1+1D Hadrons Minimize their Biparton Renyi Free Energy

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- Going to lower dimensions simplifies things too.

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- Eigenvalue Problem → Optimization Problem.

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- (Assumed one flavor of quarks in fundamental representations.)
- Will discuss the underlying principle behind our ansatz.

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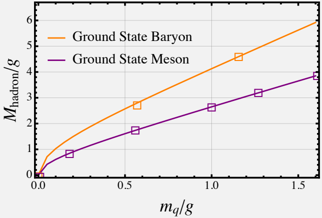
- Similar calculation can be done for baryons.

Results: 't Hooft Model with $N_c = 3$



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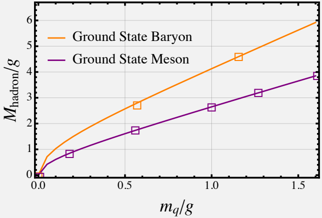

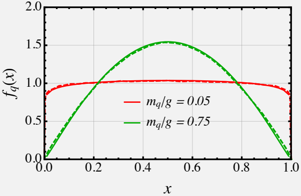

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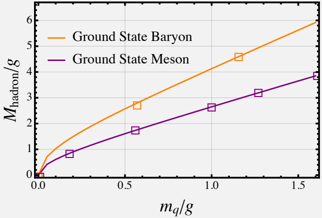

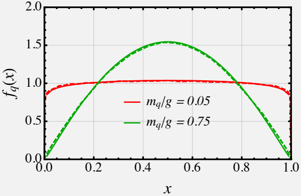

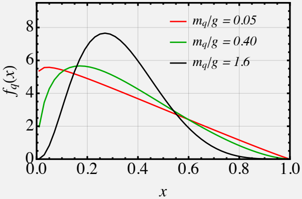

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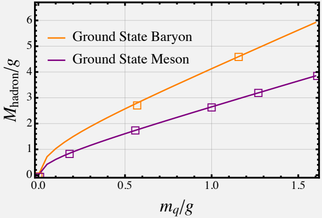

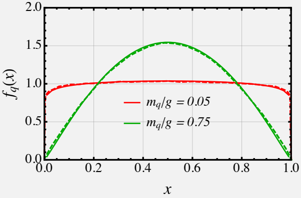

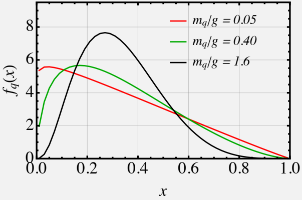

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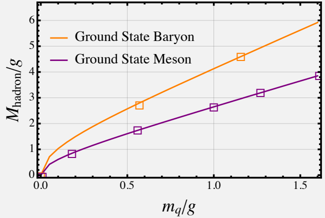

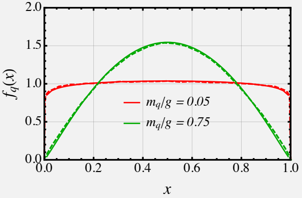

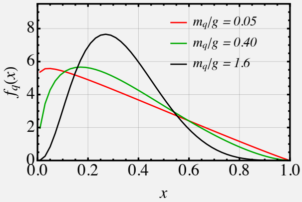

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- Where did this template function come from?

An Analogy

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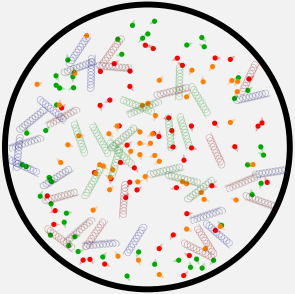

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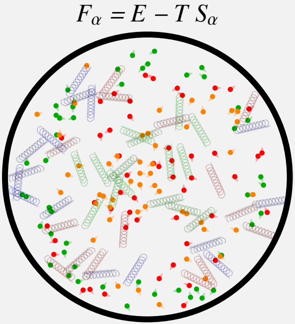

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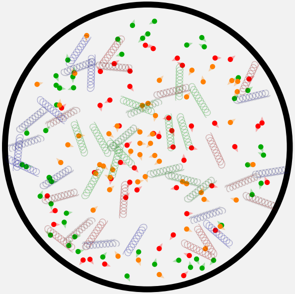

