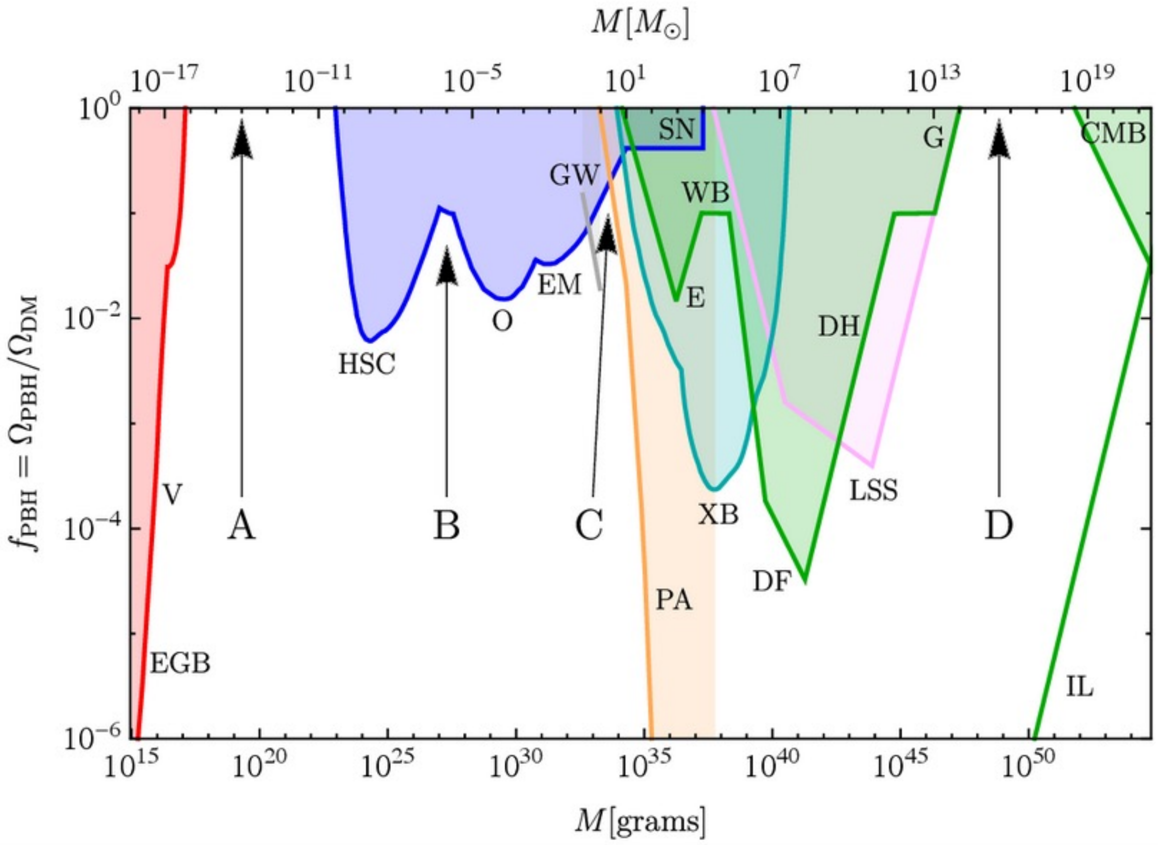
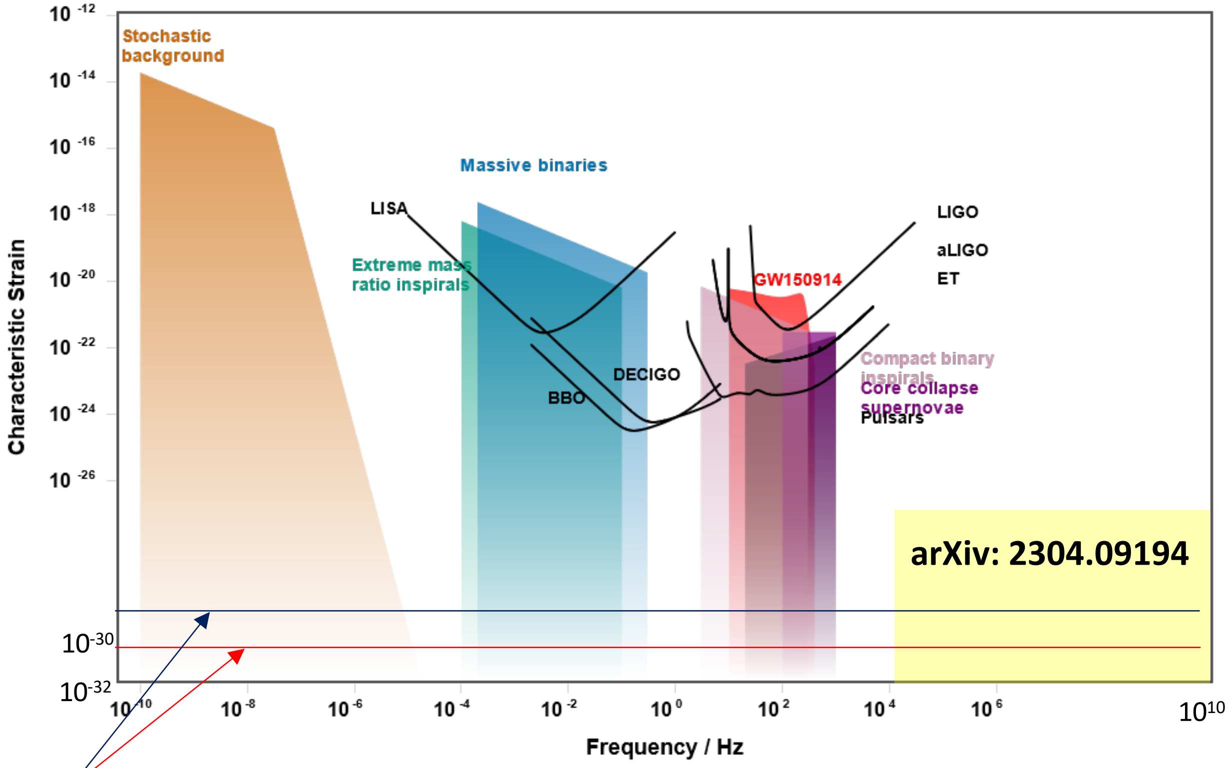


Image: Carr and Kuhnel 2020



Enhanced Magnetic Conversion (projected sensitivity)

Image credit: http://gwplotter.com / S. Rao et al. / K. Oide, G. Franchetti & F. Zimmermann / P. Chen.

