Role of dimension-eight operators in an EFT for the 2HDM

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The Standard Model and SMEFT

- The Standard Model (SM) is the renormalizable field theory of $SU(3) \times SU(2) \times U(1)$ gauge theory describing electroweak and strong interactions among the known fields
- Has three generations of quarks and leptons
- Has one Higgs doublet
- Has no right-handed neutrinos
- The Standard Model Effective Field Theory (SMEFT) includes non-renormalizable operators with the same field content

	spin	$SU(3)$	$SU(2)$	$U(1)$
H	0	$\mathbf{1}$	$\mathbf{2}$	1
ℓ_L	$\frac{1}{2}$	$\mathbf{1}$	$\mathbf{2}$	-1
e_R	$\frac{1}{2}$	$\mathbf{1}$	$\mathbf{1}$	-2
q_L	$\frac{1}{2}$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{3}$
u_R	$\frac{1}{2}$	$\mathbf{3}$	$\mathbf{1}$	$\frac{4}{3}$
d_R	$\frac{1}{2}$	$\mathbf{3}$	$\mathbf{1}$	-$\frac{2}{3}$

What SMEFT looks like (at dimension 6)

- So-called “Warsaw basis”, Grzadkowski et al, J. High Energ. Phys. 2010, 85 (2010).

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I\varphi W^I_{\mu\nu}$	$Q^{(1)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q^{(3)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I\widetilde\varphi\, W^I_{\mu\nu}$	$Q^{(1)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q^{(3)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I\varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Table 2: Dimension-six operators other than the four-fermion ones.

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q^{(1)}_{qq}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q^{(3)}_{qq}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q^{(1)}_{lq}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q^{(3)}_{lq}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q^{(1)}_{qu}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q^{(1)}_{ud}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q^{(8)}_{qu}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q^{(8)}_{ud}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q^{(1)}_{qd}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q^{(8)}_{qd}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$
$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating			
Q_{ledq}	$(\bar l^j_p e_r)(\bar d_s q^j_t)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d^\alpha_p)^T C u^\beta_r\right]\left[(q^{\gamma j}_s)^T C l^k_t\right]$		
$Q^{(1)}_{quqd}$	$(\bar q^j_p u_r)\varepsilon_{jk}(\bar q^k_s d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q^{\alpha j}_p)^T C q^{\beta k}_r\right]\left[(u^\gamma_s)^T C e_t\right]$		
$Q^{(8)}_{quqd}$	$(\bar q^j_p T^A u_r)\varepsilon_{jk}(\bar q^k_s T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q^{\alpha j}_p)^T C q^{\beta k}_r\right]\left[(q^{\gamma m}_s)^T C l^n_t\right]$		
$Q^{(1)}_{lequ}$	$(\bar l^j_p e_r)\varepsilon_{jk}(\bar q^k_s u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d^\alpha_p)^T C u^\beta_r\right]\left[(u^\gamma_s)^T C e_t\right]$		
$Q^{(3)}_{lequ}$	$(\bar l^j_p \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q^k_s \sigma^{\mu\nu} u_t)$				

Table 3: Four-fermion operators.

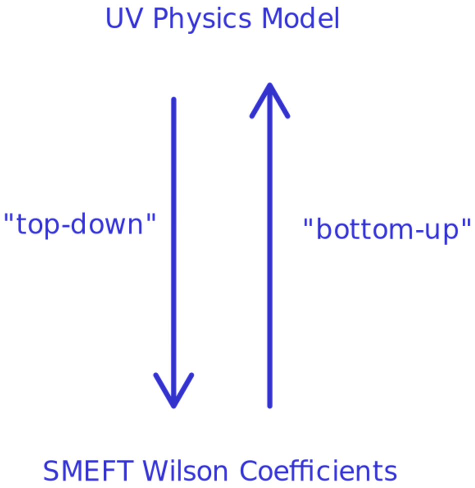
- 59 B-conserving operators not including flavor
- 2499 (!) B-conserving operators with flavor structure

Dimension 8 operators

- Murphy, JHEP 10 (2020) 174, and Li et al., Phys.Rev.D 104 (2021) 1, 015026, wrote down a complete basis of dimension 8 SMEFT operators
- There are **44807** operators when including flavor structure
  - Beyond feasible to include all dimension 8 operators in any bottom-up analysis
- Tiny sample of some dim 8 operators (one of many tables on many pages):

