Mass-varying gauge boson that couples to the dark energy field

from the perspective of gauged quintessence

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based on the ongoing research with Kunio Kaneta, Hye-Sung Lee, and Jiheon Lee

Overview

I. Introduction

Quintessence

- Dynamic dark energy model with a scalar field proposed by Ratra and Peebles.

[Bharat Ratra and P. J. E. Peebles PRD37(1988)3406]

- A scalar rolls down a potential slowly in the present universe.
- Equation of state assuming slow roll condition $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$

$$w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \approx -1$$

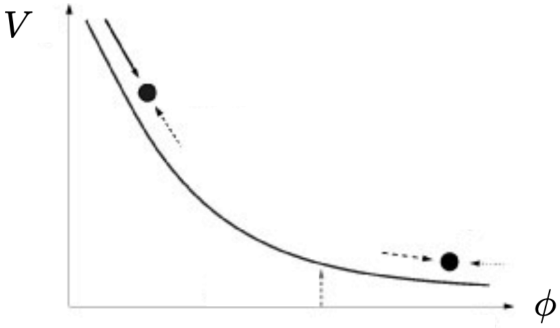
- Its potential energy is identified as the dark energy.
- The mass of quintessence is given as

$$m_\phi^2 = \frac{\partial^2 V}{\partial \phi^2}$$

Tracking Behavior

- The initial value of ϕ does not really matter. Only the potential determines the the present time value of and its equation of state (addressing the cosmological coincidence problem).

[Steinhardt, Wang, Zlatev PRL82(1999)896]



Gauged Quintessence

- The gauged quintessence model includes complex scalar $\Phi = \phi e^{i\eta}/\sqrt{2}$ and $U(1)_{\text{dark}}$ gauge boson $\mathbb{X}_\mu$. Φ is charged under the $U(1)_{\text{dark}}$ gauge symmetry and ϕ behaves as dark energy.

[KK, HL, JL, and **JY** JCAP02(2023)005]

- Under the unitary gauge, $\eta = 0$ and $X_\mu = \mathbb{X}_\mu + \frac{1}{g_X}\partial_\mu \eta$, the Lagrangian of gauged quintessence model is given by

$$\mathcal{L} \supset \sqrt{-g}\Big[-\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - V_0(\phi) - \frac{1}{2}(g_X\phi)^2 X_\mu X^\mu \Big]$$

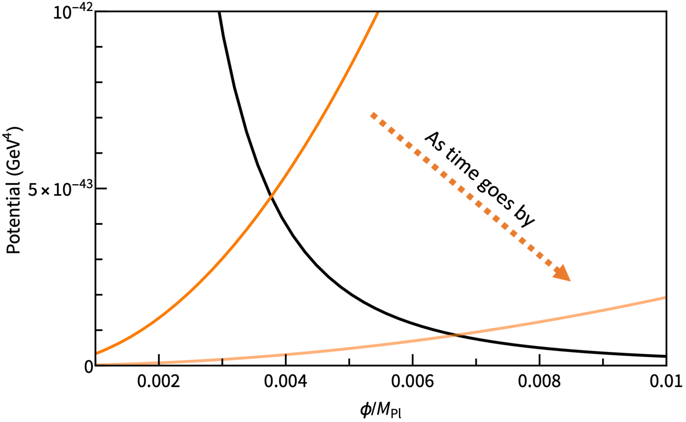
where g_X is the dark gauge coupling constant.

- We chose $V_0(\phi)$ to be the inverse power potential,

[Bharat Ratra and P. J. E. Peebles PRD37(1988)3406]

$$V_0(\phi) = \frac{M^{\alpha+4}}{\phi^\alpha}, \quad \alpha > 0$$

Potential


10^{-42}
5×10^{-43}
0
Potential (GeV^4)
0.002
0.004
0.006
0.008
0.01
$\phi/M_{\rm Pl}$
As time goes by

Mass-varying Effect

- The masses of ϕ and X are given as

$$m_\phi^2 = \frac{\partial^2 V}{\partial \phi^2}, \quad m_X^2 = g_X^2 \phi^2$$

  where V is effective potential of ϕ.
- When the tracking and rolling of quintessence begin, m_X increases.
- Since its mass changes, X does not behave like usual CDM so X is not assumed to be the dominant dark matter component.
- Nevertheless, its mass-varying behavior might imply interesting phenomena so we hope X accounts for sufficient energy density at the present.