Primordial Black Holes from DM First Order Phase Transition

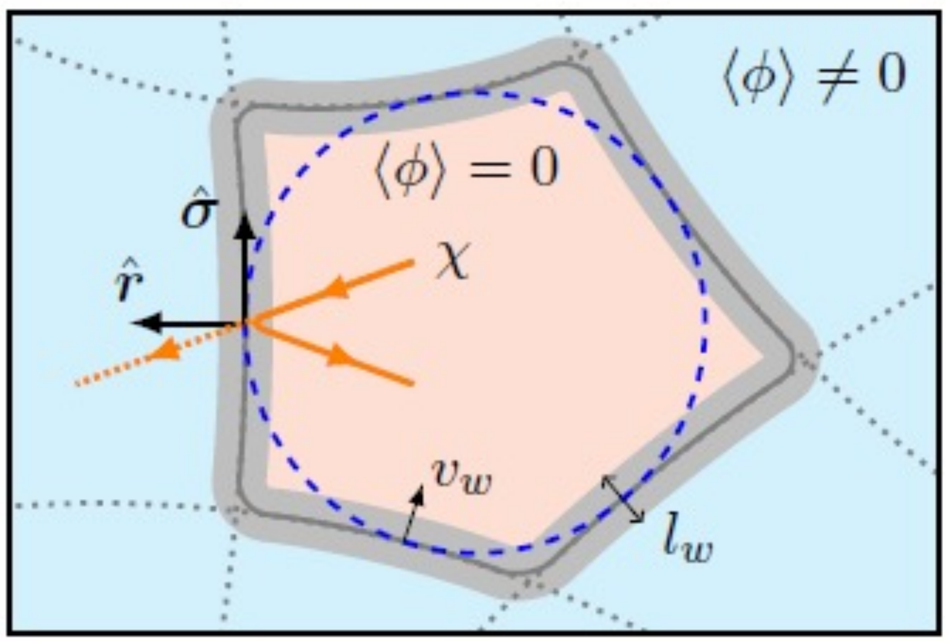
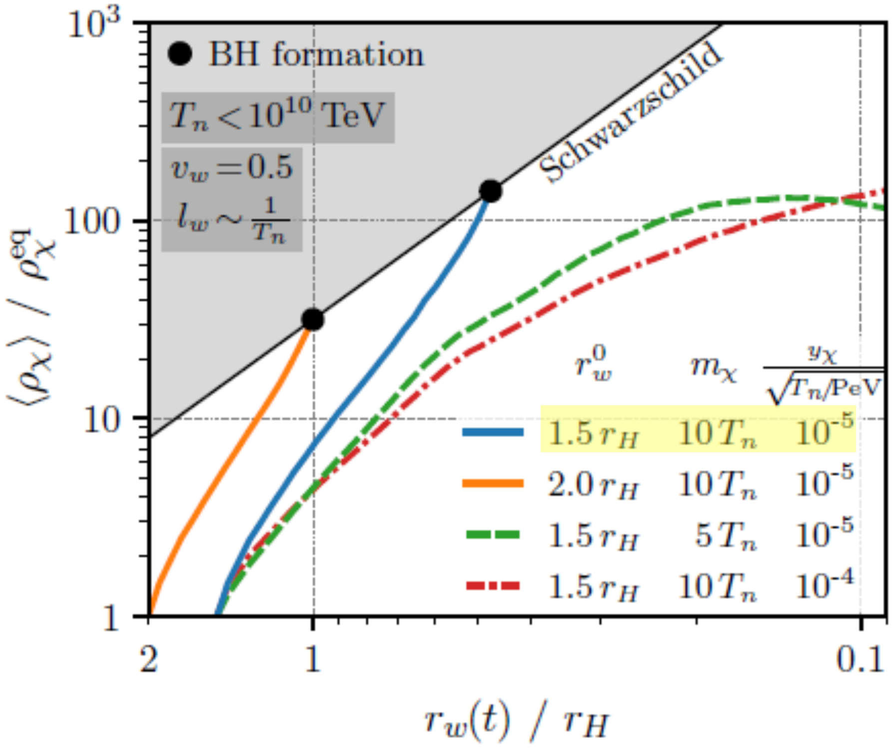


Image Credit: Baker, Breitbach, Kopp, and Mittnacht 2021



Baker, Breitbach, Kopp, and Mittnacht 2021

- χ is our dark matter particle and is weakly coupled through gravity (See Lagrangian left)

$$\mathcal{L} \supset \bar{\psi} \partial\!\!\!/ \psi - y_\psi \phi \bar{\psi} \psi + \mathcal{L}_{SM,\chi} \qquad y_\psi \sim 10^{-4} \sqrt{\frac{T_\star}{10^6 \text{GeV}}}$$

- φ & ψ is our "Phase Transition" sector and in our case ψ particles are being trapped

- We assume a %100 trapping rate

- Our Initial conditions are set to form a PBH following the criteria of above paper

$$M_{\text{PBH}} \simeq 16.3 \times \left(\frac{10^{15}\ \text{GeV}}{T_\star} \right)^2 \left(\frac{R_\star}{H_\star^{-1}} \right)^2 \text{g}$$

Black Hole Thermodynamics

$$T_{\rm PBH}(t) = \frac{M_{\rm Pl}^2}{8\pi M_{\rm PBH}(t)}$$

$$\frac{d^2 u_\chi(E,t)}{dt dE} = \frac{g_\chi}{8\pi^2} \frac{E^3}{e^{E/T_{\rm PBH}(t)} \pm 1}$$

$$\frac{d^2 N_\chi}{dt dE} = \frac{4\pi r_{\rm S}^2}{E} \frac{d^2 u_\chi}{dt dE}$$

$$N_\chi = \frac{120\,\zeta(3)}{\pi^3} \frac{g_\chi}{g_\star(T_{\rm PBH})} \frac{M_{\rm PBH}^2(t_i)}{M_{\rm Pl}^2}, \qquad T_{\rm PBH}(t_i) > m_\chi$$

$$N_\chi = \frac{15\,\zeta(3)}{8\pi^5} \frac{g_\chi}{g_\star(T_{\rm PBH})} \frac{M_{\rm Pl}^2}{m_\chi^2}, \qquad T_{\rm PBH}(t_i) < m_\chi$$

