Constructing Operator Basis in Supersymmetry

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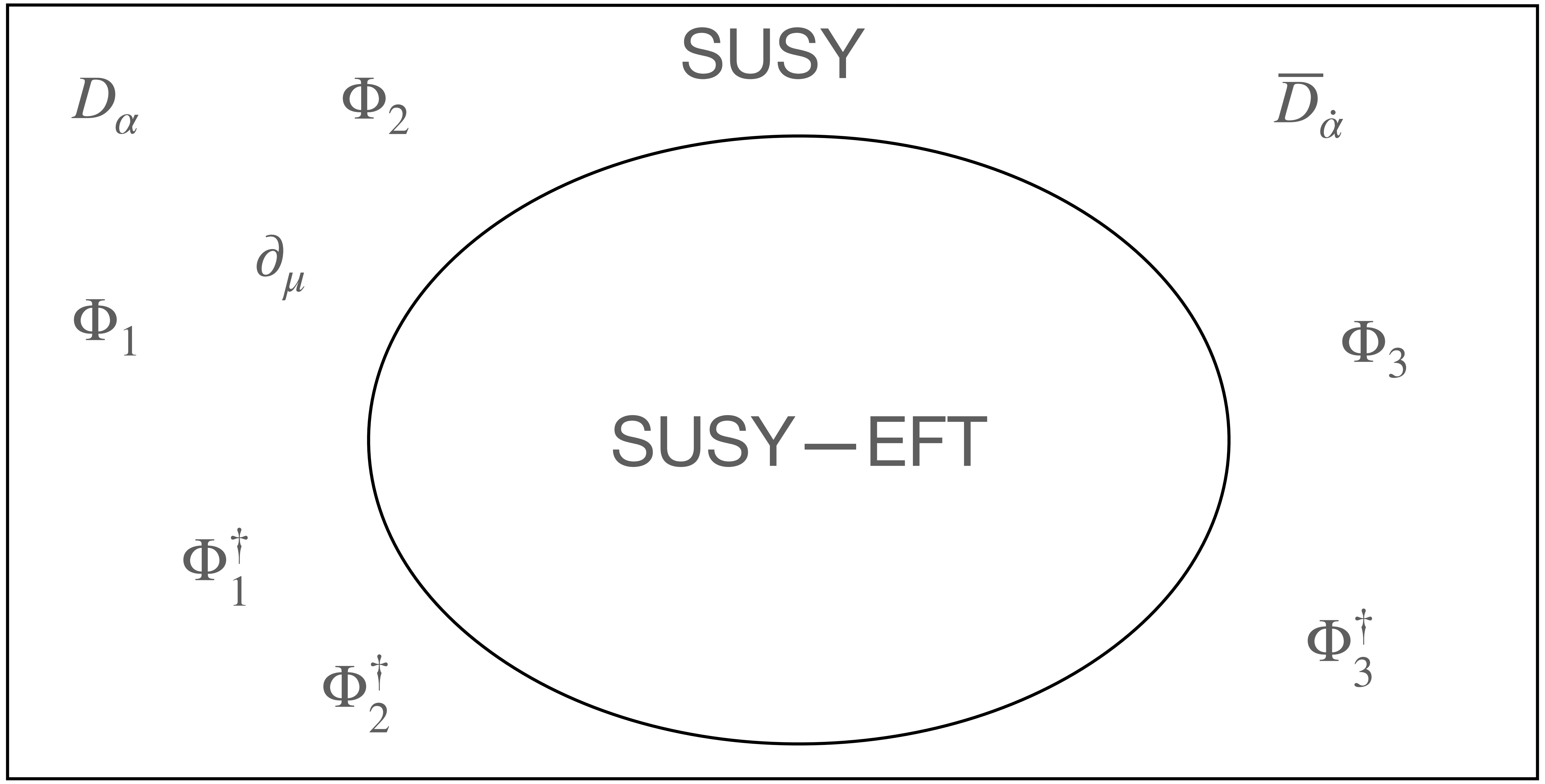


—Standard Model Effective Field Theory (SMEFT)

We should include all terms that respect symmetries of the theory into the Lagrangian.