Hubble Tension

- To relieve Hubble tension, $w(DE) < -1$ is favored in the recent era.
  [Bum-Hoon Lee *et al* JCAP04(2022)004]
- In the gauged quintessence model,

$$w_{\rm eff}(\widetilde{DE}) = -1 + \frac{1}{\rho_{\widetilde{DE}}}\left((1+w_\phi^0)\rho_\phi + \left(\frac{m_X}{m_X^0} - 1\right)\frac{\rho_X^0}{a^3}\right)$$

  where $\rho_{\widetilde{DE}} = \rho_\phi + \rho_X - \rho_X^0 a^{-3}$ and superscript 0 implies the value in the present universe.
- Since $m_X/m_X^0 < 1$ in the gauged quintessence model, $\omega_{\rm eff}(\widetilde{DE})$ can be smaller than -1 and Hubble tension might be alleviated.
- This effect is significant when ρ_X^0 is large.

Misalignment Mechanism

- The misalignment mechanism is a mechanism for retaining the correct relic density of dark matter.
  - Homogeneous condensate due to inflation.
  - Coherent oscillation.
- The misalignment mechanism does not work well for the vector field.

[Kazunori Nakayama JCAP10(2019)019]

Assuming homogeneity, the equation of motion and the energy density of the vector boson $X_\mu = (0,0,0,X)$ are given as

$$\ddot{X} + H\dot{X} + m_X^2 X = 0, \quad \rho_X = \frac{1}{2a^2}(\dot{X}^2 + m_X^2 X^2)$$

- Due to the scale factor, ρ_X becomes tiny after inflation.

$$\frac{a_{\rm end}}{a_{\rm ini}} = e^{60} \quad \Rightarrow \quad \frac{\rho_X(a_{\rm end})}{\rho_X(a_{\rm ini})} \sim e^{-120} \quad (60\ e\text{-folding inflation})$$

Motivation

- Mass-varying effect may compensate for the suppression of ρ_X, in the gauged quintessence model.
- Can the misalignment mechanism provide sufficient ρ_X^0 in the gauged quintessence model?

II. Dynamics

Equation of Motion, Energy Density

- The equations of motion for ϕ and X are given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0, \quad \ddot{X} + H\dot{X} + m_X^2 X = 0$$

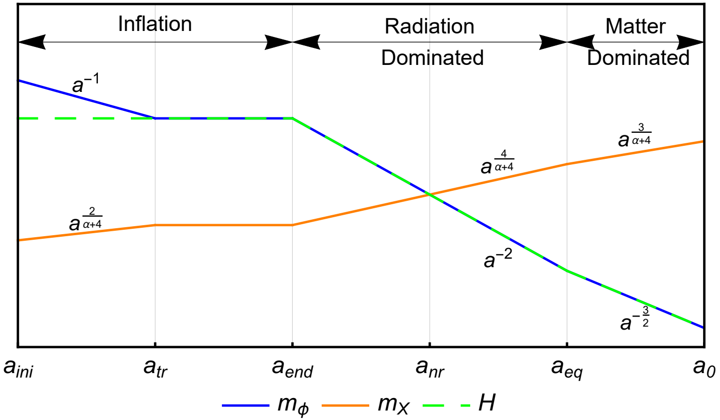
where $m_X = g_X \phi$.

- The energy density of X is given as

$$\rho_X = \frac{1}{2a^2}(\dot{X}^2 + m_X^2 X^2)$$

Reference Scale Factor (Reference Time)

- Roughly speaking, dynamics of X and ϕ are determined by relative size of m and H (whether $m > H$ or $m < H$)



Dynamics of Dark Gauge Boson

$H \gg m_X, \quad a < a_{\rm nr}$	$H \ll m_X, \quad a > a_{\rm nr}$
Fixed by Hubble friction $X \approx$ constant	Coherent oscillation $X \propto (m_X a)^{-\frac{1}{2}}$
$\rho_X \propto m_X^2 a^{-2}$	$\rho_X \propto m_X a^{-3}$

- For the constant mass case, the energy density is highly suppressed.
- If m_X increases over time, its suppression may be compensated.

Dynamics of Quintessence

$m_\phi \gg H, \quad a < a_{\rm tr}$	$m_\phi \lesssim H, \quad a > a_{\rm tr}$
ϕ follows the minimum of V	Tracking solution
$\phi \propto a^{\frac{2}{\alpha+4}}$	$\phi \propto a^{\frac{3(1+w_b)}{\alpha+4}}$

- ϕ is proportional to a^k with $k \geq 0$.
- Therefore, $m_X = g_X \phi$ increases over time as expected.