Dark Matter Relic Abundance

$$Y_\chi = \frac{3}{4}\frac{g_\star(T_i)}{g_{\star,s}(T_i)}\beta_{\rm PBH}N_\chi\frac{T_i(M_{\rm PBH})}{M_{\rm PBH}}$$

$$\Omega_\chi = \frac{m_\chi Y_\chi}{\rho_c}s(t_0)$$

m_χ ranges from (10^{-3}- 10^{19}) GeV

$$\rho_c(t_0) = 1.0537\times10^{-5}\,h^2\ \mathrm{GeV\ cm^{-3}}$$

$$s(t_0) = 2891.2\,(T_0/2.7255\mathrm{K})^3\,\mathrm{cm^{-3}}$$

First Order Phase Transition

$$Y_{\chi,\rm FOPT} = \frac{3}{4}\beta_{\rm PBH}N_\chi\frac{T_\star}{M_{\rm PBH}}$$

$$T_\star \simeq 7.1\times10^{14}\sqrt{32.8+\left(\frac{R_\star}{H_\star^{-1}}\right)^4}\left(\frac{R_\star}{H_\star^{-1}}\right)\left(\frac{1\ \mathrm{g}}{M_{\rm PBH}}\right)^{\frac{1}{2}}\ \mathrm{GeV}$$

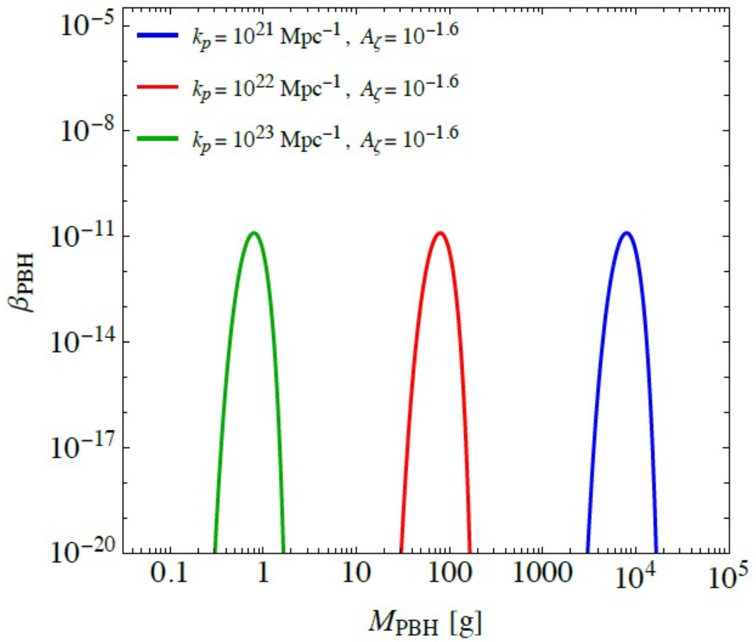
For $R_\star/H_\star^{-1}$= 1.5 this gives a prefactor value of ≈ 6.6×10^{15}

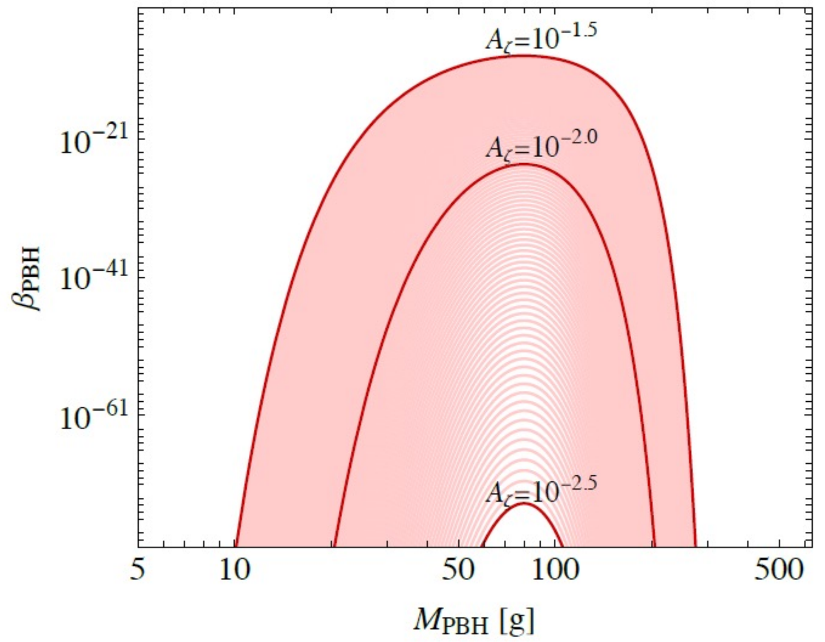
Curvature Perturbations

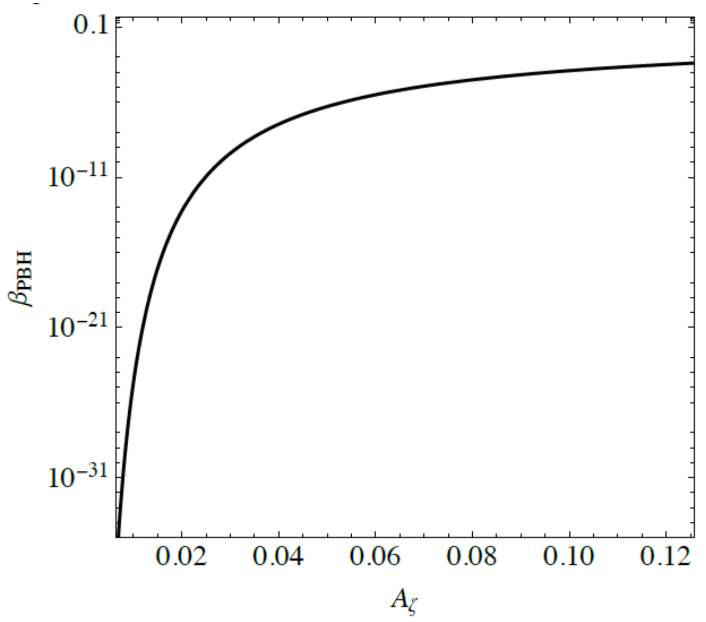
$$Y_{\chi,\zeta} = \frac{3}{4}\beta_{\rm PBH}N_\chi\frac{T_i(M_{\rm PBH})}{M_{\rm PBH}}$$

$$T_{i,\zeta} \simeq 4.3\times10^{15}\left(\frac{\gamma}{0.2}\right)^{\frac{1}{2}}\left(\frac{106.75}{g_\star(T_i)}\right)^{\frac{1}{4}}\left(\frac{1\ \mathrm{g}}{M_{\rm PBH}}\right)^{\frac{1}{2}}\ \mathrm{GeV}$$

