Resolving Combinatorial Ambiguities with Quantum Algorithms

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Based on ongoing work in collaboration with
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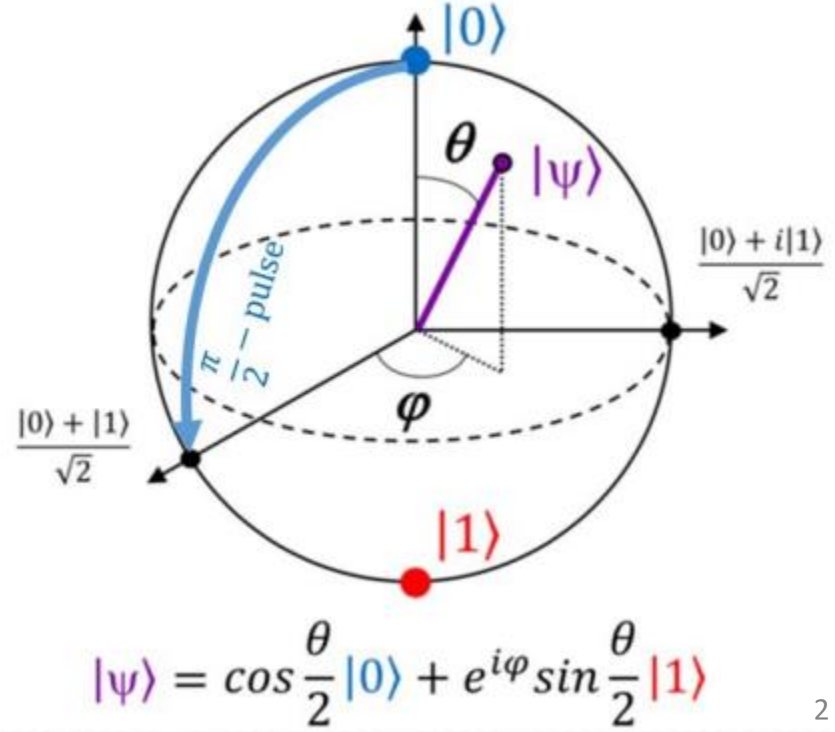
Quantum Algorithms

- The standard argument for advantage of quantum computation is that certain quantum algorithms outperforms their classical counterpart exponentially.
- This is not true in general for all computational tasks.
- It would require higher precision and much more qubits than currently available technology to solve a problem that cannot be done with classical computers.
- We should look for tasks that can be done with smaller number of qubits.



Quantum Computing within HEP

- There are many problems in high energy physics that involves quantum mechanics whose numerical answers are of interest to us but difficult to compute classically or analytically.
- e.g. Scattering process with entanglement, simulation of parton shower/hadronization, non-perturbative calculations.

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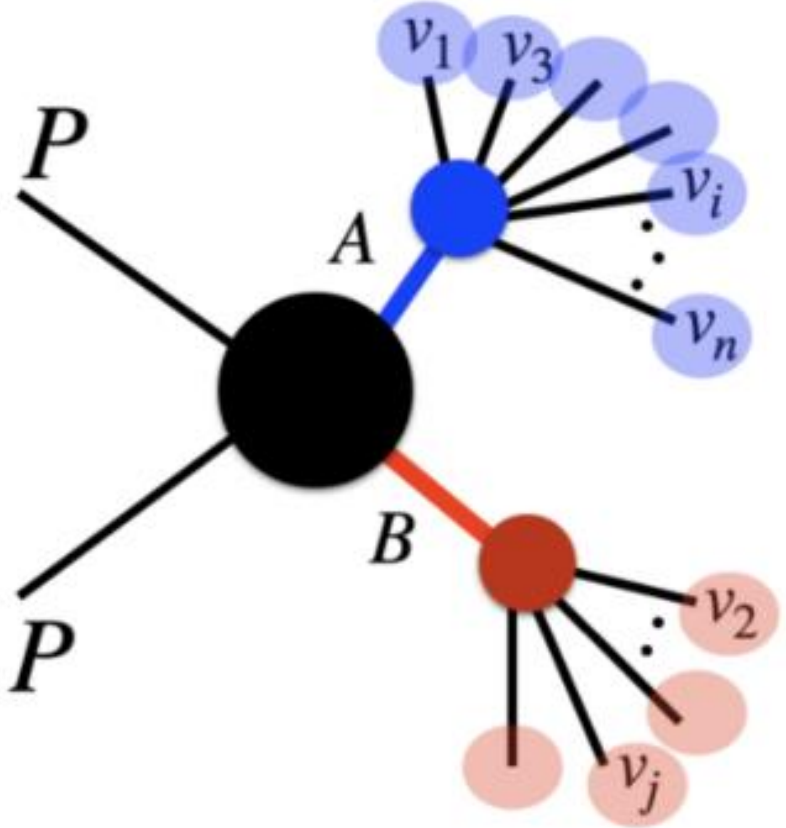
- In principle, some of these problems does not require a lot of qubits to model.
- This provides us an opportunity to utilize the qubits that are available to us now.