	$8 : XH^4D^2$
$Q^{(1)}_{WH^4D^2}$	$(H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H) W^I_{\mu\nu}$
$Q^{(2)}_{WH^4D^2}$	$(H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H) \widetilde{W}^I_{\mu\nu}$
$Q^{(3)}_{WH^4D^2}$	$\epsilon^{IJK}(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^J D^\nu H) W^K_{\mu\nu}$
$Q^{(4)}_{WH^4D^2}$	$\epsilon^{IJK}(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^J D^\nu H) \widetilde{W}^K_{\mu\nu}$
$Q^{(1)}_{BH^4D^2}$	$(H^\dagger H)(D^\mu H^\dagger D^\nu H) B_{\mu\nu}$
$Q^{(2)}_{BH^4D^2}$	$(H^\dagger H)(D^\mu H^\dagger D^\nu H) \widetilde{B}_{\mu\nu}$

Top-down vs bottom-up approaches



- Bottom-up approach starts with arbitrary Wilson coefficients, tries to get to UV model
  - E.g. experiments fits to Wilson coefficients, then attempts to explain what model any deviations could come from
- Top-down approach starts with UV model, then matches onto SMEFT to get Wilson coefficients in SMEFT
  - This is the sort of analysis I will talk about with the 2HDM

Two Higgs doublet model

- Two Higgs doublet models (2HDMs) are extremely popular scalar sector extensions
- Doublets don't mess up electroweak precision
- Most of the literature focuses on the case of (softly broken) Z_2 symmetry, to remove tree-level flavor-changing neutral currents, and with no CP violation in the scalar sector
- There are multiple different "types" which have different Yukawa relations

Two Higgs doublet model Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm kin} &= (D_\mu H_1)^\dagger (D^\mu H_1) + (D_\mu H_2)^\dagger (D^\mu H_2) \\
V &= Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + \left(Y_3 H_1^\dagger H_2 + \mathrm{h.c.} \right) \\
&+ \frac{Z_1}{2} \left(H_1^\dagger H_1 \right)^2 + \frac{Z_2}{2} \left(H_2^\dagger H_2 \right)^2 + Z_3 \left(H_1^\dagger H_1 \right) \left(H_2^\dagger H_2 \right) + Z_4 \left(H_1^\dagger H_2 \right) \\
&+ \left\{ \frac{Z_5}{2} \left(H_1^\dagger H_2 \right)^2 + Z_6 \left(H_1^\dagger H_1 \right) \left(H_1^\dagger H_2 \right) \right. \\
&+ \left. Z_7 \left(H_2^\dagger H_2 \right) \left(H_1^\dagger H_2 \right) + \mathrm{h.c.} \right\} \\
\mathcal{L}_Y &= -\lambda_u^{(1)} \bar{u}_R \tilde{H}_1^\dagger q_L - \lambda_u^{(2)} \bar{u}_R \tilde{H}_2^\dagger q_L - \lambda_d^{(1)} \bar{d}_R H_1^\dagger q_L - \lambda_d^{(2)} \bar{d}_R H_2^\dagger q_L \\
&- \lambda_l^{(1)} \bar{e}_R H_1^\dagger l_L - \lambda_l^{(2)} \bar{e}_R H_2^\dagger l_L + \mathrm{h.c.}
\end{aligned}
$$

- Y_2 will correspond to the heavy scale Λ^2

Particle content of the 2HDM

- By performing a field redefinition such that only H_1 gets a vev (the so-called Higgs basis) the doublets break down as follows:

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} \left(v + \sin(\beta - \alpha)\, h_{125} + \cos(\beta - \alpha)\, H_0 + iG_0 \right) \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} \left(\cos(\beta - \alpha)\, h_{125} - \sin(\beta - \alpha)\, H_0 + iA \right) \end{pmatrix}$$

- h_{125} is the 125 GeV light scalar state, H_0, A, H^+ are the heavy scalar states, G_0, G^+ are the Goldstones
- The mixing $\beta - \alpha$ changes the couplings of h_{125} to other Standard Model particles

Types of 2HDMs

- The Yukawas in the Higgs basis can be written as:

$$\lambda_f^{(1)} = \frac{\sqrt{2}}{v} m_f, \qquad \lambda_f^{(2)} = \frac{\eta_f}{\tan\beta} \lambda_f^{(1)}$$

- For the different types of 2HDM, η_f takes the following values:

	Type-I	Type-II	Type-L	Type-F
η_u	1	1	1	1
η_d	1	$-\tan^2\beta$	1	$-\tan^2\beta$
η_l	1	$-\tan^2\beta$	$-\tan^2\beta$	1