β_{PBH} for Curvature Perturbations: Dependency on A_ζ, k_p, and M_{PBH}







$$M_{PBH}(k) = \gamma\, M_H(k) = 4 \times 10^3 \left(\frac{\gamma}{0.2}\right)\left(\frac{k}{10^{21}\mathrm{Mpc}^{-1}}\right)^{-2} \mathrm{g}$$

$\delta_c \approx 1/3$ and is the critical value for PBHs to form

$\sigma_0 \equiv$ Variance of the power spectrum

$k_p \equiv$ Peak and $R \sim k_p^{-1}$

Curvature Perturbations - Map

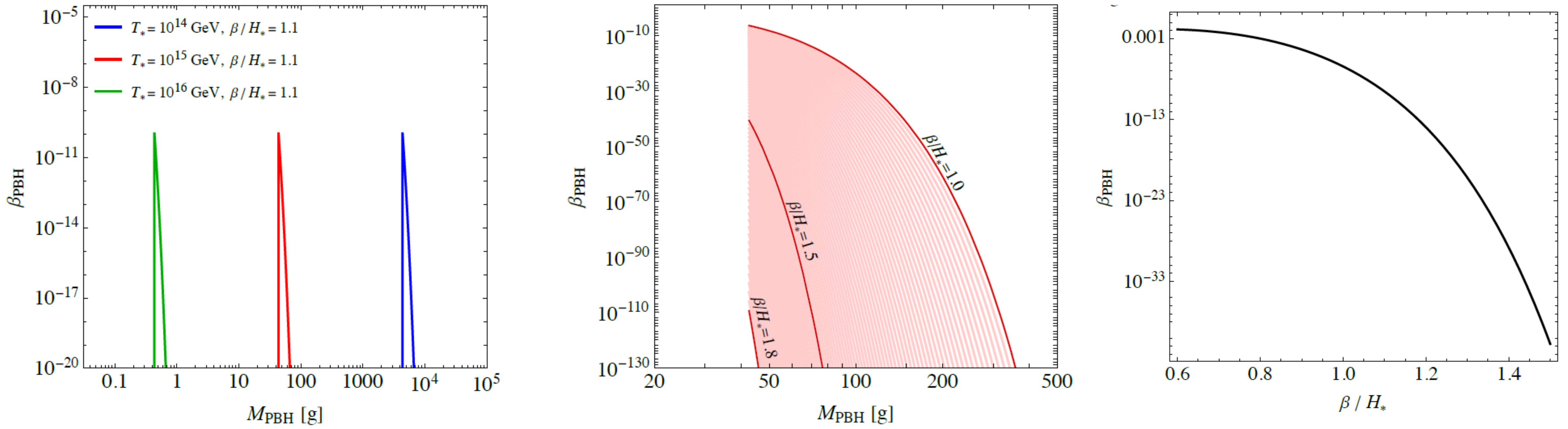
$$\{\Omega_\chi h^2, m_\chi\} \rightarrow \{M_{PBH}, \beta_{PBH}\} \rightarrow \{A_\zeta, k_p\} \rightarrow \{h_c, f_{GW}\}$$

$$P_\zeta(k) = A_\zeta\, \delta(\log k - \log k_p)$$

$$\sigma_0^2(k) = A_\zeta \frac{16}{81}\left(\frac{k_p}{k}\right)^4 \exp\left[-\left(\frac{k_p}{k}\right)^2\right]$$

$$\beta_{PBH} = \frac{\gamma}{2}\,\mathrm{Erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_0}\right)$$

β_{PBH} for FOPT: Dependency on $\beta/H_\star$ and M_{PBH}



$$\beta_{PBH} = \frac{M_{PBH} R_\star}{2\rho_{rad}} \frac{I_\star^4 \beta^4}{192 v_w^3} e^{4\beta R_\star/v_w - I_\star e^{\beta R_\star / v_w}} \left(1 - e^{-I_\star e^{\beta R_\star/v_w}}\right)\Big|_{R_\star > 1.5 H_\star^{-1}}$$