Dimension-5 operator: the Weinberg operator, violates lepton number

Dimension-6 operators: Warsaw basis

Higher dimensional operators are usually related by different redundancies, e.g. equations of motion, integration by parts, Fierz identities, etc. To find/build the basis of these operators becomes important if we want to put independent operators (give different S-matrix elements) in the Lagrangian.

What happens when we add derivatives?

—EOM and IBP relations

EOM: $\partial^2\phi \sim m^2\phi,\ i\gamma^\mu\partial_\mu\psi \sim m\psi^\dagger, \cdots$

IBP: $\mathcal{O}_1 \sim \mathcal{O}_2 + \partial\mathcal{O}_3$

Short conformal characters:

$$\bar{\chi}_{(0,0)} = P(\alpha,\beta,D)(1-D^2)$$

$$P(\alpha,\beta,D) = \left((1-D\alpha\beta)(1-\frac{D}{\alpha\beta})(1-\frac{D\alpha}{\beta})(1-\frac{D\beta}{\alpha})\right)^{-1}$$

$$\bar{\chi}_{(\frac{1}{2},0)} = P(\alpha,\beta,D)((\alpha+\frac{1}{\alpha})-D(\beta+\frac{1}{\beta})),\quad \bar{\chi}_{(0,\frac{1}{2})} = P(\alpha,\beta,D)((\beta+\frac{1}{\beta})-D(\alpha+\frac{1}{\alpha}))$$

$$\mathcal{H} = \int d\mu \frac{1}{P} PE[\sum_i \phi_i \chi_{R,i}] + \Delta H$$

Operators less than dimension 5.

- $d\mu$: project out Lorentz scalars
- $\frac{1}{P}$: remove IBP
- Short conformal characters: remove EOM

EOM in Supersymmetry

For a chiral superfield Φ, equation of motion is given by $\partial_\alpha^2 \Phi \sim m\Phi^\dagger$. We can verify this relation by expanding Φ in components, i.e. $\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$, where $y^\mu = x^\mu + i\theta\sigma^m\bar{\theta}$, and we get $\partial_\alpha^2\phi = m\phi$, $i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\psi^\alpha = m\psi^\dagger_{\dot{\alpha}}$.

$$\begin{pmatrix} \Phi \\ \partial_\alpha\Phi \\ \partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_{\dot{\beta}}\partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \cdots \end{pmatrix} = \begin{pmatrix} \Phi \\ \partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_{\dot{\beta}}\partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_{\dot{\gamma}}\partial_\gamma\partial_{\dot{\beta}}\partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \cdots \end{pmatrix} + \begin{pmatrix} \partial_\alpha\Phi \\ \partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_\gamma\partial_{\dot{\beta}}\partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_{\dot{\tau}}\partial_\gamma\partial_{\dot{\beta}}\partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \cdots \end{pmatrix}$$

Indices are chosen to be symmetric combinations

Free of EOM

Bosonic Part $\xleftrightarrow{\text{SUSY}}$ Fermionic Part

$\bar{\chi}_{(0,0)} = P(\alpha,\beta,D)(1-D^2)$

$\bar{\chi}_{(\frac{1}{2},0)} = P(\alpha,\beta,D)((\alpha+\frac{1}{\alpha}) - D(\beta+\frac{1}{\beta}))$

$$PE[\sum_i \phi_i \chi_{R,i}] = PE[\Phi\bar{\chi}_{(0,0)} + P\Phi\bar{\chi}_{(\frac{1}{2},0)}]$$