Evolution of Energy Density

	$m_X = g_X \phi$	ρ_X		
$a_{\text{ini}} < a < a_{\text{tr}}$	$\propto a^{\frac{2}{\alpha+4}}$	$\propto m_X^2 a^{-2}$	$\propto a^{-\frac{2\alpha+4}{\alpha+4}}$	$> a^{-2}$
$a_{\text{tr}} < a < a_{\text{end}}$	$\approx$ constant		$\propto a^{-2}$	
$a_{\text{end}} < a < a_{\text{nr}}$	$\propto a^{\frac{4}{\alpha+4}}$		$\propto a^{-\frac{2\alpha}{\alpha+4}}$	
$a_{\text{nr}} < a < a_{\text{eq}}$	$\propto a^{\frac{4}{\alpha+4}}$	$\propto m_X a^{-3}$	$\propto a^{-\frac{3\alpha+8}{\alpha+4}}$	$> a^{-3}$
$a_{\text{eq}} < a < a_0$	$\propto a^{\frac{3}{\alpha+4}}$		$\propto a^{-\frac{3\alpha+9}{\alpha+4}}$	

- The energy density of X decays much slower than that of minimal vector boson case.

III. Results and Discussions

Energy Density during Inflation

- We hope ρ_X to be comparable to ρ_{CDM} at the present.

$$\rho_X(a_0) \approx \rho_{\mathrm{CDM}}(a_0)$$

- Inflaton should be the dominant component during the inflation.

$$\rho_X(a_{\mathrm{ini}}) < 3M_{\mathrm{Pl}}^2 H_{\mathrm{inf}}^2$$

- The relation between $\rho_X(a_0)$ and $\rho_X(a_{\mathrm{ini}})$ can be derived by the evolution of energy density.

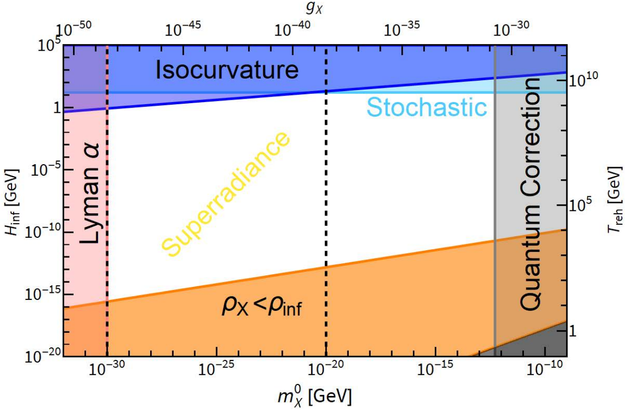
$$\rho_X(a_{\mathrm{ini}}) = \rho_X(a_0)\left(\frac{a_{\mathrm{eq}}}{a_0}\right)^{-\frac{3\alpha+9}{\alpha+4}}\left(\frac{a_{\mathrm{nr}}}{a_{\mathrm{eq}}}\right)^{-\frac{3\alpha+8}{\alpha+4}} \left(\frac{a_{\mathrm{end}}}{a_{\mathrm{nr}}}\right)^{-\frac{2\alpha}{\alpha+4}}\left(\frac{a_{\mathrm{tr}}}{a_{\mathrm{end}}}\right)^{-2}\left(\frac{a_{\mathrm{tr}}}{a_{\mathrm{ini}}}\right)^{-\frac{2\alpha+4}{\alpha+4}}$$

Energy Density during Inflation

- Each reference scale factor can be calculated.
  - $a_{\rm ini}$: $a_{\rm ini} = e^{-N} a_{\rm end}$.
  - $a_{\rm tr}$: $m_\phi(a_{\rm ini}) \left(\frac{a_{\rm tr}}{a_{\rm ini}}\right)^{-1} = H_{\rm inf}$.
  - $a_{\rm end}$: $H_{\rm inf}^2 \left(\frac{a_0}{a_{\rm end}}\right)^{-4} = \Omega_r H_0^2$.
  - $a_{\rm nr}$: $m_X(a_{\rm end}) \left(\frac{a_{\rm nr}}{a_{\rm end}}\right)^{\frac{4}{\alpha+4}} = H_{\rm inf} \left(\frac{a_{\rm nr}}{a_{\rm end}}\right)^{-2}$.
  - $a_{\rm eq}$: $\frac{a_0}{a_{\rm eq}} = \left(\frac{H_{\rm eq}}{H_0}\right)^{2/3}$.
- Combining these results, constraints for g_X and $H_{\rm inf}$ can be found for given α, M and N.