$$\rho_{rad}(T) = \frac{\pi^2}{30} g_\star(T_\star) T_\star^4$$

$I_\star = 1.238$
$v_w = 0.5$
$g_\star = 106.75$

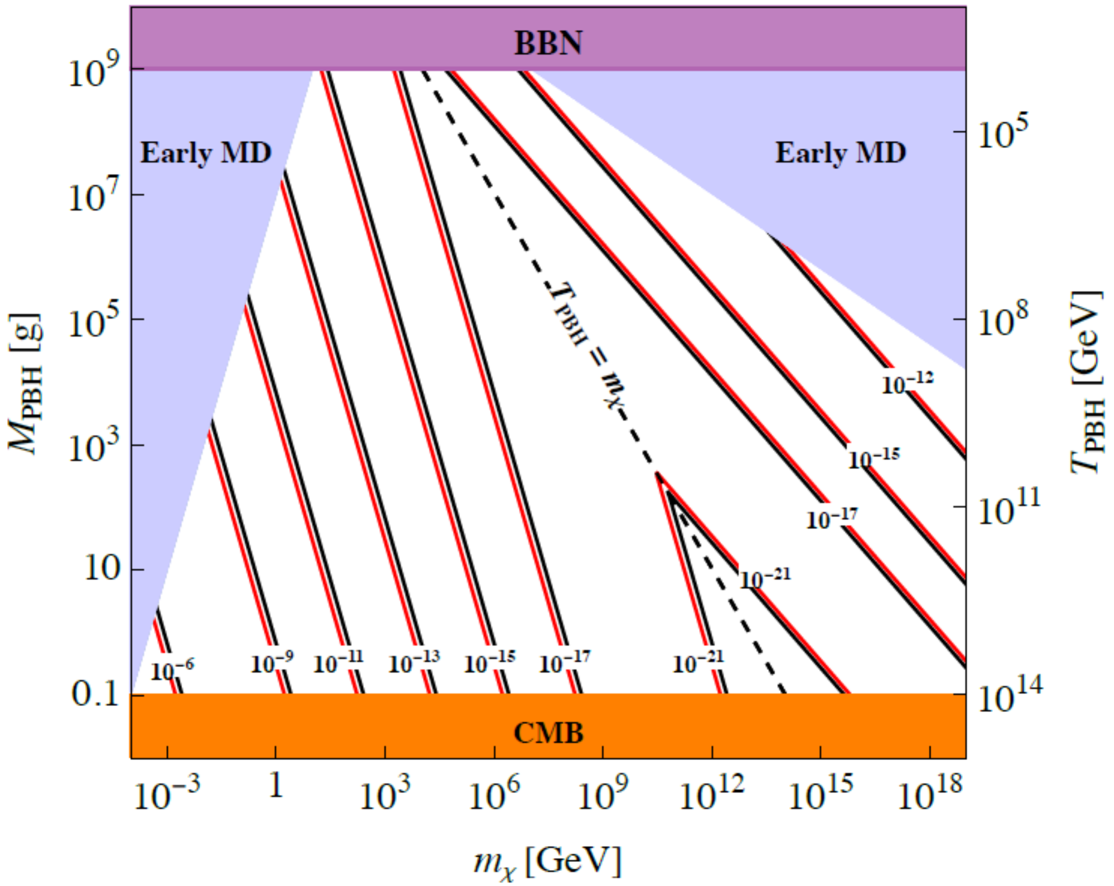
First Order Phase Transition - Map

$$\{\Omega_\chi h^2, m_\chi\} \rightarrow \{M_{PBH}, \beta_{PBH}\} \rightarrow \{\alpha, \beta, T_\star, v_w\} \rightarrow \{h_c, f_{GW}\}$$

α ≡ Energy density released during phase transition normalized by the radiation energy density

β ≡ Inverse time scale of the phase transition

β_{PBH} Contours on the Plane of M_{PBH} vs m_χ



- Red lines are for FOPT and black lines are for curvature perturbation
- Matter domination for FOPT and curvature perturbation almost overlap
- For $R_\star/H_\star^{-1} = 1.5$ the prefactor values are 5.3×10^{-13} $m_\chi < T_{PBH}$ & 4.8×10^{-21} for $m_\chi > T_{PBH}$

Curvature Perturbations

$$\beta_{PBH} = \frac{1}{g_\chi}\left(\frac{g_*(T_i)}{106.75}\right)^{1/4}\left(\frac{0.2}{\gamma}\right)^{1/2}\left(\frac{g_*(T_{PBH})}{106.75}\right) \times$$

$$\times \begin{cases} 8.0 \times 10^{-13}\left(\frac{\text{TeV}}{m_\chi}\right)\left(\frac{1\text{g}}{M_{PBH}}\right)^{1/2} & m_\chi < T_{PBH} \\ 7.1 \times 10^{-21}\left(\frac{m_\chi}{10^{15}\text{GeV}}\right)\left(\frac{M_{PBH}}{1\text{g}}\right)^{3/2} & m_\chi > T_{PBH} \end{cases}$$

$$\beta^\zeta_{PBH,crit} \simeq 2.8 \times 10^{-6}\left(\frac{g_*(T_{BH})}{106.75}\right)^{1/2}\left(\frac{0.2}{\gamma}\right)\left(\frac{1\text{g}}{M_{PBH}}\right)$$

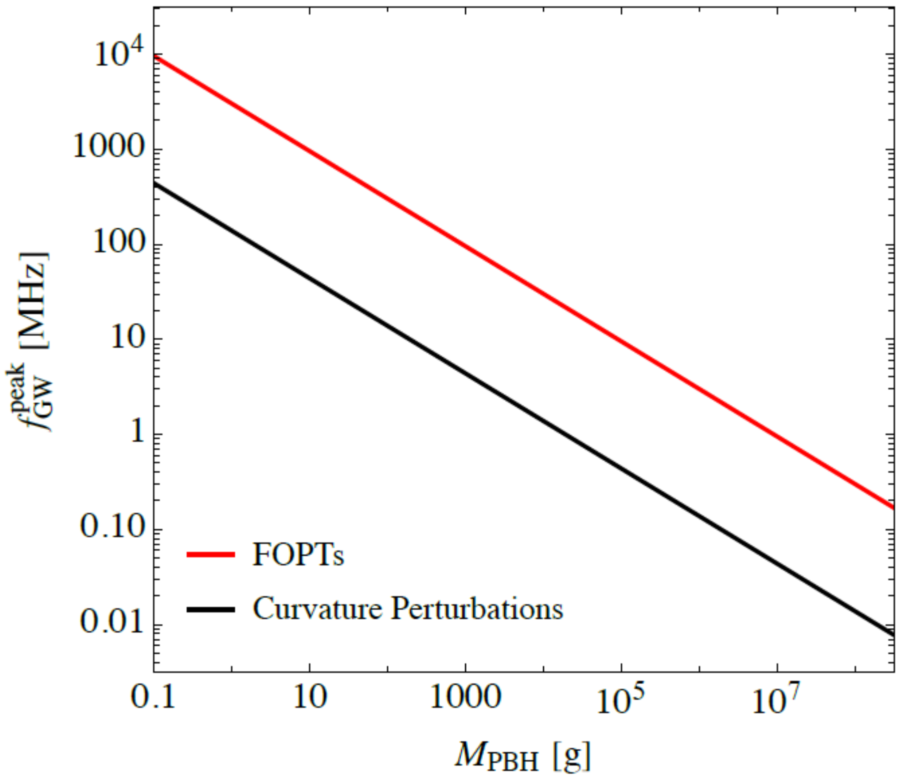
First Order Phase Transition

$$\beta_{PBH} = \frac{1}{g_\chi}\left(\frac{g_*(T_{PBH})}{106.75}\right)\left[164 + 5\left(\frac{R_\star}{H_\star^{-1}}\right)^4\right]^{-1/2}\left(\frac{R_\star}{H_\star^{-1}}\right)^{-1} \times$$