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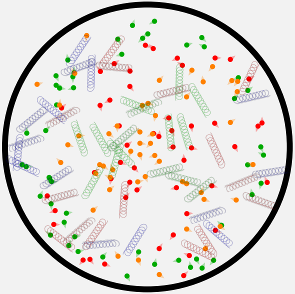

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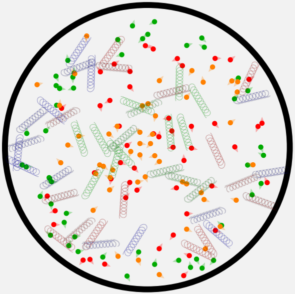

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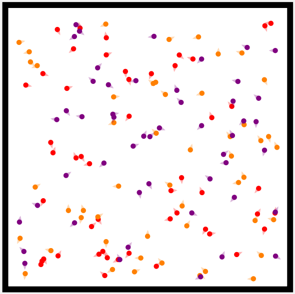

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- Probabilities from an emergent min. free energy principle.

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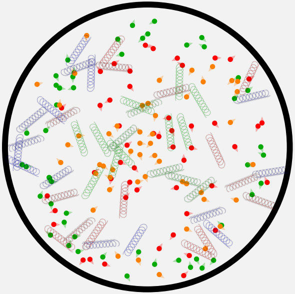

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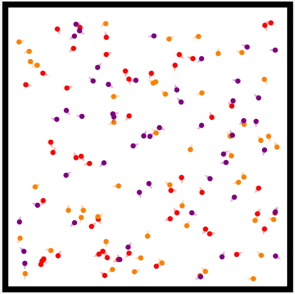

Deriving the Ansatz - Minimizing a Free Energy

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- Our ansatz minimizes Hadrons' Renyi free energy!

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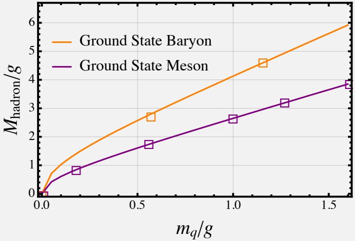

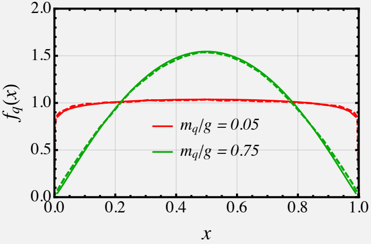

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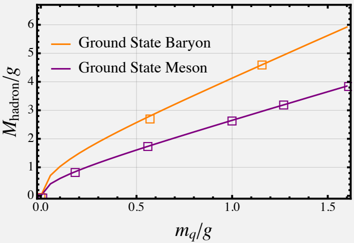

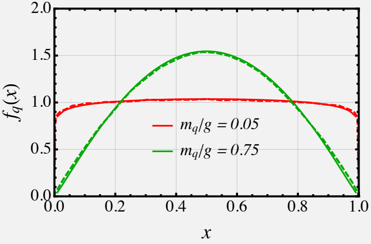

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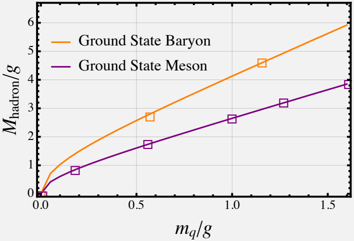
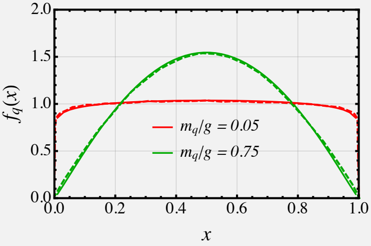




THANK YOU!

