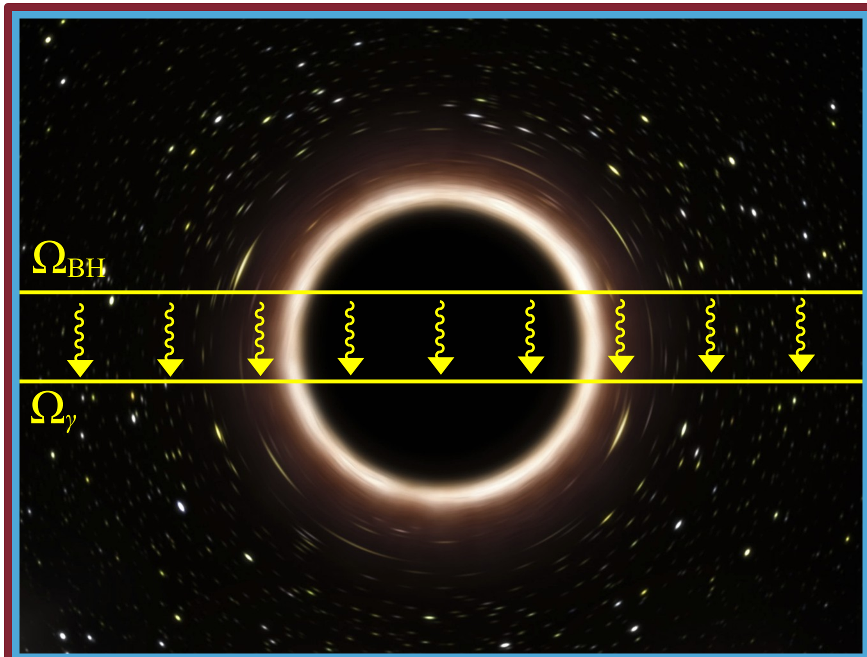
Cosmic Stasis from Primordial-Black-Hole Evaporation and Its Phenomenological Implications



Brooks Thomas

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Based on work done in collaboration with:

- **Keith R. Dienes, Fei Huang, Lucien Heurtier, Doojin Kim, and Tim M. P. Tait [arXiv:2108.02204, 2212.01369]**

PHENO 2023, May 11th, 2023

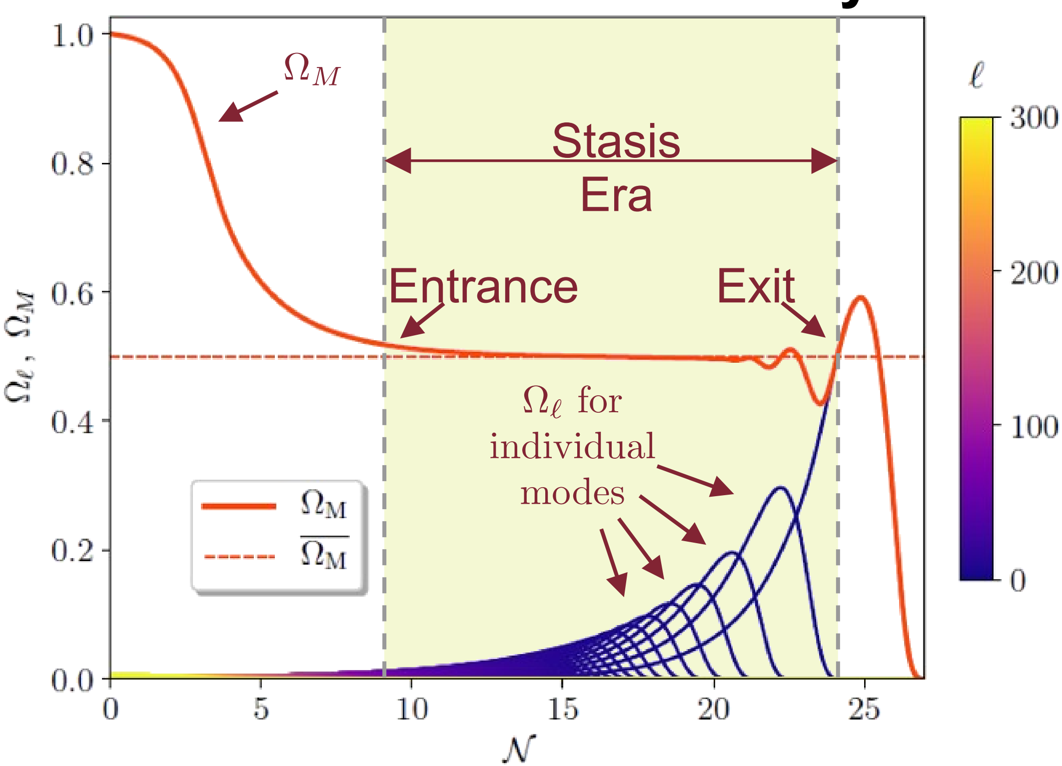
Stasis in an Expanding Universe

- **Cosmic stasis** is a phenomenon in which the abundances of multiple cosmological energy components (matter, radiation, dark energy, etc.) remain effectively constant over an extended period.

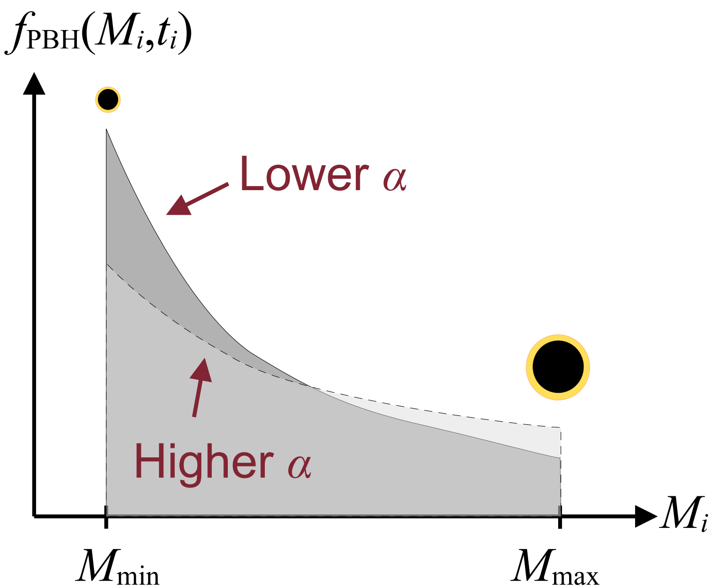
[Dienes, Huang, Heurtier, Kim, Tait, BT '21]

- Such a period of stasis requires a sustained injection of energy density from the component with a smaller equation-of-state parameter w to the component with a larger one.
- One realization of stasis involves a **tower of unstable particles** with a spectrum of lifetimes – particles whose decays provide the necessary injection of energy density from matter ($w = 0$) to radiation ($w = 1/3$).
- In this talk, I'll examine the consequences of another realization of cosmic stasis – from the evaporation of **primordial black holes**.