P: spurion of super derivative

IBP in Supersymmetry

—2 Independent IBP Relations

3 different derivatives $\partial_\mu,\ \partial_\alpha,\ \partial_{\dot\alpha}$ ⟹ ***3 IBP relations***

$$K \sim K' + \partial_\alpha X^\alpha$$
$$K \sim K' + \partial_{\dot\alpha} X^{\dot\alpha}$$
$$K \sim K' + \partial_\mu X^\mu$$

Only 2 of them are independent! $K \sim K' + \partial_\alpha(\partial_{\dot\alpha} X^{\alpha\dot\alpha}) + \partial_{\dot\alpha}(\partial_\alpha X^{\alpha\dot\alpha})$

Still, we have 2 relations and the previous 1/P factor doesn't work here. (P is not a rep here)

It is tempting to simply subtract the number of $X^\alpha, X^{\dot\alpha}$ to get the number of independent operators, because it seems like that one operator with one fewer derivative provides one IBP relation, and if we get rid of all these operators, our result is free of IBP. However it's incorrect because these IBP relations can be linearly dependent!

—Correction Space

Starting with a space $\mathcal{O}$, we define the **zeroth order equivalence relations** on $\mathcal{O}$ as follows:

$$o_1 \sim o_2 + \sum \mathcal{I}_i s_i, \quad o_i \in \mathcal{O}, s_i \in S_i^0 .$$

We call $\mathcal{S}_j^1$ the **first order correction space** if all elements in $\mathcal{S}_j^1$ satisfy the following conditions:

$$\mathcal{T}_{ij}^1 s_j \neq 0, \; and \; \mathcal{I}_i \mathcal{T}_{ij}^1 s_j = 0, \; (no\ sums\ over\ i), \forall s_j \in \mathcal{S}_j^1.$$

A **space** $\mathcal{S}_j^n$ is called the **nth-order correction** to $\mathcal{O}$ if there exist **maps**:

$\mathcal{T}_{ij}^n : S_j^n \rightarrow S_i^{(n-1)}$, such that:

$$\mathcal{T}_{ij}^n s_j \neq 0, \; and \; \mathcal{T}_{ki}^{n-1} \mathcal{T}_{ij}^n s_j = 0, \forall s_j \in S_j^n, \forall k,$$

and is denoted as $\mathcal{S}_j^n(\{\mathcal{S}_i^{n-1}\} \rightarrow \{\mathcal{S}_i^{n-2}\}), n \geq 2.$

$$\#\ of\ independent\ operators =$$

$$\#\{\mathcal{O}\} - \# \sum \{\mathcal{S}_i^0\} + \# \sum \{\mathcal{S}_i^1\} - \# \sum \{\mathcal{S}_i^2\} + \cdots$$

—SMEFT Example Revisit

e.g. $\mathscr{O}_i \sim \mathscr{O}_j + \sum_n \partial_\mu \mathscr{O}_n^\mu, \quad \mathscr{O}_i, \mathscr{O}_j \in \{X\}, \mathscr{O}_n^\mu \in \{X^\mu\}.$

Operator Space IBP Space

If we identify $\mathscr{T}^1_{11} \equiv \partial_\mu$ and $\mathscr{I}_1 \equiv \partial_\nu$.

Terminates with four total antisymmetric indices in four dimensions.

$\mathscr{I}_1 \mathscr{T}^1_{11} s = \partial_\mu \partial_\nu X^{[\mu\nu]} = 0$ $X^{[\mu\nu]}$ is the first order correction space!

$\mathscr{T}^1_{11} \mathscr{T}^2_{11} X^{[\mu\nu\rho]} = \partial_\mu \partial_\nu X^{[\mu\nu\rho]} = 0$ $X^{[\mu\nu\rho]}$ is the second order correction space!

$\mathscr{T}^2_{11} \mathscr{T}^3_{11} X^{[\mu\nu\rho\sigma]} = \partial_\mu \partial_\nu X^{[\mu\nu\rho\sigma]} = 0$ $X^{[\mu\nu\rho\sigma]}$ is the third order correction space!