Back up

- Understanding Confinement
- Homage to Lightcone
- In Praise of Lower Dimensions
- Schwinger Model
- 't Hooft Model
- Renyi and Tsallis Entropy
- (Quantum) Information and Statistical Mechanics
- Baryons
- Hamiltonian in Higher Dims.
- The Variation Parameter
- Pheno Applications

Understanding Confinement

- Formal proof of the mass gap.
- When does it take place? Effect of θ?
- What is a good order parameter?
- Is the phase transition first order or second order?
- Analytic understanding of hadron properties.
- Hadron properties: PDF, mass, spin, dipole moments, form factors

Homage to Lightcone

- See hep-ph/9705477 for further details.
- Three dimensional surface in spacetime formed by a plane wave front advancing with the velocity of light.
- Maximal number of "kinematical" Poincare generators.
- Hadrons Hilbert space decomposes into constituent partons Fock space.
- The vacuum is simple. No state with zero momentum and arbitrary number of particles.
- The wavefunction is frame-independent: no need to boost the result when expressed in terms of x_i.
- P^+ is the Hamiltonian. Hadrons are eigenstates of the confining theory Hamiltonian.
- Hamiltonian does not include a square root.
- P^-, M, $P_\perp$, and spin are invariants of the Hamiltonian.

In Praise of Lower Dimensions

- No spins; no transverse momentum; gauge fields do not propagate.
- Running is suppressed: polynomial (as oppose to logarithmic) Q/Λ suppression.
- Gauss law: linear potential between sources of charge.
- QED_{1+1} (Schwinger) and QCD_{1+1} ('t Hooft) are examples.

Schwinger Model & Bosonization

$$S = \int d^2x \, \left(i\psi_+^\dagger(\partial_0 - \partial_1)\psi_+ + i\psi_-^\dagger(\partial_0 + \partial_1)\psi_-\right)$$

$$< \psi_+(x)\psi_+^\dagger(y) >= -\frac{i}{2\pi}\frac{1}{(x-y)-i\epsilon}$$

$$S = \int d^2x \, \frac{1}{8\pi}\left(\partial_\mu\phi\right)^2 \quad \partial^\mu\phi = 2\,\epsilon^{\mu\rho}\,\partial_\rho\tilde{\phi} \quad \phi_\pm = \frac{1}{2}\left(\phi \mp 2\,\tilde{\phi}\right)$$

$$<: e^{i\phi_+(x)} :: e^{-i\phi_+(y)} :> = \frac{\epsilon}{\epsilon + i(x-y)}$$

$$\psi_-(x) \leftrightarrow \sqrt{\frac{1}{2\pi\epsilon}} \; : e^{i\phi_-(x)} :$$

$$\bar{\psi}\,\psi \leftrightarrow -\frac{1}{2\pi\epsilon}\left(: e^{-i\phi(x)} + e^{+i\phi(x)} :\right) = -\frac{1}{\pi\epsilon} : \cos\phi(x) : \quad \overline{\iota\psi}\,\gamma^5\psi \leftrightarrow -\frac{1}{\pi\epsilon} : \sin\phi(x) : \quad \bar{\psi}\,i\gamma^\mu\partial_\mu\,\psi \leftrightarrow \frac{1}{8\pi}\left(\partial_\mu\phi\right)^2$$

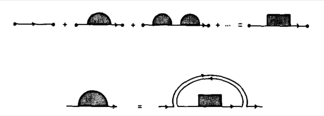
$$S = \int d^2x \, \left(\frac{1}{2}E^2 + \frac{e}{2\pi}\phi E + \frac{1}{8\pi}\left(\partial_\mu\phi\right)^2\right) \quad E = -\frac{e}{2\pi}\,\phi$$

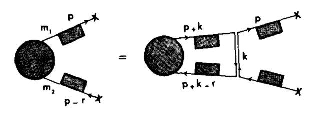
$$S = \frac{1}{8\pi}\int d^2x \, \left(\left(\partial_\mu\phi\right)^2 - \frac{e^2}{\pi}\phi^2\right)$$

With fermion mass, we get an interaction whose potential can be calculated via Bethe-Salpeter.

