Vector Spaces for Direct Detection

An Extremely Efficient Framework for Scattering Calculations

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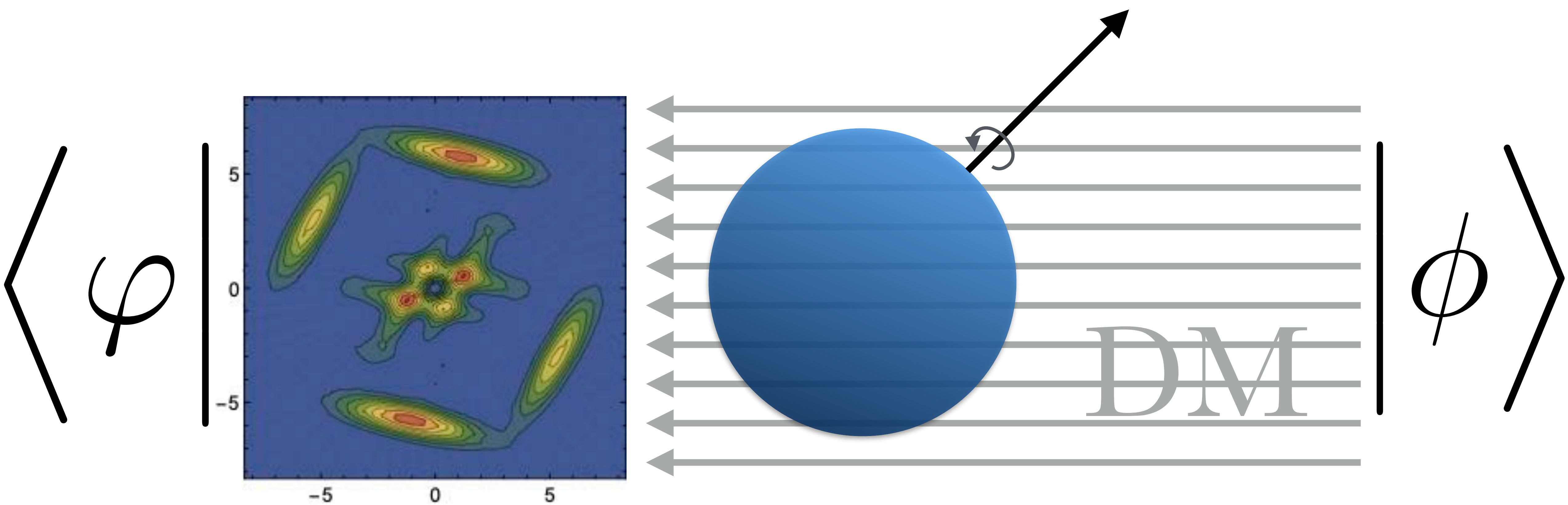
PHENO 2023



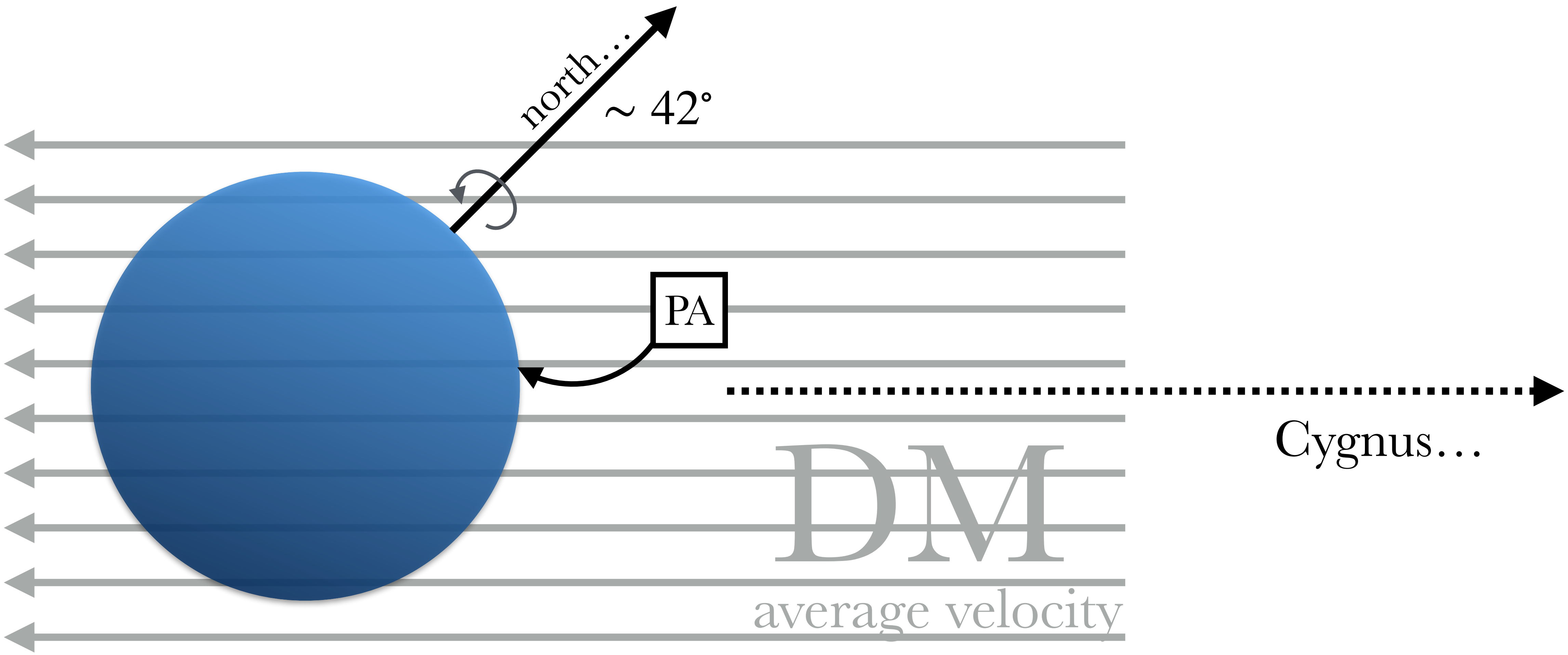
$f_s^2(\mathbf{q})$ $\quad$ $g_\chi(\mathbf{v})$