$$\times \begin{cases} 1.1 \times 10^{-11}\left(\frac{\text{TeV}}{m_\chi}\right)\left(\frac{1\text{g}}{M_{PBH}}\right)^{1/2} & m_\chi < T_{PBH}, \\ 1.0 \times 10^{-19}\left(\frac{m_\chi}{10^{15}\text{ GeV}}\right)\left(\frac{M_{PBH}}{1\text{g}}\right)^{3/2} & m_\chi > T_{PBH}. \end{cases}$$

$$\beta^{FOPT}_{PBH,crit} \simeq 1.9 \times 10^{-6}\left(\frac{1\text{ g}}{M_{PBH}}\right)\left(\frac{g_\star(T_\star)}{106.75}\right)^{\frac{1}{4}}\Big|_{R_\star/H_\star^{-1}=1.5}$$

Gravitational Waves
f_{GW}^{Peak} vs M_{PBH}



Curvature Perturbations

$$f_{GW,\zeta}^{peak} = 1.546 \times \left(\frac{k_p}{10^{21}\text{Mpc}^{-1}}\right) \text{MHz}$$

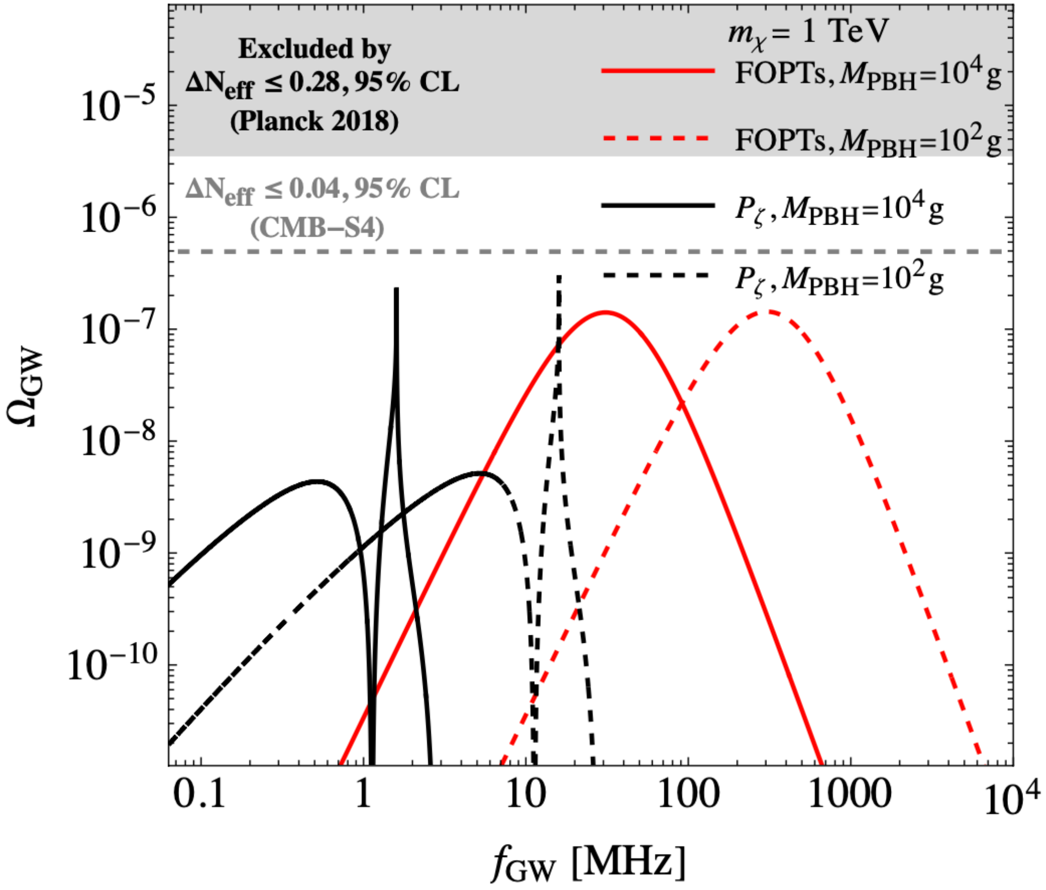
$$f_{GW,\zeta}^{peak} \simeq 1.4 \times \left(\frac{M_{PBH}}{10^4 \text{ g}}\right)^{-\frac{1}{2}} \text{MHz}$$