Stasis from Particle Decays



Initial PBH Mass Spectrum



- Let's consider a population of primordial black holes (PBHs) with the mass spectrum

$$\boxed{f_{\rm BH}(M_i,t_i) = \begin{cases} CM_i^{\alpha-1} & \text{for } M_{\min} \le M_i \le M_{\max} \\ 0 & \text{otherwise} \end{cases}}$$

- Such an **extended mass spectrum** arises naturally in scenarios in which the PBHs form after inflation via the collapse of perturbations with a scale-invariant power spectrum.

[Carr '75; Green, Liddle '97; Kim, Lee, MacGibbon '99; Bringmann, Keifer, Polarski '02; Carr et. al. '17]

- The value of α is determined by the equation-of-state parameter $w_{\rm c}$ for the universe during the epoch wherein the PBHs form.

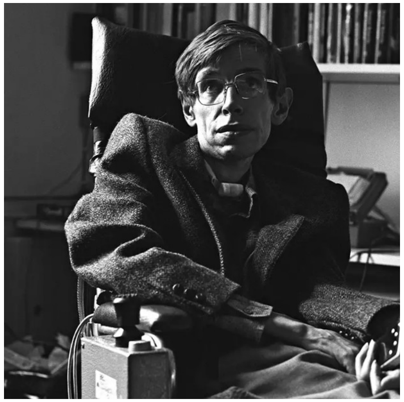
$$\boxed{\alpha = -\frac{3w_c+1}{w_c+1}} \xrightarrow{-1/3 \le w_c \le 1} \boxed{-2 \le \alpha \le 0}$$

- Observational considerations likewise place constraints on the values of $M_{\rm min}$ and $M_{\rm max}$:

[Carr, Kohri, Sendouda, Yokoyama '09; Keith, Hooper, Blinov, McDermott '20; Carr, Kohri, Sendouda, Yokoyama '21; Akrami et al. (Planck) '20]

$$\boxed{\underbrace{0.1\text{g} \lesssim}_{\text{Planck upper bound on } H_\star} M_{\min} < M_{\max} \underbrace{\lesssim 10^9\text{g}}_{\text{Heaviest PBH evaporate completely before BBN}}}$$

Evaporation



- **Hawking radiation** provides a mechanism via which energy density can be transferred from the PBHs (which behave like massive matter) to radiation. [Hawking, '74; Hawking '75]

$$\boxed{T_{\mathrm{BH}} = \frac{1}{8\pi G M} \sim 1.06\ \mathrm{GeV}\left(\frac{10^{13}\,\mathrm{g}}{M}\right)}$$

- The rate of change of the mass M of a single PBH due to this effect is

[MacGibbon, Webber, '90; MAcGibbon '91]

$$\boxed{\frac{dM}{dt} \equiv -\,\varepsilon(M)\frac{M_P^4}{M^2}}$$

Graybody factor: for this range of M, $\varepsilon(M) \approx \varepsilon$ is approximately constant.

- The time at which a PBH evaporates completely (i.e., at which $M = 0$) as a result of this effect is

$$\boxed{\tau(M_i) \equiv \frac{{M_i}^3}{3\varepsilon M_P^4}}$$

- As a result, the PBH mass spectrum subsequently evolves according to a Boltzmann equation of the form

$$\boxed{\frac{d\rho_{\mathrm{BH}}}{dt} + 3H\rho_{\mathrm{BH}} = \int_0^\infty dM \;{}_{\mathrm{BH}}(M,t)\frac{dM}{dt}}$$

Boltzmann Evolution

- The evolution of the Hubble parameter $H(t)$ is given by the Friedmann acceleration equation, which in thes case takes the form

$$\frac{dH}{dt} = -H^2 - \frac{4\pi G}{3}\left[\rho_{\rm BH}(1+3\underbrace{\cancel{w_{\rm BH}}}_{w_{\rm BH}=0}) + \rho_\gamma(1+w_\gamma)\right]$$

- Expressed in terms of the cosmological abundance $\Omega_{\rm BH} \equiv \rho_{\rm BH}/\rho_{\rm crit}$, the system of equations governing the expansion of the universe is

$$\frac{dH}{dt} = -\frac{1}{2}H^2(4-\Omega_{\rm BH})$$
$$\frac{d\Omega_{\rm BH}}{dt} = -\Gamma_{\rm BH}(t)\,\Omega_{\rm BH} + H\left(\Omega_{\rm BH} - \Omega_{\rm BH}^2\right)$$

...where we have defined

$$\Gamma_{\rm BH}(t) \equiv -\frac{\int_0^\infty f_{\rm BH}(M,t)\frac{dM}{dt}\,dM}{\int_0^\infty f_{\rm BH}(M,t)M\,dM}$$

- Alternatively, one can change variables and express this system of equations in terms of $\Omega_{\rm BH}$ and its time-averaged value $\langle\Omega_{\rm BH}\rangle$ since the time t_i at which the PBH spectrum was initially established:

$$\langle\Omega_{\rm BH}\rangle \equiv \frac{1}{t-t_i}\int_{t_i}^t dt'\,\Omega_{\rm BH}(t')$$

PBH-Induced Stasis is a Global Attractor

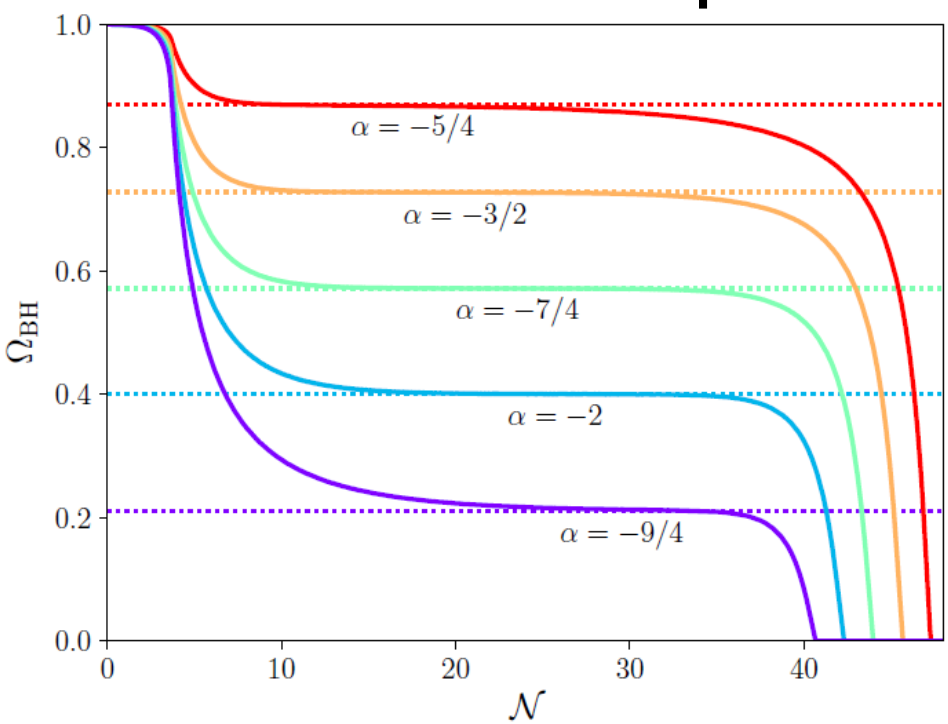
- One can show that not only do these equations admit a **stasis solution**, but that this stasis solution is a **global attractor**.