Binary Optimization Problem

- In a $2 \to 2$ process, there are usually a large number of observed particles in the final states.
- The reconstruction of the parent particle then becomes a binary combinatoric problem of assigning observed particles into one of two groups.
- In general, kinematic information are used to resolve the combinatorial problems. For instance, we can minimize the mass difference.
- We will consider top quark pair production as an example.



Quadratic Unconstrained Binary Optimization (QUBO)

- QUBO: combinatorial optimization problem with a wide range of applications in all fields.
- We can rewrite the problem as an Ising model Hamiltonian:

$$H_{\mathrm{QUBO}} = \sum_{ij} J_{ij} s_i s_j + \sum_i h_i s_i$$

- The "Spins" here will simply be which group the particles are assigned with. In particular, we can set the Hamiltonian so that the ground state balance the mass of two sides:

$$H = \left(P_1^2 - P_2^2\right)^2 \qquad J_{ij} = \sum_{k\ell} P_{ik} P_{j\ell} \qquad P_{ij} = p_i \cdot p_j$$

Adiabatic Theorem

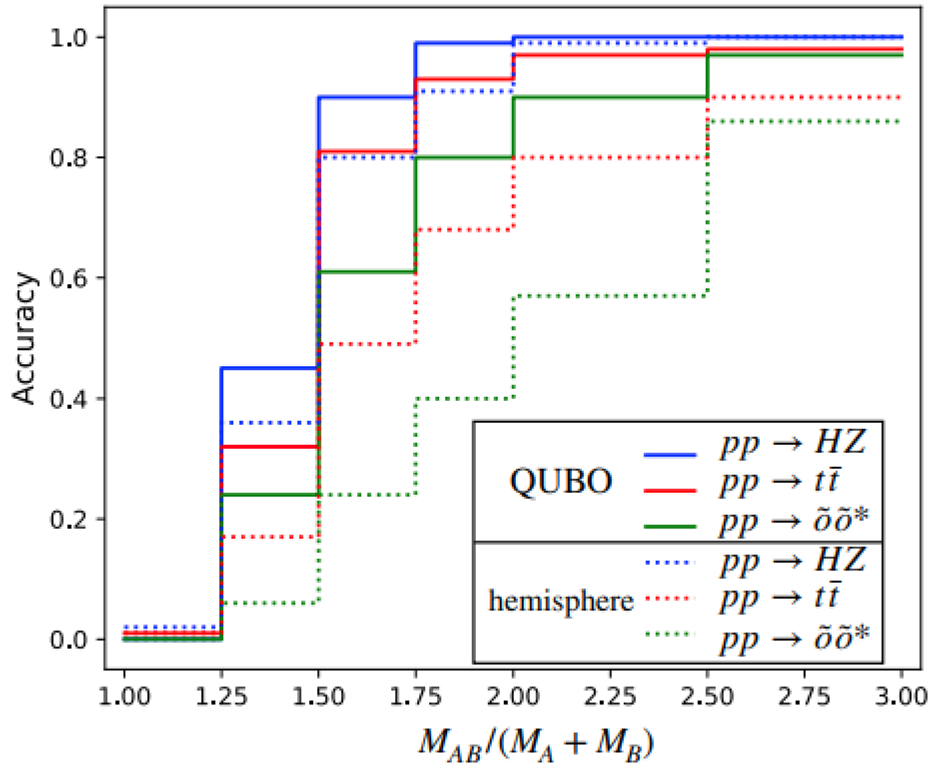
- H_0 is the initial Hamiltonian whose ground state is easy to prepare.
- H_p is the Hamiltonian whose ground state we are interested in finding.

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- Given enough time, we can evolve the ground state of the trivial Hamiltonian to the ground state of the Hamiltonian of interest.
- In quantum computing, we can take finite number of discrete steps to achieve this evolution from one Hamiltonian to another.