Diagram:

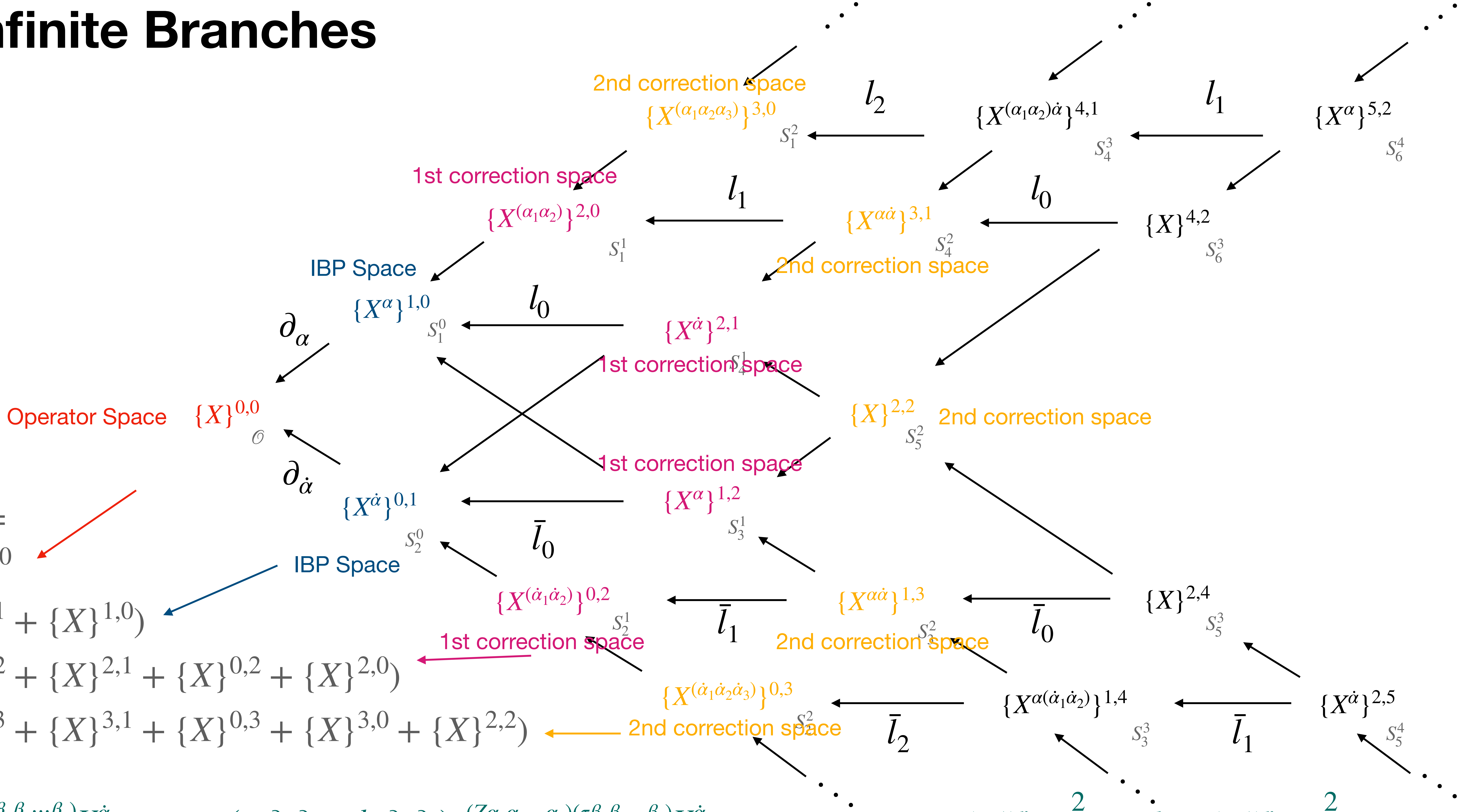
$$\{X\} \xleftarrow{\partial_\mu} \{X^\mu\} \xleftarrow{\partial_\nu} \{X^{[\mu\nu]}\} \xleftarrow{\partial_\rho} \{X^{[\mu\nu\rho]}\} \xleftarrow{\partial_\sigma} \{X^{[\mu\nu\rho\sigma]}\}$$