[Barrow, Copeland, Liddle '91; Dienes, Huang, Heurtier, Kim, Tait, BT '22]

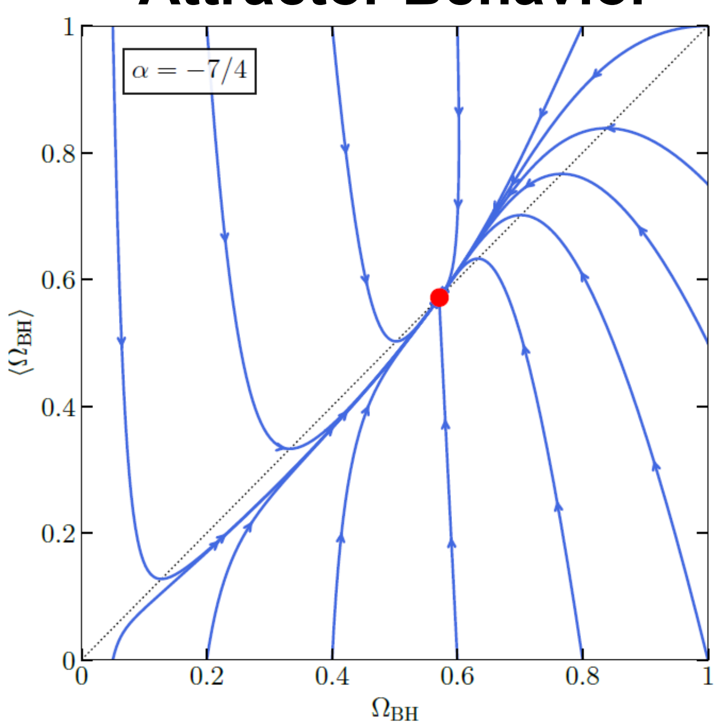
- The effective equation-of-state parameter $\overline{w}$ for the universe as a whole during the stasis epoch and the PBH abundance $\Omega_{\rm BH}$ are determined by the value of α:

$$\boxed{\overline{w} = -\frac{\alpha+1}{\alpha+7}} \qquad \boxed{\overline{\Omega}_{\rm BH} = \frac{4\alpha+10}{\alpha+7}}$$

Stasis from PBH Evaporation



Attractor Behavior



Stasis as a (Finite) Cosmological Epoch

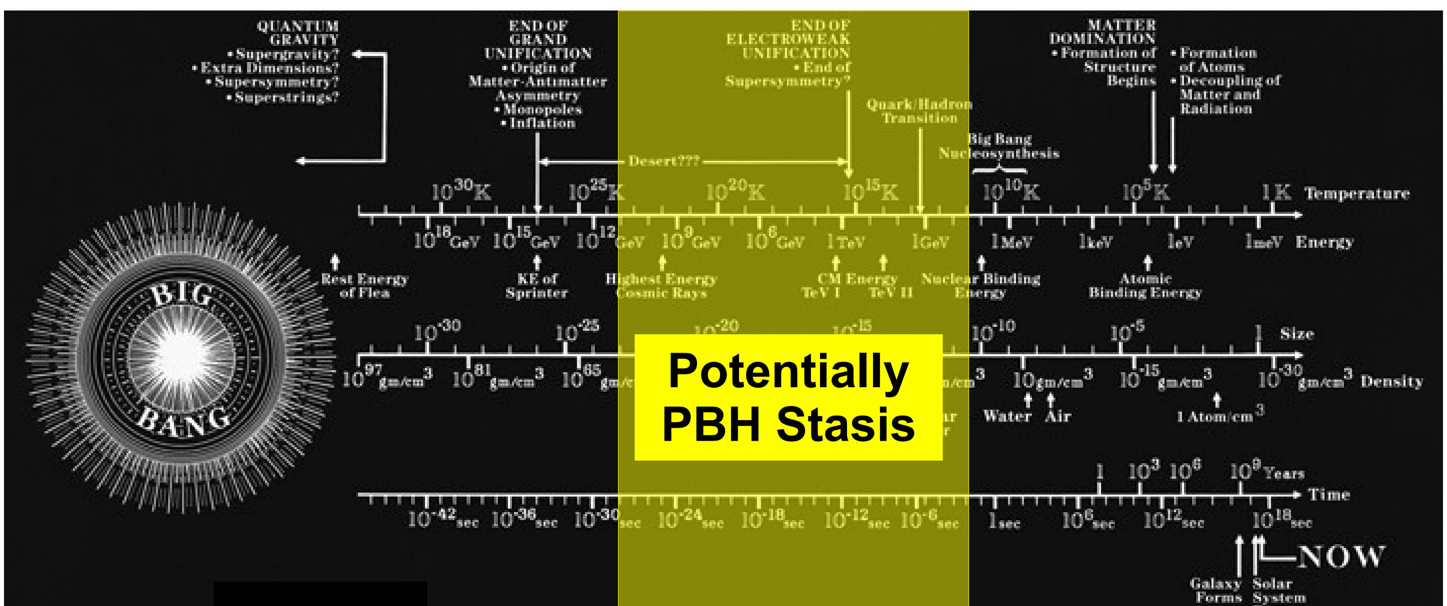
- The duration of this PBH-induced stasis epoch, expressed in terms of the number of e-folds of cosmic expansion that it spans, is given by

$$\boxed{\mathcal{N}_s \approx \log\left[\frac{a(\tau(M_{\rm max}))}{a(\tau(M_{\rm min}))}\right] \approx \frac{\alpha+7}{3}\log\left(\frac{M_{\rm max}}{M_{\rm min}}\right)}$$

- For $M_{\rm min} = 0.1$ g at its minimum and $M_{\rm max} = 10^9$ g at its maximum, this yields a stasis epoch of duration

$$\boxed{\mathcal{N}_s \lesssim 23\left(\frac{\alpha+7}{3}\right)}$$

- This is a significant duration indeed – potentially spanning a range of temperatures $\mathcal{O}(\mathrm{MeV}) \lesssim T \lesssim \mathcal{O}(10^{11}\ \mathrm{GeV})$!