Why Directionality for DM Direct Detection?

Distinguish (***DM***) **signal** from (***SM***) **background:**

- **sub-GeV**: experiments at low energies *(eV)* have larger backgrounds
- **nuclear recoil**: zero-background experiments… until they encounter irreducible neutrino background (see e.g. 2208.09002)

Directionality: if scattering rate depends on detector orientation, scattering rate **modulates** every 23 hours 56 minutes



Newly Challenging Rate Calculation:

Astrophysics

Particle Physics (DM-SM)

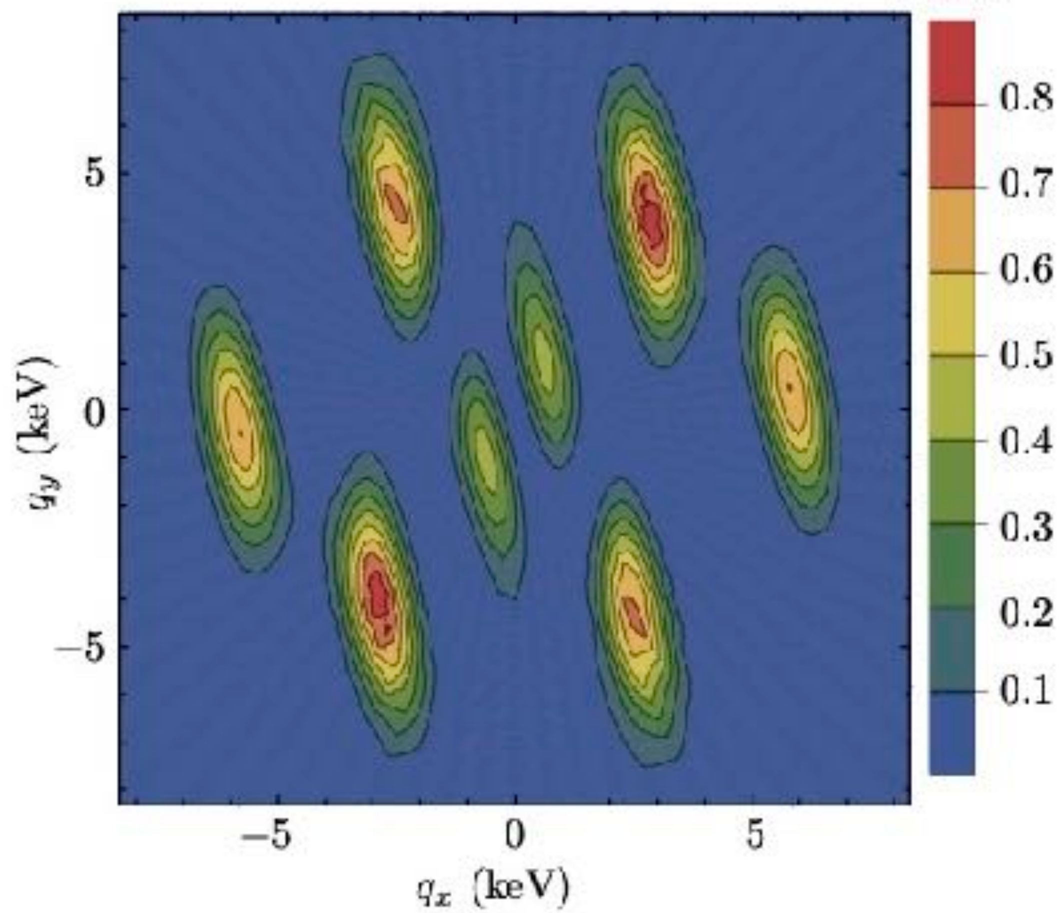
$$R_s = N_{\rm SM} n_\chi \bar{\sigma}_0 \int \frac{d^3q}{4\pi\mu_{\chi\rm SM}^2} \int d^3v\, g_\chi(\mathbf{v}) \times \boxed{\delta\left(\omega_s + \frac{q^2}{2m_\chi} - \mathbf{q}\cdot\mathbf{v}\right) F_{\rm DM}^2(q)} \times \boxed{f_s^2(\mathbf{q})}$$