Order: $1 \quad D \quad D^2 \quad D^3 \quad D^4$

$$\sum D^n \chi_{X^{[\mu_1\mu_2\cdots\mu_n]}} = 1 - D(\alpha + \frac{1}{\alpha})(\beta + \frac{1}{\beta}) + D^2[(1 + \alpha^2 + \frac{1}{\alpha^2}) + (1 + \beta^2 + \frac{1}{\beta^2})] - D^3(\alpha + \frac{1}{\alpha})(\beta + \frac{1}{\beta}) + D^4 = \frac{1}{P}$$

Operator Space - IBP Space + First order correction space - Second order correction space + Third order correction space

—6 Infinite Branches



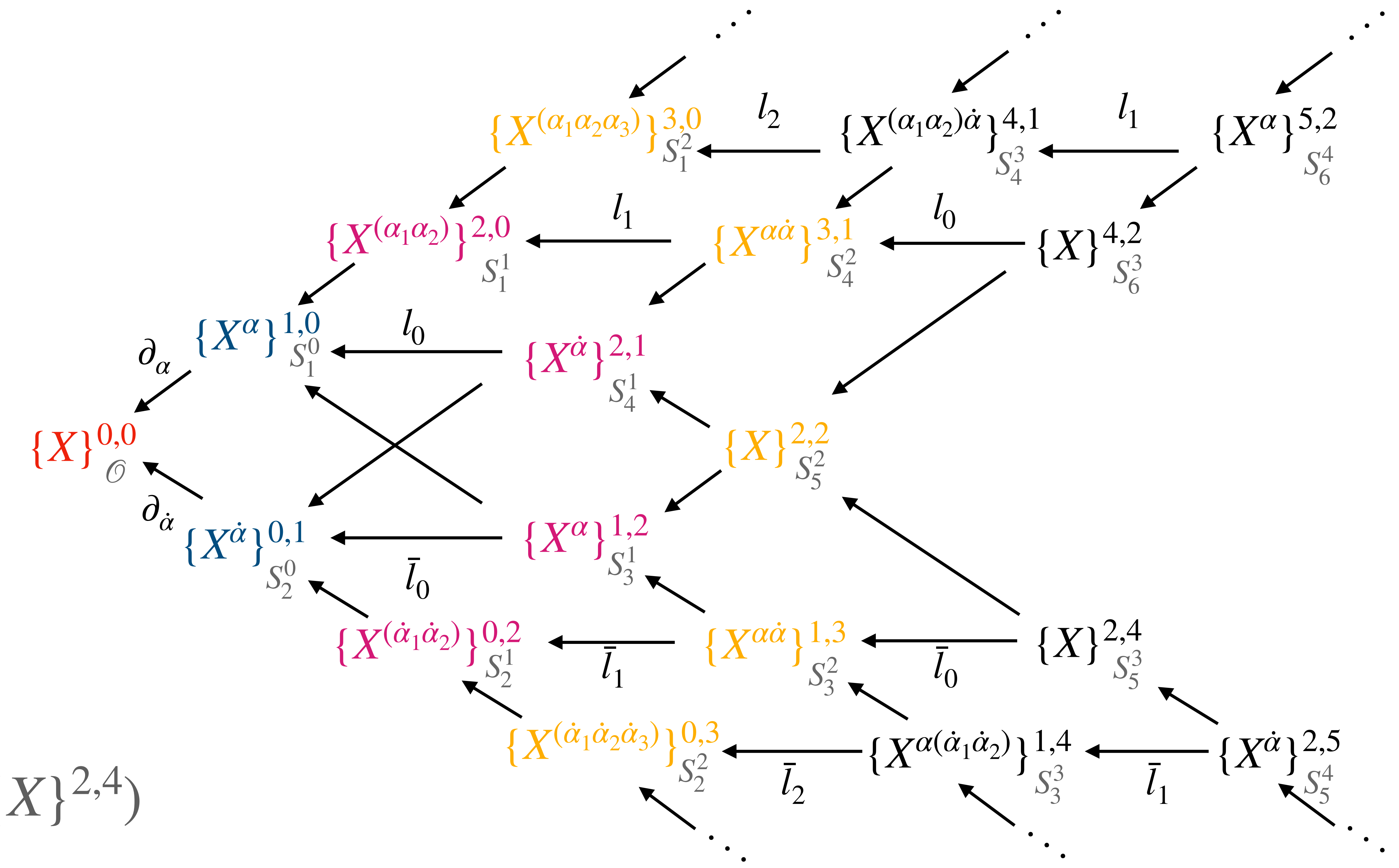
$$
\begin{aligned}
number &= \\
&= \#\{X\}^{0,0} \\
&-\#(\{X\}^{0,1} + \{X\}^{1,0}) \\
&+\#(\{X\}^{1,2} + \{X\}^{2,1} + \{X\}^{0,2} + \{X\}^{2,0}) \\
&-\#(\{X\}^{1,3} + \{X\}^{3,1} + \{X\}^{0,3} + \{X\}^{3,0} + \{X\}^{2,2}) \\
&\cdots
\end{aligned}
$$