't Hooft Model

In the large-N limit, only planar diagrams with no fermion loops matter.





$$\psi(p,r) = -\frac{4g^2}{(2\pi)^2 i}(p_- - r_-)p_-\left[M_2^2 + 2(p_+ - r_+)(p_- - r_-) + \frac{g^2}{\pi\lambda}|p_- - r_-| - i\epsilon\right]^{-1}$$

$$\times\left[M_1^2 + 2p_+p_- + \frac{g^2}{\pi\lambda}|p_-| - i\epsilon\right]^{-1}\iint\frac{\psi(p+k,r)}{k^2}\,\mathrm{d}k_+\,\mathrm{d}k_-\,, \qquad (15)$$

$$\varphi(p_-,r) = \int\psi(p_+,p_-,r)\,\mathrm{d}p_+\,,$$

$$\mu^2\varphi(x) = \left(\frac{\alpha_1}{x} + \frac{\alpha_2}{1-x}\right)\varphi(x) - \mathrm{P}\int_0^1\frac{\varphi(y)}{(y-x)^2}\,\mathrm{d}y$$

Renyi vs. Tsallis

$$S_\alpha(\rho) = \frac{1}{1-\alpha} \ln\left(\mathrm{Tr}\left[\rho^\alpha\right]\right).$$

$$S_\alpha(\rho) = \frac{1}{1-\alpha} \left(\mathrm{Tr}\left[\rho^\alpha\right] - 1\right).$$

- Usually used with $\alpha > 0$.
- Converge to von Neumann entropy at $\alpha \to 1$.
- Non-negative (for discrete distributions).
- Subadditivity and strong subadditivity not followed (except in special cases).
- Various applications in: complex system, blackhole physics, astrophysics, nuclear theory, condensed matter.

(Quantum) Information and Statistical Mechanics

- Assume we are not given the pdf of a random variable x, rather the expected value of some functions of them $f_i(x)$.
- What is the expected value of an arbitrary function $g(x)$?
- The MaxEnt principle is not an application of laws of physics, rather a method of reasoning with no arbitrary assumptions beyond what we know.
- Any suitable measure of uncertainty works.
- When measure of uncertainty is the Shannon entropy, we find exponential distributions.
- Jaynes argues this approach applies thanks to *typicality and sharply-peaked distributions*; its prerequisite is having a many-body system.

A Baryon Wavefunction in 1+1D

Start with $N = 3$. Hilbert space decomposed into qqq Fock spaces

$$|\psi\rangle = \sum_{ijk=0}^{1} p_{i,j,l}\delta_{i+j+l,1}|ijl\rangle$$

Two-partons' reduced density matrix:

$$\bar{\rho} = \sum_{l} \mathcal{N}_l \rho_{2,l},$$

$$\begin{aligned}
\rho_{2,l} &\equiv \sum_{ij=0}^{1-l}\sum_{\bar{i}\bar{j}=0}^{1-l} \mathbf{p}_{i,j,l}\mathbf{p}^{*}_{\bar{i},\bar{j},l}\delta_{i+j,1-l}\delta_{\bar{i}+\bar{j},1-l}|ij\rangle\langle\overline{ij}|, \\
\mathcal{N}_l &\equiv \sum_{i,j=0}^{1-l} |p_{i,j,l}|^2\delta_{i+j,1-l}, \quad \mathbf{p}_{i,j,l} \equiv \frac{p_{i,j,l}}{\sqrt{\mathcal{N}_l}}.
\end{aligned}$$

$\rho_{2,l}$ is a fixed momentum 2-parton reduced density matrix.

A Baryon Wavefunction in 1+1D

For a fixed momentum l, it is a pure state:

$$\begin{aligned}\rho_{2,l} &= |\psi_l^{(2)}\rangle\langle\psi_l^{(2)}| \\ |\psi_l^{(2)}\rangle &= \sum_{ij=0}^{1-l} \mathbf{p}_{i,j,l}\delta_{i+j,1-l}|ij\rangle.\end{aligned}$$

Similar to the meson case now. Tracing one remaining quark out:

$$\rho_{1,l} = \sum_i |\mathbf{p}_{i,1-i-l,l}|^2 |i\rangle\langle i|.$$

We now calculate the Renyi entropy of all $\rho_{1,l}$ matrices. Adding to this the kinetic energy of the remaining two quarks, we find the fixed momentum biparton free energy.