SM Detector Physics

What would have been a 2d integral is now a 6d integral

Repeat for every…

- **DM mass and** F_{DM}
- **velocity distribution**
- **detector form factor**
- **detector orientation**



Newly Challenging Rate Calculation:

Repeat for every…

- **DM mass and F_{DM}** $\longrightarrow (50 \times m_\chi) \times (2 \times F_{DM})$
- **velocity distribution** $\longrightarrow$ astro uncertainties; simulations…
- **detector form factor** $\longrightarrow$ many possible target materials; imprecise SM physics modeling
- **detector orientation** $\longrightarrow$ $SO(3)$: a 3d space of orientations

For a total of: $10^2 \times 10^3 \times 10^2 \times 10^3 \sim 10^{10}$ **6d integrals**

Only $5 \cdot 10^5$ minutes in a year.

Computational expense: about 10^2 **CPU-centuries**

Factorizing the Rate Calculation:

$$R_s = N_{\mathrm{SM}} n_\chi \bar{\sigma}_0 \int \frac{d^3q}{4\pi\mu_{\chi\mathrm{SM}}^2} \int d^3v\, g_\chi(\mathbf{v}) \times \delta\Big(\Delta E + \frac{q^2}{2m_\chi} - \mathbf{q}\cdot\mathbf{v}\Big) F_{\mathrm{DM}}^2(q) \times f_s^2(\mathbf{q})$$

$$R_s = \frac{N_{\mathrm{SM}} n_\chi \bar{\sigma}_0}{4\pi\mu_{\chi\mathrm{SM}}^2} \langle g_\chi | \phi_v \rangle \cdot \left\langle \phi_v \middle| \delta\Big(\Delta E + \frac{q^2}{2m_\chi} - \mathbf{q}\cdot\mathbf{v}\Big) F_{\mathrm{DM}}^2(q) \middle| \varphi_q \right\rangle \cdot \langle \varphi_q | f_s^2 \rangle$$

1. Define basis functions, $|nlm\rangle = r_n(q)\, Y_{lm}(\hat{q})$, with spherical harmonics Y_{lm}
2. Projections of g_χ and f_s^2 onto each basis $\longrightarrow$ **vectors**
3. Kinematic operator (incl. m_χ) $\longrightarrow$ **matrix** connecting ***(v, q)*** spaces
4. Scattering rate is given by **matrix multiplication**

Difficult integrals $\langle g_\chi | \phi_v \rangle$ and $\langle \varphi_q | f_s^2 \rangle$ need to be done **once** (per model)

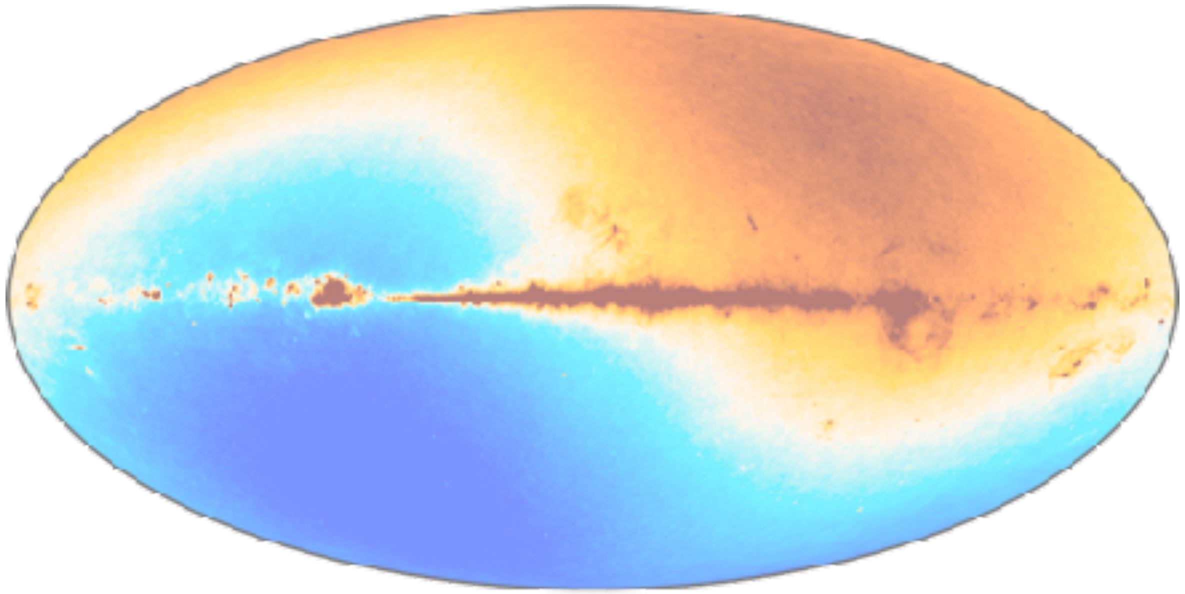
For some choices of radial basis functions, can **evaluate matrix analytically**