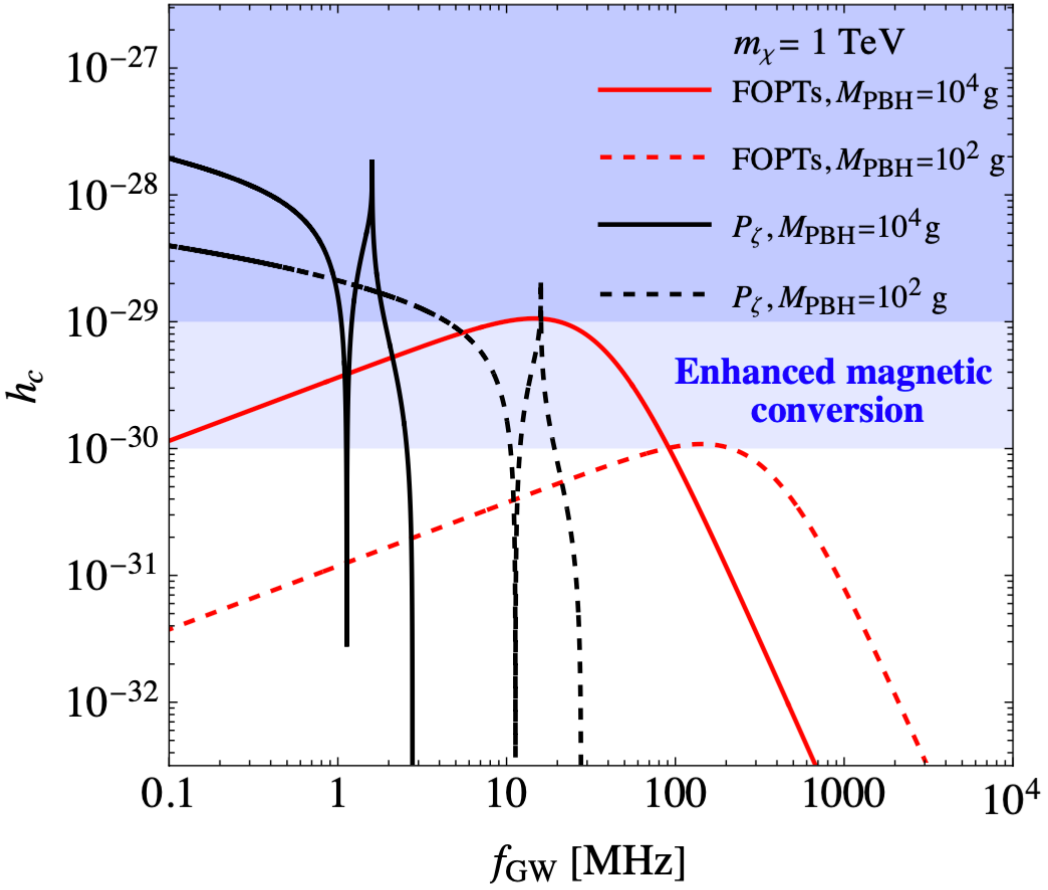
First Order Phase Transition

$$f_{GW,sw}^{peak} \simeq 1.9 \times 10^2 \left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_\star}\right)\left(\frac{T_\star}{10^{15}\text{ GeV}}\right)\left(\frac{g_\star(T_\star)}{106.75}\right)^{\frac{1}{6}} \text{MHz}$$

$$T_\star \simeq 7.1 \times 10^{14}\sqrt{32.8 + \left(\frac{R_\star}{H_\star^{-1}}\right)^4}\left(\frac{R_\star}{H_\star^{-1}}\right)\left(\frac{1\text{ g}}{M_{PBH}}\right)^{\frac{1}{2}} \text{GeV}$$

For $\beta/H_\star$ = 1.2, v_w = 0.5 and $R_\star/H_\star^{-1}$ = 1.5 gives the following peak frequency: $f_{GW,sw}^{peak} \simeq 30.1 \times \left(\frac{M_{PBH}}{10^4\text{ g}}\right)^{-\frac{1}{2}}$ MHz





Curvature Perturbations

$$\Omega_{\mathrm{GW},0}(f_{\mathrm{GW}}) = 0.39\left(\frac{g_\star(\eta_s)}{106.75}\right)^{-\frac{1}{3}} \Omega_{\mathrm{rad},0}\,\Omega_{\mathrm{GW},\zeta}(\eta_s, k)$$

$$\Omega_{\mathrm{rad},0} = 8.5\times 10^{-5}$$

$$\Delta\rho_{\mathrm{rad}} = \frac{\pi^2}{30}\frac{7}{4}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\mathrm{eff}} T_{\mathrm{rad}}^4$$

First Order Phase Transition

$$\Omega_{\mathrm{GW,sw}}(f_{\mathrm{GW}}) \simeq 5.3\times 10^{-6}\left(\frac{\beta}{H_\star}\right)^{-1}\left(\frac{\kappa\alpha}{1+\alpha}\right)^{2}\left(\frac{106.75}{g_\star(T_\star)}\right)^{\frac{1}{3}} v_w\,\Upsilon\,\times$$

$$\times\left(\frac{f_{\mathrm{GW}}}{f_{\mathrm{GW}}^{\mathrm{peak}}}\right)^{3}\left(\frac{7}{4+3(f_{\mathrm{GW}}/f_{\mathrm{GW}}^{\mathrm{peak}})}\right)^{\frac{7}{2}}$$

$$h_c = \frac{1}{f_{\mathrm{GW}}}\sqrt{\frac{3\,H_0^2\,\Omega_{\mathrm{GW}}}{4\,\pi^2}}$$

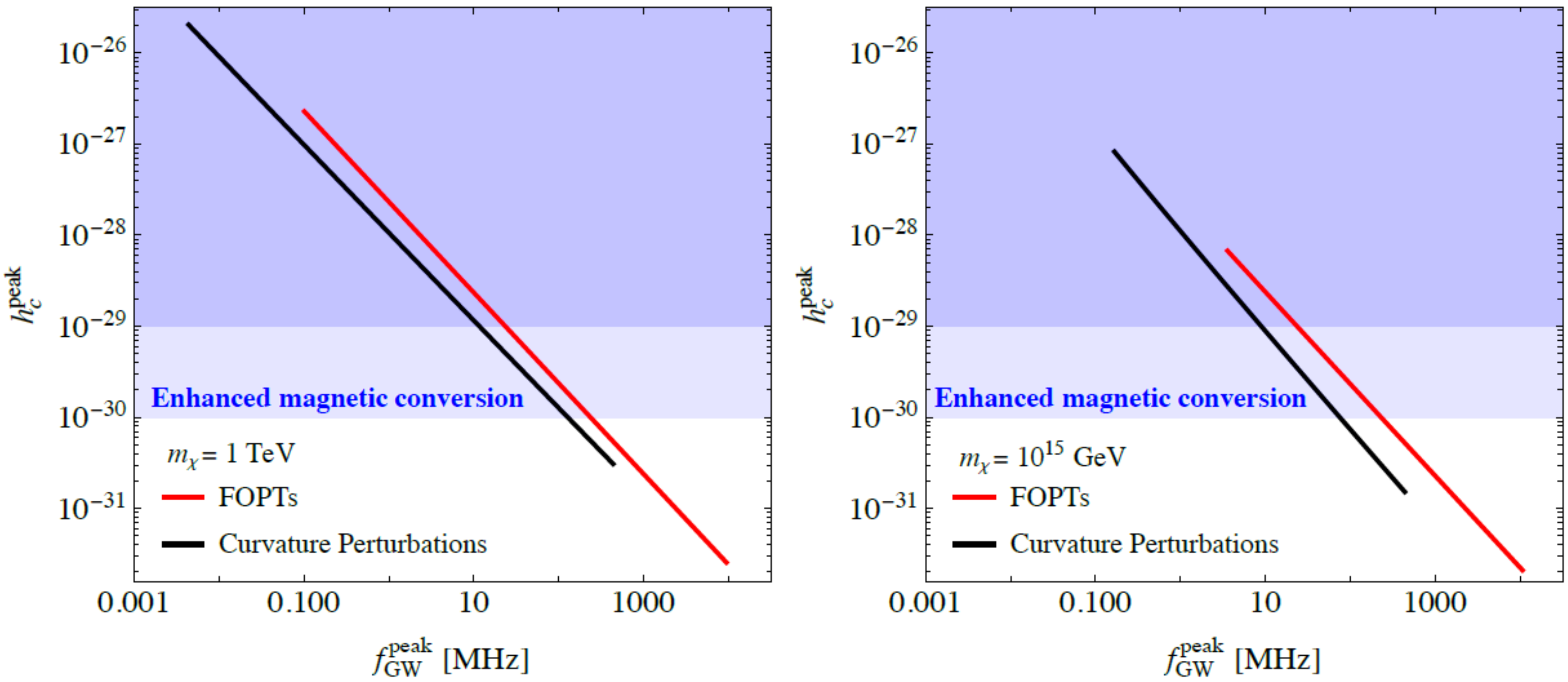
$$\Upsilon = 1 - \frac{1}{\sqrt{1+2\tau_{\mathrm{sw}}H_\star}}$$