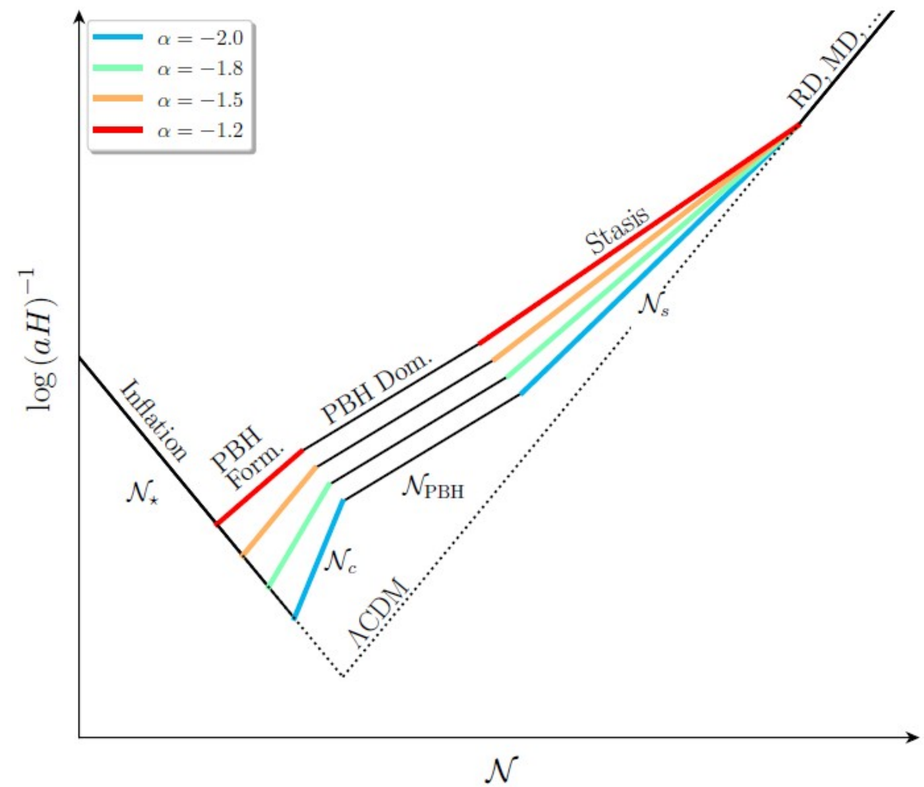


Thus, events such as the electroweak phase transition could have occurred during such a stasis epoch!

Cosmic Expansion History

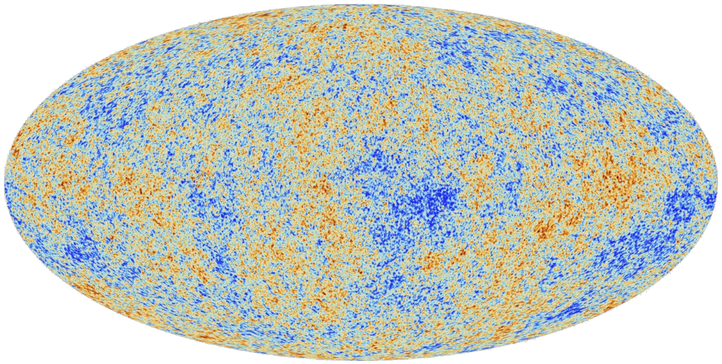
- In cosmologies involving an epoch of PBH-induced stasis, the cosmological timeline includes a series of several different epochs after cosmic inflation ends. Sequentially, these are:
  - The epoch during which the **PBHs are generated**, wherein the equation-of-state parameter w_c determines α.
  - An epoch during which the PBHs come to dominate the energy density of the universe. This epoch is **matter-dominated** ($w = 0$).
  - The **stasis epoch**, which begins once the lightest PBHs begin to evaporate, and wherein $w = \overline{w}$.

Comoving Hubble Horizon



  - The usual **RD epoch** with $w = 1/3$, which begins after the heaviest PBHs evaporate and stasis ends. Once this epoch begins, the expansion history coincides with that of the standard cosmology.

Inflationary Observables



- In the simplest inflationary scenarios, primordial perturbations of the inflaton field give rise to the pattern of inhomogeneities observed in the cosmic microwave background (CMB).
- However, **modifications of the cosmological timeline** beween the end of inflation and last scattering can alter predictions for CMB observables.
- The primary such observables are the **tensor-to-scalar ratio** r and **spectral index** n_s that characterize the primordial perturbation spectrum.
- For example, in single-field, slow-roll models of inflation, these obserables are directly related to the slow-roll parameters ε and η:

$$\boxed{\begin{aligned} n_s &= 1 - 6\epsilon + 2\eta \\ r &= 16\epsilon \end{aligned}}$$

where

$$\boxed{\epsilon \equiv \frac{M_P^2}{16\pi}\left[\frac{V'(\phi_\star)}{V(\phi_\star)}\right]^2 \qquad \eta \equiv \frac{M_P^2}{8\pi}\left|\frac{V''(\phi_\star)}{V(\phi_\star)}\right|}$$

- The quantity $\phi_\star$ denotes the value of the inflaton field at the time at which a perturbation with wavenumber equal to the pivot scale $k_\star$ exits the horizon. Following Planck, we take $k_\star = 0.002\ \mathrm{Mpc}^{-1}$. [Akrami et al. (Planck) '20]

Inflationary Observables

- I order to determine $\phi_\star$ we note that in the slow-roll approximation, the Hubble parameter $H_\star$ and scale factor $a_\star$ at the time at which this same mode exist the horizon are related to $\phi_\star$ by

$$H_\star^2 \approx \frac{8\pi V(\phi_\star)}{3M_P^2}$$

and

$$\log\left(\frac{a_{\rm end}}{a_\star}\right) = \frac{8\pi}{M_P^2}\int_{\phi_{\rm end}}^{\phi_\star} \frac{V(\phi)}{V'(\phi)}d\phi$$

- Combining these relations yields the integro-differential equation

$$\frac{8\pi}{M_P^2}\int_{\phi_{\rm end}}^{\phi_\star} \frac{V(\phi)}{V'(\phi)}d\phi = \frac{1}{2}\log\left(\frac{8\pi a_{\rm now}^2 V(\phi_\star)}{3M_P^2 k_\star^2}\right) - \log\left(\frac{a_{\rm now}}{a_{\rm end}}\right)$$

...which can be solved for a given form of $V(\phi)$.