Quantum Annealing

- D-wave has built Quantum Annealing that solves optimization problem by transferring the original optimization to a hardware, that allows nearest neighbor interaction of qubits.
- The performance of Annealer depends on the size of gap between energy eigenstate.
- It is hardware dependent compared to gate-based quantum computing.

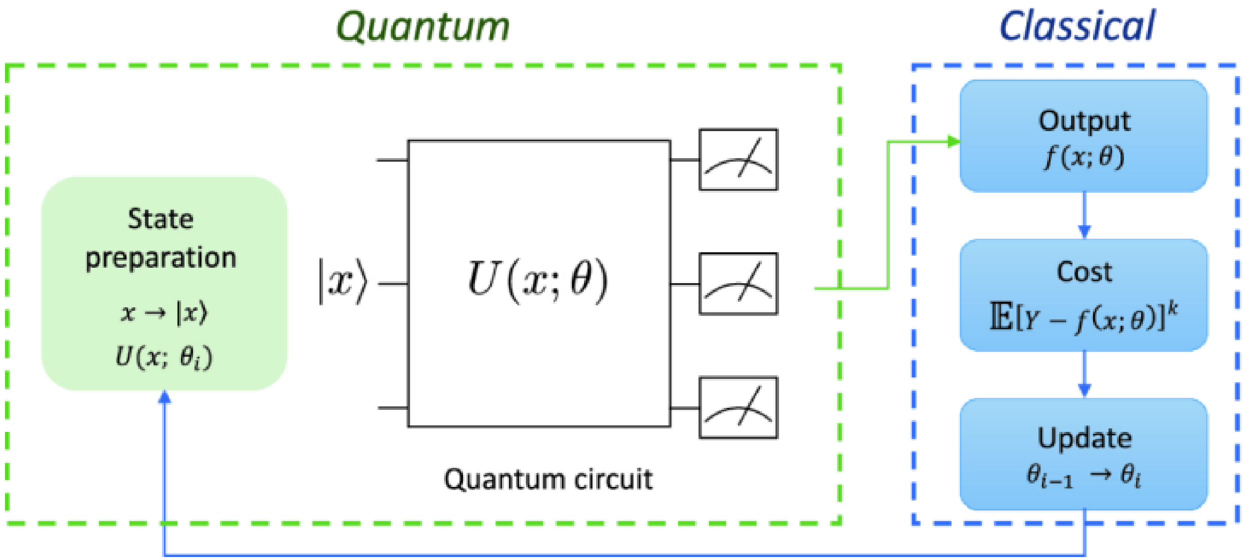


2111.07806 Kim, Ko, Park, Park

Variational Quantum Algorithms

- VQA is a hybrid strategy of parameterizing a quantum circuit and then minimize the expectation value of certain operators classically.
- VQAs have emerged as the leading strategy to obtain quantum advantage on NISQ (Noisy Intermediate-Scale Quantum) devices.

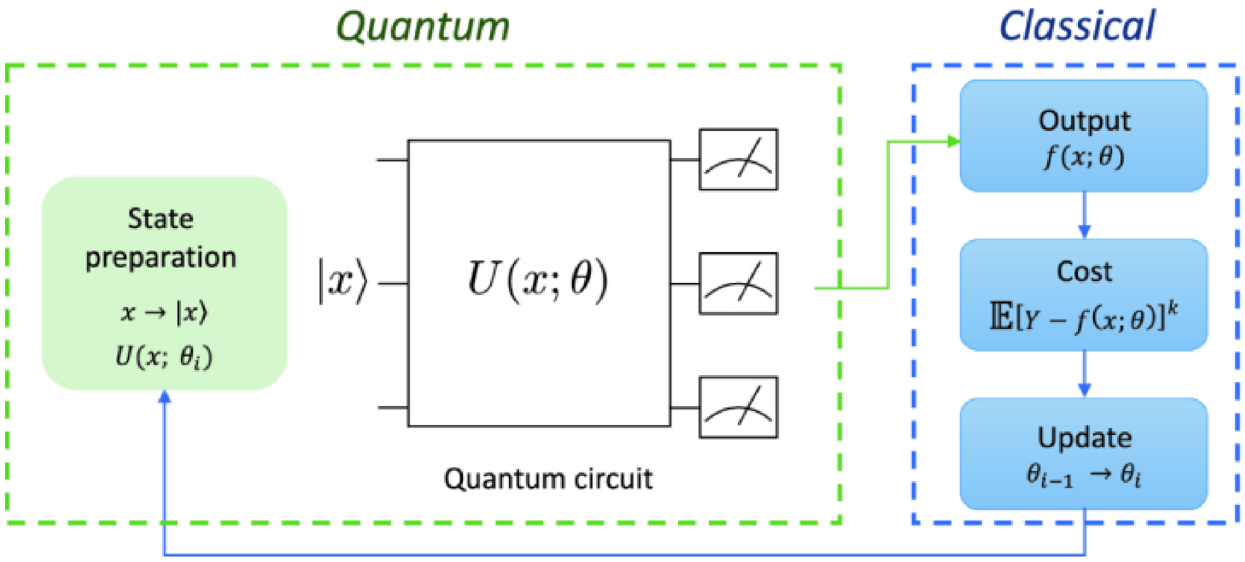
$$E(\vec{\theta}) = \langle \psi(\vec{\theta}) \,|\, H \,|\, \psi(\vec{\theta}) \rangle$$