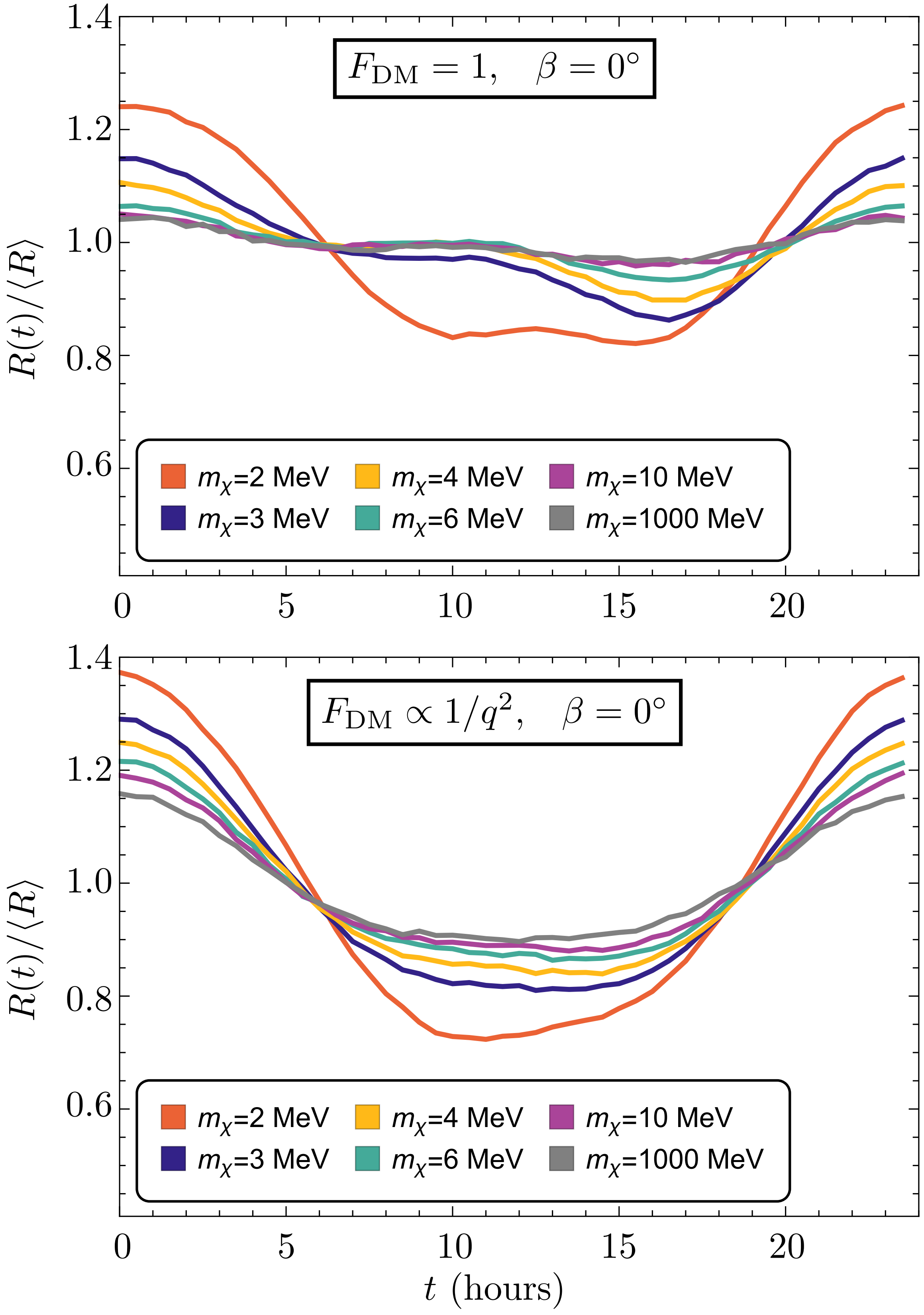
$$\left\langle n\ell m \middle| \delta\Big(\Delta E + \frac{q^2}{2m_\chi} - \mathbf{q}\cdot\mathbf{v}\Big) F_{\mathrm{DM}}^2(q) \middle| n'\ell'm' \right\rangle \propto \delta_{\ell\ell'}\delta_{mm'}\mathcal{I}_{nn'}^{(\ell)}$$

Applications

2103.08601

- Which detector orientations **maximize** or **minimize** a **modulation signal?**
- Propagate astro/materials uncertainties through the rate calculation
- Extract physics information (e.g. m_χ) from details of a modulation signal
- Compare statistical power of different target materials
- Search for substructures in DM velocity distribution





Conclusion

- "Vector space" rate calculation is faster by **many orders of magnitude** for complicated analyses

For every…	# of integrals…
- **DM mass and F_{DM}**	$\longrightarrow$ 0 (matrix $I^{(\ell)}_{n,n'}$ has analytic solution)
- **velocity distribution**	$\longrightarrow$ N_v 3d integrals $\langle g_\chi \vert nlm \rangle$
- **detector form factor**	$\longrightarrow$ N_q 3d integrals $\langle nlm \vert f_s^2 \rangle$
- **detector orientation**	$\longrightarrow$ 0 (Rotation matrices act on Y_{lm})
For a total of:	$10^3 N_v + 10^2 N_q \sim 10^6$ **3d integrals**