- $\tan\beta$ is the ratio of vevs from the Z_2 symmetric basis

Matching the 2HDM to dimension 8 at tree level

- $F_{n,m}$ denotes terms suppressed by $1/\Lambda^{(n-4)}$ of operator dimension m

$$
\begin{aligned}
F_{6,2} &= |Y_3|^2(H_1^\dagger H_1), \\
F_{6,4} &= Y_3\lambda_u^{(2)*}H_1^\dagger\widehat{q}_L u_R + Y_3\lambda_d^{(2)}\bar{d}_R H_1^\dagger q_L + Y_3 Z_6^*(H_1^\dagger H_1)^2 + \text{h.c.}, \\
F_{6,6} &= (H_1^\dagger H_1)\left[|Z_6|^2(H_1^\dagger H_1)^2 + \left\{Z_6\lambda_u^{(2)*}H_1^\dagger\widehat{q}_L u_R + Z_6\lambda_d^{(2)}\bar{d}_R H_1^\dagger q_L + \text{h.c.}\right\}\right] + 4\text{F}
\end{aligned}
$$

$$
\begin{aligned}
F_{8,4} &= |Y_3|^2(D_\mu H_1)^\dagger(D^\mu H_1) - (H_1^\dagger H_1)^2\left[|Y_3|^2 Z_{34} + \frac{1}{2}(Y_3)^2 Z_5^* + \frac{1}{2}(Y_3^*)^2 Z_5\right], \qquad (22a) \\
F_{8,6} &= \{Y_3 Z_6^* + Y_3^* Z_6\}(H_1^\dagger H_1)(D_\mu H_1)^\dagger(D^\mu H_1) + \{Y_3 Z_6^*(D_\mu H_1)^\dagger H_1 + \text{h.c.}\}\,\partial^\mu(H_1^\dagger H_1) \\
&\quad + \left\{Y_3^*\lambda_u^{(2)}\left(D_\mu(\widehat{q}_L u_R)\right)^\dagger(D^\mu H_1) + Y_3^*\lambda_d^{(2)*}\left(D_\mu(\bar{d}_R q_L)\right)^\dagger(D^\mu H_1) + \text{h.c.}\right\} \\
&\quad - (H_1^\dagger H_1)^3\left[Y_3 Z_{34} Z_6^* + Y_3 Z_5^* Z_6 + \text{h.c.}\right] \\
&\quad - (H_1^\dagger H_1)\left[H_1^\dagger\widehat{q}_L u_R\left(Y_3 Z_{34}\lambda_u^{(2)*} + Y_3^* Z_5\lambda_u^{(2)*}\right) + \bar{d}_R H_1^\dagger q_L\left(Y_3 Z_{34}\lambda_d^{(2)} + Y_3^* Z_5\lambda_d^{(2)}\right) + \text{h.c.}\right] \qquad (22b) \\
F_{8,8} &= |Z_6|^2(H_1^\dagger H_1)^2(D_\mu H_1)^\dagger(D^\mu H_1) + 2|Z_6|^2(H_1^\dagger H_1)\partial_\mu(H_1^\dagger H_1)\partial^\mu(H_1^\dagger H_1) \\
&\quad - (H_1^\dagger H_1)^4\left[Z_{34}|Z_6|^2 + \frac{1}{2}Z_5^* Z_6^2 + \frac{1}{2}Z_5(Z_6^*)^2\right] \\
&\quad - (H_1^\dagger H_1)^2\left[H_1^\dagger\widehat{q}_L u_R\left(Z_{34}Z_6\lambda_u^{(2)*} + Z_5 Z_6^*\lambda_u^{(2)*}\right) + \bar{d}_R H_1^\dagger q_L\left(Z_{34}Z_6\lambda_d^{(2)} + Z_5 Z_6^*\lambda_d^{(2)}\right) + \text{h.c.}\right] \\
&\quad + \left\{\left[Z_6^*\lambda_u^{(2)}\left(D_\mu(\widehat{q}_L u_R)\right)^\dagger + Z_6^*\lambda_d^{(2)*}\left(D_\mu(\bar{d}_R q_L)\right)^\dagger\right]\left[\partial^\mu(H_1^\dagger H_1)H_1 + (H_1^\dagger H_1)(D^\mu H_1)\right] + \text{h.c.}\right\} \\
&\quad + 4\text{F}, \qquad (22c)
\end{aligned}
$$

- We ignore the 4-fermion operators

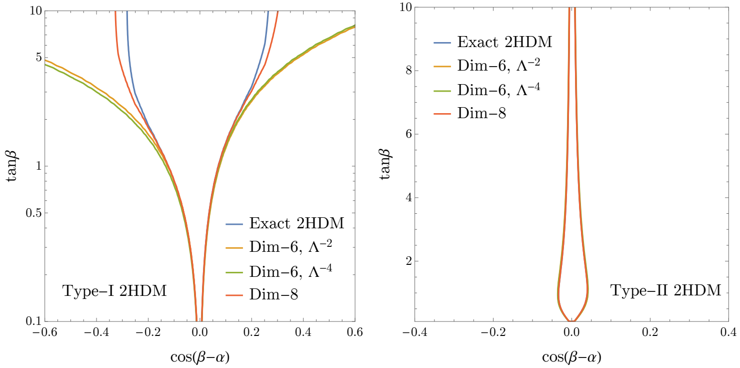
Physical parameters in the 2HDM and power counting