Results

- The misalignment mechanism can provide a sufficient amount of dark gauge boson in the gauged quintessence model.



$\alpha = 1, \ M = 2.2 \times 10^{-6}\mathrm{GeV}, \ N = 60$

Reference Scale Factor

- It is assumed that
  - $m_\phi \gg H$ initially, and $m_\phi = H$ during inflationary era.
  - $m_X \ll H$ initially, and $m_X = H$ during radiation dominated era.
- There are other possible scenarios such as
  - $m_\phi = H$ after inflation
  - $m_X = H$ during matter dominated era
- We have checked other scenarios for $\alpha = 1$ but they were rejected or not interesting because of various reasons.
- However, we are interested in the case of $\alpha < 1$ and other scenarios may become interesting.

IV. Summary

Summary

- Gauged quintessence model is a $U(1)$ charged quintessence model.
- The dynamics of gauged quintessence model with misalignment mechanism was studied.
- In the gauged quintessence model, a dark gauge boson whose amount is comparable to CDM can be produced, unlike the constant mass vector dark matter case.
- More researches on dark energy sector including gauged quintessence model is warrented.

Thank you for listening

Backup Slides

Misalignment Mechanism

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- Due to the inflation, a homogeneous condensate can be achieved.

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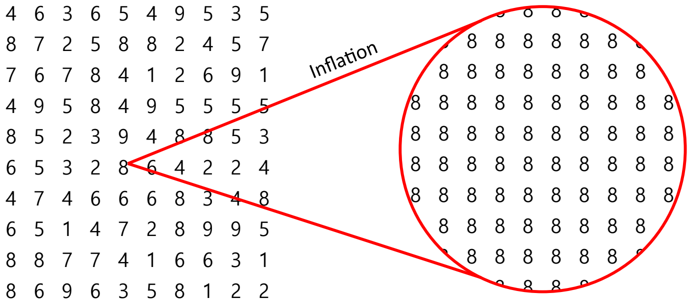
4	6	3	6	5	4	9	5	3	5
8	7	2	5	8	8	2	4	5	7
7	6	7	8	4	1	2	6	9	1
4	9	5	8	4	9	5	5	5	5
8	5	2	3	9	4	8	8	5	3
6	5	3	2	8	6	4	2	2	4
4	7	4	6	6	6	8	3	4	8
6	5	1	4	7	2	8	9	9	5
8	8	7	7	4	1	6	6	3	1
8	6	9	6	3	5	8	1	2	2

Misalignment Mechanism

- Misalignment mechanism is a mechanism for retaining correct relic density of dark matter.
- Due to the inflation, a homogeneous condensate can be achieved.



Misalignment Mechanism

- Misalignment mechanism is a mechanism for retaining correct relic density of dark matter.
  - Homogeneous condensate due to inflation.
  - Coherent oscillation.
- For example, the equation of motion and the energy density of scalar field φ is given as

$$\ddot{\varphi} + 3H\dot{\varphi} + m_\varphi^2 \varphi = 0, \quad \rho_\phi = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2} m_\phi^2 \varphi^2$$

- φ is frozen and ρ_ϕ is constant during inflation due to the large Hubble friction. As H becomes smaller than m_φ, φ begins coherent oscillation and $\rho_\varphi \propto a^{-3}$ like usual cold dark matter.

Isocurvature Fluctuation

- Although inflation generates spatial homogeneity, quantum fluctuation of X can exist.
- This quantum fluctuation generates isocurvature fluctuation.
- Since the production of transverse mode is suppressed, only the longitudinal mode of X contributes to the isocurvature fluctuation.
- Isocurvature fluctuation is given by the power spectrum of longitudinal mode. The power spectrum is given as

$$\langle X(\vec{k}) X^*(\vec{k}') \rangle \equiv \frac{2\pi^2}{k^3} \mathcal{P}_X(\vec{k}) (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}')$$

Isocurvature Fluctuation

- Iscourvature fluctuation is given as

$$S_{\rm CDM}(\vec{k}) \approx \frac{\delta\rho_X(\vec{k})}{\rho_X + \rho_{\rm CDM}} \approx \frac{2\sqrt{\mathcal{P}_X(\vec{k}, t^{\rm inf})}/X^{\rm inf}}{1 + \rho^{\rm lss}_{\rm CDM}/\rho^{\rm lss}_X},$$

  where the $\delta\rho_X$ is evaluated at the uniform density slicing, and the "inf" ("lss") denotes the values at the end of inflation (last scattering surface).
- CMB spectrum constraints $S_{\rm CDM} \lesssim 9 \times 10^{-6}$ at $k = 0.05$ Mpc, it suggests upper bound of $H_{\rm inf}$.