$$\tau_{\mathrm{sw}} \simeq \frac{R_\star}{\bar{U}_f} \qquad \bar{U}_f^2 = \frac{3}{4}\kappa\alpha$$

$\alpha = 0.8$ for $v_w = 0.5$ following Alves et. al. (2019)

Calculated $\kappa = 0.688$ following Guo, Sinha, Vagie, and White (2021)

h_c^{peak} vs f_{GW}^{Peak}



Curvature Perturbations $f^{\rm peak}_{\rm GW,sw} \simeq 30.1 \times \left(\frac{M_{\rm PBH}}{10^4\ {\rm g}}\right)^{-\frac{1}{2}}$ MHz

$$h_c = \frac{1}{f_{\rm GW}}\sqrt{\frac{3\,H_0^2\,\Omega_{\rm GW}}{4\,\pi^2}}$$

First Order Phase Transition $f^{\rm peak}_{\rm GW,\zeta} \simeq 1.4 \times \left(\frac{M_{\rm PBH}}{10^4\ {\rm g}}\right)^{-\frac{1}{2}}$ MHz.

Summary

- A Map shown from the DM relic abundance to GW frequency and strain

$$\{\Omega_\chi h^2, m_\chi\} \rightarrow \{M_{\rm PBH}, \beta_{\rm PBH}\} \rightarrow \{A_\zeta, k_p\} \rightarrow \{h_c, f_{\rm GW}\}$$

$$\{\Omega_\chi h^2, m_\chi\} \rightarrow \{M_{\rm PBH}, \beta_{\rm PBH}\} \rightarrow \{\alpha, \beta, T_\star, v_w\} \rightarrow \{h_c, f_{\rm GW}\}$$

- Showed the sensitivity of both formation mechanisms to A_ζ, and $\beta/H_\star$

- DM relic abundance can be generated from χ particle

- Showed the GW signals can be distinguished and if observed can determine what formation mechanism mechanism, either FOPT or Curvature Perturbation, can produce PBHs and then observed dark matter abundance.

Thank you!

Primordial Black Holes from Curvature Perturbations

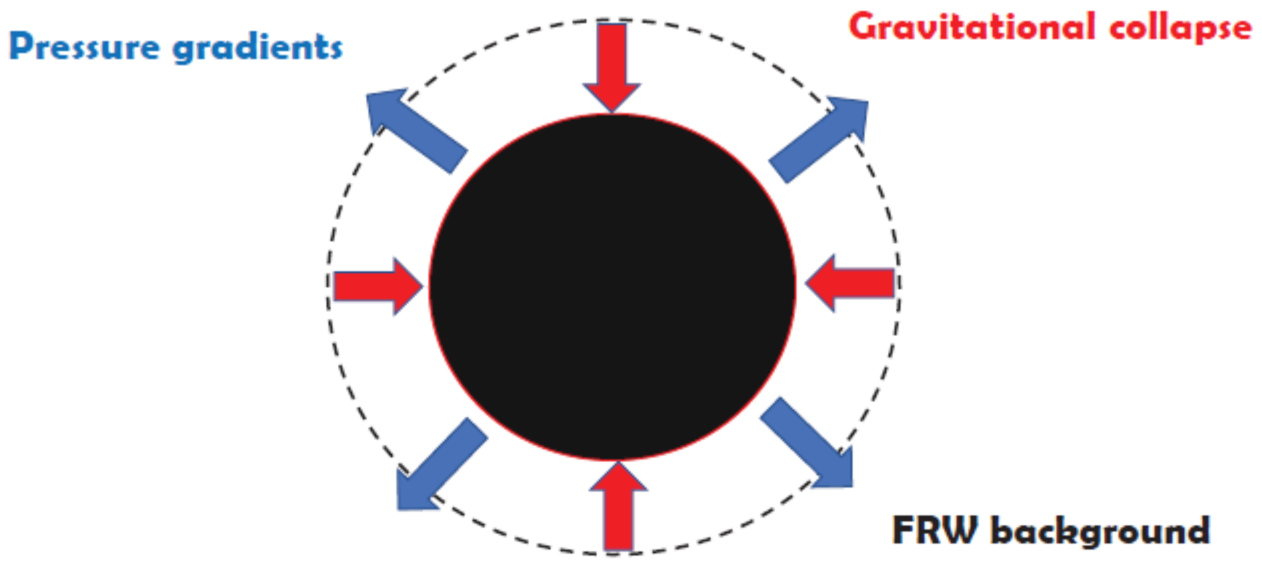


Image: Escriva 2022

- Curvature perturbations within a given horizon R have a probability of collapsing into a PBH
- The Mass of the PBH depends on the horizon size.

$$M_{\rm PBH}(k) = \gamma\, M_H(k) = 4\times 10^{3}\left(\frac{\gamma}{0.2}\right)\left(\frac{k}{10^{21}{\rm Mpc}^{-1}}\right)^{-2}\ {\rm g}$$