Variational Quantum Algorithms

- This approach can keep the quantum circuit depth relatively shallow hence reduce the effect caused by noise.
- Classical optimization can be done through either global optimization scheme, or a learning method similar to the ones used in neural networks.

$$E(\vec{\theta}) = \langle \psi(\vec{\theta}) | H | \psi(\vec{\theta}) \rangle$$



Quantum Approximate Optimization Algorithm (QAOA)

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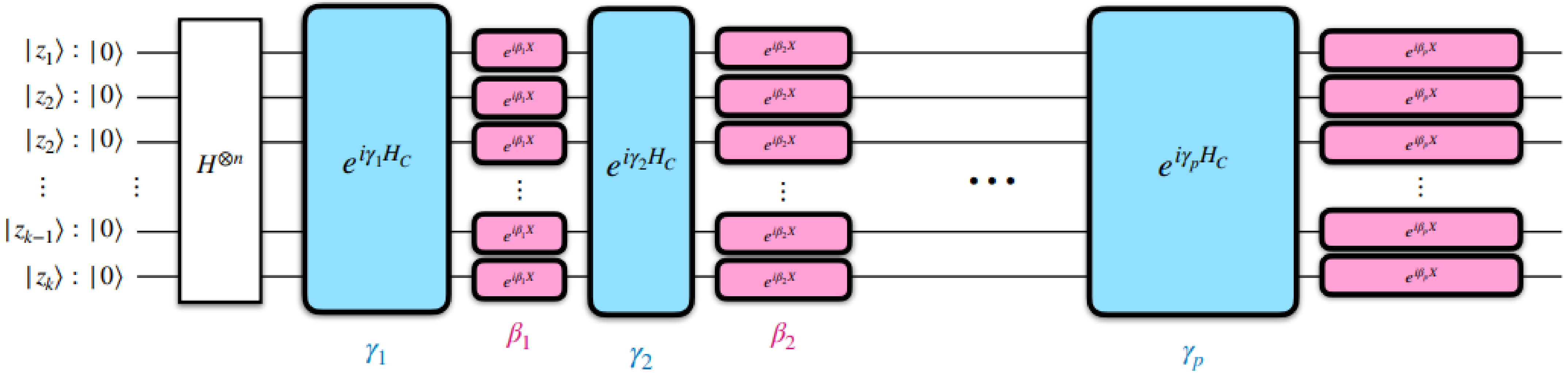
- We parameterized the circuit using optimizable coefficients to the initial Hamiltonian and the problem Hamiltonian.

