Learning about the early universe from dips in the gravitational wave spectrum

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Causality-limited GWs

- For a source of GWs that is active for a duration of $1/\beta$, the causality-limited part of the GWs are waves with $\lambda \gg 1/\beta$.

- Einstein equation:

$$h''(\mathbf{k},\tau) + 2\mathcal{H}h'(\mathbf{k},\tau) + k^2 h(\mathbf{k},\tau) = J(\mathbf{k},\tau)$$

- General solution[1]:

$$h(\mathbf{k},\tau) = \int d\tau' \frac{e^{-\mathcal{H}(\tau-\tau')}}{\sqrt{k^2-\mathcal{H}^2}} \sin\Big((\tau-\tau')\sqrt{k^2-\mathcal{H}^2}\Big) J(\mathbf{k},\tau')$$

Evolution of GWs in a fluid with a general equation of state

- The scale factor $a(\tau) \propto \tau^n$ and the conformal Hubble rate $\mathcal{H} = \frac{n}{\tau}$ with $n = 2/(1+3w)$.
- For the sub-horizon modes ($k \gg \mathcal{H}_*$), we find [1]

$$h(k,\tau) \approx \frac{a(\tau_*) J_*}{a(\tau) k} \sin(k(\tau - \tau_*))$$

- For modes that were super-horizon at the time of production after horizon entry, we have the solution [1]

$$h(k,\tau) \approx \frac{\Gamma(n - \frac{1}{2}) J_* \tau_*}{2\sqrt{\pi}} \left(\frac{2}{k\tau}\right)^n \cos\left(k\tau - \frac{n\pi}{2}\right)$$

Power spectrum

- The observable that is relevant for GW detectors is

$$\Omega_{\mathsf{GW}}(k) \equiv \frac{d\Omega_{\mathsf{GW}}}{d\log k}$$

- Sub-horizon modes:P_h is proportional to k^{-2}.

$$\implies \Omega_{\mathsf{GW}}(k) \propto k^5 P_h \propto k^3$$

- Super-horizon modes: P_h is proportional to k^{-2n}.

$$\implies \Omega_{\mathsf{GW}}(k) \propto k^5 P_h \propto k^{3-2\left(\frac{1-3w}{1+3w}\right)}$$

Scaling of $\Omega_{\mathsf{GW}}(k)$ vs k

There is a kink in the spectrum at horizon crossing for various cosmologies except for $n = 1$.

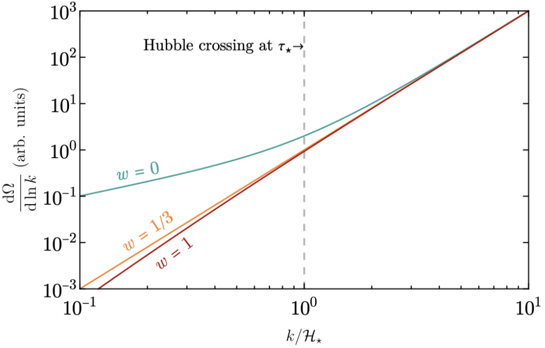

Figure 1: Scaling of $\Omega_{\mathsf{GW}}(k)$ versus $k/\mathcal{H}_*$ for different equations of state w [1].

WCSM

- The Weak-Confined Standard Model (WCSM) is a phase in which the $\mathrm{SU}(2)$ component of the electroweak force is strongly coupled.

- This can be achieved by using a scalar field $\hat{\varphi}$ which undergoes a phase transition during the early stages of the universe.

$$\frac{1}{g_{\text{eff}}^2} = \frac{1}{g^2} - \frac{\langle\hat{\varphi}\rangle}{M}$$

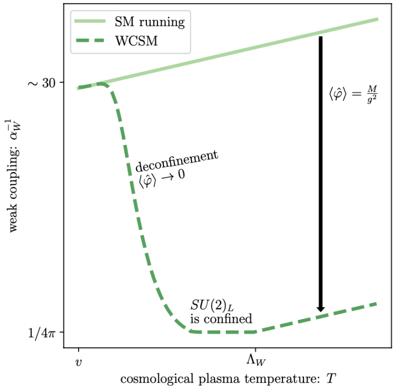

Figure 2: The WCSM phase [2].

Change in w with T

- The spectrum of the WCSM phase contains several composite states. The number density of these states get exponentially suppressed as the universe cools.
- As a result, w of the fluid pervading the universe changes with temperature.