- In order to illustrate how r and n_s are modified in cosmologies involving an epoch of PBH-induced stasis, it is useful to work in the context of a concrete model for the inflaton potential… or two. We'll choose

1 **Polynomial potentials**:

$$V(\phi) \sim |\phi|^p$$

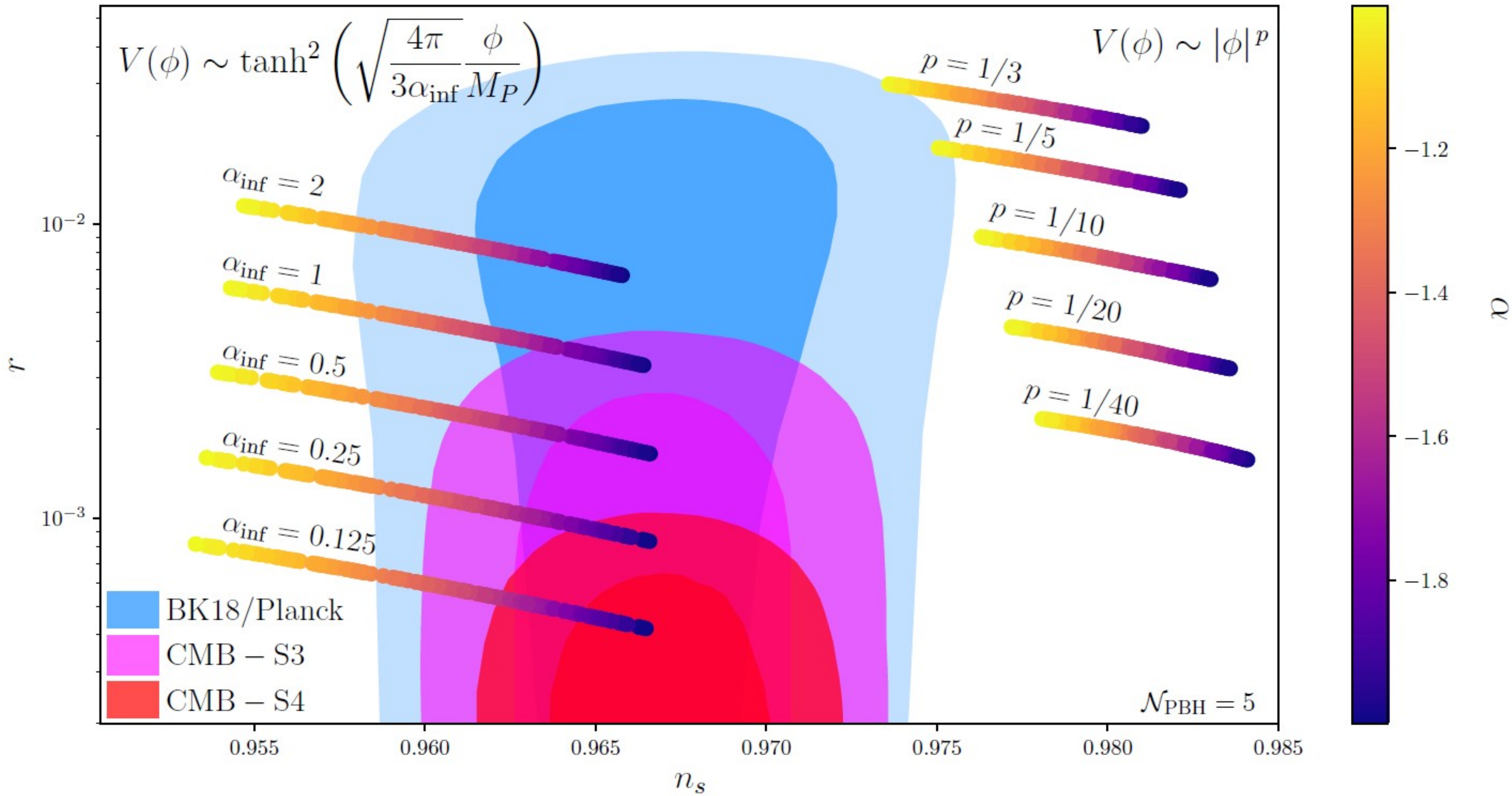
2 **T-Model α-attractors**:

[Kallosh, Linde '13]

$$V(\phi) \sim \tanh^{2n}\left(\sqrt{\frac{4\pi}{3\alpha_{\rm inf}}}\frac{\phi}{M_P}\right)$$

Inflationary Observables: Results

- In general, the modifications of the cosmological timeline associated with PBH-induced stasis serve to increase r and decrease n_s.



- As a result, depending on the inflationary model in question, tensions between the predictions for these observables and CMB data may be **<u>either eased or exacerbated</u>**.

Gravitational-Wave Background

- The cosmological modifications associated with a PBH-induced stasis epoch affect the **gravitational-wave (GW) background** in several ways.
- Perhaps most importantly, the modified expansion history alters the contribution to the GW background generated by other sources.
- For concreteness, we'll consider the simple case of a **stochastic GW background** which is homogeneous, isotropic, Gaussian, and unpolarized.
- The differential GW energy density per logarithmic comoving wavenumber k for this case is:

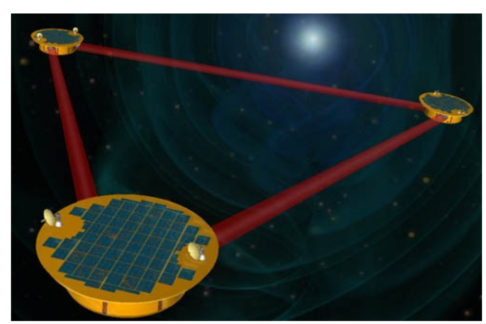
[Caprini, Figueroa '18]

$$\frac{d\rho_{\mathrm{GW}}(a)}{d\log k} = \frac{k^2 h_k^2(a)}{16\pi G a^2}$$

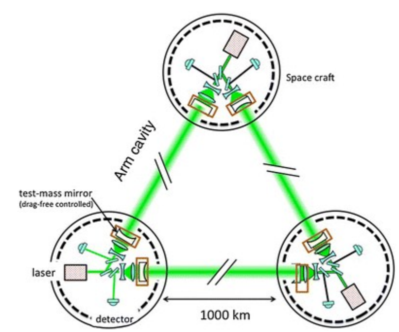
- The differential amplitude $h_k(a)$ depends on when the pertubation mode re-enters the horizon:

$$h_k(a) = \frac{a_k}{a} h_k(a_k)$$

Advanced LIGO



LISA



DECIGO

Gravitational-Wave Background

- During an epoch wherein w is constant, the wavenumber k which enters the horizon at scale factor a_k scales with a_k according to the relation

$$\boxed{k = a_k H_k \propto a_k^{-(1+3w)/2}}$$

- This implies that: $\boxed{\frac{d\rho_{\rm GW}(a)}{d\log k} \propto a^{-4} h_k^2(a_k) k^{\xi(w)}}$ where $\boxed{\xi(w) \equiv \frac{2(3w-1)}{(3w+1)}}$

- In the standard cosmology, wherein the universe remains radiation-dominated ($w = 1/3$) from the end of reheating until matter-radiation equality, $\xi(w) = 0$ throughout the entire duration.