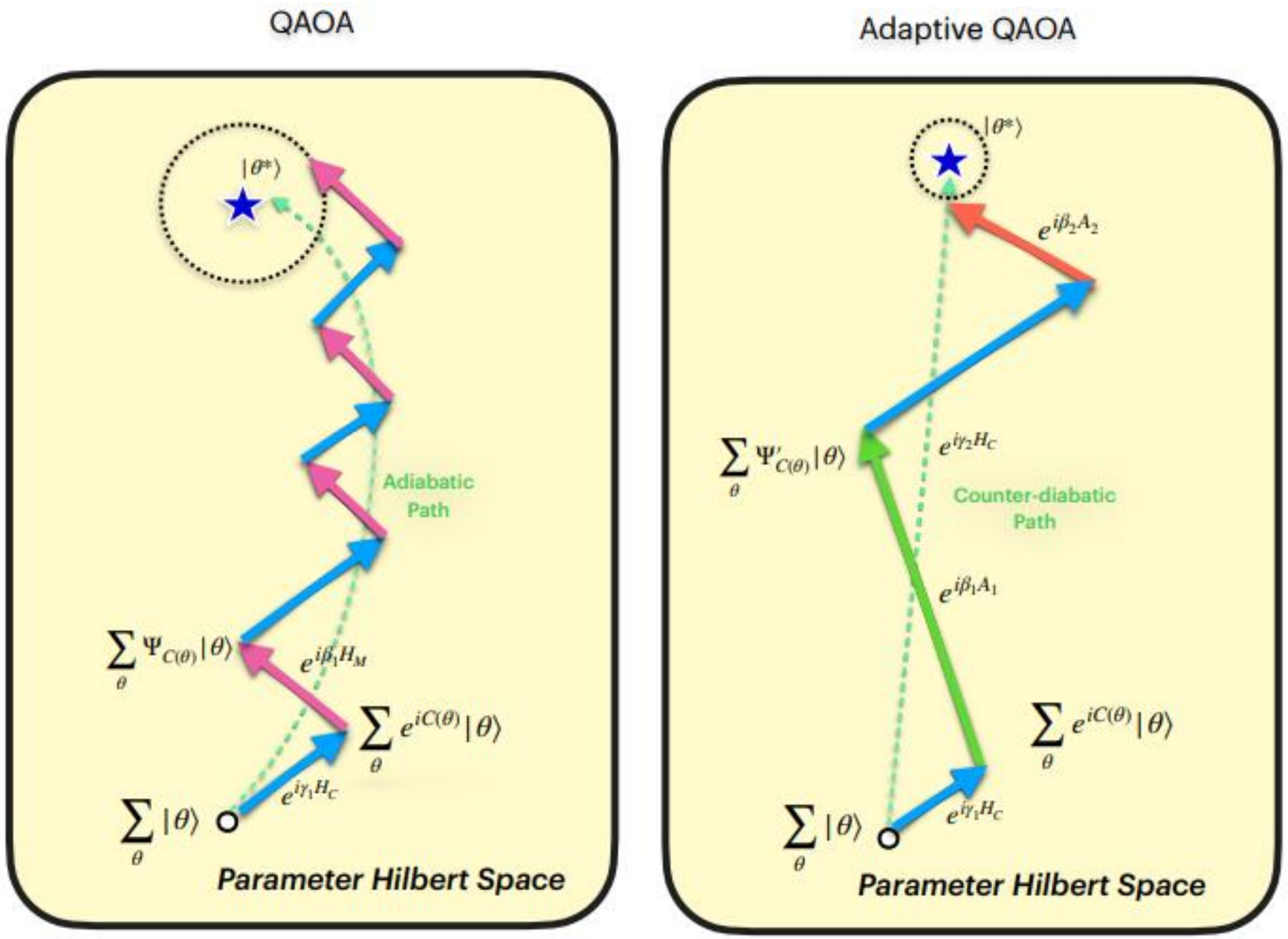
$$|\gamma, \beta\rangle = \prod_{j=1}^{p} \exp\left[-i\beta_j \hat{H}_M\right] \exp\left[-i\gamma_j \hat{H}_P\right] |+\rangle$$