Old way: about 10^2 **CPU-centuries**

New way: **CPU-days** to tabulate $\vert g_\chi \rangle$ and $\vert f_s^2 \rangle$

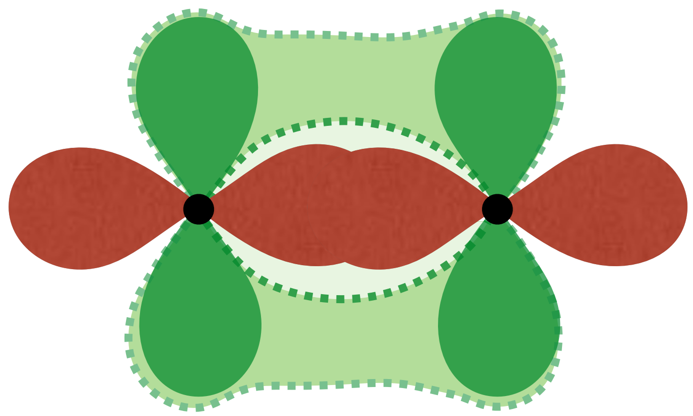
Minutes/hours for 10^{10} point calculation

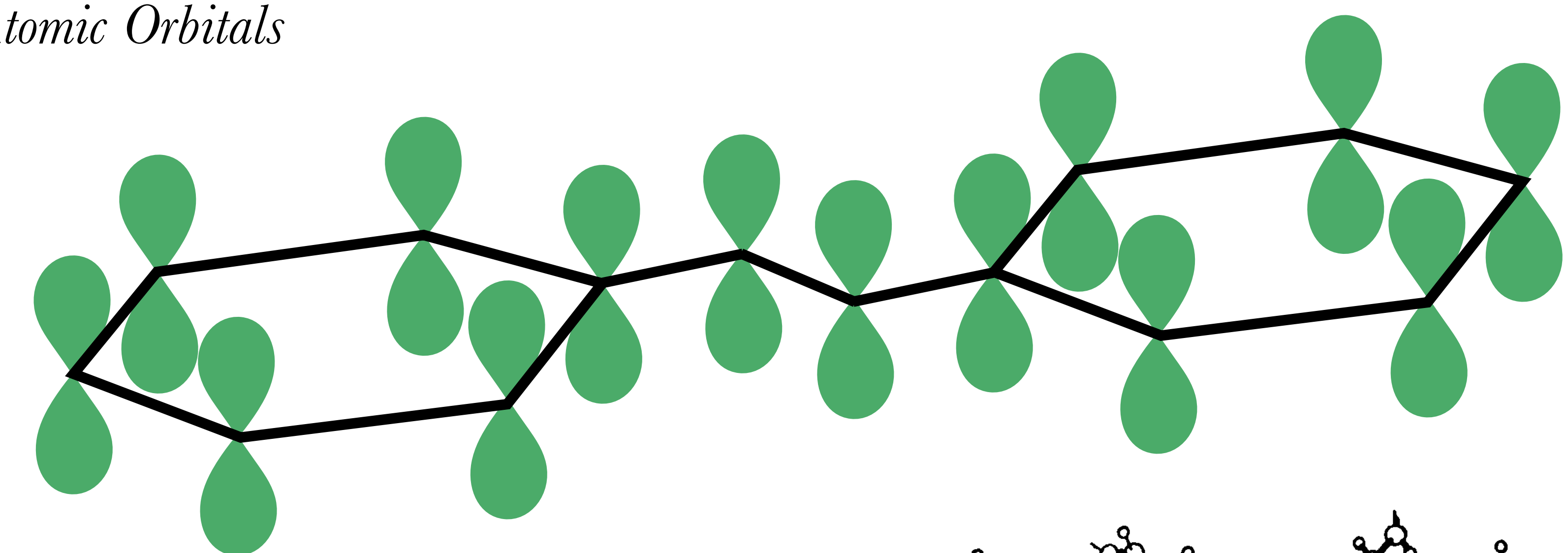
- **Coming soon to github and arXiv: VSDM and 2305.XXXXX**

How to calculate DM–molecule scattering:

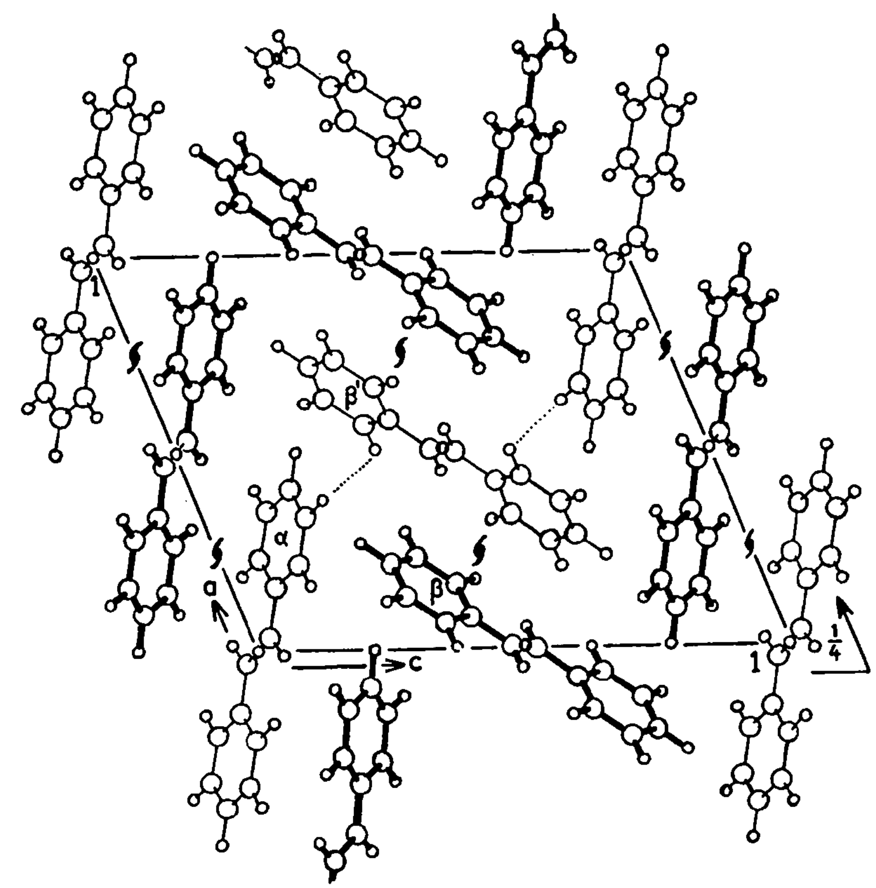
see arXiv:2103.08601

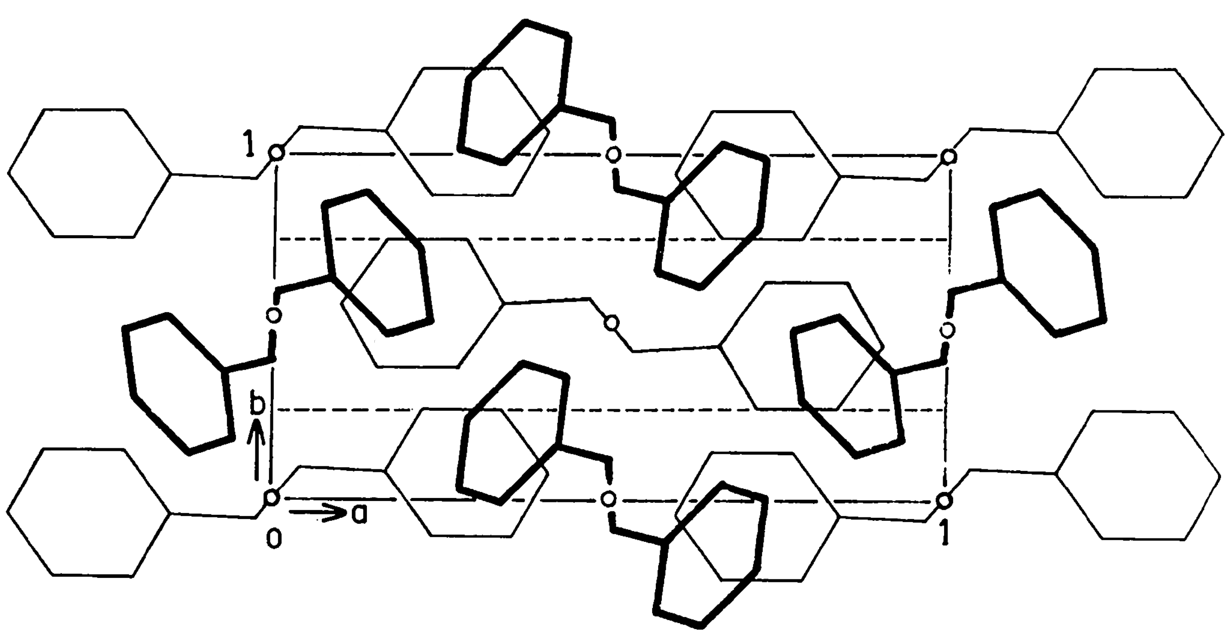
LCAO: Linear Combinations of Atomic Orbitals