- Thus, the resulting present-day GW spectrum – or, more precisely, the differential present-day GW abundance per unit physical frequency f – is flat (i.e., f-independent) and given by [Caprini, Figueroa '18]

$$\boxed{\frac{d\Omega_{\rm GW}^{\rm sc}}{d\log f} = \Omega_\gamma(a_{\rm now}) \left(\frac{g_{\star S}(T_{\rm eq})}{g_{\star S}(T_k)}\right)^{4/3} \frac{g_\star(T_k)}{24\pi^2} \frac{H_\star^2}{M_P^2}}$$

Gravitational-Wave Background

- By contrast, cosmology involving a PBH-induced stasis epoch with $w = w$ – as well as a PBH-production epoch with $w = w_c$ and a PBH-dominated epoch with $w = 1$ – can **differ significantly** from this result.
- In particular, in such a modified cosmology, the corresponding present-day GW spectrum is given by

$$\frac{d\Omega_{\mathrm{GW}}}{d\log f} = \frac{d\Omega_{\mathrm{GW}}^{\mathrm{sc}}}{d\log f} \times \begin{cases} 1 & f \le f_s \\ \left(\frac{f}{f_s}\right)^{\xi(\overline{w})} & f_s < f \le f_{\mathrm{PBH}} \\ \left(\frac{f_{\mathrm{PBH}}}{f_s}\right)^{\xi(\overline{w})} \left(\frac{f}{f_{\mathrm{PBH}}}\right)^{-2} & f_{\mathrm{PBH}} < f \le f_f \\ \left(\frac{f_{\mathrm{PBH}}}{f_s}\right)^{\xi(\overline{w})} \left(\frac{f}{f_{\mathrm{PBH}}}\right)^{-2} \left(\frac{f}{f_f}\right)^{\xi(w_c)} & f_f < f \le f_{\mathrm{end}} \\ 0 & f_{\mathrm{end}} < f \ , \end{cases}$$

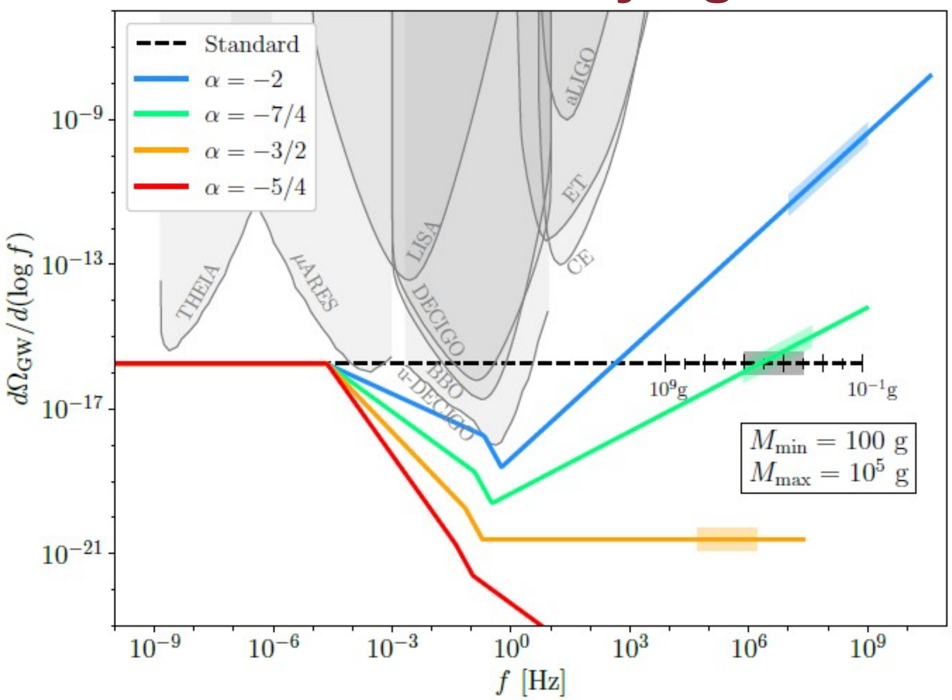
Spectrum obtained in the standard cosmology for the same $H_\star$

Piecewise function with different power-law exponents within different frequency intervals corresponding to different

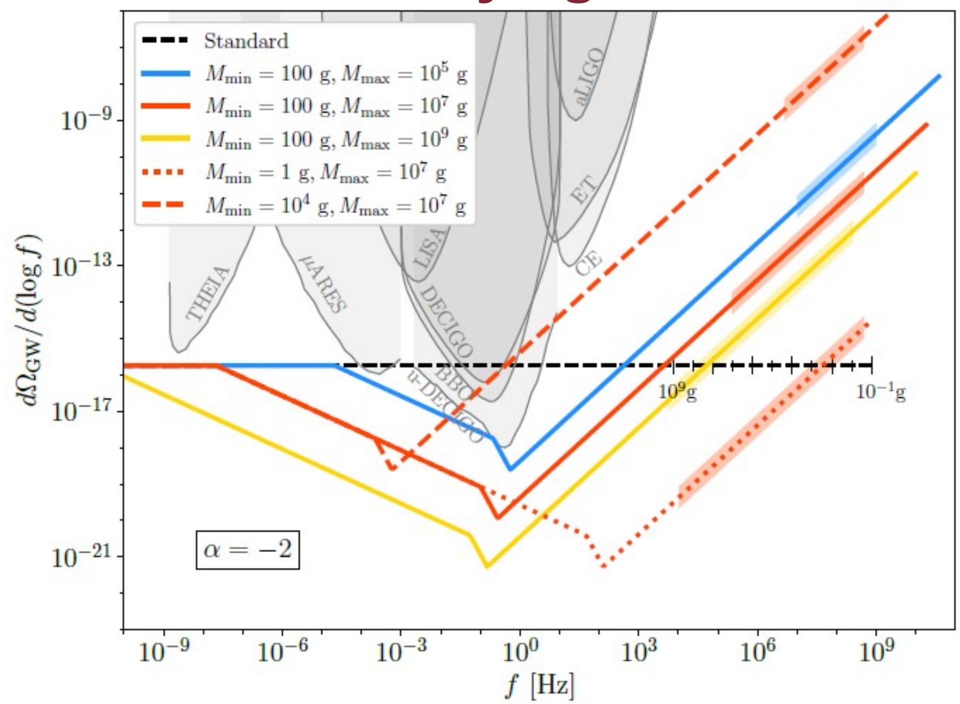
Gravitational-Wave Background: Results

- Given the sensitivities of planned, proposed, and existing **gravitational-wave observatories**, these modifications can have significant implications for the detection of the stochastic GW background.