Fluctuation of Quintessence

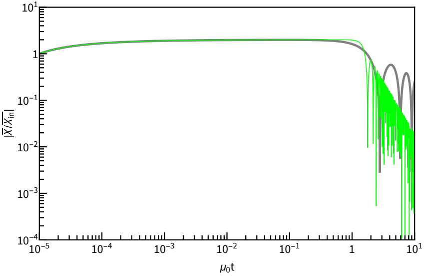
- There also exists quantum fluctuation of quintessence.
- In the usual quintessence model, there is no minimum of potential.
- In this case, quantum fluctuation might drive the value of ϕ to the infinity during inflation.
- In the gauged quintessence model, this effect can be prohibited with assist of V_{gauge}.
- This effect might suggest another constraint to this model.

Other Constraints

- CMB B-mode constraints $H_{\text{inf}} < 10^{14}\text{GeV}$ but this constraint is weaker than isocurvature constraint.
- Gauge coupling is bounded above for the quantum correction from it to be suppressed. If there exists a way to cancel or diminish quantum correction, this constraint might not work.
- Lyman-α spectrum disfavor extremely light dark matter and it suggests lower bound of gauge coupling.
- Weak gravity conjecture might suggest lower bound of g_X but it is weaker than Lyman-α constraint.

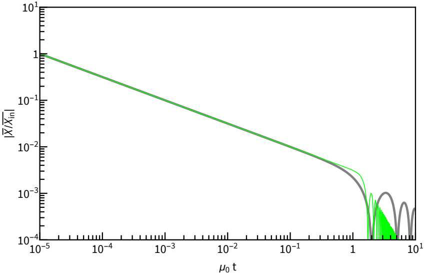
Coherent Oscillation

- Dynamics of X/a at the radiation dominated era ($w_B = 1/3$).
  - Oscillation starts when $t = 1/\mu_0$ which coincides with $m_X \sim H$.
  - Gray (green) line corresponds to the constant mass (mass increasing) case. Due to $\sqrt{m_X}$ suppression, green line is steeper than gray one.



Kinetic energy dominated

constant $\to m_X^{-1/2} a^{-3/2}$



Potential energy dominated

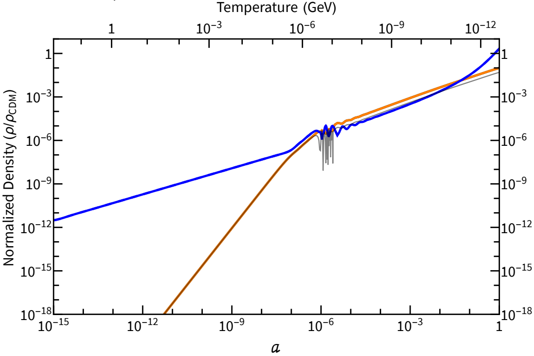
$a^{-1} \to m_X^{-1/2} a^{-3/2}$

Reference Scale Factor (Reference Time)

- $a_{\rm ini}$: Inflation begins.
- $a_{\rm tr}$: $H = m_\phi$, ϕ becomes frozen.
- $a_{\rm end}$: Inflation ends and radiation dominated era begins.
  (Instant reheating is assumed.)
- $a_{\rm nr}$: $H = m_X$, X becomes non-relativistic.
- $a_{\rm eq}$: Radiation-matter equality.
- $a_0 = 1$: Present universe.

Evolution of Energy Density

- ρ_X can be comparable to $\rho_{\rm CDM}$ in the recent universe even if ρ_X is much smaller than $\rho_{\rm CDM}$ in the earlier universe.



Evolution of normalized energy densities of ϕ and X in the potential energy dominated case. Blue, orange, and black lines correspond to energy densities of ϕ, X, and potential energy of X. $\alpha = 1$, $g_X = 10^{-45}$, $M = 2.2 \times 10^{-6}$ and $\rho_X^0/\rho_{\rm CDM}^0 = 0.1$ is chosen.