- It was proposed to solve classical problems such as the maxcut problem

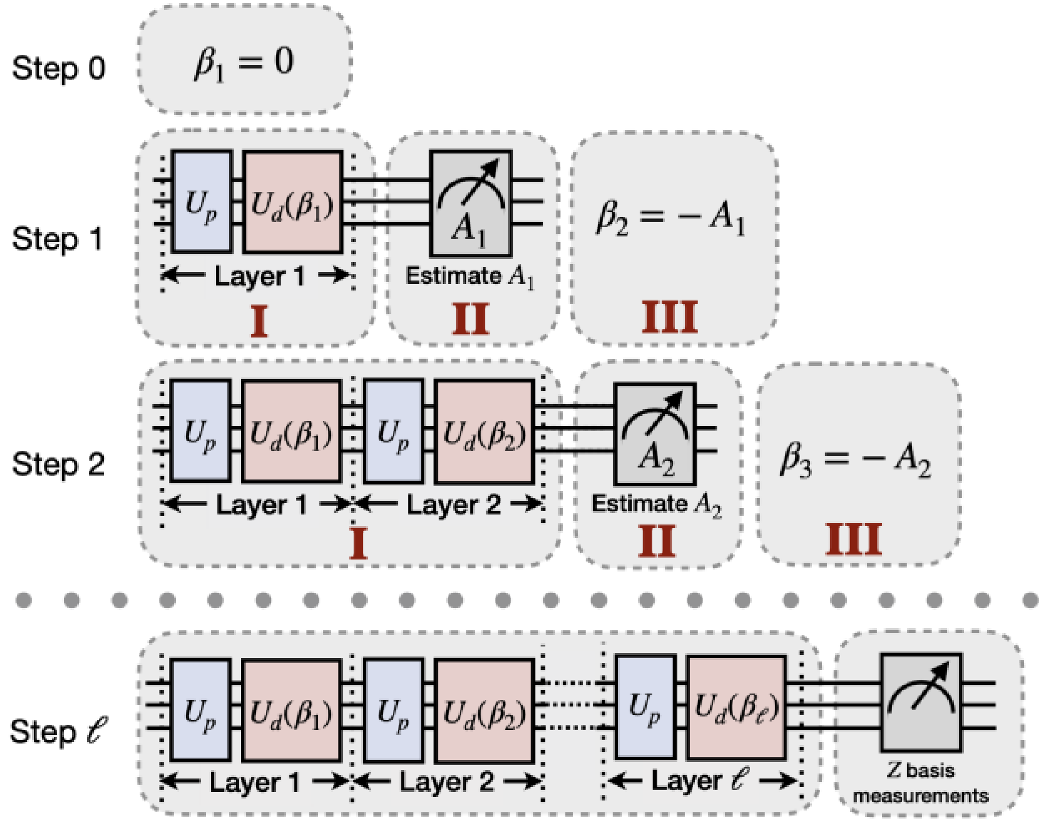


Variations On QAOA

- ADAPT-QAOA 2005.10258

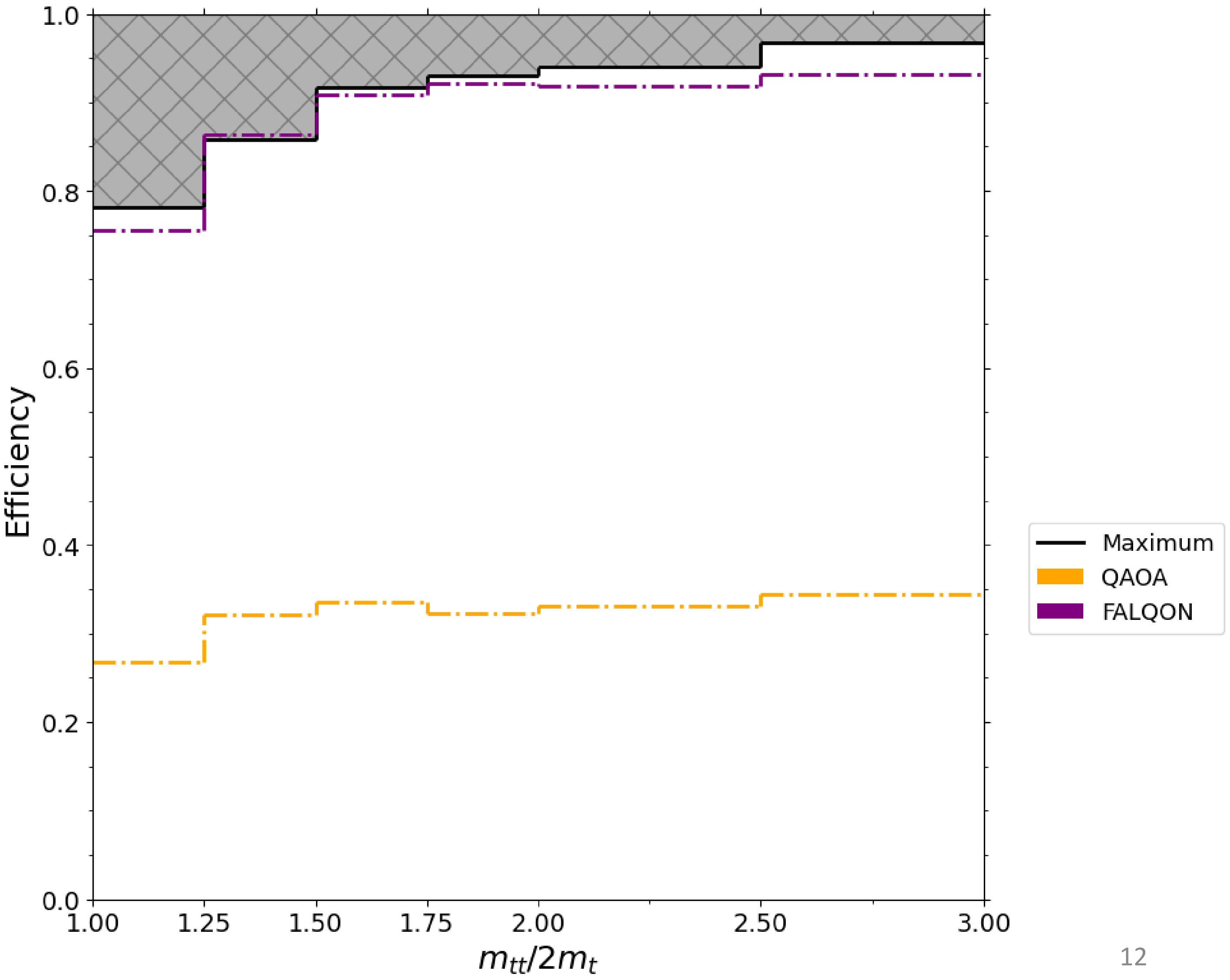


- Feedback-based ALgorithm for Quantum OptimizatioN (FALQON) 2103.08619



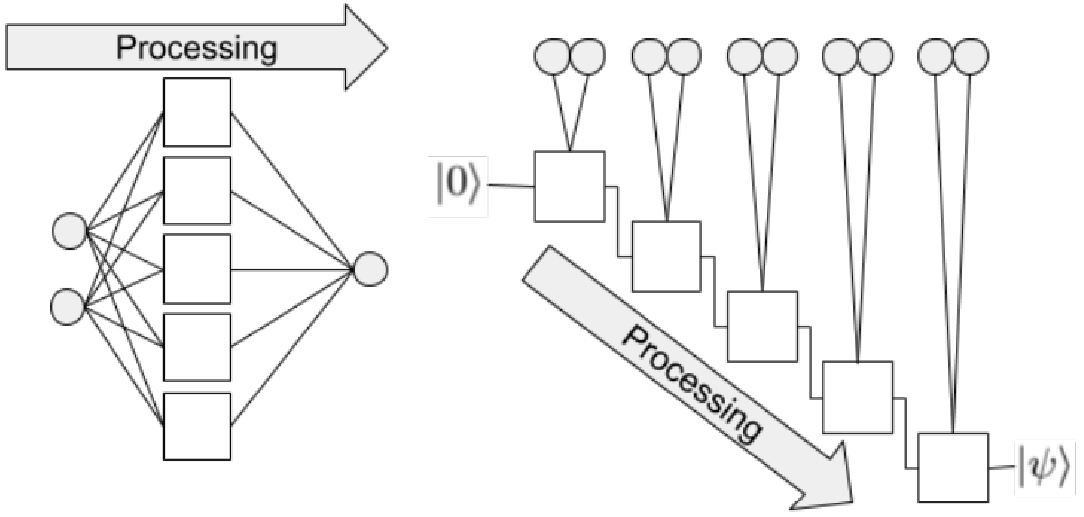
Preliminary results

- Our preliminary study shows that quantum algorithms can solve the combinatoric problem at relatively high efficiency, similar to the result by minimize the problem Hamiltonian exactly.
- Not every approximate algorithm will minimize this particular Hamiltonian correctly.



Quantum Machine Learning

- Machine learning in the context of quantum computing has some differences compare to its classical counterpart. As information from 1 qubit cannot be copied to multiple qubits in general.
- Universal approximation theorem can still hold if we take a slightly different approach.
- In principle, data reuploading routine can achieve similar performance as classical neural network when the number of parameters are small.



(a) Neural network (b) Quantum classifier

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Summary

- Quantum computing may provide unique solutions to some of the HEP problems without requiring a large number of qubits.
- We used some available quantum algorithm to study combinatoric problems that typically appears in collider phenomenology.
- Performance of quantum algorithm in finding the correct solution seem to outperform some traditional kinematic methods and are relatively comparable to classical machine learning methods.
- Quantum machine learning can also be used to solve high energy physics problem without needing too many qubits.