- Practically all 2HDM limit plots are in terms of $\tan\beta$ and $\cos(\beta-\alpha)$; we really want to change to these from the Lagrangian parameters after we do the matching
- We will also take the decoupling limit:

$$m_A^2 \sim m_{H_0}^2 \sim m_{H^\pm}^2 \sim Y_2 \equiv \Lambda^2 \gg v^2, \qquad m_h^2 \simeq v^2$$

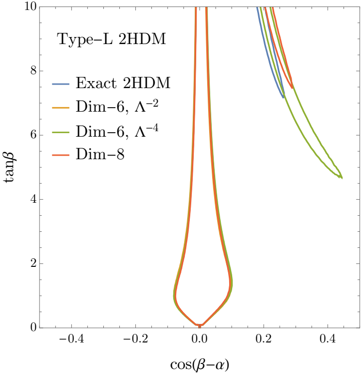
- Decoupling requires $\cos(\beta-\alpha) \sim v^2/\Lambda^2$
- Keeping a consistent power counting during the conversion is key: we matched up to $\mathcal{O}(\Lambda^{-4})$, so we should only keep expressions in terms of physical parameters up to $\mathcal{O}(\Lambda^{-4})$

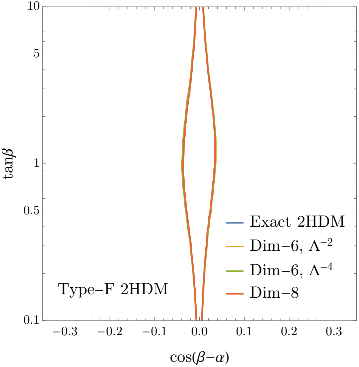
Are the dimension 8 terms relevant?



- Limits for exact 2HDM, dimension 6 expansion, dimension 6 expansion including squared terms, and dimension 8 expansion
- Type-I is not reproduced well until dimension 8!
- Type-II is already well-constrained even with just dimension 6 matching, in contrast

Dimension 8 effects for the other types





- Type-L and Type-F are well-described by dimension 6 except for the second, disconnected region in type-L, where the EFT contribution to lepton Yukawa couplings dominates over the SM contribution

Why does type-I need dimension 8?

- In the Type-I model, all the Yukawas of the heavy doublet were suppressed by $\tan\beta$, and the high $\tan\beta$ region is where dimension 8 is important
- The 2HDM also changes the couplings of the 125 GeV Higgs to W and Z bosons; where is that in the matching?
- That comes only from the following Wilson coefficient at dimension 8

$$\frac{C^{(1)}_{\mathcal{H}^6}}{\Lambda^4} = -\cos(\beta-\alpha)^2\,(\sqrt{2}G_F)^2$$

- This Wilson coefficient corresponds to the dimension 8 operator $(H^\dagger H)^2 D_\mu H^\dagger D^\mu H^\dagger$
- So, for all types, the 2HDM doesn't change hWW and hZZ couplings at dimension 6, which are important for constraining the Type-I model

Is this generic?

- There is a similar dimension 6 operator $H^\dagger H D_\mu H^\dagger D^\mu H^\dagger$
  - It matches onto $(H^\dagger H)\Box(H^\dagger H)$ and other dimension 6 operators in the Warsaw basis using field redefinitions
- Both this operator and the dimension 8 operator $(H^\dagger H)^2 D_\mu H^\dagger D^\mu H^\dagger$ have similar effects
  - hWW and hZZ couplings
  - Momentum-dependent hhh couplings
- The 2HDM happens to be a model that doesn't generate this dimension 6 operator
- Other models *can* generate the dimension 6 operator at tree-level, like scalar or vector triplets or singlets

Conclusions

- A top-down analysis of the 2HDM shows that including dimension 8 operators can be necessary, since the hWW and hZZ coupling changes are missing at dimension 6
- Even so, going to dimension 8 is opening Pandora's box of 44807 additional Wilson coefficients
- Some general SMEFT takeaways:
  - SMEFT has more subtleties than one might think
  - You can't be sure that dimension 6 matching is good enough for a model without checking
  - Determining the UV model from measurements of Wilson coefficients requires accurately attributing effects to the correct operators

Thank you!