$$(l_n)_{\dot\alpha}^{(\alpha_1\alpha_2\cdots\alpha_n)(\tau\beta_1\beta_2\cdots\beta_n)} X^{\dot\alpha}_{(\alpha_1\alpha_2\cdots\alpha_n)} = (a_n\partial_{\dot\alpha}\partial_Z + b_n\partial_Z\partial_{\dot\alpha})\epsilon^{(Z\alpha_1\alpha_2\cdots\alpha_n)(\tau\beta_1\beta_2\cdots\beta_n)} X^{\dot\alpha}_{(\alpha_1\alpha_2\cdots\alpha_n)}$$

$$a_n = (-1)^n\frac{2}{(n+1)!}, \quad b_n = (-1)^n\frac{2}{(n+2)n!}$$

—Summation

$$number =$$
$$= \#\{X\}^{0,0}$$
$$-\#(\{X\}^{0,1} + \{X\}^{1,0})$$
$$+\#(\{X\}^{1,2} + \{X\}^{2,1} + \{X\}^{0,2} + \{X\}^{2,0})$$
$$-\#(\{X\}^{1,3} + \{X\}^{3,1} + \{X\}^{0,3} + \{X\}^{3,0} + \{X\}^{2,2})$$
$$+\#(\{X\}^{1,4} + \{X\}^{4,1} + \{X\}^{0,4} + \{X\}^{4,0} + \{X\}^{4,2} + \{X\}^{2,4})$$
$$\cdots$$



$$\sum P^p Q^q \chi_{X^{p,q}}$$

P,Q represent two super derivatives

$$= 1$$
$$-(Px + Qy)$$
$$+(PQ^2x + P^2Qy + P^2(x^2-1) + Q^2(y^2-1))$$
$$-(PQ^3xy + P^3Qxy + P^3(x^3-2x) + Q^3(y^3-2y) + P^2Q^2)$$
$$\cdots$$

This becomes the 1/P factor in supersymmetry, and when we put this into Hilbert series, it will automatically remove all IBP redundancies.

$$\mathcal{H} = \int d\mu \frac{1}{P_{new}} PE[\sum_i \phi_i \chi_{R,i}] + \Delta H$$

In practical use, we will truncate this infinite series.

Conclusion

- Short conformal characters remove all EOM relations
- Correction space
- 1/P factor removes all IBP relations
- 6 infinite branches
- Gauge interactions
- Superconformal approach?

Thank you!