Effect of Varying α



Effect of Varying $M_{\min}$ and $M_{\max}$



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Detector Sensitivities

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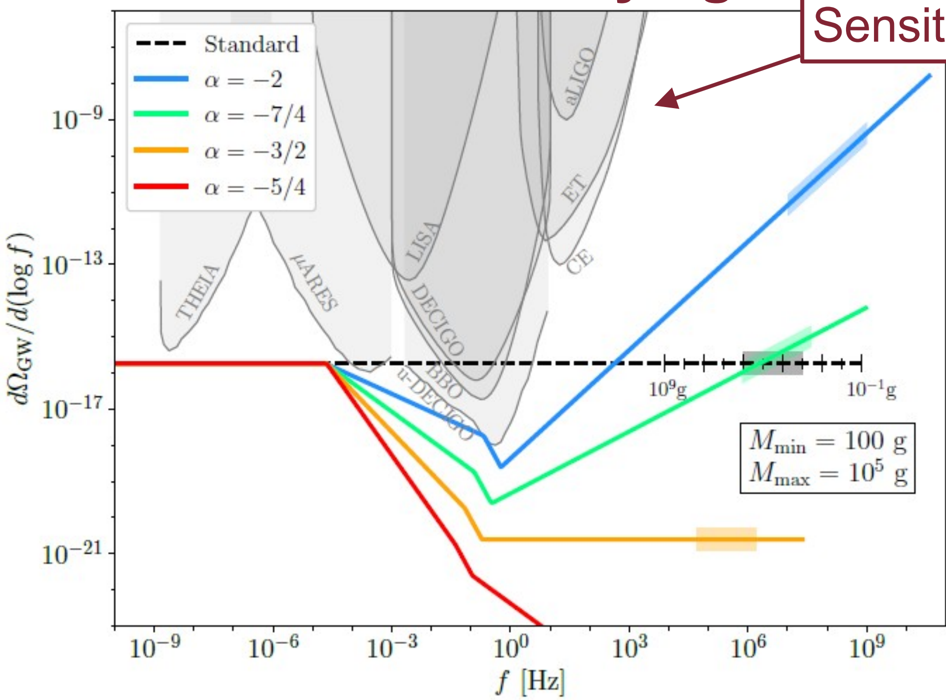
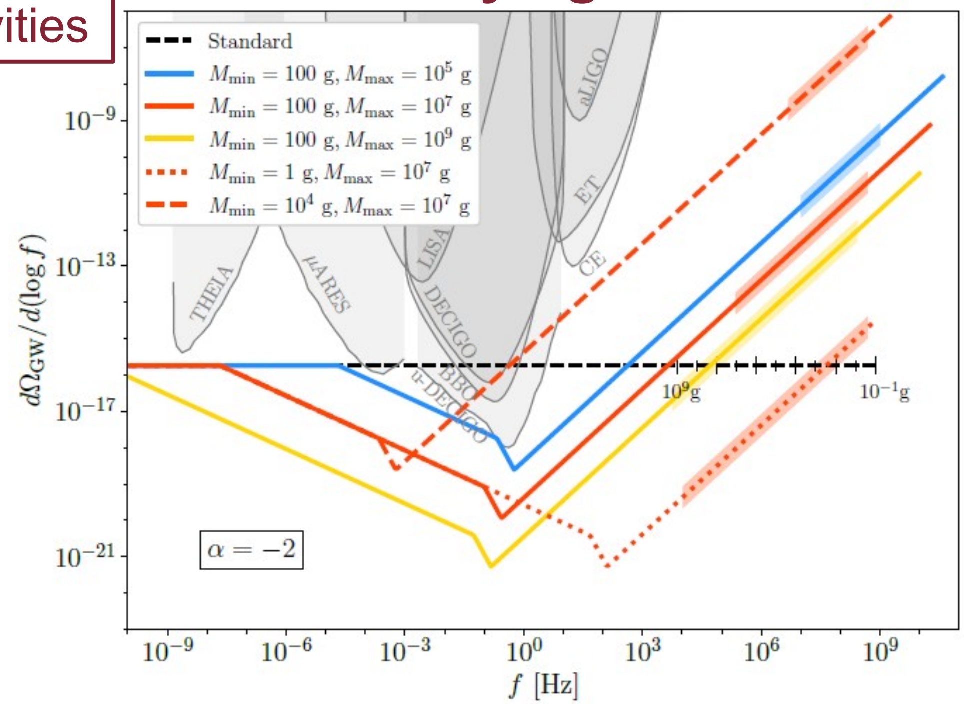


Effect of Varying $M_{\rm min}$ and $M_{\rm max}$



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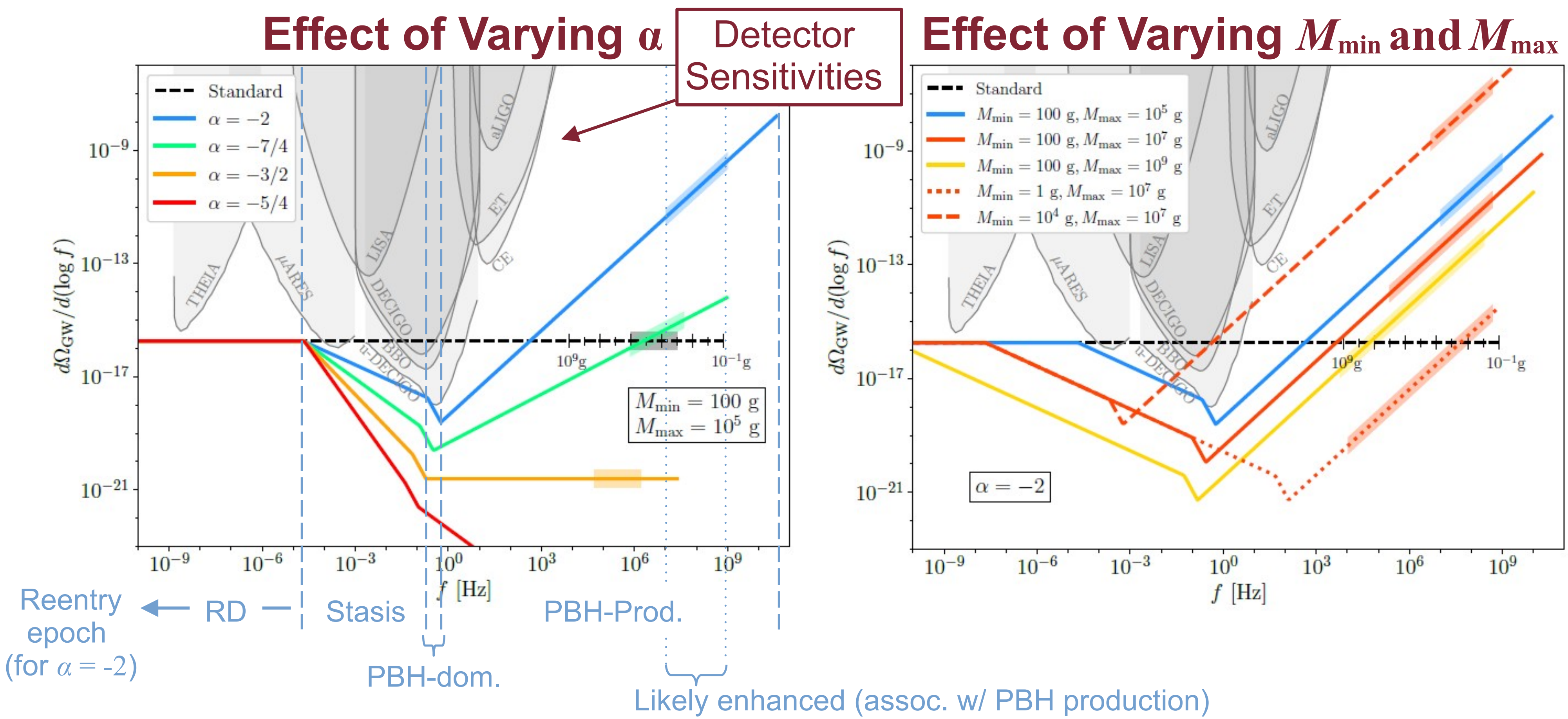