Pion	#	Squared Mass $\times 16\pi^2/\Lambda_\mathrm{w}^2$
Π_3^0	3	$-4C_G g_s^2$
	6	$-4C_G g_s^2 + 2C_\mathrm{Yuk}\lvert V_{tb}\rvert^4 y_t^4$
$\Pi_2^\pm$	8	$-\frac{3}{2}C_G g_s^2 - \frac{1}{2}C_A e_Q^2 + \frac{1}{2}C_Z g_s^2 + C_\mathrm{Yuk}\lvert V_{tb}\rvert^4 y_t^4 - \sqrt{C_W^2 g_s^4 + C_\mathrm{Yuk}^2\lvert V_{tb}\rvert^8 y_t^8}$
	8	$-\frac{3}{2}C_G g_s^2 - \frac{1}{2}C_A e_Q^2 + \frac{1}{2}C_Z g_s^2 + C_\mathrm{Yuk}\lvert V_{tb}\rvert^4 y_t^4 + \sqrt{C_W^2 g_s^4 + C_\mathrm{Yuk}^2\lvert V_{tb}\rvert^8 y_t^8}$
	12	$-\frac{3}{2}C_G g_s^2 - \frac{1}{2}C_A e_Q^2 + \frac{1}{2}C_Z g_s^2 + C_W g_s^2$
	4	$-\frac{3}{2}C_G g_s^2 - \frac{1}{2}C_A e_Q^2 + \frac{1}{2}C_Z g_s^2 - C_W g_s^2$
$\Pi_1^\pm$	6	$-2C_A e_Q^2 - \frac{2}{3}C_Z g_s^2$
Π_1^0	2	$2C_\mathrm{Yuk}\lvert V_{tb}\rvert^2 y_t^4$
	4	$2C_\mathrm{Yuk}\lvert V_{tb}\rvert^2 y_t^2 y_\tau^2$
	1	$2C_\mathrm{Yuk}\lvert V_{cs}\rvert^4 y_c^4$
	2	$-\frac{1}{2}C_\mathrm{Yuk}\mathrm{Re}[V_{ts}]^2 y_t^2 y_\tau^2$
	1	$-2C_\mathrm{Yuk}\mathrm{Re}[V_{ts}]^2 y_t^2 y_\tau^2$
	1	$6C_\mathrm{Yuk}\mathrm{Re}[V_{cs}]\mathrm{Re}[V_{td}]\frac{\mathrm{Re}[V_{cs}]\mathrm{Re}[V_{td}]-\mathrm{Re}[V_{cd}]\mathrm{Re}[V_{ts}]}{\mathrm{Re}[V_{ts}]^2} y_c^2 y_\mu^2$
	2	0

Figure 3: Mass spectrum for the pions.

δw in the WCSM phase

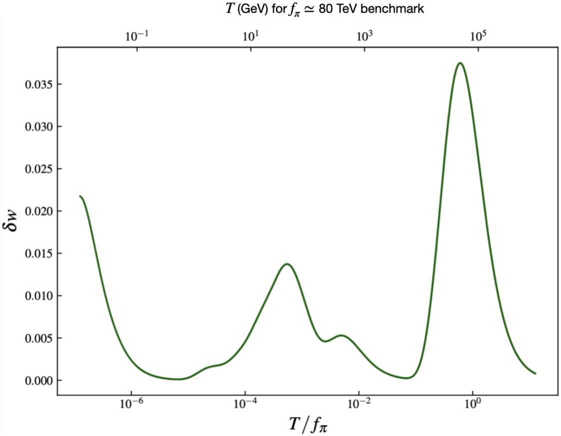

Figure 4: Typical behaviour of δw $(= w - 1/3)$ with respect to temperature during the WCSM phase.

Model and approximations

- We consider gravitational waves emanating from first-order phase transitions in the early universe for our calculations.
- The latent heat of the phase transition percolates into the cosmic fluid leading to sound waves with power spectrum

$$h^2\Omega_{\mathsf{sw}}(f) = 2.65 \times 10^{-6} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} v_w S_{\mathsf{sw}}(f)$$

$$S_{\mathsf{sw}}(f) = (f/f_{\mathsf{sw}})^3 \left(\frac{7}{4 + 3(f/f_{\mathsf{sw}})^2}\right)$$

The evolution of h

- With $\delta w \neq 0$, the conformal Hubble rate

$$\mathcal{H} \approx \frac{1}{\tau + \frac{3}{2}\int_{\tau_*}^{\tau} d\tau' \delta w(\tau')}$$

- Plugging this in the Einstein equation allows us to solve for h numerically.

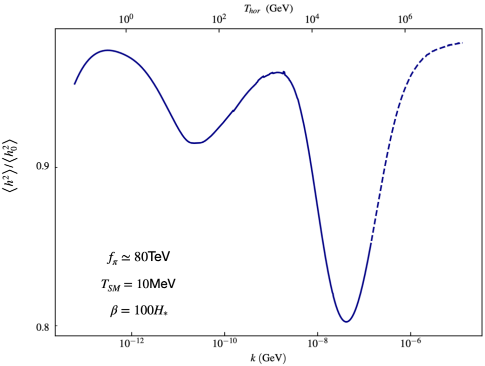

Figure 5: Ratio of $\langle h^2\rangle$ with respect to $\langle h_0^2\rangle$ for different values of k.

LISA sensitivity to signal

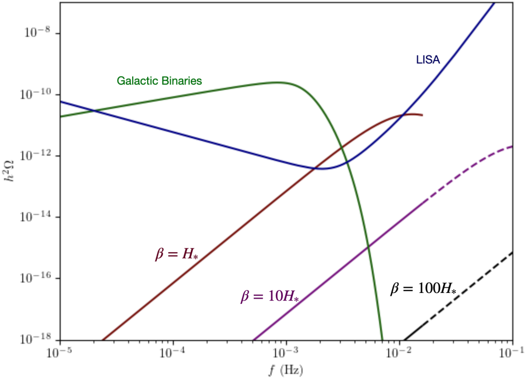

Figure 6: Sensitivity of LISA to the signal along with a background signal of galactic binaries. Data for LISA was obtained from [3] and the fit for the galactic binaries background was obtained from [4].

References I

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[2] J. Berger, A.J. Long and J. Turner, *A phase of confined electroweak force in the early Universe*, Phys. Rev. D 100, 055005 (2019), [arXiv:1906.05157].

[3] https://github.com/robsci/GWplotter/tree/master/data

[4] C. Gowling and M. Hindmarsh, *Observational prospects for phase transitions at LISA: Fisher matrix analysis*, JCAP 10 (2021) 039, [arXiv:2106.05984].

Questions?