A Complication: trans-stilbene crystals form unit cell with 4 components

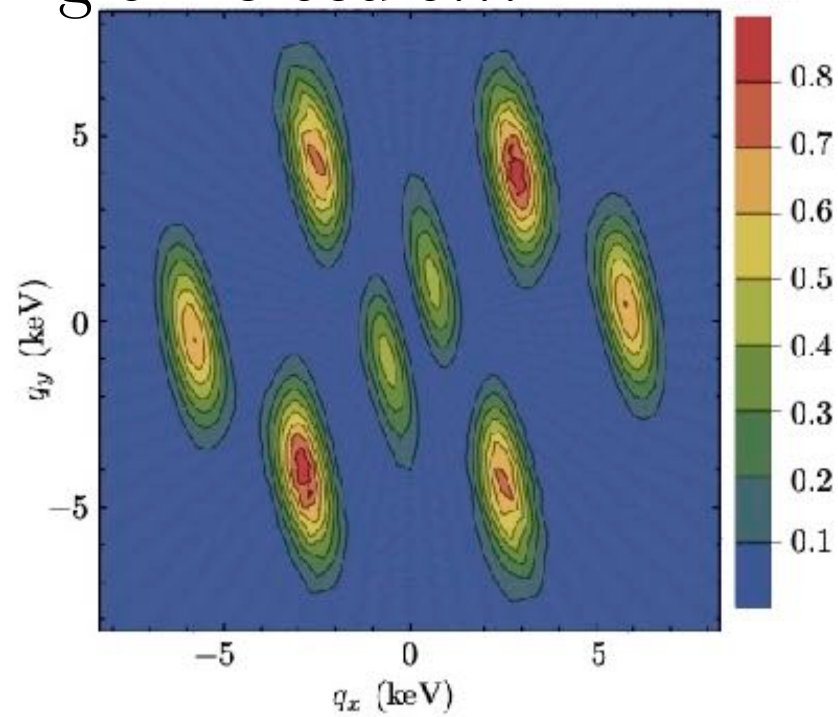




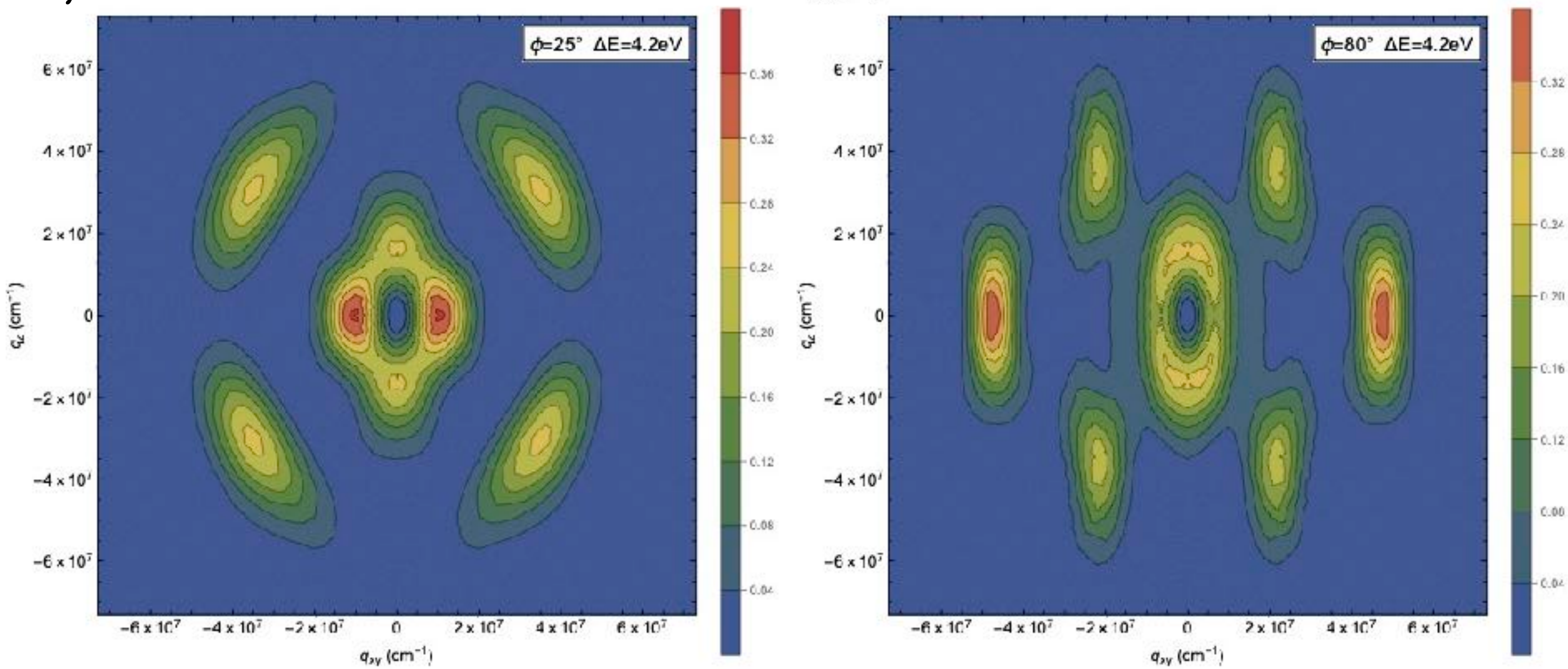
III. Results

arXiv:2103.08601