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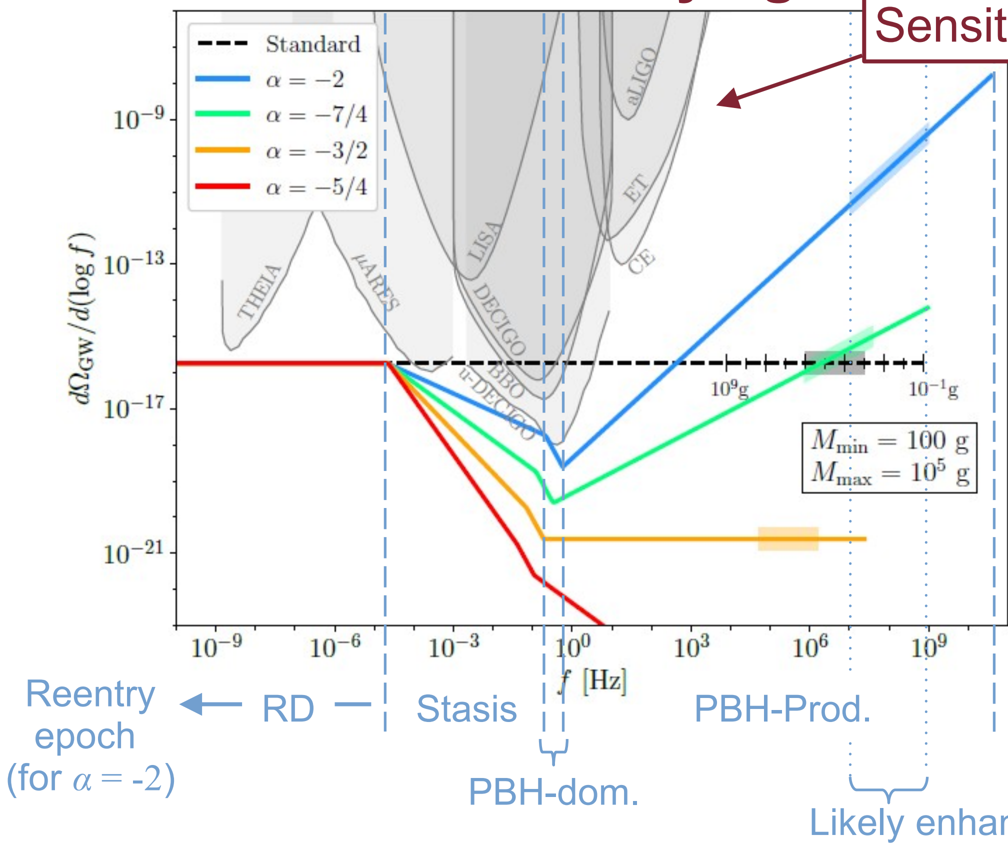

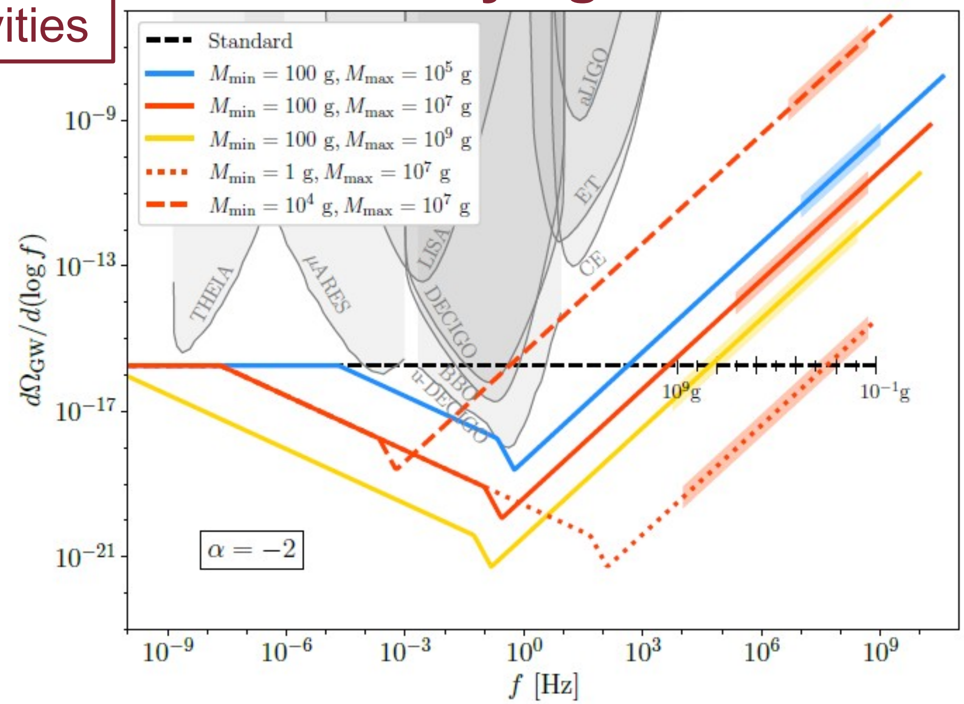

The Upshot: A GW signal can be amplified – or hidden – as a result of PBH-induced stasis. Correlations between slopes in different regions provide an observational handle on α, $M_{\rm min}$, and $M_{\rm max}$.

Summary

- ***Stable, mixed-component cosmological eras*** – *i.e.* ***stasis eras*** – are a viable cosmological possibility – and one that arises naturally from a population of evaporating PBHs with an extended mass spectrum.
- Such a population of PBHs can give rise to a stasis epoch that spans a **significant number of e-folds** of expansion – and a corresponding range of temperatures that can extend from just above the BBN scale to as high as $T \sim 10^{11}$ GeV.
- Such a period of PBH-induced stasis can have significant implications for observational cosmology:
  - It can significantly shift predictions for CMB observables that provide a window into cosmic inflation.
  - It can alter the present-day GW spectrum within the frequency range relevant for future GW detectors in a characteristic way.
  - In addition, if additional, light particle species are present in the theory which behave as dark radiation, particles of these species are generically produced via Hawking radiation. The production of such particles is constrained by observation. [Dienes, Huang, Heurtier, Kim, Tait, BT '22]