Baryon's Biparton Renyi Free Energy

Summing all these fixed-l free energies, we get the full Biparton Renyi Energy of the system.

$$\frac{\delta F_\alpha}{\delta|\phi(x,z)|^2} = 0 \Longrightarrow |\phi(x,z)|^2 = \left(\frac{m_q^2}{\mathcal{T}^2}\right)^{\frac{1}{\alpha-1}} \Big[x\ z\ (1-x-z)\Big]^{\frac{1}{1-\alpha}}.$$

The PDF is obtained by integrating over one of the fractional momentum variables:

$$f_q(x) = 3\left(\frac{m_q^2}{\mathcal{T}^2}\right)^{\frac{1}{\alpha-1}} \int_0^{1-x} dz\ \ (x\ z\ (1-x-z))^{\frac{1}{1-\alpha}}\ .$$

We will use this as the template function of a variation method for baryons.

Hamiltonian in Higher Dims.

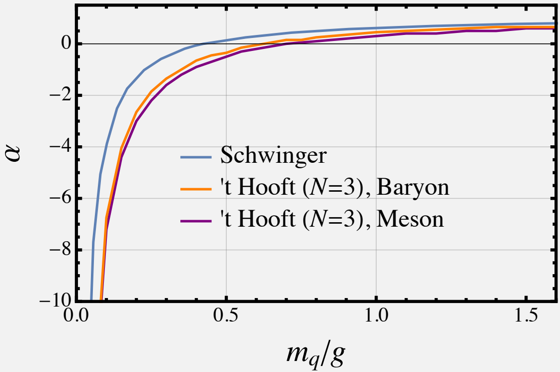
- Gauge bosons are dynamic.
- There is transverse momentum.

$$
\begin{aligned}
P^- &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \left(\bar{\psi}_+ \gamma^+ \frac{m^2 + (i\nabla_\perp)^2}{i\partial^+}\psi_+ - A^{a\mu}\left(i\nabla_\perp\right)^2 A^a_\mu\right) \\
&+ g_s \int dx^- d^2\mathbf{x}_\perp\, \bar{\psi}_+ \gamma^\mu T^a \psi_+ A^a_\mu + \\
&+ \frac{g_s^2}{4}\int dx^- d^2\mathbf{x}_\perp\, c^{abc} c^{ade} A^b_\mu A^c_\nu A^{d\mu} A^{e\nu} \\
&+ \frac{g_s^2}{2}\int dx^- d^2\mathbf{x}_\perp\, \bar{\psi}_+ \gamma^+ T^a \psi_+ \frac{1}{\left(i\partial^+\right)^2}\bar{\psi}_+ \gamma^+ T^a \psi_+ \\
&+ \frac{g_s^2}{2}\int dx^- d^2\mathbf{x}_\perp\, \bar{\psi}_+ \gamma^\mu T^a A^a_\mu \frac{\gamma^+}{i\partial^+}\left(T^b A^b_\nu \gamma^\nu \psi_+\right),
\end{aligned}
$$

$$
M^2 = \sum_n \int \left[dx_i\right]\left[d^2\mathbf{k}_{\perp i}\right] \sum_{a=1}^{n}\left(\frac{\mathbf{k}^2_{\perp a} + m_a^2}{x_a}\right)\left|\psi_n(x_i, \mathbf{k}_{\perp i})\right|^2 + (\text{interactions}),
$$

$$
\begin{aligned}
M^2 &= \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\frac{\mathbf{k}_\perp^2}{x(1-x)}\left|\psi(x, \mathbf{k}_\perp)\right|^2 + (\text{interactions}) \\
&= \int_0^1 \frac{dx}{x(1-x)}\int d^2\mathbf{b}_\perp\, \psi^*(x, \mathbf{b}_\perp)\left(-\nabla^2_{\mathbf{b}_\perp}\right)\psi(x, \mathbf{b}_\perp) + (\text{interactions}).
\end{aligned}
$$

The Variation Parameter



Pheno Applications and Future Directions

- Calculating PDFs and masses for other representations, more flavors, higher dimensions, ...
- Calculating fragmentation functions.
- Form factors relevant for flavor physics.
- Jet properties.
- Dipole moments.
- Confining dark sectors: abundance and detection.