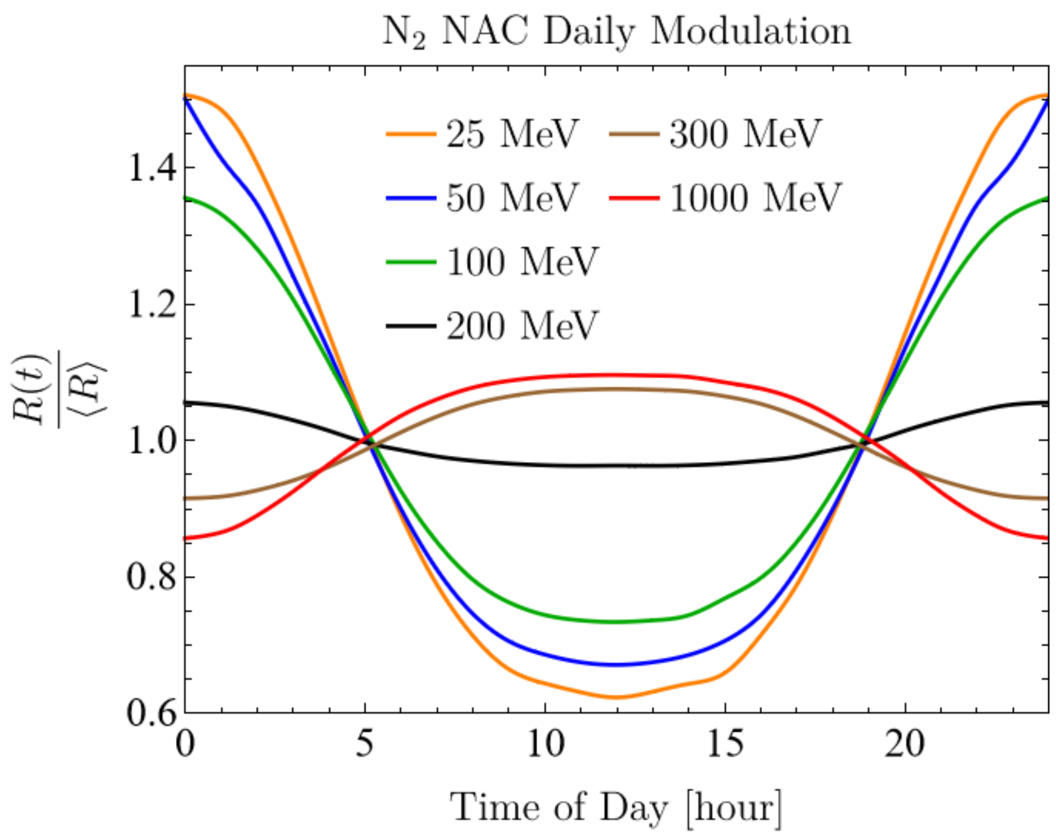
single molecule…

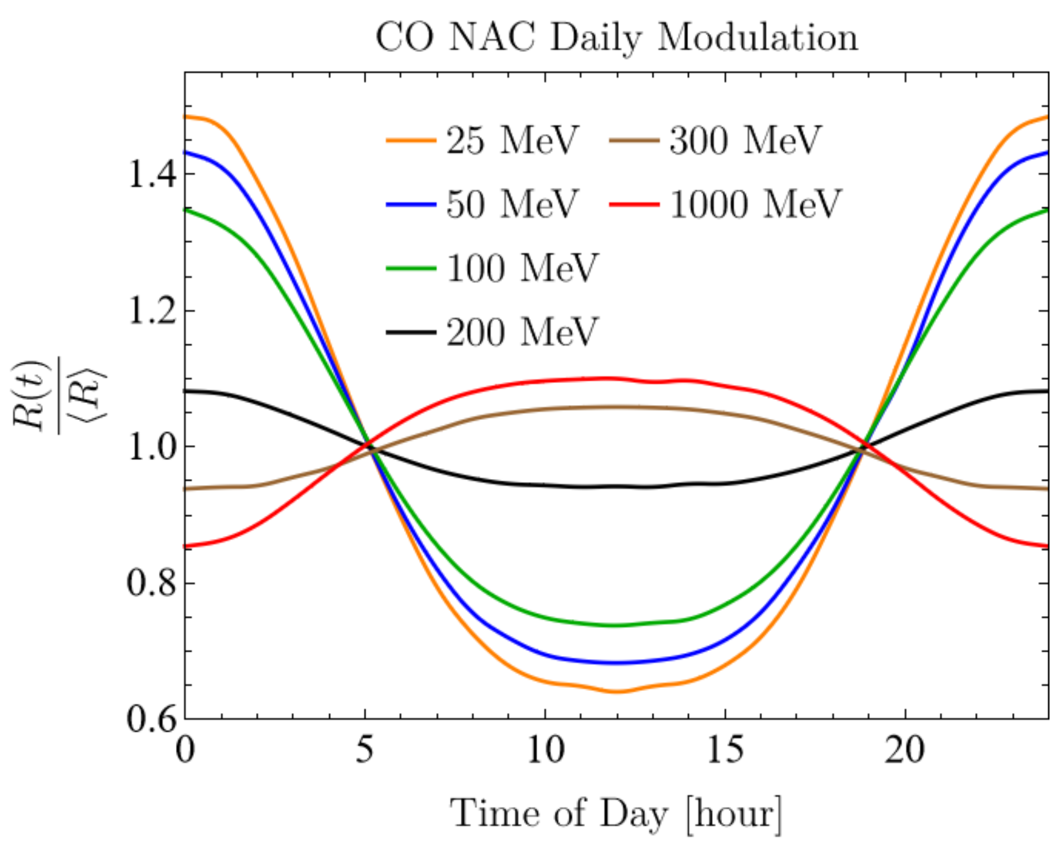


Crystal Form Factor



Results: Diatomic Molecules CO and N_2 (2208.09002)





- **NAC Migdal Effect** is the important one